Electronic supplement:

Efficiency and Market Power Analysis of Alternative Reserve Market Design Proposals: a Case Study of Greece

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1. Structure

This is a technical report which provides additional derivations and results regarding the publication of Papavasiliou and Avila, "Efficiency and Market Power Analysis of Alternative Reserve Market Design Proposals: a Case Study of Greece", published in Utilities Policy. The supplement is organized as follows. In section 2 we describe the methodology that has been considered for the analysis of the production simulation model (section 3.1 of the main paper). We describe the precise mathematical models for the co-optimization and sequential clearing designs, as well as a description of the estimation of energy clearing prices and storage modeling in the context of reserves-only markets. In section 3 we provide a detailed overview of the considered case study of section 4.1 of the main paper, as well as a more detailed analysis of certain results. Section 4 presents the market power analysis, see section 3.2, 4.2 and 5.2 of the main paper. The analysis presented in section 4 of this appendix culminates in the comparison of three models:

- Perfect competition in energy and reserves
- Monopoly in co-optimization
- Monopoly in sequential clearing

2. Methodological description

In this section we describe the model that we have developed to quantitatively assess the impacts of the IPTO proposal on the operating costs of the Greek system. The model and the methodology of the analysis are based on a corresponding model that we have developed for a pan-European study (CORE region) prepared on behalf of ACER [1]. In subsection 2.1 we describe the co-optimization model, while in subsection 2.2 we describe the sequential design. Note that, for ease of exposition, hydro modeling is not considered in these subsections but rather presented in subsection 2.3. The nomenclature of the models can be found in subsection 2.4.

2.1 Co-optimization model

The co-optimisation model jointly optimises the allocation of energy and reserve in a single multi-product day-ahead auction. Upward and downward aFRR and mFRR are thus traded jointly with energy in this model. This model accounts directly for the fact that energy and reserve are mutually exclusive when representing the technical minimum and maximum constraints.

The objective of this model is to satisfy energy demand and reserve requirements at least cost. Cost is measured as the sum of involuntary load shedding plus fixed min load and startup costs of units plus variable fuel costs. Note that the co-optimisation model internalizes the opportunity cost of allocating generation capacity for reserve. As described earlier, the hydro modelling is ignored in the present subsection and later described in subsection 2.3. We formulate the day-ahead co-optimization model as follows:

$$\min \sum_{t \in T} \sum_{g \in G} MC_g \cdot p_{gt} + VOLL \cdot\ ls_t + \sum_{t \in T} \sum_{g \in G} S_g \cdot z_{gt} \tag{2.1}$$

$$\sum_{g \in G} p_{gt} + ls_t - ps_t = D_t, t \in T \tag{2.2}$$

$$\sum_{g \in G} s_{gt}^{+/-aFRR} \geq R^{+/-aFRR} \tag{2.3}$$

$$\sum_{g \in G} s_{gt}^{+/-\ aFRR} + \sum_{g \in G} s_{gt}^{+/-\ mFRR} \geq\ R^{+/-aFRR} + R^{+/-mFRR}, t \in T \tag{2.4}$$

$$s_{gt}^{+/-mFRR} \leq \min\left(P_g^{+}, DT_z^{mFRR} \cdot R_g^{+/-}\right), g \in G, t \in T \tag{2.5}$$

$$s_{gt}^{+/-aFRR} \leq \min\left(P_g^{+}, DT_z^{aFRR} \cdot R_g^{+/-}\right), g \in G, t \in T \tag{2.6}$$

$$p_{gt} + s_{gt}^{+mFRR} + s_{gt}^{+aFRR} \leq P_g^{+} \cdot w_{gt}, g \in G, t \in T \tag{2.7}$$

$$p_{gt} - s_{gt}^{-mFRR} - s_{gt}^{-aFRR} \geq P_g^{-} \cdot w_{gt}, g \in G, t \in T \tag{2.8}$$

$$p_{gt} - p_{gt-1} + s_{gt}^{+mFRR} + s_{gt}^{+aFRR} \leq 60 \cdot R_g^{+}, g \in G, t \in T \tag{2.9}$$

$$p_{gt-1} - p_{gt} + s_{gt}^{-mFRR} + s_{gt}^{-aFRR} \leq 60 \cdot R_g^{-}, g \in G, t \in T \tag{2.10}$$

$$\sum_{q=t-UT_g+1}^{t} z_{gq} \leq w_{gt}, g \in G, t \geq UT_g \tag{2.11}$$

$$\sum_{q=t+1}^{t+Dt_g} z_{gq} \leq 1 - w_{gt}, g \in G, t \leq N - DT_g \tag{2.12}$$

$$z_{gt} \leq 1, g \in G, t \in T \tag{2.13}$$

$$z_{gt} \geq w_{gt} - w_{gt-1}, g \in G, t \in T \tag{2.14}$$

$$p_{gt}, z_{gt}, s_{gt}, ls_t \geq 0, w_{gt} \in \{0,1\} \tag{2.15}$$

Equation (2.1) corresponds to the objective function of the co-optimization model. It consists of the marginal cost of production, the cost for not serving demand and start-up costs. The load balance constraint is defined in equation (2.2), where we assume that D_t is the net demand after subtracting renewable resources (solar and wind). Constraint (2.3) clears the aFRR market, while constraint (2.4) describes the demand for frequency restoration reserve as the sum of an inelastic requirement for aFRR and an inelastic requirement for mFRR. Constraints (2.5) and (2.6) model the technical limits for the delivery of reserve. Constraint (2.7) enforces the maximum generation capacity limit and constraint (2.8) imposes minimum generation limits for all generators. Constraints (2.9)-(2.10) model the ramp up/down capabilities. Finally, constraints (2.11)-(2.15) model the start-up and minimum up/down times of the generators. These constraints correspond to the convex hull of minimum up/down time polytopes in the absence of ramp constraints [2], and therefore enable the mixed integer programming solvers that are used in our work to converge more rapidly since they tend to produce tighter bounds in the linear programming relaxation of the unit commitment problem.

2.2 Sequential clearing model

The sequential clearing design is represented by a day-ahead reserve market model which is followed by a day-ahead energy market model. The reserve market model aims at minimising the sum of fixed costs plus the opportunity cost of generators for offering reserve instead of energy. We commence by describing the opportunity cost of the thermal generators and the associated reserve market model in subsection 2.2.1. In subsection 2.2.2 we describe the energy clearing module that follows the reserve market. Finally, in subsection 2.2.3 we describe the day-ahead energy price estimation required for the calculation of the opportunity cost of the generators.

2.2.1 Reserves clearing model

The reserve market model aims at clearing reserves by breaking the coupling between energy and reserves. To do so, we need to estimate, for each generator, the opportunity cost for offering reserve instead of energy. In the context of this work, we have opted to follow a Lagrange relaxation approach as a way to estimate said opportunity cost and in turn provide a modeling framework for the reserves clearing market. Starting from the co-optimization model, we can relax the load balance constraint (2.2) and arrive to the subsequent model:

$$r(\lambda) = \sum_{t\in T} \lambda_t \cdot D_t + \min \sum_{g\in G} OC_g\left(s_g, w_g; \lambda\right) + \sum_{t\in T}\sum_{g\in G} S_g z_{gt} \tag{2.16}$$

$$\sum_{g\in G} s_{gt}^{+/-aFRR} \geq R^{+/-aFRR} \tag{2.17}$$

$$\sum_{g\in G} s_{gt}^{+/-\,aFRR} + \sum_{g\in G} s_{gt}^{+/-\,mFRR} \geq\ R^{+/-aFRR} + R^{+/-mFRR}, t \in T \tag{2.18}$$

$$s_{gt}^{+/-mFRR} \leq \min\left(P_g^{+}, DT_z^{mFRR} \cdot R_g^{+/-}\right), g \in G, t \in T \tag{2.19}$$

$$s_{gt}^{+/-aFRR} \leq \min\left(P_g^{+}, DT_z^{aFRR} \cdot R_g^{+/-}\right), g \in G, t \in T \tag{2.20}$$

$$P_g^{-} \cdot w_{gt} + s_{gt}^{+mFRR} + s_{gt}^{+aFRR} + s_{gt}^{-mFRR} + s_{gt}^{-aFRR} \leq P_g^{+} \cdot w_{gt}, g \in G, t \in T \tag{2.21}$$

$$\sum_{q=t-UT_g+1}^{t} z_{gq} \leq w_{gt} \tag{2.22}$$

$$\sum_{q=t+1}^{t+Dt_g} z_{gq} \leq 1 - w_{gt}\,, g \in G, t \leq N - DT_g \tag{2.23}$$

$$z_{gt} \leq 1, g \in G, t \in T \tag{2.24}$$

$$z_{gt} \geq w_{gt} - w_{gt-1}, g \in G, t \in T \tag{2.25}$$

$$z_{gt}, s_{gt}\ \geq 0, w_{gt} \in \{0,1\} \tag{2.26}$$

The objective function is described in equation (2.16) and consists of the sum of opportunity costs (OC_g terms) plus fixed costs. Before describing the opportunity cost let us describe the constraints of the model. Constraints (2.17)-(2.18) define the reserve requirements for both aFRR and mFRR. Constraints (2.19)-(2.20) constrain the amount of reserve that can be made available by a unit by taking into account the ramp specifications of the generators. Constraint (2.21) ensures that the offered reserved is feasible with respect to minimal and maximal generation capabilities. Finally, constraints (2.22)-(2.26) model the start-up, and min up/down times of the generators.

The OC_g function defines the opportunity cost for offering reserve instead of energy. Note that it depends on the energy price λ, on the commitment status of the generators and on the offered reserve. Mathematically, it is described by the following optimization problem:

$$OC_g\left(s_g, w_g; \lambda\right) =\ \min \sum_{t\in T} \left(MC_g - \lambda_t\right) \cdot p_{gt} \tag{2.27}$$

$$p_{gt} + s_{gt}^{+mFRR} + s_{gt}^{+aFRR} \leq P_g^{+} \cdot w_{gt}, g \in G, t \in T \tag{2.28}$$

$$p_{gt} - s_{gt}^{-mFRR} - s_{gt}^{-aFRR} \geq P_g^{-} \cdot w_{gt}, g \in G, t \in T \tag{2.29}$$

$$p_{gt} - p_{gt-1} + s_{gt}^{+mFRR} + s_{gt}^{+aFRR} \leq 60 \cdot R_g^{+}, g \in G, t \in T \tag{2.30}$$

$$p_{gt-1} - p_{gt} + s_{gt}^{-mFRR} + s_{gt}^{-aFRR} \leq 60 \cdot R_g^{-}, g \in G, t \in T \tag{2.31}$$

$$p_{gt} \geq 0 \tag{2.32}$$

We note that, due to the presence of ramp constraints, the cost function cannot be decomposed per period. This leads to a cost function that is coupled over time periods and leads to a very optimistic model of sequential market clearing, because it is richer than the bidding language that is available in existing European reserve markets. Therefore, to arrive to a decomposable, per period, opportunity cost function we consider a relaxation of the ramp constraints. This leads to the following formulation of the opportunity cost, which is the one we employ within the simulations:

$$OC_{gt}(s_{gt}, w_{gt}; \lambda t) = \begin{cases} 0 & if\ w_{gt} = 0 \\ (MC_g - \lambda_t) \cdot (P_g^- + s_{gt}^{-mFRr} + s_{gt}^{-aFRR}) & if\ w_{gt} = 1, (MC_g - \lambda_t) \geq 0 \\ (MC_g - \lambda_t) \cdot (P_g^+ - s_{gt}^{+mFRR} - s_{gt}^{+aFRR}) & if\ w_{gt} = 1, (MC_g - \lambda_t) < 0 \end{cases}$$

Note that in case the generator is profitable in the energy market, the opportunity cost function discounts the profits from running the generator in the energy market. On the other hand, if the generators is not profitable, the cost function asks for damages of running the generator to be compensated.

2.2.2 Energy clearing model

The energy market clearing model trades energy while fixing the allocation of reserve to the solution that is obtained in the reserve market model. As such, the model's structure follows the same description as in the co-optimization model (2.1)-(2.15), with the added feature that the reserve is no longer a decision variable, but a parameter fixed to the outcome of the reserve market model.

2.2.3 Price forecast error model

An important parameter in the representation of the IPTO proposal is the errors of the participants in the forecast of the energy price. As discussed in subsection 2.2.1, the energy price defines the opportunity cost of providing reserve. This opportunity cost is driven by the difference between the energy market price and the marginal cost of a unit. Therefore, the estimation of the opportunity cost in the reserve market requires the estimation of the energy price that follows, and this can precisely distort coordination in the scheduling of generation units when the energy and reserve auctions are separated. Overestimation of the energy market price implies an overestimation of the opportunity cost of a unit, and therefore overly conservative bids in the reserve market, and vice versa. In the present subsection we discuss the procedure that we have followed in order to emulate the price forecasts of agents.

We consider the efficient energy clearing price of the system as the energy clearing price of the co-optimization design. In consequence, we use this energy clearing price as the base price for our simulations. We add Gaussian noise to this base price in order to emulate price forecast errors that agents may introduce in actual operations.

The base energy clearing price is computed by considering a linear relaxation of the co-optimization model (2.1-2.15) and calculating the dual multiplier of the load balance constraint (2.2). The forecast errors of Figure 1 are added to this base value. These errors are normally distributed with a mean of 0 €/ MWh and a standard deviation of 3% of the base value. Figure 1 presents the mean value (orange line), the 25-75% percentages (in the boxes), as well as extreme values (with circles) that are those that are beyond the range of the lines.

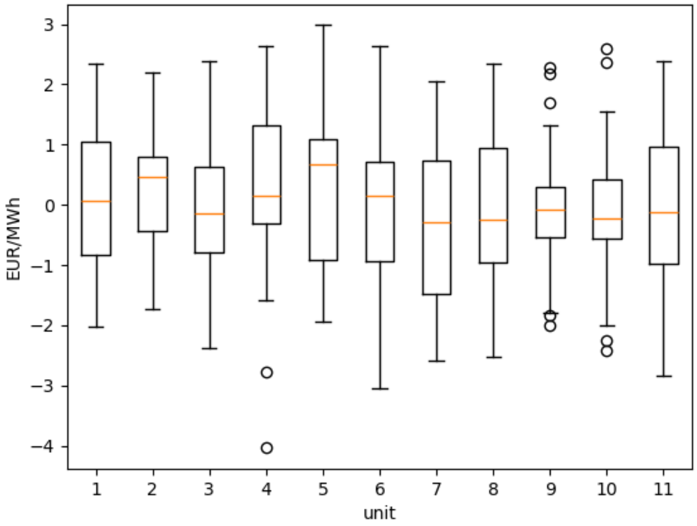

Figure 1: Statistical behavior of energy price forecast errors in the modeling of the IPTO design proposal.

2.3 Hydro model

The present subsection describes the hydro modelling approach for the co-optimization and sequential designs. In particular, we describe the different approaches that we have considered in order to model storage within the reserve market.

2.3.1 Co-optimization hydro modeling

We model the hydro unis within the co-optimization design by performing the following modifications to the model described in equations (2.1-2.17). First, we add the hydro production and demand in the load balance constraint (2.2):

$$\sum_{g \in G} p_{gt} + ls_t - ps_t + \sum_{g \in G_H} p_{gt} - d_{gt} \ = D_t, t \in T$$

We proceed by adding the ability to cover reserve with hydro resources by adding reserve allocation variables for hydro resources to constraints (2.3) and (2.4). Finally, we add technical constraints to model hydro units as follows:

$$s_{gt}^{+/-mFRR} \leq \min\left(P_g^+, DT_z^{mFRR} \cdot R_g^{+/-}\right), g \in G_H, t \in T \tag{2.33}$$

$$s_{gt}^{+/-aFRR} \leq \min\left(P_g^+, DT_z^{aFRR} \cdot R_g^{+/-}\right), g \in G_H, t \in T \tag{2.34}$$

$$p_{gt} + s_{gt}^{+mFRR} + s_{gt}^{+aFRR} \leq P_g^+, g \in G_H, t \in T \tag{2.35}$$

$$d_{gt} + s_{gt}^{-mFRR} + s_{gt}^{-aFRR} \leq D_g^+, g \in G_H, t \in T \tag{2.36}$$

$$p_{gt} - p_{gt-1} + s_{gt}^{+mFRR} + s_{gt}^{+aFRR} \leq 60 \cdot R_g^+, g \in G_H, t \in T \tag{2.37}$$

$$p_{gt-1} - p_{gt} + s_{gt}^{-mFRR} + s_{gt}^{-aFRR} \leq 60 \cdot R_g^-, g \in G_H, t \in T \tag{2.38}$$

$$d_{gt} - d_{gt-1} + s_{gt}^{-mFRR} + s_{gt}^{-aFRR} \leq 60 \cdot R_g^+, g \in G_H, t \in T \tag{2.39}$$

$$d_{gt-1} - d_{gt} + s_{gt}^{+mFRR} + s_{gt}^{+aFRR} \leq 60 \cdot R_g^-, g \in G_H, t \in T \tag{2.40}$$

$$e_{gt} = e_{gt-1} + \eta \cdot d_{gt-1} - p_{gt-1}, g \in G_H, t \in T \tag{2.41}$$

$$e_{gt} \leq E_g, g \in G_H, t \in T \tag{2.42}$$

Constraints (2.33-2.40) represent technical limitations related to maximum generation capacity and ramp rates. Constraint (2.41) models the storage balance constraint, while constraint (2.42) enforces a maximum storage capacity.

2.3.2 Sequential clearing: optimistic hydro modeling

As for thermal assets, the sequential clearing design requires the estimation of an opportunity cost for hydro storage. Relaxing the load balance constraint (2.2) leads to the following description of the opportunity cost for hydro storage.

$$OC_g\left(s_g, w_g; \lambda\right) = \min \sum_{t \in T} -\lambda_t \cdot \left(p_{gt} - d_{gt}\right) \tag{2.43}$$

$$p_{gt} + s_{gt}^{+mFRR} + s_{gt}^{+aFRR} \leq P_g^+, g \in G_H, t \in T \tag{2.44}$$

$$d_{gt} + s_{gt}^{-mFRR} + s_{gt}^{-aFRR} \leq P_g^+, g \in G_H, t \in T \tag{2.45}$$

$$p_{gt} - p_{gt-1} + s_{gt}^{+mFRR} + s_{gt}^{+aFRR} \leq 60 \cdot R_g^+, g \in G_H, t \in T \tag{2.46}$$

$$p_{gt-1} - p_{gt} + s_{gt}^{-mFRR} + s_{gt}^{-aFRR} \leq 60 \cdot R_g^-, g \in G_H, t \in T \tag{2.47}$$

$$d_{gt} - d_{gt-1} + s_{gt}^{-mFRR} + s_{gt}^{-aFRR} \leq 60 \cdot R_g^+, g \in G_H, t \in T \tag{2.48}$$

$$d_{gt-1} - d_{gt} + s_{gt}^{+mFRR} + s_{gt}^{+aFRR} \leq 60 \cdot R_g^-, g \in G_H, t \in T \tag{2.49}$$

$$e_{gt} = e_{gt-1} + \eta \cdot d_{gt-1} - p_{gt-1}, g \in G_H, t \in T \tag{2.50}$$

$$e_{gt} \leq E_g, g \in G_H, t \in T \tag{2.51}$$

$$p_{gt}, e_{gt} \geq 0 \tag{2.52}$$

Note that the opportunity cost couples the reserve allocation of different reserve products over the time horizon into a single high-dimensional function. This level of expressiveness which is not present in current EU market designs. As opposed to the case of thermal generators, where one can break inter-temporal dependencies by mild assumptions, the situation for storage is considerably more involved. The function is heavily coupled due to the storage dynamics, and there is no natural way to break said interdependency. For this reason, we have opted to consider a "Seq idealized" scenario which accepts this opportunity cost function and embeds it within the reserve clearing module.

2.3.3 Sequential clearing: historical hydro modeling

In view of the opportunity cost calculation for storage described in subsection 2.3.2, one could fix the production $p_{gt}$and demand $d_{gt}$to some values defined in advance. In fact, once these values are fixed, the opportunity cost function described through equations (2.43)-(2.52) is no longer an optimization problem, but rather a mere feasibility problem. In this sense, once the production and demand are fixed, one could just add these feasibility constraints to the reserve clearing module to obtain a reserve allocation. One modeling approach is then o consider the production p_{gt} and demand d_{gt} as parameters obtained through historical records.

2.3.4 Sequential clearing: hydro response modeling

Following the logic presented in subsection 2.3.3, one can instead fix the production p_{gt} and demand d_{gt} to some values which react to the energy price. Concretely, the following optimization problem is solved for each agent:

$$\min \sum_{t\in T} -\lambda_t \cdot (p_{gt} - d_{gt})$$
$$p_{gt} \leq P_g^+, g \in G_H, t \in T$$
$$d_{gt} \leq P_g^+, g \in G_H, t \in T$$
$$p_{gt} - p_{gt-1} \leq 60 \cdot R_g^+, g \in G_H, t \in T$$
$$p_{gt-1} - p_{gt} \leq 60 \cdot R_g^-, g \in G_H, t \in T$$
$$d_{gt} - d_{gt-1} \leq 60 \cdot R_g^+, g \in G_H, t \in T$$
$$d_{gt-1} - d_{gt} \leq 60 \cdot R_g^-, g \in G_H, t \in T$$
$$e_{gt} = e_{gt-1} + \eta \cdot d_{gt-1} - p_{gt-1}, g \in G_H, t \in T$$
$$e_{gt} \leq E_g, g \in G_H, t \in T$$
$$p_{gt}, e_{gt} \geq 0$$

The obtained optimal values p_{gt}, d_{gt} are then fixed in the opportunity cost function described in equations (2.43)-(2.52) , which is embedded into the reserve clearing module.

2.4 Nomenclature

Sets

G: set of generators
G_H: set of hydro generators
T: set of hourly time steps over a 24-hour horizon

Variables

p_{gt}: power production of unit g
g_{gt}: demand for hydro unit g
ls_t: load shedding at time period t
ps_t: production shedding at time period t
w_{gt}: commitment of unit g at time period t
z_{gt}: start-up of unit g at time period t
$s_{gt}^{mFRR/aFRR}$: reserve allocation for unit g at period t

Parameters

MC_g: marginal cost of generator g
S_g: start-up cost of generator g
D_t: energy demand at time period t
$R^{mFRR/aFRR}$: reserve requirement
P_g^+/P_g^-*:* technical maximum/minimum of generator g
R_g^+/R_g^-*:* ramp up/down rate of generator g
$DT_z^{mFRR/aFRR}$*:* delivery time of the reserve products for zone z
UT_g/DT_g: minimum up/down time of unit g
$VOLL$: value of lost load

3. Case study

3.1 Case study description

The present section aims at providing a detailed overview of the considered instance of the Greek system. The test case aims at emulating an operational year, using for this purpose the available data from 2024. Table 1 summarizes the main characteristics of the system. Note that the underlying data is built by relying on publicly vailable sources. The generation mix and network data are sourced from PyPSA [3]. The ENTSOE transparency platform [4] is employed to gather (i) cross-border flows (ii) load profiles and renewable production profiles and (iii) contracted reserves. Note that we have considered four reserve products:

mFFR and aFRR both in the upward and downward direction. The mean reserve requirement is presented in Table 1.

	Features	Data
Modeling area	Greece	
Transmission network	• Nodal • Based on B -θ formulation of load flows • Cross-border flows are set equal to historical data	• PyPSA [3] • Cross-border flows of 2024 are collected from ENTSOE transparency platform [4]
Thermal generation	• Lignite and gas generators	• PyPSA [3]
Hydro generation	• Hydro storage modeled	• PyPSA [3]
RES generation	• Solar and wind generation	• Historical profiles from 2024 gathered from ENTSOE transparency platform [4]
Load	• Annual time series	• Data obtained from PyPSA [3] • Data updated to 2024 using ENTSOE transparency platform [4]
Reserve products	• upward mFRR: 649.7 MW • downward mFRR: 223.1 MW • upward aFRR: 595 MW • downward aFRR: 126.9 MW	• Reserve requirements for 2024 are collected from the ENTSOE transparency platform [4]

Table 1: Overview of the characteristics of the model used for the quantitative analysis.

The provided overview is complemented in Table 2 by presenting the generation mix of the modeled instance of the Greek power system. We note that all technologies, except solar and wind, are assumed to be able to offer reserve in the market.

Technology	Installed capacity (MW)
Lignite	3330
Gas	7750
Hydro pumped storage	699
Hydro reservoir	2566
Solar	6700
Wind	5065

Table 2: Generation mix of the simulated Greek power network.

Simulating an entire operational year with hourly resolution leads to a large mixed-integer formulation, in turn leading to computationally intense calculations. To decrease the computational burden, we opted to select a set of representative days to approximate a full operational year. The procedure that we use is as follows. We commence by calculating the net demand (the load minus the injection of renewables). This leads to 365 daily profiles which are clustered using the k-medoids algorithm. The advantage of this algorithm is that it allows us to obtain a cluster representative which corresponds to an actual day (as opposed to the k-means algorithm). The year is clustered into 24 clusters of days, and their corresponding representatives are as follows: 2024-01-17, 2024-02-09, 2024-02-26, 2024-03-21, 2024-04-11, 2024-04-24, 2024-05-23, 2024-06-11, 2024-06-12, 2024-06-20, 2024-07-10, 2024-07-13, 2024-08-11, 2024-08-15, 2024-09-05, 2024-10-07, 2024-10-21, 2024-10-25, 2024-11-05, 2024-11-12, 2024-11-27, 2024-12-03, 2024-12-18, 2024-12-27.

3.2 Renewable energy curtailments

Our results suggest that both co-optimization and sequential clearing designs are prone to renewable curtailments. As indicated in Figure 2, whenever there are curtailments under co-optimization, then there are also curtailments under the sequential design, which is consistent with the observation that the sequential design results in higher costs. In the figure below, we consider the sequential clearing configuration that results in the lowest percentage of inefficiencies (50% hydro response).

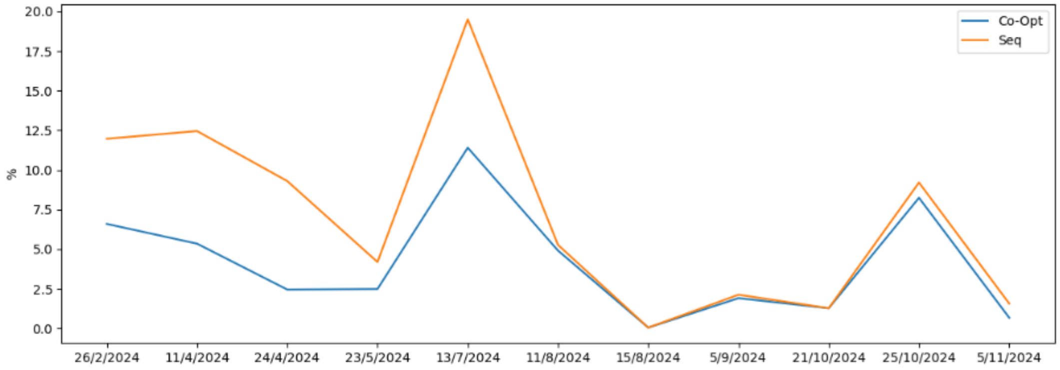

Figure 2: Renewable curtailments under the co-optimization and sequential designs, for the sample days of the study.

In 2025, an average of 6.6% of renewable energy production in Greece was curtailed (approximately 1.9 TWh out of 26.9 TWh). The results underscore an important potential benefit of co-optimization, which would be a reduction in renewable energy curtailments.

3.3 Analysis of cost differences

In this section we analyze the cost differences between co-optimization and the sequential designs, in particular the (100% hydro response) design. We are specifically interested in the 8.77% cost differential documented in the main paper.

As presented in Figure 3, the major cause of the inefficiencies relates to the costs of operating units at their technical minimum. In the left part of Figure 4 we observe the costs of running units at their technical minimum for each hour. Note that the largest difference occurs during specific periods (hours 15-20). Furthermore, the right part of figure 4 indicates the sum of technical minima over all committed units for every hour of the day. The day which is simulated in Figures 3 and 4 is January 17, 2024. We note from the right part of figure 4 that sequential clearing schedules an extra 1500 MW of technical minima relative to co-optimization between hours 15-20. The reason for this excessive commitment of thermal units under the sequential design is that during these hours the system requires roughly 1800 MW of reserve. In the co-optimization setting this reserve requirement is, in part, covered by hydro. In the sequential design the hydro energy profile is scheduled aggressively (see Figure 6 of the paper). In fact, the hydro dispatch under the sequential clearing is roughly 1000 MW higher as compared to co-optimization. This happens to be problematic as the system must satisfy its reserve requirements. Since hydro has been dispatched in a greedy fashion under the sequential design, the thermal plants are committed in order to cover these reserve requirements.

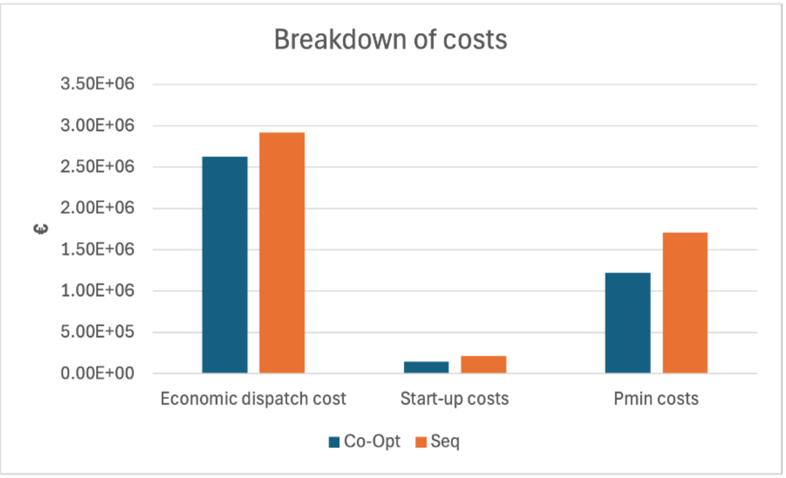

Figure 3: Breakdown of costs under (Co-Opt) and the (100% hydro response) sequential design. The simulated day is January 17, 2024.

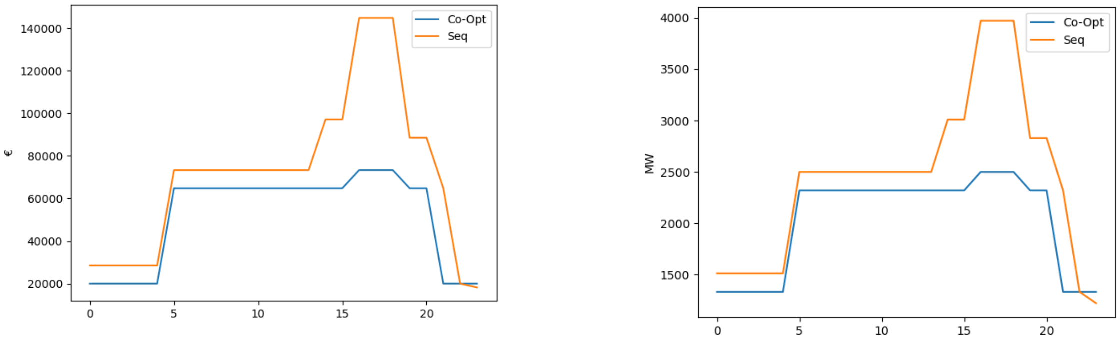

Figure 4: Left: costs (in €) of running units at their technical minimum. Right: sum of technical minimum across all thermal units committed for the day. The simulated day is January 17, 2024. The sequential design which is specifically simulated is (100% hydro response).

4. Market power model

4.1 Notation

We proceed with defining variables and parameters used in our modeling. Additional parameters and variables appearing in the text are defined when they first appear. Dual variables are defined by being indicated to the left of the constraints that they correspond to.

Variables

p_f: fringe energy supply

p_d: dominant firm energy supply

r_d: dominant firm reserve supply

dR: reserve demand

Parameters

$VR(\cdot)$: inverse demand function of TSO for reserve

$V(\cdot)$: inverse demand function for energy

$MC_f(\cdot)$: marginal cost function of fringe supplier

MC_d: marginal cost of dominant firm

D: inelastic demand for energy

P_d^+: generation capacity of dominant firm

P_f^+: generation capacity of fringe

A_d, B_d: intercept and slope of linear inverse demand function for energy

A_f, B_f: intercept and slope of linear marginal cost function of fringe producers

A_{rd}, B_{rd}: intercept and slope of linear inverse demand function for reserve

4.2 Fringe competitors in the energy-only market

4.2.1 Perfect competition model

A perfect competition model of co-optimization of energy and reserves can be formulated as follows:

$$\max_{p_f, p_d, r_d, dR} \int_0^{dR} VR(x)dx - \int_0^{p_f} MC_f(x)dx - MC_d \cdot p_d$$

$$(\lambda E): D - p_f - p_d = 0$$

$$(\lambda R): dR - r_d = 0$$

$$(\mu_d): p_d + r_d \leq P_d^+$$

$$p_f \geq 0, p_d \geq 0, r_d \geq 0, dR \geq 0$$

The KKT conditions can be expressed as follows:

$$0 \leq p_f \perp MC_f(p_f) - \lambda \geq 0$$

$$0 \leq r_d \perp \mu_d - \lambda R \geq 0$$

$$0 \leq p_d \perp \mu_d - \lambda E + MC_d \geq 0$$

$$0 \leq dR \perp -VR(dR) + \lambda R \geq 0$$

$$0 \leq \mu_d \perp P_d^+ - p_d - r_d \geq 0$$

Assuming that $p_d > 0$ and $r_d > 0$ and $p_f > 0$, the following hold:

$$\lambda E = MC_f(p_f)$$

$$\lambda R = VR(dR)$$

$$\lambda R = \lambda E - MC_d = \mu_d$$

Applying this to an illustrative model, let us consider the following input data:

$$MC_d = 10 \frac{€}{\text{MWh}}$$

$$MC_f(x) = x$$

$$P_d^+ = 150$$

$$D = 100$$

$$VR(x) = 1000 - 10x$$

To obtain the analytical solution, note from the above KKT conditions that

$$\lambda R = 1000 - 10 \cdot r_d = 1000 - 10 \cdot (150 - p_d) = 1000 - 10 \cdot (150 - 100 + p_f)$$

$$\lambda R = \lambda E - MC_d = \lambda E - 10 = p_f - 10$$

Plugging the second equality into the first, we have that

$$1000 - 10 \cdot (50 + p_f) = p_f - 10 \Rightarrow 500 - 10 \cdot p_f = p_f - 10 \Rightarrow 510 = 11 \cdot p_f$$

The analytical solution is finally obtained as

$$p_d = 53.64, p_f = 46.36, r_d = 96.36, \lambda R = 36.4, \lambda E = 46.36$$

4.2.2 Monopoly in co-optimization

If the monopoly is operating in a co-optimized market, we express the profit maximization problem as follows:

$$\max_{p_d, r_d} VR(r) \cdot r_d + (MC(D - p_d) - MC_d) \cdot p_d$$

$$(\mu): p_d + r_d \leq P_d^+$$

The KKT conditions are as follows:

$$0 \leq p_d \perp MC'_f(D - p_d) \cdot p_d - \left(MC_f(D - p_d) - MC_d\right) + \mu \geq 0$$

$$0 \leq r \perp -VR'(r_d) \cdot r_d - VR(r_d) + \mu \geq 0$$

The formulation of this model as an MPEC is as follows:

$$\max_{p_d, r_d, p_f, dR, \lambda E, \lambda R} \lambda R \cdot r_d - (\lambda E - MC_d) \cdot p_d$$

With upper-level constraints:

$$p_d + r_d \leq P_d^+$$

And lower-level constraints (which are the KKT conditions of energy market clearing given p_d and r_d):

$$D - p_d - p_f = 0$$

$$dR - r_d = 0$$

$$0 \leq p_f \perp MC_f\left(p_f\right) - \lambda E \geq 0$$

$$0 \leq dR \perp -VR(dr) + \lambda R \geq 0$$

Note that these constraints correspond to the following lower-level market clearing problem given decisions p_d and r:

$$\max_{p_f, dR} \int_0^{dR} VR(x)dx - \int_0^{p_f} MC_f(x)dx$$

$$(\lambda E): D - p_d - p = 0$$

$$(\lambda R): dR - r_d = 0$$

Going back to the illustrative example, the equilibrium solution (exploiting the fact that the dominant firm does not use its entire capacity) is

$$p_d = 45, \lambda E = 55$$

$$r_d = 50, \lambda R = 500$$

4.2.3 Monopoly in sequential clearing

The sequential clearing monopoly model decides on reserve first, according to the following model:

$$\max_{r_d \geq 0} VR(r_d) \cdot r_d + OP(r_d)$$

Where $OP(r_d)$ is defined as follows:

$$OP(r_d) = \max_{p_d \geq 0} (MC(D - p_d) - MC_d) \cdot p_d$$

$$p_d + r_d \leq P^+$$

Plugging the second problem into the first one, we have that the first problem is equivalent to

$$\max_{r_d \geq 0} VR(r_d) \cdot r_d + \max_{p_d \geq 0} (MC(D - p_d) - MC_d) \cdot p_d$$

$$p_d + r_d \leq P_d^+$$

This suggests that the sequential clearing and co-optimized models are equivalent for the case where there are no fringe suppliers in the reserve market. In order to confirm whether this is the case or not, we proceed by first considering the lower-level energy market model given the production decision of the dominant firm.

$$\min_{p_f \geq 0} \int_0^{p_f} MC_f(x)dx$$

$$(\lambda E): D - p_f - p_d = 0$$

The KKT conditions are:

$$D - p_f - p_d = 0$$

$$0 \leq p_f \perp MC_f(p_f) - \lambda E \geq 0$$

The profit maximization problem in the energy market, given a decision r_d of the dominant player in the reserve market, is expressed as:

$$OpProf: OP(r_d) = \max_{p_d \geq 0, \lambda E, p_f} (\lambda E - MC_d) \cdot p_d$$

$$(\pi): p_d + r_d \leq P_d^+$$

$$D - p_f - p_d = 0$$

$$0 \leq p_f \perp MC_f(p_f) - \lambda E \geq 0$$

This is, in general, a non-convex optimization problem (because of the complementarity constraint), so in principle there is not much one can say about the structure of the value function $OP(r_d)$. But it is easy to get an understanding of the structure from an intuitive standpoint, at least in the case of a linear marginal cost function for fringe suppliers. If the amount of reserve is such that the leftover capacity is greater than the one that equalizes marginal revenue to marginal cost then the profit becomes constant, for lower reserve level the profit is quadratic in reserve.

For the analytical example, the first order condition in the energy market equalizes marginal revenue to marginal cost, thus:

$$-MC_f'(D-p_d)\cdot p_d + MC_f(D-p_d) - MC_d = 0$$

Plugging in the numerical values of the example, we have

$$-1\cdot p_d + D - p_d - MC_d = 0 \Rightarrow p_d = \frac{100-10}{2} = 45$$

For $P_d^+ = 150$, as long as the reserve is greater than 105 then the optimal reserve becomes r_d. This creates an energy price which is linearly increasing in reserve, and multiplied by the optimal production (which is equal to the nominal capacity minus reserve), which creates a quadratic function in r_d. The results are presented in the following figure. The form of the curve aligns with what is anticipated by the theory.

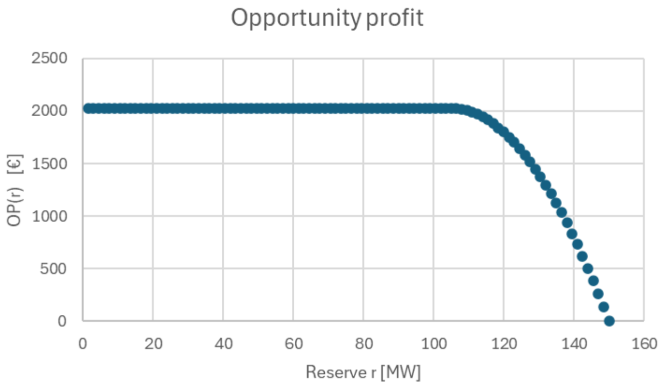

Figure 5: Opportunity profit function for the sequential clearing model.

In order to develop an algorithm for tackling the full model, which can be used with a much richer set of features, we now propose an approximation of the opportunity profit function that linearly approximates each segment of the opportunity profit function, with a binary variable indicating which segment we are in. Given a reserve value R, we can count on a first-order oracle that produces $OP(R)$ as the objective function value of $(OpProf)$ and its derivative (sensitivity) as $-\pi$, where π is the dual multiplier that implicates the variable r_d in the first constraint of $(OpProf)$. We can then approximate the opportunity profit function as follows:

$$\sum_{n=1}^{NA} u_n = 1$$

$$\theta_n \leq M \cdot u_n, n = 1, \dots, NA$$

$$\theta_n \leq OP_n + OPD_n \cdot (r - R_n), n = 1, \dots NA$$

$$R_n - \Delta - (R_n - \Delta) \cdot (1 - u_n) \leq r \leq R_n + (P_d^+ - R_n) \cdot (1 - u_n), n = 1, \dots, NA$$

Here, NA is the number of segments over which the opportunity profit function is approximated, OP_n is the value of the opportunity profit in segment n, OPD_n is the derivative of the opportunity profit in segment n, and u_n is a binary variable indicating whether segment n is the one that we are in. The first constraint requires that we pick exactly one segment. The second constraint upper bounds the opportunity profit (by a sufficiently large constant, M) and effectively limits the contribution of linear segment n to zero if we are not in that segment (i.e. $u_n = 0$). The third constraint effectively describes the linear behavior of the opportunity profit within a segment, given the information furnished by the oracle. The fourth constraint limits the reserve in the upper and lower limits of the chosen interval[1]. The total opportunity profit is then $\sum_{n=1}^{NA} \theta_n$.

Given a reserve provision r_d by the dominant firm, the reserve market clears according to the following model:

$$\max_{dR \geq 0} \int_0^{dR} VR(x) dx$$

$$(\lambda R): dR - r_d = 0$$

The KKT conditions of this model are expressed as follows:

$$dR = r_d$$

$$0 \leq dR \perp \lambda R - VR(dR) \geq 0$$

We can also add an explicit constraint that the offer of the monopolist should not exceed its available capacity.

Combining all of the above elements, the first-stage reserve market profit maximization problem of the monopolist is formulated as follows:

$$\max_{r_d, \lambda R, dR} \lambda R \cdot r_d + OP(r_d)$$

$$r_d \leq P_d^+$$

---

[1] The width of the interval is $\Delta = \frac{P_d^+}{NA}$.

$$dR = r_d$$

$$0 \leq dR \perp \lambda R - VR(dR) \geq 0$$

Which is further expressed equivalently as follows:

$$\max_{r_d, \lambda R, \theta, u} \lambda R \cdot r_d + \sum_{n=1}^{NA} \theta_n$$

$$r_d \leq P_d^+$$

$$dR = r_d$$

$$0 \leq dR \perp \lambda R - VR(dR) \geq 0$$

$$\sum_{n=1}^{NA} u_n = 1$$

$$\theta_n \leq M \cdot u_n, n = 1, \dots, NA$$

$$\theta_n \leq OP_n + OPD_n \cdot (r_d - R_n), n = 1, \dots NA$$

$$r_d \geq 0, u \in \{0,1\}$$

This is a mixed integer program with complementarity constraints. Once the reserve price is decided by this model, we can then proceed to the solution of a standard MPEC, which represents the lower-level energy market, in order to fully characterize the equilibrium.

We provide a proof of concept in our illustrative example. Indeed, the model chooses an amount of reserve in the first stage equal to $r_d = 50$ MW, and the reserve price becomes $\lambda R =$ 500 €/MWh. As predicted by the theoretical analysis of section 4.2.2, this is exactly the outcome predicted by our closed-form analysis. The difference is that we now have a coding base for testing generalizations in the model which can allow us to hopefully eventually uncover differences in the two designs.

4.3 Fringe competitors in both energy and reserve markets

We proceed with adding fringe competitors in the reserve market. In order to make this analysis meaningful, we need to introduce a capacity constraint on the fringe competitors in order to avoid a collapse of the reserve price. And this in turn necessitates introducing price elasticity in the energy market, so that the energy market can clear no matter how much reserve is committed from fringe competitors.

4.3.1 Perfect competition model

The model is expressed as follows:

$$\max_{p,p_d,r_d,r_f,dR,d} \int_0^d V(x)dx + \int_0^{dR} VR(x)dx - \int_0^{p_f} MC_f(x)dx - MC_d \cdot p_d$$

$$(\lambda E): d - p_d - p_f = 0$$

$$(\lambda R): dR - r_d - r_f = 0$$

$$(\mu_d): p_d + r_d \leq P_d^+$$

$$(\mu_f): p_f + r_f \leq P_f^+$$

$$p_f \geq 0, p_d \geq 0, d \geq 0, r_d \geq 0, r_f \geq 0, dR \geq 0$$

We now include the r_f variable (provision of reserve by fringe competitors) in the second constraint, as well as a capacity constraint on the fringe units, since otherwise the competition in the reserve market becomes too strong and this probably eliminates any meaningful outcomes for our analysis. Moreover, we introduce a demand curve in the energy market, denoted as $V(x)$.

For the numerical values of the illustrative example, we assume that the fringe competitors have the same capacity as the dominant player, i.e. $P_f^+ = 150$. Moreover, we introduce a demand curve $V(x) = 900 - 6x$. We also modify the inverse demand function for reserve to $VR(x) = 1500 - 10x$, so that the total installed capacity in the system exactly coincides with the sum of the intersection of the inverse demand curves for energy and reserves with the quantity axis. This pushes the system at the margin of being scarce in terms of installed capacity. The valuation of the energy demand curve starts off lower than that of the reserve, and lands to zero at 150 MW. We find that $d = 148.3$ MW, $p_d = 138.33$ MW, $p_f = 10$ MW, $dR = 150$ MW, $r_d = 11.21$ MW, $r_f = 138.79$ MW, $\lambda E = 10$ €/MWh, $\lambda R = 0$ €/MWh. In order to understand the solution, it is easiest to first understand why the reserve price collapses: there is a sufficient supply of fringe suppliers, thus abundant access to expensive fringe suppliers who can fill the reserve requirement. We then proceed to analyze the dispatch of energy: the question is whether the dominant firm is cheap enough to be used in the optimal solution. This is indeed the case, and it is in fact partially activated, so fringe suppliers are used up to a marginal cost of 10 €/MWh (thus 10 MW from fringe supply) while the rest of the energy demand is filled from the dominant firm and the consumption of energy stops at a valuation of 10 €/MWh. The fact that the dominant firm is marginal sets the energy price to 10 €/MWh, taking also into account the fact that the reserve price is 0 €/MWh. Contrasting this to the case without fringe suppliers in the reserve market, we note that reserve prices collapse, and energy prices also drop markedly.

4.3.2 Monopoly in co-optimization

The formulation of this model as an MPEC is as follows:

$$\max_{p_d, r_d, p_f, dR, d, \lambda E, \lambda R, \mu_f} \lambda R \cdot r_d + (\lambda E - MC_d) \cdot p_d$$

With upper-level constraints:

$$p_d + r_d \leq P_d^+$$

And lower-level constraints (which are the KKT conditions of energy market clearing given p_d and r_d):

$$d - p_d - p_f = 0$$

$$dR - r_d - r_f = 0$$

$$0 \leq \mu_f \perp P_f^+ - p_f - r_f \geq 0$$

$$0 \leq p_f \perp MC_f\left(p_f\right) - \lambda E + \mu_f \geq 0$$

$$0 \leq r_f \perp -\lambda R + \mu_f \geq 0$$

$$0 \leq dR \perp -VR(dr) + \lambda R \geq 0$$

$$0 \leq d \perp -V(d) + \lambda E \geq 0$$

Note that these constraints correspond to the following lower-level market clearing problem given decisions p_d and r_d:

$$\max_{p_f, r_f, dR} \int_0^d V(x)dx + \int_0^{dR} VR(x)dx - \int_0^{p_f} MC_f(x)dx$$

$$(\lambda E): d - p_d - p = 0$$

$$(\lambda R): dR - r_d - r_f = 0$$

$$\left(\mu_f\right): p_f + r_f \leq P_f^+$$

$$p_f \geq 0, r_f \geq 0, dR \geq 0, d \geq 0$$

Running this model against the illustrative example with a fringe capacity limit of $P_f^+ = 150$ MW, we find that $d = 100.7$ MW, $p_d = 69.2$ MW, $p_f = 31.5$ MW, $dR = 123.5$ MW, $r_d = 5$ MW, $r_f = 118.5$ MW, $\lambda E = 296.18$ €/MWh, $\lambda R = 264.71$ €/MWh. The profit is calculated as:

$$(\lambda E - MC_d) \cdot p_d + \lambda R \cdot r_d = (296.18 - 10) \cdot 69.2 + 264.71 \cdot 5 = 21170.11 \text{ €}.$$

Even if the capacity constraint of the dominant firm is not binding, this does not mean that the energy and reserve market can be considered independently, because the fringe supplier is capacity constrained. We explain what we mean exactly in section 4.5.

4.3.3 Monopoly in sequential clearing

We proceed by first considering the lower-level energy market model given the production decision of the dominant firm *and* the reserve decision of the fringe competitors.

$$\max_{p_f \geq 0, d \geq 0} \int_0^d V(x)dx - \int_0^{p_f} MC_f(x)dx$$

$$(\lambda E): d - p_f - p_d = 0$$

$$(\mu_f): p_f + r_f \leq P_f^+$$

The KKT conditions are:

$$d - p_f - p_d = 0$$

$$0 \leq \mu_f \perp P_f^+ - p_f - r_f \geq 0$$

$$0 \leq p_f \perp MC_f(p_f) + \mu_f - \lambda E \geq 0$$

$$0 \leq d \perp -V(d) + \lambda E \geq 0$$

The profit maximization problem in the energy market, given a decision r_d of the dominant player in the reserve market and a decision r_f of the fringe agents in the reserve market, is expressed as:

$$OpProf: OP(r_d, r_f) = \max_{p_d \geq 0, d \geq 0, \lambda E, p_f, \mu_f} (\lambda E - MC_d) \cdot p_d$$

$$(\pi): p_d + r_d \leq P_d^+$$

$$d - p_f - p_d = 0$$

$$0 \leq \mu_f \perp P_f^+ - p_f - r_f \geq 0$$

$$0 \leq p_f \perp MC_f(p_f) + \mu_f - \lambda E \geq 0$$

This is, in general, a non-convex optimization problem (because of the complementarity constraints), so in principle there is not much one can say about the structure of the value function $OP(r_d, r_f)$. And the analysis is now complexified, since it is no longer easy to get an understanding of the structure of the opportunity profit function from an intuitive standpoint. The tricky aspect of the analysis is that the fringe reserve supply is now dependent on the decisions of the monopolist, and then enters this opportunity profit function through non-convex complementarity constraints. On an intuitive level, this captures the complex tradeoff that the monopolist faces between implicitly affecting his competition in the energy market by how he acts in the reserve market which precedes it.

In order to gain an intuitive understanding of the phenomena at hand, we now proceed to plot $OP(r_d, r_f)$ in a three-dimensional plot (for different values of the variables (r_d, r_f)). We will then generalize the analysis of section 4.2.3 by resorting to a Taylor approximation of $OP(r_d, r_f)$ and using that as input to the monopolist problem in the reserve market clearing. The opportunity profit function is depicted in the following figure.

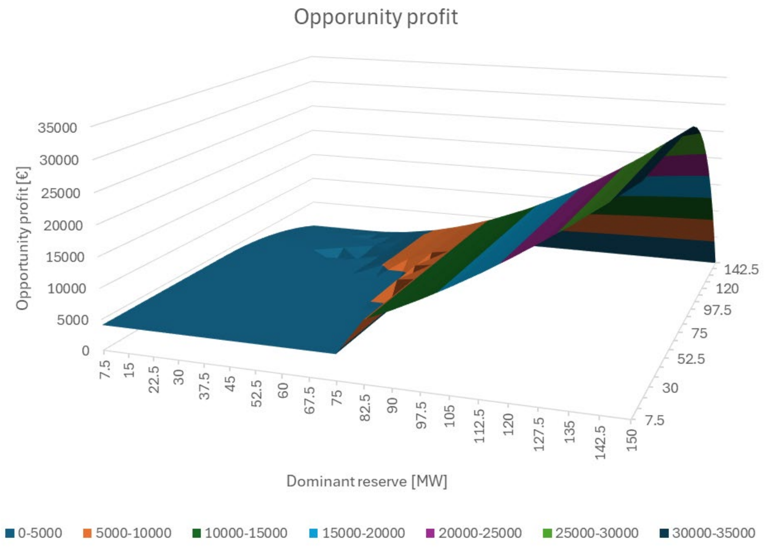

Figure 6: Opportunity profit as a function of reserve committed by the dominant firm and the fringe competitor.

The results indicate non-convexity. Whereas the profits are concave for a given level of fringe reserve, the behavior of the profit function along the fringe reserve axis is not as “good”. For low fringe reserve levels, the profits in the energy market are low, intuitively due to more intense competition. Then they accelerate, then increase but at less accelerated pace. For instance, fixing the dominant firm reserve at 7.5 MW, the profit as a function of fringe reserve capacity is depicted in figure 6.

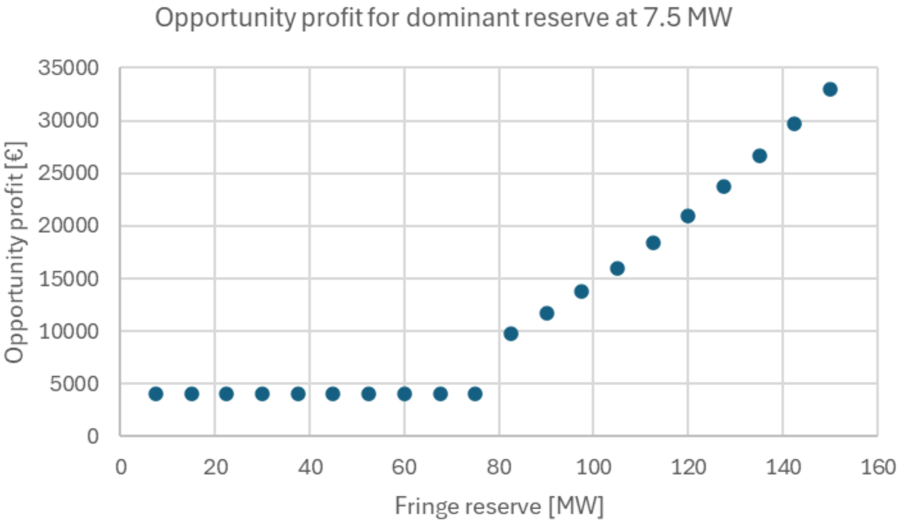

Figure 7: The non-convex behavior of opportunity profit given dominant reserve $r_d = 7.5$ MW.

At this point, and in order to arrive to a result as efficiently as possible, we opt for a zeroth order oracle in order to simulate the outcome of the reserve market under the sequential design. The reason is the following:

- Gurobi cannot solve the $(OpProf)$ problem since its constraint matrix is not positive semidefinite.
- ipopt cannot solve the $(OpProf)$ problem because it gets stuck at a locally infeasible solution.
- knitro can solve the $(OpProf)$ problem but it is not clear if/how it can compute dual multipliers.

We thus opt for knitro, but with a zeroth-order approximation of the opportunity profit function. This model is precisely formulated as follows:

$$\sum_{n=1}^{NAD} ud_n = 1$$

$$\sum_{n=1}^{NAF} uf_n = 1$$

$$w_{nm} \leq ud_n, n = 1, \ldots, NAD, m = 1, \ldots, NAF$$

$$w_{nm} \leq uf_m, n = 1, \ldots, NAD, m = 1, \ldots, NAF$$

$$w_{nm} \geq ud_n + uf_m - 1, n = 1, \ldots, NAD, m = 1, \ldots, NAF$$

$$RD_n - \mathit{\Delta} D - (RD_n - \mathit{\Delta} D) \cdot (1 - ud_n) \leq r_d \leq RD_n + (P_d^+ - RD_n) \cdot (1 - ud_n), n = 1, \ldots, NAD$$

$$RF_n - \mathit{\Delta} F - (RF_n - \mathit{\Delta} F) \cdot (1 - uf_n) \leq r_f \leq RF_n + (P_f^+ - RF_n) \cdot (1 - uf_n), n = 1, \ldots, NAF$$

Here, RD_n and RF_n corresponds to the value of reserve committed by the dominant firm and fringe suppliers respectively. The first and second constraints force us to choose one reserve value for the dominant firm and one reserve value for the fringe competition. The third, fourth and fifth constraint linearize the product of ud_n and uf_m, resulting in a variable w_{nm} which indicates whether we are in point (n, m) of the grid or not. The last two constraints confine the choice of dominant and fringe reserves to the proper ranges, with $\mathit{\Delta} D = \frac{P_d^+}{NAD}$ and $\mathit{\Delta} F = \frac{P_f^+}{NAF}$. The opportunity profit is given by $OP_{nm} \cdot w_{nm}$, where OP_{nm} is the opportunity profit evaluated at point (n, m) of the grid of figure 6.

Suppose that the market anticipates an energy market clearing price λE. Then the opportunity cost of a slice dx of fringe capacity is given by $\max\left(\lambda E - MC_f\left(P_f^+ - x\right), 0\right)$. Suppose that MC_f is strictly increasing, thus invertible. Then the total provision of reserve is "free" for a capacity up to the capacity for which the marginal cost equals the energy price, i.e. the free capacity solves the following equation:

$$MC_f\left(P_f^+ - R_{f,0}^+\right) = \lambda E \Rightarrow P_f^+ - R_{f,0}^+ = MC_f^{-1}(\lambda E) \Rightarrow R_{f,0}^+ = P_f^+ - MC_f^{-1}(\lambda E).$$

Concretely, given a linear marginal cost function $MC_f(x) = A_f + B_f \cdot x$, we have that

$$MC_f^{-1}(\lambda E) = \frac{\lambda E - A_f}{B_f} \Rightarrow R_{f,0}^+ = P_f^+ - \frac{\lambda E - A_f}{B_f}.$$

From that point onwards, the opportunity cost of supplying x MW of reserve is $\lambda E - MC_f\left(P_f^+ - x\right)$. For a linear marginal cost function, this opportunity cost is expressed as:

$$\lambda E - \left(A_f + B_f \cdot \left(P_f^+ - x\right)\right).$$

As a sanity check, we can confirm that this opportunity cost function is increasing in x. At $P_f^+ - \frac{\lambda E - A_f}{B_f}$, we have that this marginal cost is zero. Denoting $r1_f$ as the additional reserve capacity above $P_f^+ - \frac{\lambda E - A_f}{B_f}$, we have that the above opportunity cost as a function of $r1_f$ can be expressed as follows:

$$\lambda E-\left(A_f+B_f\cdot\left(P_f^+-\left(P_f^+-\frac{\lambda E-A_f}{B_f}+r1_f\right)\right)\right)=\lambda E-\left(A_f+B_f\cdot\left(\frac{\lambda E-A_f}{B_f}-r1_f\right)\right)$$

$$=\lambda E-\left(\lambda E-B_f\cdot r1_f\right)=B_f\cdot r1_f.$$

This actually makes a lot of sense since the first unit from the stack of non-zero opportunity cost starts off at zero opportunity cost whereas every additional MW follows the slope of the energy marginal cost curve.

To account for the possibility where $P_f^+-\frac{\lambda E-A_f}{B_f}<0$, the above opportunity cost can be generalized as follows:

$$\lambda E-\left(A_f+B_f\cdot\left(P_f^+-\left(\max\left(P_f^+-\frac{\lambda E-A_f}{B_f},0\right)+r1_f\right)\right)\right).$$

So if it turns out that if $P_f^+-\frac{\lambda E-A_f}{B_f}<0$ then the above expression becomes

$$\lambda E-\left(A_f+B_f\cdot\left(P_f^+-\left(\max\left(P_f^+-\frac{\lambda E-A_f}{B_f},0\right)+r1_f\right)\right)\right)=\lambda E-\left(A_f+B_f\cdot\left(P_f^+-r1_f\right)\right)$$

$$=\lambda E-\left(A_f+B_f\cdot P_f^+\right)+B_f\cdot r1_f.$$

Given a reserve provision r_d by the dominant firm and a price λE in the energy market, we thus propose that the reserve market clears according to the following model:

$$\max_{dR\geq 0,r_f\geq 0,r0_f\geq 0,r1_f\geq 0}\int_0^{dR}VR(x)dx-\int_0^{r1_f}\left(\max\left(\lambda E-\left(A_f+B_f\cdot P_f^+\right),0\right)+B_f\cdot x\right)dx$$

$$(\lambda R): dR-r_d-r_f=0$$

$$(\delta): r_f-r0_f-r1_f-=0$$

$$\left(\mu R0_f\right): r0_f\leq\max\left(P_f^+-\frac{\lambda E-A_f}{B_f},0\right)$$

$$\left(\mu R_f\right): r_f\leq P_f^+$$

Here, $r0_f$ is the fringe reserve that has zero opportunity cost, and $r1_f$ is the fringe reserve that has non-zero opportunity cost. The KKT conditions are expressed as follows:

$$dR-r_d-r_f=0$$

$$r_f-r0_f-r1_f=0$$

$$0 \leq \mu R0_f \perp \max\left(P_f^+ - \frac{\lambda E - A_f}{B_f}, 0\right) - r0_f \geq 0$$

$$0 \leq \mu R_f \perp P_f^+ - r_f \geq 0$$

$$0 \leq dR \perp -VR(dR) + \lambda R \geq 0$$

$$0 \leq r_f \perp \delta - \lambda R + \mu R_f \geq 0$$

$$0 \leq r0_f \perp -\delta + \mu R0_f \geq 0$$

$$0 \leq r1_f \perp \max\big(\lambda E - \big(A_f + B_f \cdot P_f^+\big), 0\big) + B_f \cdot r1_f - \delta \geq 0$$

We can also add an explicit constraint that the offer of the monopolist should not exceed its available capacity.

Combining all of the above elements, the first-stage reserve market profit maximization problem of the monopolist is formulated as follows:

$$\max_{r_d, dR, r_f, r0_f, r1_f, \lambda R, \delta, \mu R0_f, \mu R_f} \lambda R \cdot r_d + OP\big(r_f, r_d\big)$$

$$r_d \leq P_d^+$$

$$dR - r_d - r_f = 0$$

$$r_f - r0_f - r1_f = 0$$

$$0 \leq \mu R0_f \perp \max\left(P_f^+ - \frac{\lambda E - A_f}{B_f}, 0\right) - r0_f \geq 0$$

$$0 \leq \mu R_f \perp P_f^+ - r_f \geq 0$$

$$0 \leq dR \perp -VR(dR) + \lambda R \geq 0$$

$$0 \leq r_f \perp \delta - \lambda R + \mu R_f \geq 0$$

$$0 \leq r0_f \perp -\delta + \mu R0_f \geq 0$$

$$0 \leq r1_f \perp \max\big(\lambda E - \big(A_f + B_f \cdot P_f^+\big), 0\big) + B_f \cdot r1_f - \delta \geq 0$$

This problem is further expressed equivalently as follows:

$$\max_{r_d, dR, r_f, r0_f, r1_f, \lambda R, \delta, \mu R0_f, \mu R_f, ud, uf, w, \lambda E} \lambda R \cdot r_d + \sum_{n=1}^{NAD} \sum_{m=1}^{NAF} OP_{nm} \cdot w_{nm}$$

$$r_d \leq P_d^+$$

$$dR - r_d - r_f = 0$$

$$r_f - r0_f - r1_f = 0$$

$$0 \leq \mu R0_f \perp \max\left(P_f^+ - \frac{\lambda E - A_f}{B_f}, 0\right) - r0_f \geq 0$$

$$0 \leq \mu R_f \perp P_f^+ - r_f \geq 0$$

$$0 \leq dR \perp -VR(dR) + \lambda R \geq 0$$

$$0 \leq r_f \perp \delta - \lambda R + \mu R_f \geq 0$$

$$0 \leq r0_f \perp -\delta + \mu R0_f \geq 0$$

$$0 \leq r1_f \perp \max\big(\lambda E - \big(A_f + B_f \cdot P_f^+\big), 0\big) + B_f \cdot r_f - \delta \geq 0$$

$$\sum_{n=1}^{NAD} ud_n = 1$$

$$\sum_{n=1}^{NAF} uf_n = 1$$

$$w_{nm} \leq ud_n, n = 1, \dots, NAD, m = 1, \dots, NAF$$

$$w_{nm} \leq uf_m, n = 1, \dots, NAD, m = 1, \dots, NAF$$

$$w_{nm} \geq ud_n + uf_m - 1, n = 1, \dots, NAD, m = 1, \dots, NAF$$

$$RD_n - \mathit{\Delta} D - (RD_n - \mathit{\Delta} D) \cdot (1 - ud_n) \leq r_d \leq RD_n + (P_d^+ - RD_n) \cdot (1 - ud_n), n = 1, \dots, NAD$$

$$RF_n - \mathit{\Delta} F - (RF_n - \mathit{\Delta} F) \cdot (1 - uf_n) \leq r_f \leq RF_n + \big(P_f^+ - RF_n\big) \cdot (1 - uf_n), n = 1, \dots, NAF$$

$$\lambda E = \sum_{n=1}^{NAD} \sum_{m=1}^{NAF} \mathit{\Lambda} E_{nm} \cdot w_{nm}$$

$$r_d \geq 0, ud \in \{0,1\}, uf \in \{0,1\}, w \in \{0,1\}$$

Note that the second-to-last constraint has been introduced in order to ensure that the price that occurs in the energy market clearing stage is consistent with what is decided in the reserve market clearing stage. Here, $\mathit{\Lambda} E_{nm}$ is the energy market price which corresponds to choice (n, m) of the two-dimensional grid of figure 6. This parameter is computed in problem $OpProf$.

Running this model against the illustrative example with a fringe capacity limit of $P_f^+ = 150$ MW, we find that $d = 96.7$ MW, $p_d = 51.7$ MW, $p_f = 45$ MW, $dR = 123.0$ MW, $r_d = 22.5$ MW, $r_f = 45$ MW, $\lambda E = 320$ €/MWh, $\lambda R = 270.46$ €/MWh.

The code is giving lower profit in the sequential case than in the co-optimized case. We discuss this finding in the end of the previous subsection (section 4.3.2) as well as in the last paragraph of section 4.7.

4.4 Analytical derivation of energy-only monopoly

In this section we derive analytically the optimal monopoly solution for an energy-only market with fringe suppliers and an elastic demand.

The market equilibrium, if we consider the energy market in isolation, as a function of dominant and fringe competition decisions, is

$$p_d + p_f = d$$

$$A_d - B_d \cdot d = \lambda E$$

$$A_f + B_f \cdot p_f = \lambda E$$

Substituting out the fringe production, we have

$$p_f = \frac{\lambda E - A_f}{B_f}$$

And similarly for the demand

$$d = \frac{A_d - \lambda E}{B_d}$$

Substituting into the power balance, we have

$$p_d + \frac{\lambda E - A_f}{B_f} = \frac{A_d - \lambda E}{B_d}$$

Thus, the dependence of energy price as a function of dominant firm production can be expressed as follows:

$$\lambda E \cdot \left(\frac{1}{B_f} + \frac{1}{B_d}\right) = \frac{A_d}{B_d} + \frac{A_f}{B_f} - p_d \Rightarrow \lambda E = \frac{\frac{A_d}{B_d} + \frac{A_f}{B_f} - p_d}{\frac{1}{B_f} + \frac{1}{B_d}} = \frac{B_f \cdot A_d + B_d \cdot A_f - B_d \cdot B_f \cdot p_d}{B_f + B_d}$$

The profit of the monopolist is

$$\lambda E(p_d) \cdot p_d - MC_d \cdot p_d$$

And the first order condition with respect to p_d gives

$$\lambda E'(p_d) \cdot p_d + \lambda E(p_d) = MC_d$$

Which further implies that

$$-\frac{B_d \cdot B_f}{B_f + B_d} p_d + \frac{B_f \cdot A_d + B_d \cdot A_f - B_d \cdot B_f \cdot p_d}{B_f + B_d} = MC_d$$

The constants in this expression are as follows, given the numerical settings of the problem ($A_d = 900, B_d = 6, A_f = 0, B_f = 1, MC_d = 10$):

$$\frac{B_d \cdot B_f}{B_f + B_d} = \frac{6 \cdot 1}{6 + 1} = \frac{6}{7} = 0.857$$

$$\frac{B_f \cdot A_d + B_d \cdot A_f}{B_f + B_d} = \frac{900 \cdot 1 + 0 \cdot 6}{6 + 1} = \frac{900}{7} = 128.57$$

$$-2 \cdot 0.857 p_d + 128.57 = 10 \Rightarrow p_d = \frac{128.57 - 10}{2 \cdot 0.857} = 69.18$$

This would result in a market price of

$$\lambda E = \frac{B_f \cdot A_d + B_d \cdot A_f - B_d \cdot B_f \cdot p_d}{B_f + B_d} = 128.57 - 0.857 \cdot 69.18 = 69.23 \text{ €/MWh}$$

This implies a demand of

$$d = \frac{A_d - \lambda E}{B_d} = \frac{900 - 69.23}{6} = 138.5 \text{ MW}$$

And a fringe supply of

$$p_f = \frac{\lambda E - A_f}{B_f} = \frac{69.23 - 0}{1} = 69.23$$

With indeed production matching demand:

$$p_d + p_f = 69.18 + 69.23 = 138.4 = d$$

The profit would then be

$$(\lambda E - MC_d) \cdot p_d = (69.23 - 10) \cdot 69.18 = 4097.5 \text{ €}$$

4.5 Analytical derivation of co-optimized energy and reserves monopoly

The major change in the case of energy and reserve is that the energy price is no longer equal to fringe marginal cost (i.e. their difference being zero), but instead that their difference is equal to the reserve price. The set of equations that characterize the equilibrium (for the case where the dominant reserve supply is not binding) are:

$$p_d + p_f = d$$

$$A_d - B_d \cdot d = \lambda E$$

$$\lambda E - A_f - B_f \cdot p_f = \lambda R$$

$$\lambda R = A_{rd} - B_{rd} \cdot dR$$

$$dR = r_f + r_d = \left(P_f^+ - p_f\right) + r_d$$

The strategy in section 4.4 was to derive an energy price as a function of dominant reserve decisions, by substituting out demand and fringe supply as a function of energy price and plugging that into the energy balance constraint. We can no longer do this because fringe energy supply does not depend on energy price alone.

In section 4.4 we started with three equalities and four unknowns ($p_d, p_f, d, \lambda E$), so one degree of freedom, and derived a function of the unknowns with respect to p_d which we choose as our degree of freedom, with a specific focus on $\lambda E(p_d)$ (but essentially all other unknowns also become functions of p_d, it is just that we care only about $\lambda E(p_d)$ in the analysis of the profit maximization of the monopoly). We now have five equalities in seven unknowns ($p_d, p_f, d, \lambda E, dR, r_d, \lambda R$), thus two degrees of freedom, and we will strive to derive two functional relations, $\lambda E(p_d, r_d)$ and $\lambda R(p_d, r_d)$.

Substituting the first equality to the second:

$$\lambda E = A_d - B_d \cdot d = A_d - B_d \cdot \left(p_d + p_f\right)$$

And the last to the fourth:

$$\lambda R = A_{rd} - B_{rd} \cdot dR = A_{rd} - B_{rd} \cdot \left(\left(P_f^+ - p_f\right) + r_d\right)$$

The third equality allows us to substitute out p_f as a function of λE and λR:

$$p_f = \frac{\lambda E - A_f - \lambda R}{B_f}$$

Substituting out p_f in the previous two equalities, we have

$$\lambda E = A_d - B_d \cdot (p_d + p_f) = A_d - B_d \cdot \left(p_d + \frac{\lambda E - A_f - \lambda R}{B_f}\right)$$

$$\lambda R = A_{rd} - B_{rd} \cdot \left((P_f^+ - p_f) + r_d\right) = A_{rd} - B_{rd} \cdot \left(P_f^+ - \frac{\lambda E - A_f - \lambda R}{B_f} + r_d\right)$$

The first equality above gives us

$$\lambda E \cdot \left(1 + \frac{B_d}{B_f}\right) = A_d - B_d \cdot p_d + \frac{B_d \cdot A_f}{B_f} + B_d \cdot \frac{\lambda R}{B_f} \Rightarrow$$

$$\lambda E = \frac{A_d - B_d \cdot p_d + \frac{B_d \cdot A_f}{B_f} + B_d \cdot \frac{\lambda R}{B_f}}{1 + \frac{B_d}{B_f}} = \frac{B_f \cdot A_d - B_f \cdot B_d \cdot p_d + B_d \cdot A_f}{B_f + B_d} + \frac{B_d}{B_f + B_d} \cdot \lambda R$$

This implies that

$$\frac{\lambda E - A_f - \lambda R}{B_f} = \frac{B_f \cdot A_d - B_f \cdot B_d \cdot p_d + B_d \cdot A_f}{B_f \cdot (B_f + B_d)} - \frac{1}{B_f + B_d} \cdot \lambda R - \frac{A_f}{B_f}$$

We finally substitute this energy price into the second equality above to obtain

$$\lambda R = A_{rd} - B_{rd} \cdot \left(\left(P_f^+ - \frac{\lambda E - A_f - \lambda R}{B_f}\right) + r_d\right) = A_{rd} - B_{rd} \cdot \left(P_f^+ - \frac{\lambda E - A_f - \lambda R}{B_f} + r_d\right) \Rightarrow$$

$$\lambda R = A_{rd} - B_{rd} \cdot \left(P_f^+ + r_d - \frac{B_f \cdot A_d - B_f \cdot B_d \cdot p_d + B_d \cdot A_f}{B_f \cdot (B_f + B_d)} + \frac{1}{B_f + B_d} \cdot \lambda R + \frac{A_f}{B_f}\right) \Rightarrow$$

$$\lambda R \cdot \left(1 + \frac{B_{rd}}{B_f + B_d}\right) = A_{rd} - B_{rd} \cdot \left(P_f^+ + r_d - \frac{B_f \cdot A_d - B_f \cdot B_d \cdot p_d + B_d \cdot A_f}{B_f \cdot (B_f + B_d)} + \frac{A_f}{B_f}\right)$$
$$= A + B \cdot p_d + C \cdot r_d$$

for some constants A, B, C which can be computed by expanding the above expression.

This is precisely the function $\lambda R(p_d, r_d)$. Substituting λR back into the energy price gives

$$\lambda E = \frac{B_f \cdot A_d - B_f \cdot B_d \cdot p_d + B_d \cdot A_f}{B_f + B_d} + \frac{B_d}{B_f + B_d} \cdot (A + B \cdot p_d + C \cdot r_d)$$

Which allows us to derive the function $\lambda E(p_d, r_d)$.

So the overall profit being maximized by the monopolist becomes

$$\lambda E(p_d, r_d) \cdot p_d + \lambda R(p_d, r_d) \cdot r_d - MC_d \cdot p_d$$

The first order conditions of this model should give the same result as the MPEC, and this explains why the co-optimization monopoly differs from the energy-only monopoly. But note that the energy quantity in both is equal, even if the energy price is not. Qualitatively what is happening is that the monopoly is not considering the two markets independently, but accounting for the joint effect of its energy and reserve decisions on the reserve market, and not only the energy market.

Computing the functions $\lambda E(p_d, r_d)$ and $\lambda R(p_d, r_d)$ in Julia, we get the following:

$$\lambda E(p_d, r_d) = -3.882 \cdot p_d - 3.529 \cdot r_d + 582.35$$

$$\lambda R(p_d, r_d) = -3.529 \cdot p_d - 4.112 \cdot r_d + 529.41$$

This leads to the following profit maximization problem:

$$\begin{aligned}(-3.882 \cdot p_d - 3.529 \cdot r_d + 582.35) \cdot p_d + (-3.529 \cdot p_d - 4.112 \cdot r_d + 529.41) \cdot r_d - 10 \cdot p_d \\ = -3.882 \cdot p_d^2 - 4.112 \cdot r_d^2 - 2 \cdot 3.529 \cdot p_d \cdot r_d + 582.35 \cdot p_d + 529.41 \cdot r_d \\ - 10 \cdot p_d\end{aligned}$$

The first-order conditions with respect to p_d are:

$$-2 \cdot 3.882 \cdot p_d - 2 \cdot 3.529 \cdot r_d + 582.35 - 10 = 0$$

And the first-order conditions with respect to r_d are:

$$-2 \cdot 4.112 \cdot r_d - 2 \cdot 3.529 \cdot p_d + 529.41 = 0$$

The solution to this 2x2 system is

$$p_d = 69.14$$

$$r_d = 5.04$$

This actually matches well with the mathematical programming solution. The only thing that is left to confirm for the sake of completeness is that the quadratic objective is concave, i.e. the Hessian of the objective is negative semidefinite. Indeed, the Hessian of the objective function is:

$$H = \begin{bmatrix} -2 \cdot 3.882 & -2 \cdot 3.529 \\ -2 \cdot 3.529 & -2 \cdot 4.112 \end{bmatrix} = \begin{bmatrix} -7.764 & -7.058 \\ -7.058 & -8.224 \end{bmatrix}$$

Both eigenvalues are negative, thereby verifying that the Hessian is negative definite.

4.6 Analytical derivation of sequential energy and reserves monopoly

The following conditions hold for the sequential monopoly, as they did for the co-optimized monopoly:

$$p_d + p_f = d \; (C1)$$

$$A_d - B_d \cdot d = \lambda E \; (C2)$$

$$\lambda E - A_f - B_f \cdot p_f = \lambda R \; (C3)$$

$$\lambda R = A_{rd} - B_{rd} \cdot dR \; (C4)$$

$$dR = r_f + r_d = \left(P_f^+ - p_f\right) + r_d \; (C5)$$

Indeed, all of these relations are fulfilled by the results of the mathematical programming code: $d = 96.7$ MW, $p_d = 51.7$ MW, $p_f = 45$ MW, $dR = 123.0$ MW, $r_d = 22.5$ MW, $r_f = 100.5$ MW, $\lambda E = 320$ €/MWh, $\lambda R = 270.46$ €/MWh.

But care needs to be taken as to what is considered fixed at each stage of decision making. Concretely, fringe production p_f is considered fixed at 45 MW because ex post we find that max fringe production is binding (modulo discretization error) and thus the fringe production is not responsive to price. Thus, we have that relations (C1) and (C2) yield the following relation of λE as a function of p_d:

$$\begin{bmatrix} 0 & -1 \\ 1 & B_d \end{bmatrix} \begin{bmatrix} \lambda E \\ d \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} p_d = \begin{bmatrix} -p_f \\ A_d \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} \lambda E \\ d \end{bmatrix} = -\begin{bmatrix} 0 & -1 \\ 1 & B_d \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} p_d + \begin{bmatrix} 0 & -1 \\ 1 & B_d \end{bmatrix}^{-1} \begin{bmatrix} -p_f \\ A_d \end{bmatrix} = \begin{bmatrix} -6 \\ 1 \end{bmatrix} p_d + \begin{bmatrix} 270 \\ 105 \end{bmatrix}$$

This implies that

$$\lambda E(p_d) = 270 - 6 \cdot p_d.$$

And plugging this into the first-order conditions of the monopolist in the energy market:

$$\frac{d\lambda E(p_d)}{dp_d} \cdot p_d + \lambda E(p_d) - MC_d = 0 \; (C6) \Rightarrow$$

$$-2 \cdot 6 \cdot p_d + 270 - 10 = 0 \Rightarrow$$

$$p_d = 21.7.$$

This indeed confirms the solution obtained from the code.

Reserve production decisions affect reserve prices. In particular, reserve production can migrate fringe suppliers from the energy to the reserve market. Constrains[2] (C1)-(C6) implicate the following variables: $p_f, \lambda R, \lambda E, dR, d, p_d, r_d$.

The above linear equations are expressed as follows:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & B_d & 0 \\ -B_f & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & B_{rd} & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -B_d \end{bmatrix} \begin{bmatrix} p_f \\ \lambda R \\ \lambda E \\ dR \\ d \\ p_d \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} r_d = \begin{bmatrix} 0 \\ A_d \\ A_f \\ A_{rd} \\ P_f^+ \\ MC_d \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} p_f \\ \lambda R \\ \lambda E \\ dR \\ d \\ p_d \end{bmatrix} = -\begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & B_d & 0 \\ -B_f & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & B_{rd} & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -B_d \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} r_d + \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & B_d & 0 \\ -B_f & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & B_{rd} & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -B_d \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ A_d \\ A_f \\ A_{rd} \\ P_f^+ \\ MC_d \end{bmatrix}$$

This implies the following functional relation for λR, λE, and p_d respectively:

$$\lambda R(r_d) = -2.857 \cdot r_d + 325$$

$$p_d(r_d) = -0.357 \cdot r_d + 57.92$$

$$\lambda E(r_d) = -2.143 \cdot r_d + 357.5$$

The profit maximization is expressed as follows:

$$\max_{r_d} \lambda R(r_d) \cdot r_d + (\lambda E(r_d) - MC_d) \cdot p_d(r_d) \Rightarrow$$

$$\max_{r_d} (-2.857 \cdot r_d + 325) \cdot r_d + (-2.143 \cdot r_d + 357.5 - 10) \cdot (-0.357 \cdot r_d + 57.92) \Rightarrow$$

$$\max_{r_d} - 2.09 \cdot r_d^2 + 76.82 \cdot r_d + 20127.2$$

Indeed, the objective is concave. The first-order condition yields

$$-2 \cdot 2.09 \cdot r_d + 76.82 = 0 \Rightarrow r_d = 18.38$$

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[2] Constraint (C6) is written in the following form:

$$(-B_d) \cdot p_d + \lambda E(p_d) - MC_d = 0$$

which exploits the fact that fringe supply is not responsive to price in the energy market, and amounts to a linear expression.

If we select this amount of dominant reserve, then the following solution to the other variables occurs:

$$\begin{bmatrix} p_f \\ \lambda R \\ \lambda E \\ dR \\ d \\ p_d \end{bmatrix} = \begin{bmatrix} 45.63 \\ 272.49 \\ 318.11 \\ 122.75 \\ 96.98 \\ 51.35 \end{bmatrix}$$

In this case, the profit becomes

$$\lambda R(r_d) \cdot r_d + (\lambda E(r_d) - MC_d) \cdot p_d(r_d) = 272.5 \cdot 18.38 + (318.11 - 10) \cdot 51.35 = 20830$$

The reserve fringe supply becomes $r_f = dR - r_d = 122.75 - 18.38 = 104.37 = P_f^+ - p_f = 150 - 45.63$.

We come back to the ampl code. The chosen reserve values correspond to the 3rd slot of dominant reserve values and the 14th slot of fringe reserve values. The opportunity profit for these values is $OP_{3,14} = 16016.7$ €. Comparing to the analytical solution:

$$(318.11 - 10) \cdot 51.35 = 15821 \text{ €}.$$

This is reasonably close, given the discretization approximation error.

The solution of the math program must obey the following values:

$$r0_f = 0$$

$$r1_f = r_f = 104.37$$

$$\mu R_f = 0$$

$$\delta - \lambda R + \mu R_f = 0 \Rightarrow \delta = \lambda R - \mu R_f = \lambda R = 272.49$$

$$\begin{aligned} \delta = \max\left(\lambda E - \left(A_f + B_f \cdot P_f^+\right), 0\right) + B_f \cdot r_f &= \max(318.11 - (0 + 1 \cdot 150), 0) + 1 \cdot 104.37 \\ &= 168.11 + 104.37 = 272.48 \end{aligned}$$

Any $\mu R0_f \geq \delta$ positive is an acceptable solution. So we can choose $\mu R0_f = 272.49$. Indeed, forcing $w_{3,14} = 1$ in the math programming code gives the best profit that we have been able to find, namely 22101.9 €. Interestingly, not forcing w results in a feasible but suboptimal solution (i.e. $w_{6,12} = 1$ with a profit of 21113.3 €).

4.7 Comparative analysis and conclusions

In the following table we present the results of our analytical model for the specific values that have been used in the illustrative example.

Section 4.2 (no fringe in reserve)	**Perfect competition**	**Co-optimization monopoly**	**Sequential monopoly**
Energy price [€/MWh]	46.36	55	55
Reserve price [€/MWh]	36.36	500	500
Dominant firm profit [€]	5454.6	27025	27025
Section 4.3 (fringe in reserve)	**Perfect competition**	**Co-optimization monopoly**	**Sequential monopoly**
Energy price [€/MWh]	10	296.18	320
Reserve price [€/MWh]	0	264.71	270.46
Dominant firm profit [€]	0	21117.4	22101.9
Welfare [€]	178558	168000	166280

Table 3: aggregate results of the different designs on the different evolutions of the illustrative example.

Comparing the monopoly models of section 4.3, we do indeed find that the sequential design offers a slight deterioration in market power (in the sense of higher energy and reserve prices, and higher dominant firm profits). Welfare also decreases in the case of sequential clearing.

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