Additional material: Application of Hierarchical TSO-DSO Coordination in European Pilot Projects

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1 Distribution monolithic

In this section, we provide an analytical formulation of the distribution dispatch model used in our TSO-DSO coordination platform. It should be noted that, for the sake of clarity, we introduce the F^{IL}_{nt} parameter to formulate the interface line active power injection into the distribution system root node, as this value was derived according to the methodology analytically presented in [1]. In all other nodes and examined periods, the respective parameter is equal to zero.

$$min \sum_{e\in\mathcal{E}}\sum_{b\in\mathcal{B}}\sum_{t\in\mathcal{T}}\left[P^{up}_{etb}\cdot p^{up}_{etb} - P^{dn}_{etb}\cdot p^{dn}_{etb}\right] \tag{1}$$

$$p^{up}_{etb} \leq Q^{up}_{etb} \quad \forall e\in\mathcal{E}, t\in\mathcal{T}, b\in\mathcal{B} \quad (\beta^{up}_{etb}) \tag{2}$$

$$p^{dn}_{etb} \leq Q^{dn}_{etb} \quad \forall e\in\mathcal{E}, t\in\mathcal{T}, b\in\mathcal{B} \quad (\beta^{dn}_{etb}) \tag{3}$$

$$q^{up}_{et} \leq Q^{max}_{e} \quad \forall e\in\mathcal{E}, t\in\mathcal{T} \quad (\xi^{up}_{et}) \tag{4}$$

$$q^{dn}_{et} \leq Q^{min}_{e} \quad \forall e\in\mathcal{E}, t\in\mathcal{T} \quad (\xi^{dn}_{et}) \tag{5}$$

$$\begin{aligned}\sum_{e\in\mathcal{E}}\sum_{b\in B} I_{en}\cdot p^{up}_{etb} - \sum_{e\in\mathcal{E}}\sum_{b\in\mathcal{B}} I_{en}\cdot p^{dn}_{etb} \\ + P^{inj}_{nt} + F^{IL}_{nt} - p^{inj}_{nt} = 0 \quad \forall n\in\mathcal{N}, t\in\mathcal{T} \quad (\lambda^p_{nt})\end{aligned} \tag{6}$$

$$\begin{aligned}\sum_{e\in\mathcal{E}} I_{en}\cdot q^{up}_{et} - \sum_{e\in\mathcal{E}} I_{en}\cdot q^{dn}_{et} \\ + Q^{inj}_{nt} - q^{inj}_{nt} = 0 \quad \forall n\in\mathcal{N}, t\in\mathcal{T} \quad (\lambda^q_{nt})\end{aligned} \tag{7}$$

$$\sum_{n'\in\mathcal{N}_n} f^p_{nn't} - p^{inj}_{nt} = 0 \quad \forall n\in\mathcal{N}, t\in\mathcal{T} \quad (l^p_{nt}) \tag{8}$$

$$\sum_{n'\in\mathcal{N}_n} f^q_{nn't} - q^{inj}_{nt} = 0 \quad \forall n\in\mathcal{N}, t\in\mathcal{T} \quad (l^q_{nt}) \tag{9}$$

$$\begin{aligned}(g_{nn'} + g^{sh}_{nn'})\cdot c_{nnt} + G_{nn'}\cdot c_{nn't} + B_{nn'}\cdot s_{nn't} - f^p_{nn't} = 0 \\ \forall (n,n')\in\mathcal{L}, n\neq n', t\in\mathcal{T} \quad (\gamma^p_{nn't})\end{aligned} \tag{10}$$

$$\begin{aligned}-(b_{nn'} + b^{sh}_{nn'})\cdot c_{nnt} + G_{nn'}\cdot s_{nn't} - B_{nn'}\cdot c_{nn't} \\ - f^q_{nn't} = 0 \quad \forall (n,n')\in\mathcal{L}, n\neq n', t\in\mathcal{T} \quad (\gamma^q_{nn't})\end{aligned} \tag{11}$$

$$(f^p_{nn't})^2 + (f^q_{nn't})^2 \leq (S^{max}_{nn'})^2 \qquad \forall (n,n') \in \mathcal{L}, n \neq n', t \in \mathcal{T} \quad (\kappa_{nn't}) \tag{12}$$

$$-c_{nnt} \leq -(V^{min}_n)^2 \quad \forall n \in \mathcal{N}, t \in \mathcal{T} \quad (v^{min}_{nt}) \tag{13}$$

$$c_{nnt} \leq (V^{max}_n)^2 \quad \forall n \in \mathcal{N}, t \in \mathcal{T} \quad (v^{max}_{nt}) \tag{14}$$

$$(c_{nn't})^2 + (s_{nn't})^2 - c_{nnt} \cdot c_{n'n't} \leq 0 \qquad \forall (n,n') \in \mathcal{L}^{\mathcal{S}}, n \neq n', t \in \mathcal{T} \quad (\sigma_{nn't}) \tag{15}$$

$$s_{nnt} = 0 \quad \forall n \in \mathcal{N}, t \in \mathcal{T} \quad (\zeta_{nt}) \tag{16}$$

$$c_{nnt} = 1 \quad n = \text{root node}, t \in \mathcal{T} \quad (\phi_{nt}) \tag{17}$$

$$c_{nn't} - c_{n'nt} = 0 \quad \forall (n,n') \in \mathcal{L}^{\mathcal{S}}, n \neq n', t \in \mathcal{T} \quad (\delta^c_{nn't}) \tag{18}$$

$$s_{nn't} + s_{n'nt} = 0 \quad \forall (n,n') \in \mathcal{L}^{\mathcal{S}}, n \neq n', t \in \mathcal{T} \quad (\delta^s_{nn't}) \tag{19}$$

2 Karush Kunn Tucker Equations

In this section we present the KKT equations of the monolithic problem with minor sign changes from [1].

1. Type 1 constraints:

   All equality constraints of the original primal problem.

2. Type 2 constraints:

   Contains all inequality constraints of the primal problem, that are complementary to one of the positive dual variables.

$$0 \leq \beta^{up}_{etb} \perp Q^{up}_{etb} - p^{up}_{etb} \geq 0 \quad \forall e \in \mathcal{E}, t \in \mathcal{T}, b \in \mathcal{B} \tag{20}$$

$$0 \leq \beta^{dn}_{etb} \perp Q^{dn}_{etb} - p^{dn}_{etb} \geq 0 \quad \forall e \in \mathcal{E}, t \in \mathcal{T}, b \in \mathcal{B} \tag{21}$$

$$0 \leq \xi^{up}_{et} \perp Q^{max}_e - q^{up}_{etb} \geq 0 \quad \forall e \in \mathcal{E}, t \in \mathcal{T} \tag{22}$$

$$0 \leq \xi^{dn}_{et} \perp Q^{min}_e - q^{up}_{etb} \geq 0 \quad \forall e \in \mathcal{E}, t \in \mathcal{T} \tag{23}$$

$$0 \leq \kappa_{nn't} \perp (S^{max}_{nn'})^2 - (f^p_{nn't})^2 - (f^q_{nn't})^2 \geq 0 \quad \forall (n,n') \in \mathcal{L}, n \neq n', t \in \mathcal{T} \tag{24}$$

$$0 \leq v^{min}_{nt} \perp -(V^{min}_n)^2 + c_{nnt} \geq 0 \quad \forall n \in \mathcal{N}, t \in \mathcal{T} \tag{25}$$

$$0 \leq v^{max}_{nt} \perp (V^{max}_n)^2 - c_{nnt} \geq 0 \quad \forall n \in \mathcal{N}, t \in \mathcal{T} \tag{26}$$

$$0 \leq \sigma_{nn't} \perp c_{nnt} \cdot c_{n'n't} - (c_{nn't})^2 - (s_{nn't})^2 \geq 0 \quad \forall (n,n') \in \mathcal{L}^{\mathcal{S}}, n \neq n', t \in \mathcal{T} \tag{27}$$

3. Type 3 constraints:

Type 3 constraints are associated to the non-negative primal variables c_{nnt}, p^{up}_{etb}, p^{dn}_{etb}, q^{up}_{et}, q^{dn}_{et}, that are complementary to dual inequality constraints.

$$0 \leq p^{up}_{etb} \perp P^{up}_{etb} + \beta^{up}_{etb} - \sum_{n \in N} I_{en} \cdot \lambda^{p}_{nt} \geq 0 \quad \forall e \in \mathcal{E}, t \in \mathcal{T}, b \in \mathcal{B} \tag{28}$$

$$0 \leq p^{dn}_{etb} \perp P^{dn}_{etb} + \beta^{dn}_{etb} - \sum_{n \in N} I_{en} \cdot \lambda^{p}_{nt} \geq 0 \quad \forall e \in \mathcal{E}, t \in \mathcal{T}, b \in \mathcal{B} \tag{29}$$

$$0 \leq q^{up}_{et} \perp \xi^{up}_{et} + \sum_{n \in N} I_{en} \cdot \lambda^{q}_{nt} \geq 0 \quad \forall e \in \mathcal{E}, t \in \mathcal{T} \tag{30}$$

$$0 \leq q^{dn}_{et} \perp \xi^{dn}_{et} - \sum_{n \in N} I_{en} \cdot \lambda^{q}_{nt} \geq 0 \quad \forall e \in \mathcal{E}, t \in \mathcal{T} \tag{31}$$

$$\begin{aligned} 0 \leq c_{nnt} \perp & \sum_{i \in N | i = \text{root node}} \phi_{i,t} + \sum_{n' \in N} (g_{nn'} + g^{sh}_{nn'}) \cdot \gamma^{p}_{nn't} \\ & - \sum_{n' \in N} (b_{nn'} + b^{sh}_{nn'}) \cdot \gamma^{q}_{nn't} - v^{min}_{nt} + v^{max}_{nt} \\ & - \sum_{n' \in N_n} \sigma_{nn'} \cdot c_{n'n'} - \sum_{n' \in N_n} \sigma_{n'n} \cdot c_{n'n'} \geq 0, \quad \forall n \in \mathcal{N}, t \in \mathcal{T} \end{aligned} \tag{32}$$

$$\begin{aligned} 0 \leq c_{nn't} \perp & G_{nn'} \cdot \gamma^{p}_{nn't} - B_{nn'} \cdot \gamma^{q}_{nn't} \\ & - 2 \cdot c_{nn't} \cdot \sigma_{nn't} + \delta^{c}_{nn't} - \delta^{c}_{n'nt} \geq 0, \quad \forall (n, n') \in \mathcal{L}, n \neq n', t \in \mathcal{T} \end{aligned} \tag{33}$$

4. Type 4 constraint:

Type 4 constraints correspond to the Dual equalities that are associated with the free primal variables p^{inj}_{nt}, q^{inj}_{nt}, $f^{p}_{nn't}$, $f^{q}_{nn't}$, s_{nnt}, $s_{nn't}$:

$$(p^{inj}_{nt}) \quad -\lambda^{p}_{nt} - l^{p}_{nt} = 0 \quad \forall n \in \mathcal{N}, t \in \mathcal{T} \tag{34}$$

$$(q^{inj}_{nt}) \quad -\lambda^{q}_{nt} - l^{q}_{nt} = 0 \quad \forall n \in \mathcal{N}, t \in \mathcal{T} \tag{35}$$

$$(f^{p}_{nn't}) \quad l^{p}_{nt} - \gamma^{p}_{nn't} - 2 \cdot \kappa_{nn't} \cdot f^{p}_{nn't} = 0 \quad \forall (n, n') \in \mathcal{L}, n \neq n', t \in \mathcal{T} \tag{36}$$

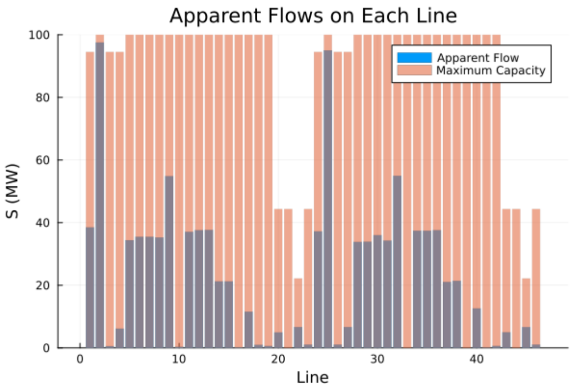
$$(f^{q}_{nn't}) \quad l^{q}_{nt} - \gamma^{q}_{nn't} - 2 \cdot \kappa_{nn't} \cdot f^{q}_{nn't} = 0 \quad \forall (n, n') \in \mathcal{L}, n \neq n', t \in \mathcal{T} \tag{37}$$

$$(s_{nnt}) \quad \zeta_{nt} = 0 \quad \forall n \in \mathcal{N}, t \in \mathcal{T} \tag{38}$$

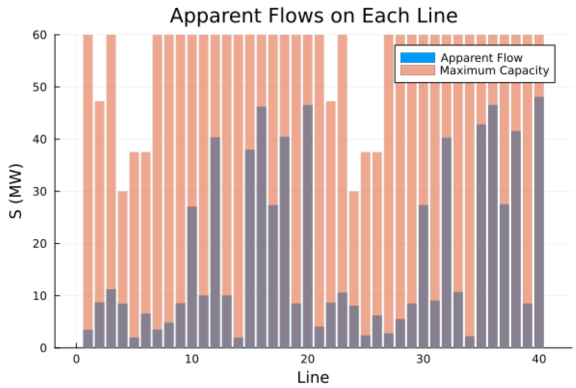
$$\begin{aligned} (s_{nn't}) \quad & B_{nn'} \cdot \gamma^{p}_{nn't} + G_{nn'} \cdot \gamma^{q}_{nn't} - 2 \cdot \sigma_{nn't} \cdot s_{nn't} + \delta^{s}_{nn't} + \delta^{s}_{n'nt} = 0 \\ & \forall (n, n') \in \mathcal{L}, n \neq n', t \in \mathcal{T} \end{aligned} \tag{39}$$

3 Additional Results

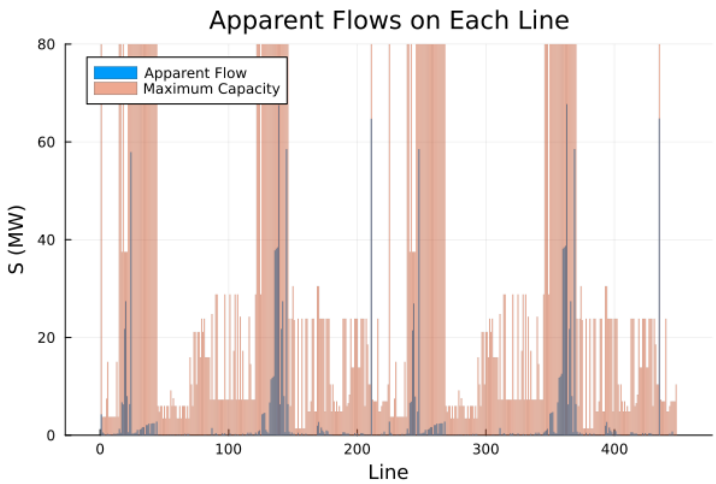
In this section, we present results related to period T1 of the three networks analyzed in the main paper. Fig. 1 illustrates the apparent flows. Notably, no violations of line capacities are observed in any of the networks. Likewise, the voltage magnitudes depicted in Figure 2 confirm that no such violations occur.



(a) SWH Apparent Flows

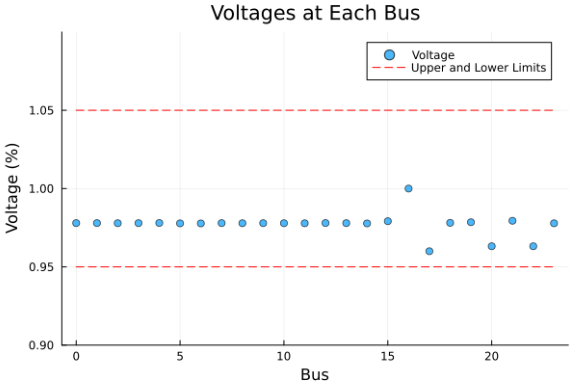


(b) SWW Apparent Flows

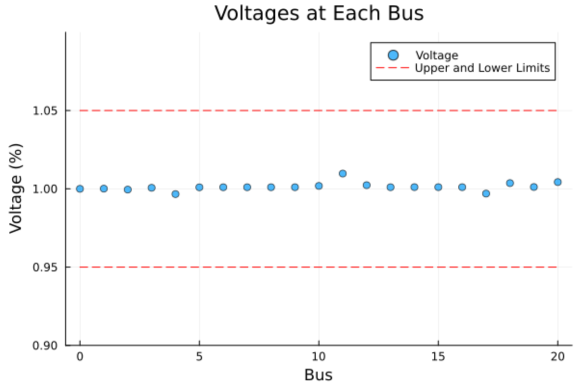


(c) EST Apparent Flows

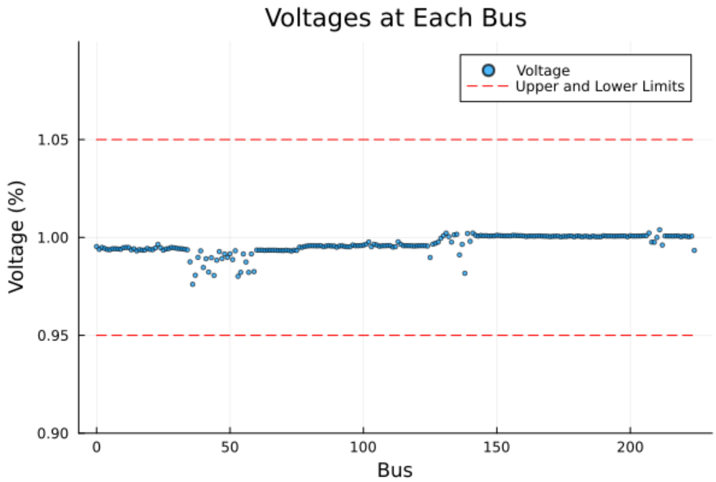
Figure 1: Apparent Flows on Each Line: Comparison of SWH, SWW, and EST networks.



(a) SWH Voltages



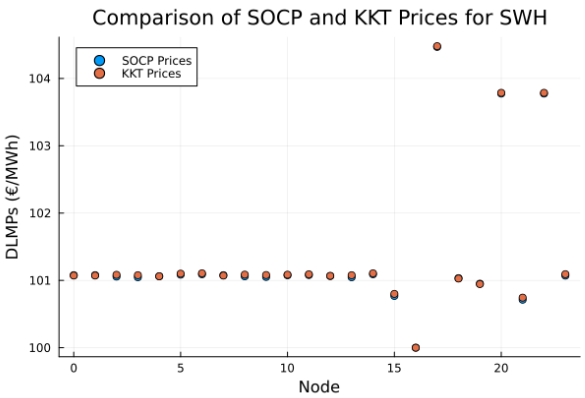
(b) SWW Voltages



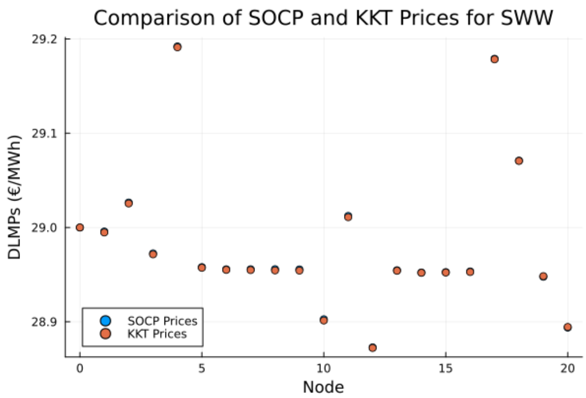
(c) EST Voltages

Figure 2: Voltages at Each Bus: Comparison of SWH, SWW, and EST networks.

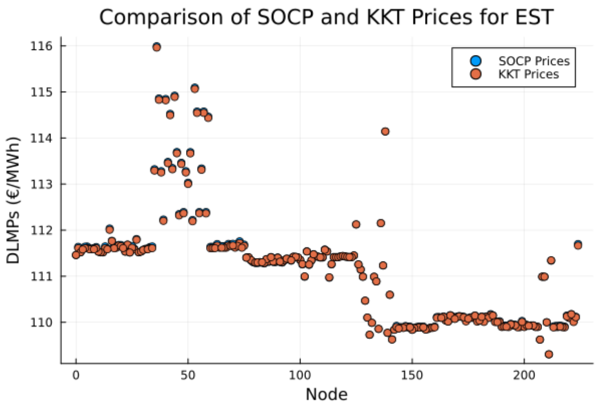
The DLMPs obtained from the proposed pricing submodule, as depicted in Fig. 3, are identical with the DLMPs generated when solving the SOCP formulation with IPOPT. The price spikes observed in all the networks occur without any binding line capacity constraint and thus, are attributed solely to line losses.



(a) SWH



(b) SWW



(c) EST

Figure 3: Comparison of DLMPs for each network.

References

[1] I. Mezghani, N. Stevens, A. Papavasiliou, and D. I. Chatzigiannis, "Hierarchical coordination of transmission and distribution system operations in european balancing markets," *IEEE Transactions on Power Systems*, vol. 38, no. 5, pp. 3990–4002, 2023.