Electronic supplement for “Market Equilibria in Cross-Border Balancing Platforms”: Optimal Strategies and Nash-Equilibria

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1 Balancing Markets

This appendix provides different stylized models of balancing markets that allow us to define various terms that are introduced in the paper unambiguously. It further allows us to justify our economic reasoning based on a well-defined underlying model. This appendix is based on the electronic supplement of [1]. We begin with an ideal single product balancing market that co-optimizes balancing energy and balancing capacity, and we continue with a representation of the existing European balancing market. This section ends with a description of the state of affairs concerning balancing markets in Europe, followed by a comparison with US-style real-time markets.

1.1 Balancing Market with Co-Optimization of Balancing Energy and Balancing Capacity

Balancing markets can be represented as a welfare maximization problem where a TSO aims at maximizing the benefit of holding reserve minus the cost of covering inelastic imbalances given BSP bids that are submitted at a given marginal cost (C) and volume (P), given BRP real-time imbalances (IMB) and given the marginal valuation for reserve by the TSO ($ORDC(\cdot)$). The BSP capacity is split between energy (p) and reserve (r) (or balancing energy and balancing capacity in EU terminology), and the duality conditions of the optimization problem unambiguously characterize the remuneration of both products. Balancing energy is remunerated at the energy price, which is the dual variable related to the market clearing constraint for energy (λ). Balancing capacity is remunerated at the reserve price, which is the dual variable related to the market clearing constraint for reserve (λ^R). Note that constraint (4) results in a coupling of the energy and reserve prices and that dr represents the remaining balancing capacity in the system, which is also the satisfied demand for reserve.[1]

$$\max_{dr,r,p} \quad \int_0^{dr} ORDC(x)dx - \sum_{i \in BSP} C_i \cdot p_i \tag{1}$$

$$s.t. \quad (\lambda): \quad \sum_{i \in BSP} p_i = \sum_{i \in BRP} IMB_i \tag{2}$$

$$(\lambda^R): \quad \sum_{i \in BSP} r_i = dr \tag{3}$$

$$(\mu_i): \quad p_i + r_i \leq P_i \qquad \forall i \in BSP \tag{4}$$

$$p, r, dr \geq 0 \tag{5}$$

An inspection of the KKT conditions related to the energy production and reserve variables and the remaining balancing capacity variable connects the price for energy and reserve to the remaining capacity in the system:

$$0 \leq r_i \perp \mu_i - \lambda^R \geq 0 \tag{6}$$

$$0 \leq p_i \perp C_i + \mu_i - \lambda \geq 0 \tag{7}$$

$$0 \leq dr \perp \lambda^R - ORDC(dr) \geq 0 \tag{8}$$

Constraint (8) links the reserve price to the value of the ORDC at the level of remaining balancing capacity in the system. Let us refer to a marginal generator as a generator that supplies both energy and reserve at the optimal solution, and let us index this generator by g. Constraints (7) and (6) show that the scarcity rent for a marginal generator g is equal to μ_g and is further equal to the reserve price. In this stylized model without ramp constraints, μ_g is equal to λ^R, and the energy price is then equal to the marginal cost of the marginal generator C_g supplemented by the reserve price λ_R. These conditions ensure that a marginal generator is indifferent between supplying reserve or energy.

Without co-optimization, it is possible to induce scarcity pricing through an ORDC, as in the original ERCOT energy-only[2] market in Texas [4]. Coherent price signals are approximated by supplementing the energy price of the energy-only market with a scarcity adder component derived from the ORDC at the level of the available balancing capacity in the system. This approach uses an implicit ORDC adder in the agents' remuneration, in contrast to the

[1]This stylised model is assumed to be convex, feasible, and to trade a single product. We thus ignore indivisible balancing bids [2] and irregular balancing market operations where the market does not clear [3]. The single-product assumption reinforces the inelastic assumption for covering the imbalance. In practice, TSOs have some flexibility regarding the demand for balancing energy as they can arbitrage between the different balancing products. This arbitrage and the submission of an elastic demand bid has been observed in TERRE, the replacement reserve platforms, and is allowed in MARI, the mFRR platform. There is an ongoing consultation on whether this feature should be included in PICASSO, the aFRR platform.

[2]An energy-only market refers in this text to a market that only trades balancing energy and not reserve / balancing capacity. It does not refer to a market without capacity remuneration mechanisms.

explicit ORDC model, where the ORDC appears in the objective function (1). The obtained prices approximate the equilibrium prices of a co-optimization model without binding ramp constraints if the energy dispatch of the energy-only market is identical to the one of the co-optimized market. The argument is only valid for markets trading a single reserve product composed of a single zone with a zero cost function for supplying reserve.

1.2 European Balancing Market

In practice, balancing markets in the EU differ from the idealized co-optimization presented in the previous section in various points. A quantitative model of the European balancing market can be described as follows:

$$\max_{p} \quad -\sum_{i \in BSP} C_i \cdot p_i \tag{9}$$

$$s.t. \quad (\lambda): \quad \sum_{i \in BSP} p_i = \sum_{i \in BRP} IMB_i \tag{10}$$

$$(\mu_i): \quad p_i \leq P_i \qquad \forall i \in BSP \tag{11}$$

$$p \geq 0 \tag{12}$$

First, TSOs do not value balancing capacity in real time and run an "energy-only" optimization problem. This absence of a market for real-time balancing capacity raises issues for the forward reservation of balancing capacity, as already pointed out in [5].

Second, the law of one price is not respected for the trading of balancing energy. BSPs' balancing energy (on the left-hand side of constraint (10)) is remunerated at the balancing price, whereas BRPs (on the right-hand side of constraint (10)) face an imbalance price that is based on the balancing price but may differ from it. In Belgium, an "alpha" component was introduced on top of the balancing price. The stated objective of such an adder is to incentivize BRPs to keep their imbalance low. ELIA's point of view is that the difference in pricing between imbalances caused by BRPs and balancing energy provided by BSPs can be justified by the different goals of the price signals:

> *"The imbalance price incentivizes BRPs to keep and/or restore system balance of their imbalance price area in accordance with the Electricity Balancing Regulation, while the balancing energy price reflects the price of the marginal bid selected in the uncongested area by the activation optimization function of the EU balancing platform"* [6].

In practice, flexible assets belonging to a BRP portfolio can move freely between active imbalance and participation in the balancing market, giving rise to arbitrage opportunities.

1.3 European Context

Balancing markets in Europe have historically been controlled by national TSOs and, as such, are characterized by a variety of designs. Table 1 recaps the state of affairs concerning balancing markets in EU countries. It includes (i) the balancing process, the organization of (ii) FCR, (iii) aFRR, and (iv) mFRR markets, (v) whether replacement reserve is used to balance the market and (vi) information on imbalance settlement. The table is built with information drawn from the ancillary services survey conducted by ENTSO-E [7], and complemented by other sources such as [8], [9] and [10]. Information that is not sourced from the survey is denoted with an asterisk.

Table 1: European balancing market state of affairs (SD: self-dispatch, CD: central-dispatch, PaB: pay-as-bid, cap.: capacity, en.: energy, proc.: procurement, ISP: integrated scheduling process)

Country		Belgium	the Netherlands	Germany	France	Poland	Italy	Denmark	Norway	Spain	EU
Bal. Process		SD Portfolio	SD Portfolio	SD Portfolio	SD Unit based	CD	CD	SD Portfolio	SD Unit based	SD Unit based	
FCR	Cap. Proc.	Market	Market	Market	Market	Hybrid	Mandatory	Market	Hybrid	Mandatory	Market* (FCR cooperation)
	Cap. Pricing	Marginal	Marginal	Marginal	Marginal	Regulated	N/A	Marginal	Marginal	None*	Marginal*
	En. Settlement	No	No	No	Yes (regulated)	Yes (marginal)	Yes	No	Yes	No	No*
aFRR	Cap. Proc.	Market	Market	Market	Market	Hybrid	ISP*	Market	Market	Market	Memb. State*
	Cap. Pricing	PaB	PaB	PaB	Marginal	Regulated	PaB*	PaB	Marginal	Marginal	Memb. State*
	En. Proc.	Market	Market	Market	Mandatory	Market	Hybrid	Hybrid	Market*	Hybrid	Market* (PICASSO)
	En. Pricing	PaB	Marginal	Marginal*	Regulated	Marginal	PaB	Regulated	Marginal	Hybrid	Marginal*
	Activation rule	Merit order	Merit order	Merit order	Pro rata	Merit order	Pro rata	Pro rata	Pro rata	Pro rata	Merit order*

mFRR	Cap. Proc.	Market	Market	Market	Market	No mFRR	ISP*	Market	Market	Mandatory*	Memb. State*
	Cap. Pricing	PaB	PaB	PaB	Marginal		PaB*	Marginal	Marginal	None*	Memb. State*
	En. Proc.	Market*	Market*	Market	Market		Hybrid	Market	Market	Market	Market* (MARI)
	En. Pricing	Marginal	Marginal	Marginal*	PaB		PaB	Marginal	Marginal	Marginal	Marginal*
RR		No	No	No	Yes	Yes	Yes	No	No	Yes	Yes* (TERRE)
Imbalance settlement	Number of position	1	1	1	1	1	2	2	2	2	
	Price scheme	Single*	Mostly single	Single	Dual*	Single	Dual (gen.) and single (con.)*	Dual (gen.) and single (con.)	Dual (gen.) and single (con.)	Dual	Single*
	Component	aFRR, mFRR	aFRR, mFRR	aFRR, mFRR	aFRR, mFRR, RR	FCR, aFRR, RR	RR	mFRR	mFRR	aFRR, mFRR, RR	aFRR, mFRR*
	GCT for internal trade	Ex-post	Ex-post	15 min	15 min	45 min	N/A	N/A	45 min	1 hour	
	Duration	15 min	15 min	15 min	30 min	1 hour	15 min*	30 min	30 min	30 min	15 min*

a) There exist two classes of balancing processes. In a central dispatch model (CD), the generation schedules and consumption schedules of facilities are determined by a TSO that runs an *Integrated Scheduling Process* (ISP). ISP are run several times per day and can be used to start up additional units, if needed. In self-dispatch models (SD), BRPs follow their own generation and consumption schedule. There is a distinction between portfolio-based SD models and unit-based models. In portfolio-based models, BRPs follow aggregated generation and consumption schedules and they set the dispatch of their facilities. In unit-based models, BRPs have to follow the individual generation schedule that they set for all of their units.

b) FCR, aFRR and mFRR capacity can be procured in a mandatory fashion, where assets connected to the grid are obliged to offer their capacity, or through a market, where agents can voluntarily offer their capacity. There also exist hybrid methods, where connected assets have to offer some of their capacity and other agents can voluntarily participate in the capacity market. Finally, capacity can be procured through an ISP, such as in Italy, with some kind of co-optimization between energy and capacity. The ISP commits and dispatches assets in order to fulfil a certain reserve requirement. The capacity can be paid often, remunerated using marginal pricing in market-based procurement, or remunerated with a regulated price in mandatory auctions. For some markets, such as FCR and mFRR in Spain, capacity is not remunerated.

c) FCR energy is often not remunerated but some countries offer an energy settlement. aFRR energy can be activated based on the merit order or on a pro-rata criterion. If the activation is performed through the merit order, then either pay-as-bid or marginal pricing is used for the balancing price. If it is based on pro-rata activation, then a regulated price is often used. There also exist pricing methods based on the avoided cost of mFRR activation (as in Spain). mFRR is always activated based on the merit order and the pricing is either pay-as-bid or marginal.

d) Replacement reserve is not used in every country.

e) The number of positions in the imbalance settlement section refers to the number of imbalance volumes a BRP is responsible for. It can either be 1 (an aggregated position for all its assets), 2 (one position for generation, and one for consumption), or more than 2. For the last option, it is often the case when every asset has its own imbalance position.

f) The imbalance pricing scheme can be single-price or dual-price. In single-price schemes (see table 2), all BRPs face the same price regardless of their position. The imbalance price only depends on the overall position of the system. If the system is long and downward activation is required, imbalances are settled at the downward activation price (MP_d), and if the system is short and upward activation is required, imbalances are settled at the upward activation price (MP_u). This upward (downward) activation price is supposed to be based on either the maximum (minimum) marginal price of some balancing product (see imbalance settlement component in the table) or their weighted average [11]. In dual pricing schemes, the imbalance price faced by a BRP depends on the direction of its imbalance with respect to the system imbalance. Dual pricing schemes can take different forms but their usual implementation is illustrated in table 3. This implementation can be referred to as *punitive dual pricing*. The imbalance price is still dependent on the position of the system, but a penalty based on a parameter k is applied for BRPs in imbalance. Short BRPs have to pay a premium to the TSO and long BRPs receive a discounted price. The imbalance price is based on the average cost of upward regulation (AP_u) for a short system and on the average cost of downward regulation (AP_d) for a long system. Under punitive dual pricing, agents are disincentivized to deviate from their schedule, even if it helps the system. This prevents decentralized balancing actions from flexible assets that cannot participate in balancing energy auctions and restricts the flexibility pool. There also exists dual pricing scheme that include a premium to account for the trend of the system imbalance.

g) The imbalance price is set based on different balancing products, depending on the country. Almost all combinations are present.

As illustrated in table 1, there is a large diversity of balancing market designs in Europe. There is a significant difference between Belgium, the Netherlands and Germany, all of which have embraced a market-based approach for almost all components of balancing markets, and between countries with a mandatory procurement of capacity or countries that use a pro-rata activation scheme for aFRR.

The last column of the table describes the trends of European balancing market integration and the different initiatives and/or regulations taken by ENTSO-E, ACER and the European Commission.

- There is an initiative to jointly procure FCR capacity on a European level called *FCR cooperation*.
- Member States are incentivized to connect to the balancing platforms MARI and PICASSO. The dispatch of aFRR and mFRR for connected countries will be market-based and the pricing of these assets will be governed by ACER's *pricing methodologies*. The adoption of PICASSO will force the transition to a merit-order activation for aFRR. The procurement of aFRR and mFRR capacity is left to the Member States under the EBGL framework.
- Countries using replacement reserve can connect to the TERRE platform.
- The regulations pushed forward by the European Commission for imbalance settlement can be found in the *imbalance settlement harmonization*

Table 2: Imbalance Settlement with single-price schemes

		System Imbalance	
		Negative (short)	Positive (long)
BRP imbalance	Negative (short)	BRP pays TSO MP_u	BRP pays TSO MP_d
	Positive (long)	TSO pays BRP MP_u	TSO pays BRP MP_d

methodology (ISHM). A single-price scheme composed of the aFRR and mFRR platform prices is favoured by the ISHM but there is some flexibility on the Member States' side concerning these rules.

An additional component of the balancing market integration is the IGCC. This is an imbalance netting methodology for the activation of aFRR.

Other differences not contained in the table relate to the following:

- Cost recovery scheme: Most capacity costs are passed through grid tariffs to all the consumers, whereas the energy activation costs are covered by the BRPs causing the imbalance, although this is not always the case. Italy and Denmark, for example, cover the aFRR energy cost through the grid tariff.
- Imbalance exemption: Some agents are not responsible for their imbalance. This is the case for renewable assets in France and Greece.[3]
- Provider: Not all assets can provide all types of reserve. In Belgium, France, Germany and the Netherlands, generators, pumped-storage, demand-side response, and batteries can provide FCR, aFRR and mFRR (and distributed generation can provide aFRR and mFRR). In Spain and Italy, all types of assets can provide mFRR but only classical generators can provide FCR and aFRR. Poland only allows classical generators to provide reserve.

The differences listed earlier reflect the TSOs' balancing philosophy which is often classified into two groups [12].

1. Reactive TSOs consider that "they should not take side in the electricity market" and that they should only activate balancing energy to react to actual imbalances and not to their forecast [13]. These TSOs rely on their BRPs to keep the balance. They generate a strong imbalance price that reflects the need of the system and BRPs are encouraged to generate imbalances that would improve the position of the system.

[3]Only renewable energy source with feed-in-tariffs are not responsible for their imbalance in France. A special entity is dedicated to managing the imbalance of portfolios that are operating under feed-in tariffs.

Table 3: Imbalance Settlement with punitive Dual-Price Scheme

		System Imbalance	
		Negative (short)	Positive (long)
BRP imbalance	Negative (short)	BRP pays TSO $AP_u \cdot (1+k)$	BRP pays TSO $AP_d \cdot (1+k)$
	Positive (long)	TSO pays BRP $AP_u \cdot (1-k)$	TSO pays BRP $AP_d \cdot (1-k)$

2. Proactive TSOs resolve their imbalance in a centralized fashion and tend to activate balancing products with longer activation times, such as replacement reserve. They intervene early in the formation of imbalances by reacting to the forecast of the system imbalance or to maintain the available capacity margin for a given look-ahead above a certain threshold. They can disincentivize decentralized self-activation by forbidding deviation after a certain point in time (although enforcing such a prohibition can be difficult in practice) and/or by setting unfavourable imbalance prices.

1.4 Comparison with US Real-Time Markets

A number of differences exists in terms of market organisation between the US and Europe. Table 4 describes these differences by using PJM as a representative US market.

The main organizational differences concern the time of pricing, the dispatching method, and the exclusion or inclusion of secondary reserve in the main RT market.

- In Europe, the imbalance price (which is considered the real-time energy price) is set at the end of the imbalance period after the imbalance has been resolved by the activation of balancing energy. In the US, the RT energy price is often done at the beginning of the interval when dispatching the assets.
- In Europe (at least for countries with portfolio-based self-dispatch), an aggregated forward position is generated in day-ahead and in intraday and agents determine themselves how to allocate this position between their assets. In the US, a dispatch is given by the system operator after solving *security constrained economic dispatch* (SCED) based on the technical and economic characteristic of the assets.
- In Europe, secondary reserve (aFRR) is part of the balancing market and used to compute the imbalance price. In the US, secondary reserve (regulation reserve in PJM) is controlled and remunerated through an auxiliary market that is distinct from the real-time market.

Table 4: European balancing market versus US real-time markets

	EU Balancing Market	US RT market
Product traded	Energy imbalance	Energy and reserve
Time of pricing	Ex-post	Ex-ante
Dispatching model	Self-dispatch	Central dispatch
Secondary reserve	aFRR included in the market	Regulation not included and part of an auxiliary market
Capacity Remuneration	DA procurement auctions	DA and RT markets
Number of RT prices	aFRR price (per 4 seconds), mFRR price (per 15 minutes) and imbalance price (per 15 minutes)	RT energy price (per 5 minutes) and RT reserve price (per 5 minutes)
Relationship between RT prices	Imbalance price is a combination of mFRR price and aFRR price	RT energy and RT Reserve are generated by an economic dispatch with co-optimization

2 Optimal Strategies

The following strategies assume that the platform price, λ_P, obtained by clearing the balancing energy auction, is strictly monotone increasing in x, which is the level of required balancing activation. These strategies also assume that the supply function, λ_P^{-1}, is differentiable almost everywhere.[4] The scarcity component λ_R is assumed to be non-decreasing in the level of demand for balancing energy. The fringe agents can consider these two prices as being exogenous.

Proposition 1 (Bidding Strategy – No Adder)**.** *The optimal strategy for a fringe agent under a "no adder" design is to bid truthfully in the balancing auction.*

Proof. The optimal strategy for a fringe agent with upward balancing capacity P^+ and marginal cost C can be found by maximizing the sum of z_B and z_I.

a) If $\mathbb{E}_\mu[\lambda_P] \leq C$, then $z_I(q) = 0$ and the total reward of the agent $R(p,q)$ is defined hereunder.

$$R(p,q) = z_B(p,q) + z_I(q) \tag{13}$$
$$= C_1(p) \cdot q \tag{14}$$

with $C_1(p)$ characterized as follows:

$$C_1(p) = \int_{\lambda_P(x)>p} (\lambda_P(x) - C) d\mu(x) \tag{15}$$

Note that the bound on the integral in $C_1(p)$ can be reformulated, since λ_P is strictly monotonic increasing and thus has a uniquely defined inverse function.

$$\int_{\lambda_P(x)>p} (\lambda_P(x) - C) d\mu(x) = \int_{x>\lambda_P^{-1}(p)} (\lambda_P(x) - C) d\mu(x) \tag{16}$$

The optimal bidding strategy of the agent can be derived from the first-order conditions with respect to p and q. Let us fix p first.

$$\frac{\partial R(p,q)}{\partial p} = C_1'(p) \cdot q \tag{17}$$
$$= -(\lambda_P(\lambda_P^{-1}(p)) - C) \cdot \mu(\lambda_P^{-1}(p)) \cdot \frac{d\lambda_P^{-1}(p)}{dp} \cdot q \tag{18}$$
$$= -(p - C) \cdot \mu(\lambda_P^{-1}(p)) \cdot \frac{d\lambda_P^{-1}(p)}{dp} \cdot q \tag{19}$$

[4] At equilibrium, the supply function may exhibit breaking points due to BSPs resorting to reactive balancing. This can cause the supply function to not be differentiable everywhere.

For fixed q, the payoff function $R(p, q)$ is increasing in $(-\infty, C]$, zero at C, and decreasing in $[C, +\infty)$[5]. Thus for any q, an optimal strategy in the balancing auction is to bid truthfully the marginal cost. Given this strategy, the payoff becomes

$$R(C, q) = C_1(C) \cdot q. \tag{20}$$

We can then determine the first-order condition with respect to q.

$$\frac{\partial R(C, q)}{\partial q} = C_1(C) \tag{21}$$

$$= \int_{\lambda_P(x) > C} (\lambda_P(x) - C) d\mu(x) \tag{22}$$

$$> 0 \tag{23}$$

We conclude that, for a fringe agent with upward balancing capacity and marginal cost C higher than $\mathbb{E}_\mu[\lambda_P]$, the optimal strategy is to bid truthfully in the balancing auction.

b) If $\mathbb{E}_\mu[\lambda_P] > C$, then $z_I(q) = (\mathbb{E}_\mu[\lambda_P] - C) \cdot (P^+ - q)$ and the total reward of the agent $R(p, q)$ is defined hereunder.

$$R(p, q) = z_B(p, q) + z_I(q) \tag{24}$$

$$= C_1 + C_2 \cdot q + C_3(p) \cdot q \tag{25}$$

with C_1, C_2, and $C_3(p)$ characterized as follows:

$$C_1 = (\mathbb{E}_\mu[\lambda_P] - C) \cdot P^+ \tag{26}$$

$$C_2 = -(\mathbb{E}_\mu[\lambda_P] - C) \tag{27}$$

$$C_3(p) = \int_{\lambda_P(x) > p} (\lambda_P(x) - C) d\mu(x) \tag{28}$$

The optimal bidding strategy of the agent can be derived from the first-order conditions with respect to p and q. The optimal bidding price p can be obtained as in case a), and is equal to the marginal cost C.

Given this strategy, the payoff becomes

$$R(C, q) = C_1 + C_2 \cdot q + C_3(C) \cdot q. \tag{29}$$

[5]This argument relies on $R(p, q)$ being continuous and $d\lambda_P^{-1}(p)/dp$ being positive where it exists, since λ_P is strictly monotonic increasing. The cases where $\lambda_P^{-1}(p)$ is not differentiable are analysed on a case by case basis.

We then have

$$\frac{\partial R(C,q)}{\partial q} = C_2 + C_3(C) \tag{30}$$

$$= -(\mathbb{E}_\mu[\lambda_P] - C) + \int_{\lambda_P(x)>C} (\lambda_P(x) - C) d\mu(x) \tag{31}$$

$$= -\Big(\int_{x \leq \lambda_P^{-1}(C)} (\lambda_P(x) - C) d\mu(x) + \int_{x > \lambda_P^{-1}(C)} (\lambda_P(x) - C) d\mu(x) \Big)$$
$$+ \int_{x > \lambda_P^{-1}(C)} (\lambda_P(x) - C) d\mu(x) \tag{32}$$

$$= - \int_{x \leq \lambda_P^{-1}(C)} (\lambda_P(x) - C) d\mu(x)) \tag{33}$$

$$> 0 \tag{34}$$

We conclude that, for a fringe agent with upward balancing capacity and marginal cost C higher than $\mathbb{E}_\mu[\lambda_P]$, the optimal strategy is to bid truthfully in the balancing auction.

□

Proposition 2 (Bidding Strategy – Adder on BRPs)**.** *The optimal strategy for a fringe agent under an "adder on BRPs" design is to bid truthfully in the balancing market if*

$$C \geq \mathbb{E}_\mu[\lambda_P + \lambda_R] - \int_{\lambda_P(x) \geq C} (\lambda_P(x) - C) d\mu(x),$$

else to perform reactive balancing.

Proof. The optimal strategy for a fringe agent with upward balancing capacity P^+ and marginal cost C can be found by maximizing the sum of z_B and z_I.

a) If $\mathbb{E}_\mu[\lambda_P + \lambda_R] \leq C$, then $z_I(q) = 0$ and the total reward of the agent $R(p,q)$ is defined hereunder.

$$R(p,q) = z_B(p,q) + z_I(q) \tag{35}$$
$$= C_1(p) \cdot q \tag{36}$$

with C_1 characterized as follows:

$$C_1 = \int_{\lambda_P(x)>p} (\lambda_P(x) - C) d\mu(x) \tag{37}$$

Note that the bound of the the integral in $C_1(p)$ can be reformulated as a function of x, since λ_P is strictly monotonic increasing and thus has an inverse function.

$$\int_{\lambda_P(x)>p}(\lambda_P(x)-C)d\mu(x)=\int_{x>\lambda_P^{-1}(p)}(\lambda_P(x)-C)d\mu(x) \tag{38}$$

The optimal bidding strategy of the agent can then be derived from the first-order conditions with respect to p and q. Let us focus on p first.

$$\frac{\partial R(p,q)}{\partial p}=C_1'(p)\cdot q \tag{39}$$

$$=-(\lambda_P(\lambda_P^{-1}(p))-C)\cdot\mu(\lambda_P^{-1}(p))\cdot\frac{d\lambda_P^{-1}(p)}{dp}\cdot q \tag{40}$$

$$=-(p-C)\cdot\mu(\lambda_P^{-1}(p))\cdot\frac{d\lambda_P^{-1}(p)}{dp}\cdot q \tag{41}$$

For fixed q, the payoff function $R(p,q)$ is increasing in $(-\infty,C]$, zero at C, and decreasing in $[C,+\infty)$. Thus, for any q, an optimal strategy is to bid truthfully the marginal cost. Given this strategy, the payoff becomes

$$R(C,q)=C_1(C)\cdot q. \tag{42}$$

We can then determine the first order condition relative to q.

$$\frac{\partial R(C,q)}{\partial q}=C_1(C) \tag{43}$$

$$=\int_{\lambda_P(x)>C}(\lambda_P(x)-C)d\mu(x) \tag{44}$$

$$>0 \tag{45}$$

We conclude that, for a fringe agent with upward balancing capacity and marginal cost C higher than $\mathbb{E}[\lambda_P+\lambda_R]$, the optimal strategy is to bid truthfully in the balancing auction.

b) If $\mathbb{E}[\lambda_P+\lambda_R]-\int_{\lambda_P(x)>C}(\lambda_P(x)-C)d\mu(x)\leq C<\mathbb{E}[\lambda_P+\lambda_R]$, then $z_I(q)=(\mathbb{E}[\lambda_P+\lambda_R]-C)\cdot(P^+-q)$ and the total reward of the agent $R(p,q)$ is defined hereunder.

$$R(p,q)=z_B(p,q)+z_I(q) \tag{46}$$

$$=C_1+C_2\cdot q+C_3(p)\cdot q \tag{47}$$

with C_1, C_2 and $C_3(p)$ characterized as follows:

$$C_1=(\mathbb{E}_\mu[\lambda_P+\lambda_R]-C)\cdot P^+ \tag{48}$$

$$C_2=-(\mathbb{E}_\mu[\lambda_P+\lambda_R]-C) \tag{49}$$

$$C_3(p)=\int_{\lambda_P(x)>p}(\lambda_P(x)-C)d\mu(x) \tag{50}$$

The optimal bidding strategy of the agent can then be derived from the first-order conditions with respect to p and q. The optimal bidding price p can be obtained as in case a), and is equal to the marginal cost C.

Given this strategy, the payoff becomes

$$R(C,q) = C_1 + C_2 \cdot q + C_3(C) \cdot q. \tag{51}$$

The next step is to examine the first-order condition with respect to q.

$$\frac{\partial R(C,q)}{\partial q} = C_2 + C_3(C) \tag{52}$$

$$= -(\mathbb{E}_\mu[\lambda_P + \lambda_R] - C) + \int_{\lambda_P(x)>C} (\lambda_P(x) - C) d\mu(x) \tag{53}$$

$$> 0 \tag{54}$$

We conclude that, for a fringe agent with upward balancing capacity P^+ and marginal cost C between $\mathbb{E}_\mu[\lambda_P+\lambda_R]$ and $\mathbb{E}_\mu[\lambda_P+\lambda_R] - \int_{\lambda_P(x)>C}(\lambda_P(x) - C)d\mu(x)$, the optimal strategy is to bid its entire capacity truthfully in the balancing auction.

c) If $C < \mathbb{E}_\mu[\lambda_P + \lambda_R] - \int_{\lambda_P(x)>C}(\lambda_P(x) - C)d\mu(x)$, then $z_I(q) = (\mathbb{E}_\mu[\lambda_P + \lambda_R] - C) \cdot (P^+ - q)$ and the total reward of the agent $R(p,q)$ is defined hereunder.

$$R(p,q) = z_B(p,q) + z_I(q) \tag{55}$$

$$= C_1 + C_2 \cdot q + C_3(p) \cdot q \tag{56}$$

with C_1, C_2 and $C_3(p)$ characterized as follows:

$$C_1 = (\mathbb{E}_\mu[\lambda_P + \lambda_R] - C) \cdot P^+ \tag{57}$$

$$C_2 = -(\mathbb{E}_\mu[\lambda_P + \lambda_R] - C) \tag{58}$$

$$C_3(p) = \int_{\lambda_P(x)>p} (\lambda_P(x) - C) d\mu(x) \tag{59}$$

The optimal bidding strategy of the agent can then be derived from the first-order conditions with respect to p and q. The optimal bidding price p can be obtained as in case a), and is equal to the marginal cost C.

Given this strategy, the payoff becomes

$$R(C,q) = C_1 + C_2 \cdot q + C_3(C) \cdot q. \tag{60}$$

The next step is to examine the first-order condition with respect to q.

$$\frac{\partial R(C,q)}{\partial q} = C_2 + C_3(C) \tag{61}$$

$$= -(\mathbb{E}[\lambda_P + \lambda_R] - C) + \int_{\lambda_P(x)>C} (\lambda_P(x) - C) d\mu(x) \tag{62}$$

$$< 0 \tag{63}$$

We conclude that, for a fringe agent with upward balancing capacity and marginal cost C lower than $\mathbb{E}[\lambda_P + \lambda_R] - \int_{\lambda_P(x)>C} (\lambda_P(x) - C) d\mu(x)$, the optimal strategy is to not participate in the balancing auction and to rather resort to reactive balancing.

□

Proposition 3 (Bidding Strategy – Adder on BRPs and BSPs)**.** *The optimal strategy for a fringe agent under an "adder on BRPs and BSPs" design is to bid its full capacity in the balancing energy market at price*

$$\lambda_P((\lambda_P + \lambda_R)^{-1}(C)).$$

Proof. The optimal strategy for a fringe agent with upward balancing capacity P^+ and marginal cost C can be found by maximizing the sum of z_B and z_I.

a) If $\mathbb{E}_\mu[\lambda_P + \lambda_R] \leq C$, then $z_I(q) = 0$ and the total reward of the agent $R(p,q)$ is defined hereunder.

$$R(p,q) = z_B(p,q) + z_I(q) \tag{64}$$

$$= C_1(p) \cdot q \tag{65}$$

with C_1 characterized as follows:

$$C_1 = \int_{\lambda_P(x)>p} (\lambda_P(x) + \lambda_R(x) - C) d\mu(x) \tag{66}$$

Note that the bound of the the integral in $C_3(p)$ can be reformulated as a function of x, since λ_P is strictly monotonic increasing and thus has an inverse function.

$$\int_{\lambda_P(x)>p} (\lambda_P(x) + \lambda_R(x) - C) d\mu(x) = \int_{x>\lambda_P^{-1}(p)} (\lambda_P(x) + \lambda_R(x) - C) d\mu(x) \tag{67}$$

The optimal bidding strategy of the agent can then be derived from the first-order conditions with respect to p and q. Let us fix p first.

$$\frac{\partial R(p,q)}{\partial p} = C_1'(p) \cdot q \tag{68}$$

$$= -(\lambda_P(\lambda_P^{-1}(p)) + \lambda_R(\lambda_P^{-1}(p)) - C) \cdot \mu(\lambda_P^{-1}(p)) \cdot \frac{d\lambda_P^{-1}(p)}{dp} \cdot q \tag{69}$$

$$= -(p + \lambda_R(\lambda_P^{-1}(p)) - C) \cdot \mu(\lambda_P^{-1}(p)) \cdot \frac{d\lambda_P^{-1}(p)}{dp} \cdot q \tag{70}$$

The unique root of $g(p) = p + \lambda_R(\lambda_P^{-1}(p)) - C$ is $\lambda_P(\lambda_I^{-1}(C))$, with $\lambda_I(x) = \lambda_P(x) + \lambda_R(x)$. This root can be interpreted as the platform price when the total system imbalance is such that the platform plus the reserve adder is equal to C.

$$g(\lambda_P(\lambda_I^{-1}(C))) = \lambda_P(\lambda_I^{-1}(C)) + \lambda_R(\lambda_P^{-1}(\lambda_P(\lambda_I^{-1}(C)))) - C \tag{71}$$

$$= \lambda_P(\lambda_I^{-1}(C)) + \lambda_R(\lambda_I^{-1}(C)) - C \tag{72}$$

$$= \lambda_I(\lambda_I^{-1}(C)) - C \tag{73}$$

$$= 0 \tag{74}$$

For fixed q, the payoff function $R(p,q)$ is increasing in $(-\infty, C']$, zero at C', and decreasing in $[C', +\infty)$ with $C' = \lambda_P(\lambda_I^{-1}(C))$. Thus, for any q, an optimal strategy in the balancing energy auction is to bid the platform price when the platform price plus the reserve adder is equal to the marginal cost of the agent. This strategy internalizes the added reward from the scarcity adder in the balancing bid such that the balancing bid is equal to the marginal cost minus the reserve adder when the platform price plus the reserve adder is equal to the marginal cost. Given this strategy, the payoff becomes

$$R(C', q) = C_1(C') \cdot q. \tag{75}$$

We can then determine the first order condition relative to q.

$$\frac{\partial R(C',q)}{\partial q} = C_1(C') \tag{76}$$

$$= \int_{\lambda_P(x) > C'} (\lambda_P(x) + \lambda_R(x) - C) d\mu(x) \tag{77}$$

$$= \int_{x > \lambda_I^{-1}(C)} (\lambda_I(x) - C) d\mu(x) \tag{78}$$

$$> 0 \tag{79}$$

We conclude that, for a fringe agent with upward balancing capacity and marginal cost C higher than $\mathbb{E}_\mu[\lambda_P + \lambda_R]$, the optimal strategy is to bid its full capacity at price $\lambda_P((\lambda_P + \lambda_R)^{-1}(C))$ in the balancing energy auction.

b) If $\mathbb{E}_\mu[\lambda_P + \lambda_R] > C$, then $z_I(q) = (\mathbb{E}_\mu[\lambda_P + \lambda_R] - C) \cdot (P^+ - q)$ and the total reward of the agent $R(p, q)$ is defined hereunder.

$$R(p, q) = z_B(p, q) + z_I(q) \tag{80}$$
$$= C_1 + C_2 \cdot q + C_3(p) \cdot q \tag{81}$$

with C_1 and $C_3(p)$ characterized as follows:

$$C_1 = (\mathbb{E}_\mu[\lambda_P + \lambda_R] - C) \cdot P^+ \tag{82}$$
$$C_2 = -(\mathbb{E}_\mu[\lambda_P + \lambda_R] - C) \tag{83}$$
$$C_3(p) = \int_{\lambda_P(x) > p} (\lambda_P(x) + \lambda_R(x) - C) d\mu(x) \tag{84}$$

The optimal bidding strategy of the agent can then be derived from the first-order conditions with respect to p and q. The optimal bidding price p can be derived as previously, and is equal to $C' = \lambda_P(\lambda_I^{-1}(C))$.

Given this strategy, the payoff becomes

$$R(C', q) = C_1 + C_2 \cdot q + C_3(C') \cdot q. \tag{85}$$

We can then determine the first-order condition with respect to q.

$$\frac{\partial R(C', q)}{\partial q} = C_2 + C_3(C') \tag{86}$$
$$= -(\mathbb{E}_\mu[\lambda_P + \lambda_R] - C) + \int_{\lambda_P(x) > C'} (\lambda_P(x) + \lambda_R(x) - C) d\mu(x) \tag{87}$$
$$= -\left(\int_{x \leq \lambda_I^{-1}(C)} (\lambda_I(x) - C) d\mu(x) + \int_{x > \lambda_I^{-1}(C)} (\lambda_I(x) - C) d\mu(x) \right)$$
$$+ \int_{x > \lambda_I^{-1}(C)} (\lambda_I(x) - C) d\mu(x) \tag{88}$$
$$= - \int_{x \leq \lambda_I^{-1}(C)} (\lambda_I(x) - C d\mu(x)) \tag{89}$$
$$> 0 \tag{90}$$

We conclude that, for a fringe agent with upward balancing capacity and marginal cost C higher than $\mathbb{E}_\mu[\lambda_P + \lambda_R]$, the optimal strategy is to bid its entire capacity at price $\lambda_P(\lambda_I^{-1}(C))$ in the balancing energy auction.

□

Proposition 4 (Bidding Strategy – RT Market for Reserve). *The optimal strategy for a fringe agent under a "RT market for reserve" design is to bid truthfully in the balancing energy market.*

Proof. The optimal strategy for a fringe agent with upward balancing capacity P^+ and marginal cost C can be found by maximizing the sum of z_B and z_I.

a) If $\mathbb{E}_\mu[\lambda_P] \leq C$, then $z_I(q) = 0$ and the total reward of the agent $R(p,q)$ is defined hereunder.

$$R(p,q) = z_B(p,q) + z_I(q) \tag{91}$$
$$= C_1 + C_2(p) \cdot q \tag{92}$$

with C_1 and $C_3(p)$ characterized as follows:

$$C_1 = \mathbb{E}_\mu[\lambda_R] \cdot P^+ \tag{93}$$
$$C_2(p) = \int_{\lambda_P(x)>p} (\lambda_P(x) - C) d\mu(x) \tag{94}$$

Maximizing $R(p,q)$ here is equivalent to case (a) of the "no adder" design, where we have shown that it is optimal to bid the entire capacity of the BSP truthfully.

b) If $\mathbb{E}_\mu[\lambda_P] > C$, then $z_I(q) = (\mathbb{E}_\mu[\lambda_P - C) \cdot (P^+ - q)$ and the total reward of the agent, $R(p,q)$, is defined hereunder.

$$R(p,q) = z_B(p,q) + z_I(q) \tag{95}$$
$$= C_1 + C_2 \cdot q + C_3(p) \cdot q \tag{96}$$

with C_1,C_2 and $C_3(p)$ characterized as follows:

$$C_1 = (\mathbb{E}_\mu[\lambda_P + \lambda_R] - C) \cdot P^+ \tag{97}$$
$$C_2 = -(\mathbb{E}_\mu[\lambda_P] - C) \tag{98}$$
$$C_3(p) = \int_{\lambda_P(x)>p} (\lambda_P(x) - C) d\mu(x) \tag{99}$$

Maximizing $R(p,q)$ here is equivalent to case (b) of the "no adder" design. The optimal strategy for an agent is to bid its entire capacity truthfully.

□

3 Nash Equilibrium for "Adder on BRPs" Design

We define the platform price for a level α of reactive balancing,

$$\lambda_P(x,\alpha) = \begin{cases} MC(x-\alpha) & \text{if } x < \alpha, \\ \text{Price indeterminacy between } MC(0) \text{ and } MC(\alpha) & \text{if } x = \alpha, \\ MC(x) & \text{else.} \end{cases} \tag{100}$$

The opportunity cost of performing reactive balancing for an agent with marginal cost C, given a level α of reactive balancing in the system, is expressed as:

$$z(\alpha, C) = (\mathbb{E}_\mu[\lambda_P(\cdot,\alpha) + \lambda_R(\cdot)] - C) - \int_{\lambda_P(x,\alpha)\geq C} (\lambda_P(x,\alpha) - C) d\mu(x).$$

Proposition 5 (Equilibrium – Adder on BRPs)**.** *If $z(\alpha, MC(\alpha))$ is continuous, there exists a unique Nash equilibrium generated by fringe agents under the "adder on BRPs" design characterized by an equilibrium level of reactive balancing, α^*, such that $0 \leq \alpha^* \leq P^{max}$, and with the other BSPs bidding truthfully. This optimal level of reactive balancing is equal to (i) 0 if $z(0, MC(0)) < 0$, (ii) P^{max} if $z(P^{max}, MC(P^{max})) > 0$ or (iii) α^* characterized by the identity*

$$z(\alpha^*, MC(\alpha^*)) = 0. \tag{101}$$

This equilibrium level of reactive balancing generates platform prices equal to $\lambda_P(x, \alpha^*)$ and is strictly monotonic increasing.

Proof. $z(\alpha, C)$ is strictly monotonic decreasing in C for a fixed α as $z(\alpha, C) < z(\alpha, C^-)$ for $C^- < C$.

$$\begin{aligned} z(\alpha, C) - z(\alpha, C^-) &= \int_{\lambda_P(x,\alpha)\leq C} (\lambda_P(x,\alpha)) - C) d\mu(x) - \int_{\lambda_P(x,\alpha)\leq C^-} (\lambda_P(x,\alpha) - C^-) d\mu(x) && (102) \\ &= \int_{C^-\leq\lambda_P(x,\alpha)\leq C} (\lambda_P(x,\alpha) - C) d\mu(x) + \int_{\lambda_P(x,\alpha)\leq C^-} (C^- - C) d\mu(x) && (103) \\ &< 0 && (104) \end{aligned}$$

This allows us to prove the stability of the optimal level of reactive balancing mentioned in points (i) to (iii). As mentioned in the main manuscript, stability refers to a level of reactive balancing for which no agent has an incentive to deviate from their decision. BSPs after α^* on the merit order prefer participating in the balancing auction and agents before α^* prefer to resort to reactive balancing.

(i) If $z(0, MC(0)) < 0$, then a level of reactive balancing equal to 0 is stable as $z(0, MC(x)) < z(0, MC(0)$ for all x in $(0, P^{max}]$. In other words, if the cheapest generator finds the balancing energy auction more profitable than resorting to reactive balancing, every other generator should also find the balancing energy auction more profitable.

(ii) If $z(P^{max}, MC(P^{max})) > 0$, then a level of reactive balancing equal to P^{max} is stable as $z(P^{max}, MC(x)) > z(P^{max}, MC(x))$ for all x in $[0, P^{max})$. In other words, if the most expensive generator finds resorting to reactive balancing more profitable than participating in the balancing energy auction, every other generator should also find resorting to reactive balancing more profitable.

(iii) If $z(\alpha^*, MC(\alpha^*)) = 0$, then a level of reactive balancing equal to α^* is stable, since $z(\alpha^*, MC(\alpha^*)) > z(\alpha^*, MC(x))$ for all x in $(\alpha^*, P^{max}]$ and $z(\alpha^*, MC(\alpha^*)) < z(\alpha^*, MC(x))$ for all x in $[0, \alpha^*)$. In other words, if the frontier agent is indifferent between resorting to reactive balancing and participating in the balancing energy auction, every cheaper (resp. more expensive) generator should find reactive balancing (resp. the balancing energy auction) more profitable than the balancing energy auction (resp. doing reactive balancing) and has no incentive to modify its behaviour.

The existence of an equilibrium is proven by the continuity of $z(\alpha, MC(\alpha))$ with respect to α and the fact that at least one condition of (i) to (iii) must be true. The proof of uniqueness relies on $z(\alpha, MC(\alpha))$ being strictly monotonic decreasing with respect to α for strictly monotonic increasing MC. We show hereunder that $z(\alpha, MC(\alpha)) < z(\alpha^-, MC(\alpha^-))$ for $\alpha^- < \alpha$.

$$
\begin{aligned}
z(\alpha^-, MC(\alpha^-)) - z(\alpha, MC(\alpha)) \\
&= \int_{x \leq \alpha^-} (MC(x - \alpha^-) - MC(\alpha^-)) d\mu(x) \\
&\quad - \int_{x \leq \alpha} (MC(x - \alpha) - MC(\alpha)) d\mu(x) && (105) \\
&= \int_{x \leq \alpha^-} (MC(x - \alpha^-) - MC(x - \alpha) - (MC(\alpha^-) - MC(\alpha))) d\mu(x) \\
&\quad - \int_{\alpha^- < x \leq \alpha} MC(x - \alpha) - MC(\alpha) d\mu(x) && (106) \\
&> 0 && (107)
\end{aligned}
$$

This bidding behavior, coupled with the balancing energy auction selecting the bid in increasing price order, results in the platform price being equal to $MC(x - \alpha)$ for $x < \alpha$ (as $x - \alpha$ MWh of downward balancing capacity has to be activated) and to $MC(x)$ if $x > \alpha$.

The optimal strategy developed in proposition 2 of the main paper is still valid even when λ_P is not a standard single-valued function but rather a *set-valued function* with a price indeterminacy at α. The first-order condition with respect to p (39) exhibits two behaviours.

1. If $p < MC(0)$ and $p > MC(\alpha)$, the supply function λ_P^{-1} is well defined and the first order condition is equal to the following:

$$
\frac{\partial R(p, q)}{\partial p} = -(p - C) \cdot \mu(\lambda_P^{-1}(p)) \cdot \frac{d\lambda_P^{-1}(p)}{dp} \cdot q. \qquad (108)
$$

2. If $MC(0) < p < MC(\alpha)$, the supply function hits a plateau (due to the agents between 0 and α resorting to reactive balancing and not participating in the energy balancing auction), and the payoff function is then

characterized as follows.

$$R(p,q) = \int_{x>\alpha} (\lambda_P(x) - C) d\mu(x) \cdot q \tag{109}$$

The first order condition with respect to p is equal to 0 for any p between $MC(0)$ and $MC(\alpha)$.

The optimal price bid for an agent can be derived as a function of the agent's marginal cost, based on the continuity of R.

1. If $C < MC(0)$, the derivative of R with respect to p is increasing in $(-\infty, C[$, zero at C, decreasing in $]C, MC(0)[$, zero in $]MC(0), MC(\alpha)[$ and decreasing in $]MC(\alpha), \infty)$. Bidding $p = C$ is optimal.

2. If $MC(0) \leq C \leq MC(\alpha)$, the derivative of R with respect to p is increasing in $(-\infty, MC(0)[$, zero in $]MC(0), MC(\alpha)[$ and decreasing in $]MC(\alpha), \infty)$. Bidding any cost between $MC(0)$ and $MC(\alpha)$ (including $p = C$) is optimal.

3. If $MC(\alpha) < p$, the derivative of R with respect to p is increasing in $(-\infty, MC(0)[$, zero in $]MC(0), MC(\alpha)[$, increasing in $]MC(\alpha), C[$, zero at C and decreasing in $]C, \infty)$. Bidding $p = C$ is optimal.

□

The uniqueness of the equilibrium in the two-zone setting is not certain due to the impact of the level of reactive balancing on λ_R and its impact on the monotonicity of z.

The "adder on BSPs" design fails to produce a Nash equilibrium if the imbalance distribution is drawn from a unit set. In other words, if BSPs have perfect information on the level of imbalance that they will face, no equilibrium can be reached for the "adder on BRPs" design. The following argument illustrates this point. Let us assume that all agents know that the system will be exposed to a level of imbalance x'. In this context, the optimal level of reactive balancing is exactly x', since all agents below x' on the merit order would prefer the imbalance price to the balancing price and all agents after x' will participate in the balancing energy auction. The platform price could be anything between be 0 and $MC(x')$ due to the discontinuity in the merit order curve.

a) If it is between 0 and $MC(x') - \lambda_R(x')$, a level x' of reactive balancing is not sustainable, as the agent at position x' on the merit order curve would rather participate in the balancing auction. But if the agent at position x' on the merit order curve participates truthfully in the balancing energy auction, they would be selected, and that platform price would have incentivized them to resort to reactive balancing.

b) If the platform price is between $MC(x') - \lambda_R(x')$ and $MC(x')$, then some agents located after x' on the merit order curve would rather do reactive balancing than participate in the balancing energy auction. The platform price with a level of reactive balancing higher than x' is not sufficient to sustain this level of self-activation.

Only a balancing energy price equal to $MC(x') - \lambda_R(x')$ would ensure a Nash equilibrium in this situation but, even if it is possible, it is not guaranteed.

Note that there is no risk of failing to produce a Nash equilibrium even if the domain of the imbalance distribution is a unit set in the 2-zones setting. The aggregation of a continuous and a non-continuous offer curve is continuous and this induces unique balancing energy prices.

4 Optimal Strategies with Reservation Cost

The following strategies are derived for profit-maximizing agents with production cost C and reservation cost K. These agents can first participate in a reservation auction followed by a balancing energy auction for activation. They can also only participate in the balancing energy auction or only resort to reactive balancing. The reservation cost is incurred once if (i) the agent is selected in the reservation auction, and/or (ii) if the agent is selected in the balancing energy auction, and/or (iii) if the agent resorts to reactive balancing. The agents offer a reservation bid (p^{DA}, q^{DA}), a balancing energy bid (p, q), and decide to self-activate ai MWh. The reservation payoff can be expressed as:

$$z_{DA}(p^{DA}, q^{DA}) = \begin{cases} \lambda_{DA} \cdot q^{DA} & \text{if } \lambda_{DA} \geq p^{DA}, \\ 0 & \text{else.} \end{cases} \tag{110}$$

Here, λ_{DA} is the reservation price. If the reservation bid is accepted, the balancing energy payoff is:

$$z_B(p, q, q^{DA}, x) = \begin{cases} \lambda_P \cdot (q + q^{DA}) & \text{if } \lambda_P(x) \geq p, \\ 0 & \text{else,} \end{cases} \tag{111}$$

and the expected balancing energy payoff is

$$z_B(p, q, q^{DA}) = \int_{\lambda_P(x) \geq p} (\lambda_P(x) - C) d\mu(x) \cdot (q + q^{DA}). \tag{112}$$

Finally, the reactive balancing payoff if the reservation bid has been accepted is the solution to the following optimization problem:

$$\max_{ai} \quad (\mathbb{E}_\mu[\lambda_{imb}] - C) \cdot ai \tag{113}$$

$$s.t. \quad ai + q + q^{DA} \leq P^+ \tag{114}$$

$$ai \geq 0 \tag{115}$$

If $C \geq \mathbb{E}_\mu[\lambda_{imb}]$, the optimal level of reactive balancing $ai^\star$ is 0, else $ai^\star$ is equal to the leftover capacity from the balancing energy and reservation auctions. The reactive balancing payoff is then described as follows:

$$z_I(q, q^{DA}) = \begin{cases} (\mathbb{E}_\mu[\lambda_{imb}] - C) \cdot (P^+ - q - q^{DA}) & \text{if } C \leq \mathbb{E}_\mu[\lambda_{imb}], \\ 0 & \text{else.} \end{cases} \tag{116}$$

The introduction of a real-time market for reserve has an impact on the real-time payoff. The capacity that was accepted in the reservation auction has to be bought back in real-time:

$$\begin{aligned} & z_B(p, q, q^{DA}, x) + z_I(ai, x) \\ &= \begin{cases} (\lambda_P(x) + \lambda_R(x) - C) \cdot (q + q^{DA} + ai) + \lambda_R(x) \cdot (P^+ - q - q^{DA} - ai) - \lambda_R(x) \cdot q^{DA} \\ \qquad \text{if } \lambda_P(x) \leq p, \\ (\lambda_P(x) + \lambda_R(x) - C) \cdot ai + \lambda_R(x) \cdot (P^+ - ai) - \lambda_R(x) \cdot q^{DA} \\ \qquad \text{else,} \end{cases} \end{aligned} \tag{117}$$

$$= \begin{cases} (\lambda_P(x) - C) \cdot (q + q^{DA} + ai) + \lambda_R(x) \cdot (P^+ - q^{DA}) & \text{if } \lambda_P(x) \leq p, \\ (\lambda_P(x) - C) \cdot ai + \lambda_R(x) \cdot (P^+ - q^{DA}) & \text{else.} \end{cases} \tag{118}$$

Both the expected imbalance and balancing payoffs can be modified to account for the real-time market for reserve:

$$z_I(q, q^{DA}) = \begin{cases} (\mathbb{E}_\mu[\lambda_P] - C) \cdot (P^+ - q - q^{DA}) & \text{if } C \leq \mathbb{E}_\mu[\lambda_P] \\ 0 & \text{else,} \end{cases} \tag{119}$$

$$z_B(p, q,^{DA}) = \int_{\lambda_P(x) \geq p} (\lambda_P(x) - C) d\mu(x) \cdot (q + q^{DA}) + \mathbb{E}_\mu[\lambda_R] \cdot (P^+ - q^{DA}). \tag{120}$$

We can now derive the optimal bidding strategy for a fringe agent with marginal cost C and reservation cost K. The reservation cost can be considered as a fixed must-run cost for keeping the plant up-and-running.

Proposition 6 (Bidding Strategy – Reservation Cost – No Adder)**.** *The optimal strategy for a fringe agent under a "no adder" design is to bid its full capacity in the day-ahead auction at a price*

$$p^{DA} = \max\left(\frac{K - \delta z_B \cdot P^+}{P^+}; 0\right),$$

with $\delta z_B = \int_{\lambda_P(x) > C} (\lambda_P(x) - C) d\mu(x)$*, and to then bid truthfully in the balancing auction. If the day-ahead bid is not accepted, the optimal action is to not activate its plant.*

Proof. (a) If $E[\lambda_P] \leq C$ then $z_I(q, q^{DA}) = 0$. The total reward of an agent selected in the day-ahead auction, $R(p, q, q^{DA})$, is defined hereunder for a given day-ahead price λ^{DA}:

$$R(p,q,q^{DA}) = z_B(p,q) + z_I(q) + z_{DA}(q^{DA}) \tag{121}$$

$$= C_1 + C_2(p)\cdot q + C_3\cdot q^{DA} + C_4(p)\cdot q^{DA} \tag{122}$$

with $C_1, C_2(p), C_3$ and $C_4(p)$ equal to:

$$C_1 = -K \tag{123}$$

$$C_2(p) = \int_{\lambda_P(x)>p} (\lambda_P(x) - C) d\mu(x) \tag{124}$$

$$C_3 = \lambda^{DA} \tag{125}$$

$$C_4(p) = \int_{\lambda_P(x)>p} (\lambda_P(x) - C) d\mu(x) \tag{126}$$

The optimal bidding strategy of the agent can be derived from the first-order condition with respect to p, q and q^{DA}. We focus on p first:

$$\frac{\partial R(p,q,q^{DA})}{\partial p} = C_2'(p)\cdot q + C_4'(p)\cdot q^{DA} \tag{127}$$

$$= -(\lambda_P(\lambda_P^{-1}(p)) - C)\cdot \mu(\lambda_P^{-1}(p))\cdot \frac{d\lambda_P^{-1}(p)}{dp}\cdot (q+q^{DA}) \tag{128}$$

$$= -(p - C)\cdot \mu(\lambda_P^{-1}(p))\cdot \frac{d\lambda_P^{-1}(p)}{dp}\cdot (q+q^{DA}) \tag{129}$$

For fixed q and q^{DA}, the payoff function $R(p,q,q^{DA})$ is increasing in $(-\infty, C]$, zero at C, and decreasing in $[C, +\infty)$. Thus, for any q and q^{DA}, an optimal strategy in the balancing auction is to bid truthfully the marginal cost.

The payoff maximization problem given this strategy is presented in equation (130). It shows that, for a positive λ^{DA}, the coefficient on q^{DA} in the objective is higher than on q. The strategy of setting q at 0 dominates all other strategies for allocating the balancing capacity between q and q^{DA}.

$$\begin{aligned} \max_{q,q^{DA}} \quad & R(C,q,q^{DA}) \\ s.t. \quad & q^{DA} + q \leq P^+ \\ & q^{DA} \geq 0 \\ & q \geq 0 \end{aligned} \tag{130}$$

Given these two strategies, the payoff becomes

$$R(C,0,q^{DA}) = C_1 + C_3\cdot q^{DA} + C_4(C)\cdot q^{DA}. \tag{131}$$

The first-order condition with respect to q^{DA} can be examined, in order to compute the optimal action with respet to q^{DA}.

$$\frac{\partial R(C,0,q^{DA})}{\partial q^{DA}} = C_2 + C_4(C) \tag{132}$$

$$= \lambda^{DA} + \int_{\lambda_P(x)>C'} (\lambda_P(x) - C)d\mu(x) \tag{133}$$

$$= \lambda^{DA} + \int_{x>\lambda_P^{-1}(C)} (\lambda_P(x) - C)d\mu(x) \tag{134}$$

$$> 0 \tag{135}$$

We conclude that, for a fringe agent with upward balancing capacity and marginal cost C higher than $\mathbb{E}[\lambda_P]$ selected in the day-ahead auction at an arbitrary day-ahead price λ^{DA}, the optimal strategy is to bid its full capacity in the DA auction and to then bid thruthfully in the balancing auction.

The payoff of an agent selected in the day ahead given this strategy is expressed as a function of a given day-ahead price as follows:

$$R^{DA}(\lambda_{DA}) = \delta z_B \cdot P^+ - K + \lambda_{DA} \cdot P^+. \tag{136}$$

The payoff for an agent not participating in the day-ahead auction can be inferred from proposition 1:

$$R^B = \begin{cases} \delta z_B \cdot P^+ - K & \text{if } K \leq \Pi_B, \\ 0 & \text{else.} \end{cases} \tag{137}$$

The optimal price bid in the day ahead can then be derived from the profit maximization problem (138). This problem is defined as a function of the stochastic day-ahead demand x, the corresponding day-ahead price $\lambda^{DA}(x)$ and the probability measure $\mu^{DA}(x)$. The total payoff is the probability-weighted sum of (i) the payoff of an agent not selected in the day-ahead auction, if $p^{DA} \geq \lambda_D A$, and (ii) the payoff of an agent selected in the day ahead, if $p^{DA} < \lambda_{DA}$.

$$Q(p^{DA}) = \max_{p^{DA}} \int_{\lambda_{DA}(x)\leq p^{DA}} R^B d\mu^{DA}(x) + \int_{\lambda_{DA}(x)>p^{DA}} R^{DA}(\lambda_{DA}(x))d\mu^{DA}(x) \tag{138}$$

If the balancing payoff is lower than the reservation cost, so $\Pi_B < K$ and $R^B = 0$, the optimal bidding strategy can be derived from the first order condition with respect to p^{DA}:

$$\frac{dQ(p^{DA})}{dp^{DA}} = -R^{DA}(\lambda_{DA}(\lambda_{DA}^{-1}(p^{DA}))) \cdot \mu^{DA}(\lambda_{DA}^{-1}(p^{DA})) \cdot \frac{d\lambda_{DA}^{-1}(p^{DA})}{dp^{DA}} \tag{139}$$

$$= -(\delta z_B \cdot P^+ - K + p^{DA} \cdot P^+) \cdot \mu^{DA}(\lambda_{DA}^{-1}(p^{DA}) \cdot \frac{d\lambda_D A^{-1}(p^{DA})}{dp^{DA}} \tag{140}$$

Condition (140) indicates the optimal price bid in the day ahead as $p^{DA} = (\delta z_B \cdot P^+ - K)/P^+$.

The same reasoning can be applied if the balancing payoff is greater than the reservation cost, so $\delta z_B \cdot P^+ > K$ and $R^B = \delta z_B \cdot P^+ - K$:

$$\frac{\partial Q(p^{DA})}{\partial p^{DA}} = (\delta z_B - K - R^{DA}(p^{DA})) \cdot \mu^{DA}(\lambda_{DA}^{-1}(p^{DA})) \cdot \frac{d\lambda_{DA}^{-1}(p^{DA})}{dp^{DA}} \tag{141}$$

$$= -p^{DA} \cdot P^+ \cdot \mu^{DA}(\lambda_{DA}^{-1}(p^{DA})) \cdot \frac{d\lambda_{DA}^{-1}(p^{DA})}{dp^{DA}} \tag{142}$$

Condition (142) specifies the optimal price bid in the day ahead as $p^{DA} = 0$.

The optimal bidding strategy in the day-ahead market if $\mathbb{E}[\lambda_P] \leq C$ is then for a unit to bid its maximal capacity at price

$$p^{DA} = \max\left(\frac{K - \delta z_B \cdot P^+}{P^+}; 0\right)$$

and to then bid truthfully in the balancing energy auction, if selected, or not activate its plant, if not selected.

(b) If $E[\lambda_P] > C$, then $z_I(q, q^{DA}) = \mathbb{E}[\lambda_P - C] \cdot (P^+ - q - q^{DA})$. Since participating in the balancing energy auction is always more profitable than resorting to reactive balancing, the reasoning developed in point (a) applies and the optimal bidding strategy in the day-ahead market is to bid the full capacity at price

$$p^{DA} = \max\left(\frac{K - \delta z_B \cdot P^+}{P^+}; 0\right)$$

and to then bid truthfully in the balancing energy auction, if selected, or not activate the plant, if not selected. □

Proposition 7 (Bidding Strategy – Reservation Cost – Adder on BRPs)**.** *The optimal strategy for a fringe agent under an "adder on BRPs" design is to bid its full capacity in the day-ahead auction at price*

$$p^{DA} = \max\left(\frac{K - \delta z_B \cdot P^+}{P^+}; \delta z_B - \delta z_I; 0\right)$$

with $\delta z_B = \int_{\lambda_P(x)>C} (\lambda_P(x) - C) d\mu(x)$ *and* $\delta z_I = (\mathbb{E}[\lambda_P + \lambda_R] - C)$*. If the DA bid has been accepted, the optimal action is to bid truthfully in the balancing auction. If the DA bid is* $\delta z_I - \delta z_B$ *and has not been accepted, the optimal action for an agent is to do reactive balancing with its full capacity. Else, the agent should not activate its plant.*

Proof. (a) If $\mathbb{E}[\lambda_P + \lambda_R] \leq C$, then $z_I(q, q^{DA}) = 0$. As participating in the balancing energy auction is always more profitable than resorting to reactive balancing, the reasoning developed in point (a) of proposition 6 applies and the optimal bidding strategy in the day-ahead market is to bid the maximal capacity at a price

$$p^{DA} = \max\left(\frac{K - \delta z_B \cdot P^+}{P^+}; 0\right)$$

and to then bid truthfully in the balancing energy auction, if selected, or not activate the plant, if not selected.

(b) If $E[\lambda_P + \lambda_R] - \int_{\lambda_P(x)>C}(\lambda_P(x) - C)d\mu x \leq C < \mathbb{E}[\lambda_P + \lambda_R]$, then $z_I(q, q^{DA}) = \mathbb{E}[\lambda_P + \lambda_R - C] \cdot (P^+ - q - q^{DA})$. Since participating in the balancing energy auction is always more profitable than resorting to reactive balancing, the reasoning developed in point (a) of proposition 6 applies and the optimal bidding strategy in the day-ahead market is to bid the maximal capacity at a price

$$p^{DA} = \max\left(\frac{K - \delta z_B \cdot P^+}{P^+}; 0\right)$$

and to then bid truthfully in the balancing energy auction, if selected, or not activate the plant, if not selected.

(c) If $C < E[\lambda_P + \lambda_R] - \int_{\lambda_P(x)>C}(\lambda_P(x) - C)d\mu x$ then $z_I(q, q^{DA}) = \mathbb{E}[\lambda_P + \lambda_R - C] \cdot (P^+ - q - q^{DA})$ and the total reward of an agent selected in the day-ahead auction, $R(p, q, q^{DA})$, is defined hereunder for an arbitrary day-ahead price λ^{DA}:

$$R(p, q, q^{DA}) = z_B(p, q) + z_I(q) + z_{DA}(q^{DA}) \tag{143}$$

$$= C_1 + C_2 \cdot q + C_3(p) \cdot q + C_4 \cdot q^{DA} + C_5(p) \cdot q^{DA} \tag{144}$$

with $C_1, C_2, C_3(p), C_4$ and $C_5(p)$ expressed as:

$$C_1 = -K + (\mathbb{E}[\lambda_P + \lambda_R] - C) \cdot P^+ \tag{145}$$

$$C_2 = -(\mathbb{E}[\lambda_P + \lambda_R] - C) \tag{146}$$

$$C_3(p) = \int_{\lambda_P(x)>p} (\lambda_P(x) - C)d\mu(x) \tag{147}$$

$$C_4 = -(\mathbb{E}[\lambda_P + \lambda_R] - C) + \lambda_{DA} \tag{148}$$

$$C_5(p) = \int_{\lambda_P(x)>p} (\lambda_P(x) - C)d\mu(x) \tag{149}$$

The same argument as in case (a) of proposition 6 can be applied for proving the optimality of bidding $p = C$ in the balancing auction. Similarly, the strategy consisting of bidding its full capacity in the reservation auction dominates all other strategies for allocating the capacity between the reservation auction and the balancing energy auction.

The first order condition with respect to q^{DA} can be examined next:

$$\frac{\partial R(C,0,q^{DA})}{\partial q^{DA}} = C_4 + C_5(C) \tag{150}$$

$$= -(\mathbb{E}[\lambda_P + \lambda_R] - C) + \lambda^{DA} + \int_{\lambda_P(x)>C} (\lambda_P(x) - C) d\mu(x) \tag{151}$$

$$= \lambda_{DA} + \delta z_B - \delta z_I \tag{152}$$

In contrast to case (a) and (b), it is not possible to ascertain the sign of $\partial R(C,O,q^{DA})\partial q^{DA}$ for any positive λ^{DA}. If $\lambda^{DA} > \delta z_I - \delta z_B$ then

$$\frac{\partial R(C,0,q^{DA})}{\partial q^{DA}} > 0 \quad \text{and} \quad q^{DA} = P^+$$

and if $\lambda^{DA} < \delta z_I - \delta z_B$ then

$$\frac{\partial R(C,0,q^{DA})}{\partial q^{DA}} < 0 \quad \text{and} \quad q^{DA} = 0.$$

The payoff for an agent selected in the day-ahead market is then dependent on the value of λ^{DA} and on the day-ahead capacity bid q^{DA}. It is characterized as follows, as a function of an arbitrary day-ahead price:

$$R^{DA}(\lambda_{DA}, q^{DA}) = q^{DA} \cdot (\lambda_{DA} + \delta z_B) + (P^+ - q^{DA}) \cdot \delta z_I - K \tag{153}$$

The payoff for an agent not participating in the day-ahead auction can be derived from proposition 2:

$$R^B = \begin{cases} \delta z_I \cdot P^+ - K & \text{if } K \leq \delta z_I \cdot P^+, \\ 0 & \text{else.} \end{cases} \tag{154}$$

The optimal price and quantity bid in the day ahead can then be derived from the profit maximization problem (155). This problem is defined as a function of the day-ahead price bid for the stochastic day-ahead demand x and the corresponding day-ahead price $\lambda^{DA}(x)$ and probability measure $\mu^{DA}(x)$. The total payoff is the probability weighted sum of (i) the payoff of an agent not selected in the day-ahead auction, if $p^{DA} \geq \lambda_{DA}$, and (ii) the payoff of an agent selected in the day ahead, if $p^{DA} < \lambda_{DA}$:

$$Q(p^{DA}, q^{DA}) = \max_{q^{DA}, p^{DA}} \int_{\lambda_{DA}(x) \leq p^{DA}} R^B d\mu^{DA}(x) + \int_{\lambda_{DA}(x) > p^{DA}} R^{DA}(\lambda_{DA}(x), q^{DA}) d\mu^{DA}(x) \tag{155}$$

If $\delta z_I \cdot P^+ < K$ then $R^B = 0$, and the optimal strategy can be derived from the first order condition with respect to p^{DA} and q^{DA}:

$$\frac{\partial Q(p^{DA}, q^{DA})}{\partial p^{DA}} = -R^{DA}(\lambda_{DA}(\lambda_{DA}^{-1}(p^{DA})), q^{DA}) \cdot \mu^{DA}(\lambda_{DA}^{-1}(p^{DA})) \cdot \frac{d\lambda_{DA}^{-1}(p^{DA})}{dp^{DA}} \tag{156}$$

$$= -R^{DA}(p^{DA}, q^{DA}) \cdot \mu^{DA}(\lambda_{DA}^{-1}(p^{DA})) \cdot \frac{d\lambda_{DA}^{-1}(p^{DA})}{dp^{DA}} \tag{157}$$

$$= -(q^{DA} \cdot (p^{DA} + \delta z_B) + (P^{+} - q^{DA}) \cdot \delta z_I - K) \cdot \mu^{DA}(\lambda_{DA}^{-1}(p^{DA})) \cdot \frac{d\lambda_{DA}^{-1}(p^{DA})}{dp^{DA}} \tag{158}$$

Equating (158) to zero leads to the following optimality condition:

$$p^{DA*} = \frac{K - P^{+} \cdot \delta z_I + q^{DA} \cdot (\delta z_I - \delta z_B)}{q^{DA}} \tag{159}$$

This allows us to define the optimal price bid p^{DA*} for any capacity bid q^{DA} different from 0. We can also derive the first-order condition with respect to q^{DA}:

$$\frac{\partial Q(p^{DA}, q^{DA})}{\partial q^{DA}} = \int_{\lambda_{DA}(x) > p^{DA}} (\lambda_{DA}(x) + \delta z_B - \delta z_I) d\mu^{DA}(x). \tag{160}$$

The next step consists in inserting p^{DA*} in equation (160):

$$\frac{\partial Q(p^{DA*}, q^{DA})}{\partial q^{DA}} = \int_{\lambda_{DA}(x) > p^{DA*}} (\lambda_{DA}(x) + \delta z_B - \delta z_I) d\mu^{DA}(x) \tag{161}$$

$$> 0 \tag{162}$$

The last inequality is due to p^{DA*} always being greater than $\delta z_B - \delta z_I$ for any q^{DA} different than zero. This leads to $q^{DA*} = P^{+}$ and $p^{DA*} = (K - P^{+} \cdot \delta z_B)/P^{+}$.

If $\delta z_I \cdot P^{+} \geq K$ then $R^B = \delta z_I \cdot P^{+} - K$ and the optimal strategy can be derived from the first-order condition with respect to p^{DA} and q^{DA}:

$$\frac{\partial Q(p^{DA}, q^{DA})}{\partial p^{DA}} = (R^B - R^{DA}(p^{DA}, q^{DA})) \cdot \mu^{DA}(\lambda_{DA}^{-1}(p^{DA})) \cdot \frac{d\lambda_{DA}^{-1}(p^{DA})}{dp^{DA}} \tag{163}$$

$$= (\delta z_I \cdot P^{+} - K - q^{DA} \cdot (p^{DA} + \delta z_B) - (P^{+} - q^{DA}) \cdot \delta z_I + K) \cdot \mu^{DA}(\lambda_{DA}^{-1}(p^{DA})) \cdot \frac{d\lambda_{DA}^{-1}(p^{DA})}{dp^{DA}} \tag{164}$$

$$= -(p^{DA} + \delta z_B - \delta z_I) \cdot q^{DA} \cdot \mu^{DA}(\lambda_{DA}^{-1}(p^{DA})) \cdot \frac{d\lambda_{DA}^{-1}(p^{DA})}{dp^{DA}} \tag{165}$$

For fixed q^{DA}, the day-ahead payoff function $R^{DA}(p^{DA}, q^{DA})$ is increasing in $(-\infty, \delta z_I - \delta z_B[$, zero at $\delta z_I - \delta z_B$ and decreasing in $]\delta z_I - \delta z_B, +\infty)$. Thus, for any q^{DA}, an optimal strategy in the reservation auction is to bid $\delta z_I - \delta z_B$.

We can also derive the first-order condition with respect to q^{DA}:

$$\frac{\partial Q(\delta z_I - \delta z_B, q^{DA})}{\partial q^{DA}} = \int_{\lambda_{DA}(x) > \delta z_I - \delta z_B} (\lambda_{DA}(x) + \delta z_B - \delta z_I) d\mu^{DA}(x) \quad (166)$$

$$\geq 0 \quad (167)$$

This shows that the optimal strategy in the day-ahead capacity auction is to bid the full capacity at the price $\delta z_I - \delta z_B$.

Combining both cases $\delta z_I \cdot P^+ < K$ and $\delta z_I \cdot P^+ \geq K$ allows us to define the optimal bid in the capacity auction if $C < \mathbb{E}[\lambda_P - \lambda_R] - \int_{\lambda_P(x) \geq C} (\lambda_P(x) - C) d\mu(x)$ as

$$p^{DA} = \max\big(\delta z_I - \delta z_B, \frac{K - \delta z_B \cdot P^+}{P^+}\big). \quad (168)$$

□

Proposition 8 (Bidding Strategy – Reservation Cost – Adder on BRPs and BSPs)**.** *The optimal strategy for a fringe agent under an "adder on BRPs and BSPs" design is to bid its full capacity in the day-ahead auction at price*

$$p^{DA} = \max\left(\frac{K - \delta z_B \cdot P^+}{P^+}; 0\right),$$

with $\delta z_B = \int_{\lambda_P(x) > \lambda_P((\lambda_P + \lambda_R)^{-1}(C))} (\lambda_P(x) - C) d\mu(x)$*, and to then bid at price* $\lambda_P((\lambda_P + \lambda_R)^{-1}(C))$ *in the balancing energy auction. If the day-ahead bid is not accepted, the optimal action is to not activate its plant.*

Proof. (a) If $E[\lambda_P] \leq C$, then $z_I(q, q^{DA}) = 0$. As internalizing the value of the adder when participating in the balancing energy auction is always more profitable than resorting to reactive balancing, the reasoning developed in point (a) in proposition 9 applies and the optimal bidding strategy in the day-ahead market is to bid the maximal capacity at price

$$p^{DA} = \max\left(\frac{K - \delta z_B \cdot P^+}{P^+}; 0\right)$$

and to then internalize the value of the adder in the balancing energy auction, if selected, or to not activate the plant, if not selected.

(b) If $E[\lambda_P] > C$, then $z_I(q, q^{DA}) = \mathbb{E}[\lambda_P + \lambda_R - C] \cdot (P^+ - q - q^{DA})$. Since internalizing the value of the adder when participating in the balancing energy auction is always more profitable than resorting to reactive balancing, the reasoning developed in point (a) in proposition 9 applies, and the optimal bidding strategy in the day-ahead market is to bid the full capacity at price

$$p^{DA} = \max\left(\frac{K - \delta z_B \cdot P^+}{P^+}; 0\right)$$

and to then internalize the value of the adder in the balancing energy auction, if selected, or not activate the plant, if not selected. □

Proposition 9 (Bidding Strategy – Reservation Cost – RT Market for Reserve)**.** *The optimal strategy for a fringe agent under a "RT market for reserve" design is to bid its full capacity in the day-ahead auction at price*

$$p^{DA} = \max\left(\frac{K - (\delta z_B + \mathbb{E}_\mu[\lambda_R]) \cdot P^+}{P^+}; \mathbb{E}_\mu[\lambda_R]\right),$$

with $\delta z_B = \int_{\lambda_P(x)>C} (\lambda_P(x) - C) d\mu(x)$ *and* $\delta z_I = \mathbb{E}[\lambda_P] - C$*. If the day-ahead bid is accepted, the optimal action is to bid truthfully in the balancing auction. If the day-ahead bid is* $\mathbb{E}_\mu[\lambda_R]$ *and has not been accepted, the optimal action for the agent is to bid truthfully in the balancing auction with its full capacity. Else, the agent should not activate its plant.*

Proof. (a) If $\mathbb{E}[\lambda_P] \leq C$ then $z_I(q, q^{DA}) = 0$ and the total reward of an agent selected in the day-ahead auction, $R(p, q, q^{DA})$, is defined hereunder for a given day-ahead price λ^{DA}:

$$R(p, q, q^{DA}) = z_B(p, q) + z_I(q) + z_{DA}(q^{DA}) \tag{169}$$

$$= C_1 + C_2(p) \cdot q + C_3 \cdot q^{DA} + C_4(p) \cdot q^{DA} \tag{170}$$

with $C_1, C_2(p), C_3$ and $C_4(p)$ equal to:

$$C_1 = -K + \mathbb{E}_\mu[\lambda_R] \cdot P^+ \tag{171}$$

$$C_2(p) = \int_{\lambda_P(x)>p} (\lambda_P(x) - C) d\mu(x) \tag{172}$$

$$C_3 = -\mathbb{E}[\lambda_R] + \lambda_{DA} \tag{173}$$

$$C_4(p) = \int_{\lambda_P(x)>p} (\lambda_P(x) - C) d\mu(x) \tag{174}$$

The same argument as in case (a) of proposition 6 can be applied for proving the optimality of bidding $p = C$ in the balancing auction. The dominance of allocating the capacity of an agent between q and q^{DA} depends on the sign of $\lambda_{DA} - \mathbb{E}_\mu[\lambda_R]$.

If $\lambda_{DA} > \mathbb{E}_\mu[\lambda_R]$, then the payoff of the real-time reserve market is less profitable than the day-ahead payoff, and setting q at 0 dominates all other strategies for allocating the balancing capacity between q and q^{DA}. If $\lambda_{DA} < \mathbb{E}_\mu[\lambda_R]$, then the real-time reserve payoff is more profitable than the day-ahead payoff and setting q^{DA} at 0 dominates all other strategies for allocating the balancing capacity between q and q^{DA}. In either case, resorting to reactive balancing is dominated by at least one of these strategies.

The payoff for an agent selected in the day ahead is then dependent on the value of λ^{DA} and on the day-ahead capacity bid q^{DA}. It is expressed as follows, as a function of a given day-ahead price:

$$R^{DA}(\lambda_{DA}, q^{DA}) = (\delta z_B + \mathbb{E}_\mu[\lambda_R]) \cdot P^+ + (\lambda_{DA} - \mathbb{E}_\mu[\lambda_R]) \cdot q^{DA} - K \tag{175}$$

The payoff for an agent not participating in the day-ahead auction can be derived from proposition 2:

$$R^B = \begin{cases} (\delta z_B + \mathbb{E}_\mu[\lambda_R]) \cdot P^+ - K & \text{if } K \leq (\delta z_B + \mathbb{E}_\mu[\lambda_R]) \cdot P^+, \\ 0 & \text{else.} \end{cases} \tag{176}$$

The optimal price and quantity bid in the day-ahead market can then be derived from the profit maximization problem (177). This problem is defined as a function of the day-ahead price bid for the stochastic day-ahead demand x and the corresponding day-ahead price $\lambda^{DA}(x)$ and probability measure $\mu^{DA}(x)$. The total payoff is the probability weighted sum of (i) the payoff of an agent not selected in the day-ahead auction, if $p^{DA} \geq \lambda_{DA}$, and (ii) the payoff of an agent selected in the day ahead, if $p^{DA} < \lambda_{DA}$.

$$Q(p^{DA}, q^{DA}) = \max_{q^{DA}, p^{DA}} \int_{\lambda_{DA}(x) \leq p^{DA}} R^B d\mu^{DA}(x) + \int_{\lambda_{DA}(x) > p^{DA}} R^{DA}(\lambda_{DA}(x), q^{DA}) d\mu^{DA}(x) \tag{177}$$

If $(\delta z_B + \mathbb{E}_\mu[\lambda_R]) \cdot P^+ < K$ then $R^B = 0$, and the optimal strategy can be derived from the first-order condition with respect to p^{DA} and q^{DA}:

$$\frac{\partial Q(p^{DA}, q^{DA})}{\partial p^{DA}} = -R^{DA}(\lambda_{DA}(\lambda_{DA}^{-1}(p^{DA})), q^{DA}) \cdot \mu^{DA}(\lambda_{DA}^{-1}(p^{DA})) \cdot \frac{d\lambda_{DA}^{-1}(p^{DA})}{dp^{DA}} \tag{178}$$

$$= -R^{DA}(p^{DA}, q^{DA}) \cdot \mu^{DA}(\lambda_{DA}^{-1}(p^{DA})) \cdot \frac{d\lambda_{DA}^{-1}(p^{DA})}{dp^{DA}} \tag{179}$$

$$= -((\delta z_B + \mathbb{E}_\mu[\lambda_R]) \cdot P^+ + (p^{DA} - \mathbb{E}_\mu[\lambda_R]) \cdot q^{DA} - K) \cdot \mu^{DA}(\lambda_{DA}^{-1}(p^{DA})) \cdot \frac{d\lambda_{DA}^{-1}(p^{DA})}{dp^{DA}} \tag{180}$$

Equating (158) to zero leads to the following optimality condition:

$$p^{DA*} = \frac{K - P^+ \cdot (\delta z_B + \mathbb{E}_\mu[\lambda_R])}{q^{DA}} + \mathbb{E}_\mu[\lambda_R] \tag{181}$$

This allows us to define the optimal price bid p^{DA*} for any capacity bid q^{DA} different from 0. We can also derive the first-order condition with respect to q^{DA}:

$$\frac{\partial Q(p^{DA}, q^{DA})}{\partial q^{DA}} = \int_{\lambda_{DA}(x) > p^{DA}} (\lambda_{DA}(x) - \mathbb{E}_\mu[\lambda_R]) d\mu^{DA}(x). \tag{182}$$

The next step consists of inserting p^{DA*} in equation (182):

$$\frac{\partial Q(p^{DA*}, q^{DA})}{\partial q^{DA}} = \int_{\lambda_{DA}(x) > p^{DA*}} (\lambda_{DA}(x) - \mathbb{E}_\mu[\lambda_R]) d\mu^{DA}(x) \tag{183}$$

$$> 0 \tag{184}$$

The last inequality is due to p^{DA*} always being greater than or equal to $\mathbb{E}_\mu[\lambda_R]$ for $K > P^+ \cdot (\delta z_B + \mathbb{E}_\mu[\lambda_R])$ for any q^{DA} different from zero and positive. This leads to $q^{DA*} = P^+$ and

$$p^{DA*} = (K - P^+ \cdot \delta z_B)/P^+.$$

If $(\delta z_B + \mathbb{E}_\mu[\lambda_R]) \cdot P^+ \geq K$ then $R^B = (\delta z_B + \mathbb{E}_\mu[\lambda_R]) \cdot P^+ - K$ and the optimal strategy can be derived from the first-order condition with respect to p^{DA} and q^{DA}:

$$\frac{\partial Q(p^{DA}, q^{DA})}{\partial p^{DA}} = (R^B - R^{DA}(p^{DA}, q^{DA})) \cdot \mu^{DA}(\lambda_{DA}^{-1}(p^{DA})) \cdot \frac{d\lambda_{DA}^{-1}(p^{DA})}{dp^{DA}} \tag{185}$$

$$= (\delta z_B + \mathbb{E}_\mu[\lambda_R]) \cdot P^+ - K - ((\delta z_B + \mathbb{E}_\mu[\lambda_R]) \cdot P^+ + (p^{DA} - \mathbb{E}_\mu[\lambda_R]) \cdot q^{DA} - K))$$
$$\cdot \mu^{DA}(\lambda_{DA}^{-1}(p^{DA})) \cdot \frac{d\lambda_{DA}^{-1}(p^{DA})}{dp^{DA}} \tag{186}$$

$$= -(p^{DA} - \mathbb{E}_\mu[\lambda_R]) \cdot q^{DA} \cdot \mu^{DA}(\lambda_{DA}^{-1}(p^{DA})) \cdot \frac{d\lambda_{DA}^{-1}(p^{DA})}{dp^{DA}} \tag{187}$$

For fixed q^{DA}, the day-ahead payoff function $R^{DA}(p^{DA}, q^{DA})$ is increasing in $(-\infty, \mathbb{E}_\mu[\lambda_R][$, zero at $\mathbb{E}_\mu[\lambda_R]$, and decreasing in $]\mathbb{E}_\mu[\lambda_R], +\infty)$. Thus, for any q^{DA}, an optimal strategy in the reservation auction is to bid $\mathbb{E}_\mu[\lambda_R]$.

We can also derive the first-order condition with respect to q^{DA}:

$$\frac{\partial Q(\mathbb{E}_\mu[\lambda_R], q^{DA})}{\partial q^{DA}} = \int_{\lambda_{DA}(x) > \mathbb{E}_\mu[\lambda_R]} (\lambda_{DA}(x) - \mathbb{E}_\mu[\lambda_R]) d\mu^{DA}(x) \tag{188}$$

$$> 0 \tag{189}$$

This shows that the optimal strategy in the day-ahead capacity auction is to bid the entire capacity at the price $\mathbb{E}_\mu[\lambda_R]$.

Combining both cases $(\delta z_B + \mathbb{E}_\mu[\lambda_R]) \cdot P^+ < K$ and $(\delta z_B + \mathbb{E}_\mu[\lambda_R]) \cdot P^+ \geq K$ allows us to define the optimal bid in the capacity auction if $C < \mathbb{E}[\lambda_P]$ as

$$p^{DA} = \max\left(\mathbb{E}_\mu[\lambda_R], \frac{K - \delta z_B \cdot P^+}{P^+}\right). \tag{190}$$

(b) If $E[\lambda_P] > C$ then $z_I(q, q^{DA}) = \mathbb{E}[\lambda_P - C] \cdot (P^+ - q - q^{DA})$. Since participating in the balancing energy auction is always more profitable than resorting to reactive balancing, the reasoning developed in point (a) applies and the optimal bidding strategy in the day-ahead market is to bid the maximal capacity at price

$$p^{DA} = \max\left(\mathbb{E}_\mu[\lambda_R], \frac{K - \delta z_B \cdot P^+}{P^+}\right).$$

If the reservation bid is accepted, the optimal action is to bid truthfully in the balancing auction. If the day-ahead bid is $\mathbb{E}_\mu[\lambda_R]$ and is not accepted, the optimal action is to bid the full capacity truthfully in the balancing energy auction. Else, the agent should not activate its plant. □

5 Aggregated Offer Curves in a Cross-Border Setting with Linear Marginal Cost

This section describes the analytical formulation characterizing the market equilibrium resulting from the aggregation of two-zone: B and D. Zone B has a merit order curve equal to $MC_B(x) = a_B x + b$ and apply one of the four designs with an ORDC equal to $ORDC(x) = a_R x$. Zone D has a merit order curve equal to $MC_D(x) = a_D x + b$.

5.1 No Adder and RT Market for Reserve:

The aggregated offer curve is defined as follows:

$$B(x) = B_B(x) \cup B_D(x) = MC_B(x) \cup MC_D(x). \tag{191}$$

This result in

$$\begin{cases} a_D x_D = a_B x_B \\ x_D + x_B = x \end{cases} = \begin{cases} x_B = \frac{a_D x}{a_D + a_B} \\ x_D = \frac{a_B x}{a_D + a_B} \end{cases} \tag{192}$$

and

$$B(x) = MC_B(\frac{x a_D}{a_D + a_B}) = \frac{a_D a_B x}{a_D + a_B} + b. \tag{193}$$

5.2 Adder on BRPs and BSPs:

Under this design, $B_B(x) = MC_B(x) - \lambda_R(x)$ so a_B is replaced by $a_B - a_R$ if $x \geq 0$.

$$B(x) = \begin{cases} \frac{a_D a_B x}{a_D + a_B} + b & \text{if } x \leq 0, \\ \frac{a_D (a_B - a_R) x}{a_D + a_B - a_R} + b & \text{else.} \end{cases} \tag{194}$$

5.3 Adder on BRPs:

Under this design,

$$B_B(x) = \begin{cases} MC_B(x - \alpha) & \text{if } x \leq \alpha, \\ MC_B(x) & \text{else.} \end{cases} \tag{195}$$

Due to the discontinuity in $B_B(x)$, three cases have to be considered: (i) $x \leq \alpha$, $\alpha < x \leq (1 + a_B/a_D)\alpha$, and (iii) $x > (1 + a_B/a_D)\alpha$. The result are presented here are for an arbitrary α.

$$x_B = \begin{cases} \frac{a_D(x-\alpha)}{a_D+a_B} & \text{if } x \leq \alpha \\ 0 & \text{if } \alpha < x \leq (1+a_B/a_D)\alpha \\ \frac{a_D x}{a_B+a_D} - \alpha & \text{if } x > (1+a_B/a_D)\alpha \end{cases} \tag{196}$$

$$x_D = \begin{cases} \frac{a_B(x-\alpha)}{a_D+a_B} & \text{if } x \leq \alpha \\ x-\alpha & \text{if } \alpha < x \leq (1+a_B/a_D)\alpha \\ \frac{a_B x}{a_B+a_D} & \text{if } x > (1+a_B/a_D)\alpha \end{cases} \tag{197}$$

$$B(x) = \begin{cases} \frac{a_B a_D(x-\alpha)}{a_D+a_B} + b & \text{if } x \leq \alpha \\ a_D(x-\alpha)+b & \text{if } \alpha < x \leq (1+a_B/a_D)\alpha \\ \frac{a_B a_D x}{a_B+a_D} + b & \text{if } x > (1+a_B/a_D)\alpha \end{cases} \tag{198}$$

6 Analytical Platform Prices with Congestion

This section revisits the analytical formulation of the market equilibrium resulting from the connection of zones B and D with an interconnector capacity of F MW. As previously, the merit order curves in zone B and D are $MC_B(x) = a_B x + b$ and $MC_D(x) = a_D x + b$. The ORDC in zone B is equal to $ORDC(x) = a_R x$. The demand for balancing energy is denoted as x^{BRP} and the activated balancing energy as x^{BSP}. The platform price in zone B and D is denoted as $\lambda_{P,B}$ and $\lambda_{P,D}$ respectively.

6.1 No Adder and RT Market for Reserve

The offer curve in zone B and D is equal to the merit order curve. If there is no congestion,

$$x_B^{BSP,uncon} = K_D \cdot (x_B^{BRP} + x_D^{BRP}) \tag{199}$$

$$x_D^{BSP,uncon} = K_B \cdot (x_B^{BRP} + x_D^{BRP}). \tag{200}$$

with $K_B = a_B/(a_B + a_D)$ and $K_D = a_D/(a_B + a_D)$. The interconnector is congested when

$$F \leq x_B^{BSP,uncon}(x_B^{BRP}, x_D^{BRP}) - x_B^{BRP} \text{ or } F \leq x_B^{BRP} - x_B^{BSP,uncon}(x_B^{BRP}, x_D^{BRP}). \tag{201}$$

Note that, in a two-zone setting, this is equivalent to

$$F \leq x_D^{BSP,uncon}(x_B^{BRP}, x_D^{BRP}) - x_D^{BRP} \text{ or } F \leq x_D^{BRP} - x_D^{BSP,uncon}(x_B^{BRP}, x_D^{BRP}). \tag{202}$$

These constraints allow us to compute the activated balancing energy in both zones for all cases: no congestion, congestion B to D, and congestion D to

B.

$$x_B^{BSP} = \begin{cases} x_B^{BRP} + F & \text{if } F \leq K_D \cdot (x_B^{BRP} + x_D^{BRP}) - x_B^{BRP} \\ x_B^{BRP} - F & \text{if } F \leq x_B^{BRP} - K_D \cdot (x_B^{BRP} + x_D^{BRP}) \\ K_D \cdot (x_B^{BRP} + x_D^{BRP}) & \text{else} \end{cases} \tag{203}$$

$$x_D^{BSP} = \begin{cases} x_D^{BRP} - F & \text{if } F \leq K_D \cdot (x_B^{BRP} + x_D^{BRP}) - x_B^{BRP} \\ x_D^{BRP} + F & \text{if } F \leq x_B^{BRP} - K_D \cdot (x_B^{BRP} + x_D^{BRP}) \\ K_B \cdot (x_B^{BRP} + x_D^{BRP}) & \text{else.} \end{cases} \tag{204}$$

The platform price in both zones can then be expressed as follows:

$$\lambda_{P,B}(x_B^{BRP}, x_D^{BRP}) = B_B(x_B^{BSP}) = \begin{cases} MC_B(x_B^{BRP} + F) & \text{if } F \leq K_D \cdot (x_B^{BRP} + x_D^{BRP}) - x_B^{BRP} \\ MC_B(x_B^{BRP} - F) & \text{if } F \leq x_B^{BRP} - K_D \cdot (x_B^{BRP} + x_D^{BRP}) \\ MC_B(K_D \cdot (x_B^{BRP} + x_D^{BRP})) & \text{else} \end{cases} \tag{205}$$

$$\lambda_{P,D}(x_B^{BRP}, x_D^{BRP}) = B_D(x_D^{BSP}) = \begin{cases} MC_D(x_D^{BRP} - F) & \text{if } F \leq K_D \cdot (x_B^{BRP} + x_D^{BRP}) - x_B^{BRP} \\ MC_D(x_D^{BRP} + F) & \text{if } F \leq x_B^{BRP} - K_B \cdot (x_B^{BRP} + x_D^{BRP}) \\ MC_D(K_D \cdot (x_B^{BRP} + x_D^{BRP})) & \text{else} \end{cases} \tag{206}$$

This equilibrium is illustrated in figure 1.

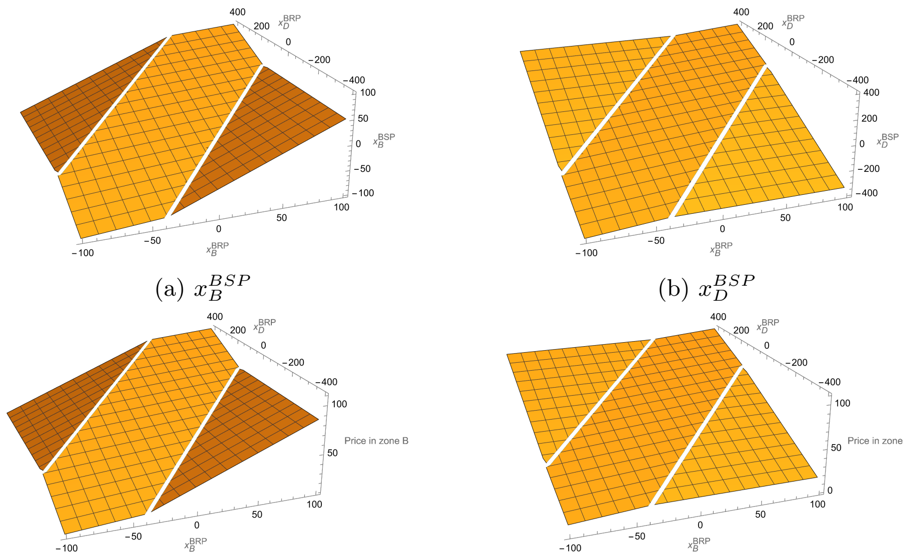
6.2 Adder on BRPs and BSPs

If there is no congestion, the offer curves in zone B and D are respectively equal to the merit order curve in zone B minus the ORDC and the merit order curve in zone D. The activated balancing energy can then be expressed as follows:

$$x_B^{BSP} = \begin{cases} K_D \cdot (x_B^{BRP} + x_D^{BRP}) & \text{if } x_B^{BRP} + x_D^{BRP} \leq 0 \\ K_D^R \cdot (x_B^{BRP} + x_D^{BRP}) & \text{else} \end{cases} \tag{207}$$

$$x_D^{BSP} = \begin{cases} K_B \cdot (x_B^{BRP} + x_D^{BRP}) & \text{if } x_B^{BRP} + x_D^{BRP} \leq 0 \\ K_B^R \cdot (x_B^{BRP} + x_D^{BRP}) & \text{else.} \end{cases} \tag{208}$$

Here, $K_B^R = (a_B - a_R)/(a_D + a_B - a_R)$ and $K_D^R = a_D/(a_B + a_D - a_R)$. From there, the same procedure as for the "no adder" design can be reproduced to generate six cases: no congestion, congestion from D to B and congestion from B to D for either negative or positive demand for balancing energy. The equilibrium is illustrated in figure 2.



(a) x_B^{BSP} (b) x_D^{BSP}

(c) $\lambda_{P,B}$ (d) $\lambda_{P,D}$

Figure 1: Illustration of the activated balancing energy and the price at equilibrium under the "no adder" and "RT market for reserve" designs for $a_B = 1/2$, $a_D = 1/8$, $b = 60$ and $F = 50$

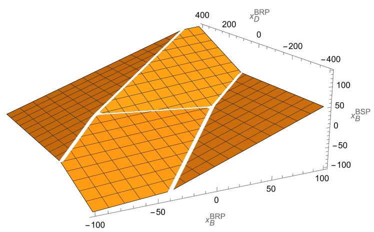
6.3 Adder on BRPs

If there is no congestion, the offer curves in zone B and D can be expressed as follows:

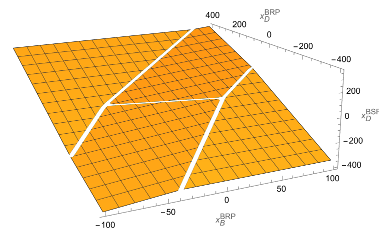
$$x_B^{BSP} = \begin{cases} K_D \cdot (x_B^{BRP} + x_D^{BRP} - \alpha) & \text{if } x_B^{BRP} + x_D^{BRP} \leq \alpha \\ 0 & \text{if } \alpha \leq x_B^{BRP} + x^{BRP_D} \leq (1 + a_B/a_D)\alpha \\ K_D \cdot (x_B^{BRP} + x_D^{BRP}) - \alpha & \text{else} \end{cases} \tag{209}$$

$$x_D^{BSP} = \begin{cases} K_B \cdot (x_B^{BRP} + x_D^{BRP} - \alpha) & \text{if } x_B^{BRP} + x_D^{BRP} \leq \alpha \\ x_B^{BRP} + x_D^{BRP} - \alpha & \text{if } \alpha \leq x_B^{BRP} + x^{BRP_D} \leq (1 + a_B/a_D)\alpha \\ K_B \cdot (x_B^{BRP} + x_D^{BRP}) & \text{else.} \end{cases} \tag{210}$$

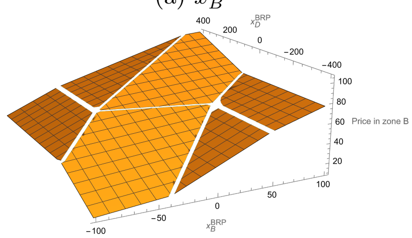
The same procedure as previously can be reproduced to generate nine cases: no congestion, congestion from D to B and congestion from B to D for either a demand for balancing energy lower than α, a demand for balancing energy between α and $(1 + a_B/a_D)\alpha$ and a demand for balancing energy greater than $(1+a_B/a_D)\alpha$. Figure 3 illustrates these results. Note that, for the non-congested case when $\alpha \leq x_B^{BRP} + x_D^{BRP} \leq (1 + a_B/a_D)\alpha$, there is no activated balancing energy from zone B and we need to use the offer curve and the activated



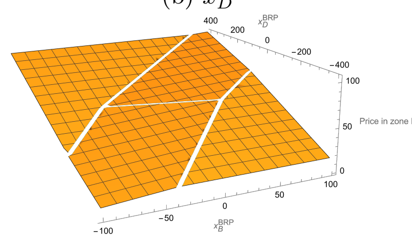
(a) x_B^{BSP}



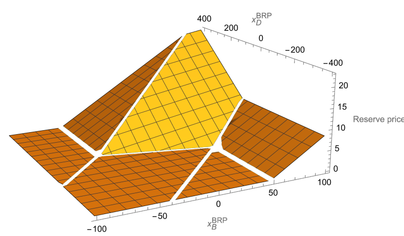
(b) x_D^{BSP}



(c) $\lambda_{P,B}$



(d) $\lambda_{P,D}$



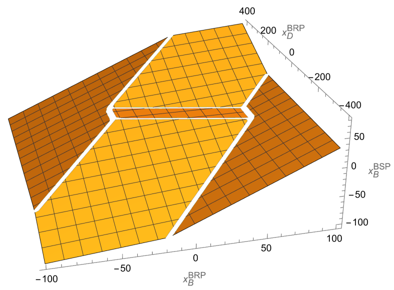
(e) λ_R

Figure 2: Illustration of the activated balancing energy and prices at equilibrium under the "adder on BRPs and BSPs" design for $a_B = 1/2$, $a_D = 1/8$, $a_R = 1/6$, $b = 60$ and $F = 50$

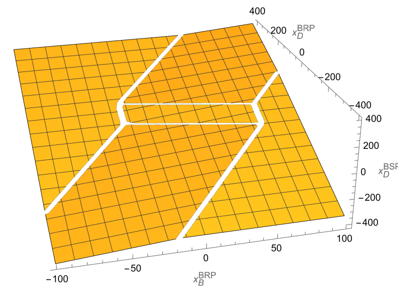
balancing energy in zone D to obtain the platform price.

7 Complete Results

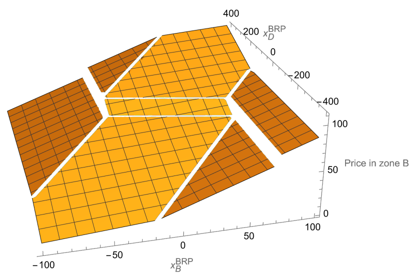
The complete results for the third example are displayed in tables 5 to 8. The complete results for the fourth example are displayed in tables 9 to 12. Branches 1, 2, 3 and 4 correspond to aggregated demand for balancing energy drawn from distributions $\mathcal{U}[0,100]+\mathcal{U}[0,400]$, $\mathcal{U}[0,100]+\mathcal{U}[-400,0]$, $\mathcal{U}[-100,0]+\mathcal{U}[0,400]$, and $\mathcal{U}[-100,0]+\mathcal{U}[-400,0]$.



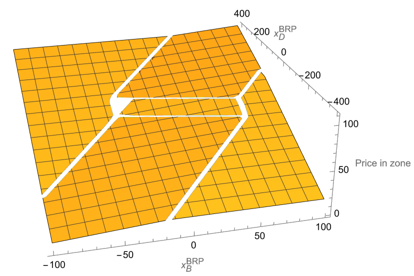
(a) x_B^{BSP}



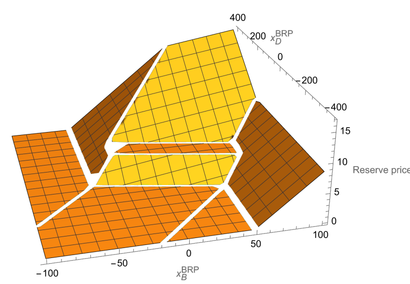
(b) x_D^{BSP}



(c) $\lambda_{P,B}$



(d) $\lambda_{P,D}$



(e) λ_R

Figure 3: Illustration of the activated balancing energy and prices at equilibrium under the "adder on BRPs" design for $a_B = 1/2$, $a_D = 1/8$, $a_R = 1/6$, $\alpha = 20$, $b = 60$ and $F = 50$

Table 5: Example 3 – No adder

		$\mathbb{E}_\mu[\lambda_P]$	$\mathbb{E}_\mu[\lambda_R]$	$\mathbb{E}_\mu[x^{BSP}]$	α	Welfare	Activation cost	Producer payoff	Producer surplus	Consumer surplus	Capacity cost
Branch 1	Zone B	85.00	0.00	50.00	0.00	-3566.69	3766.67	4533.33	766.67	-4333.35	0.00
	Zone D	85.00	0.00	200.00	0.00	-15266.69	15066.66	18133.32	3066.66	-18333.35	0.00
Branch 2	Zone B	45.00	0.00	-30.00	0.00	-1966.69	-1433.34	-1066.67	366.67	-2333.35	0.00
	Zone D	45.00	0.00	-120.00	0.00	9133.31	-5733.34	-4266.68	1466.66	7666.65	0.00
Branch 3	Zone B	75.00	0.00	30.00	0.00	4033.32	2166.67	2533.33	366.67	3666.65	0.00
	Zone D	75.00	0.00	120.00	0.00	-14866.69	8666.66	10133.32	1466.66	-16333.35	0.00
Branch 4	Zone B	35.00	0.00	-50.00	0.00	2433.32	-2233.34	-1466.67	766.67	1666.65	0.00
	Zone D	35.00	0.00	-200.00	0.00	8733.31	-8933.34	-5866.68	3066.66	5666.65	0.00
Full tree	Zone B	60.00	0.00	0.00	0.00	233.32	566.67	1133.33	566.67	-333.35	0.00
	Zone D	60.00	0.00	0.00	0.00	-3066.69	2266.66	4533.32	2266.66	-5333.35	0.00

Table 6: Example 3 – Adder on BRPs and BSPs

		$\mathbb{E}_\mu[\lambda_P]$	$\mathbb{E}_\mu[\lambda_R]$	$\mathbb{E}_\mu[x^{BSP}]$	α	Welfare	Activation cost	Producer payoff	Producer surplus	Consumer surplus	Capacity cost
Branch 1	Zone B	82.73	11.36	68.18	0.00	-3736.94	5516.53	6942.14	1425.62	-4818.20	-344.35
	Zone D	82.73	11.36	181.82	0.00	-15223.16	13443.52	15977.95	2534.43	-17757.59	0.00
Branch 2	Zone B	44.96	0.19	-29.70	0.00	-1964.64	-1413.36	-1044.91	368.46	-2344.71	11.62
	Zone D	44.96	0.19	-120.30	0.00	9130.92	-5752.97	-4287.75	1465.21	7665.70	0.00
Branch 3	Zone B	73.60	7.01	41.21	0.00	3831.26	3152.75	3832.77	680.02	3920.43	-769.20
	Zone D	73.60	7.01	108.79	0.00	-14724.90	7740.84	8954.40	1213.56	-15938.46	0.00
Branch 4	Zone B	35.00	0.00	-50.00	0.00	2433.32	-2233.34	-1466.67	766.67	1666.65	0.00
	Zone D	35.00	0.00	-200.00	0.00	8733.31	-8933.34	-5866.68	3066.66	5666.65	0.00
Full tree	Zone B	59.07	4.64	7.42	0.00	140.75	1255.64	2065.83	810.19	-393.96	-275.48
	Zone D	59.07	4.64	-7.42	0.00	-3020.96	1624.51	3694.48	2069.97	-5090.93	0.00

Table 7: Example 3 – Adder on BRPs

		$\mathbb{E}_\mu[\lambda_P]$	$\mathbb{E}_\mu[\lambda_R]$	$\mathbb{E}_\mu[x^{BSP}]$	α	Welfare	Activation cost	Producer payoff	Producer surplus	Consumer surplus	Capacity cost
Branch 1	Zone B	84.41	9.12	17.75	36.96	-3588.99	4113.92	5172.68	1058.76	-4785.32	137.57
	Zone D	84.41	0.00	195.29	0.00	-15273.33	14748.40	17779.42	3031.01	-18304.35	0.00
Branch 2	Zone B	45.00	0.14	-30.00	0.00	-1966.67	-1433.33	-1066.67	366.67	-2343.75	10.42
	Zone D	45.00	0.00	-120.00	0.00	9133.33	-5733.33	-4266.67	1466.67	7666.67	0.00
Branch 3	Zone B	74.55	5.60	16.11	17.47	3979.03	2403.86	2840.06	436.20	3897.91	-355.07
	Zone D	74.55	0.00	116.42	0.00	-14837.97	8455.08	9924.83	1469.75	-16307.73	0.00
Branch 4	Zone B	35.00	0.00	-50.00	0.00	2433.33	-2233.33	-1466.67	766.67	1666.67	0.00
	Zone D	35.00	0.00	-200.00	0.00	8733.33	-8933.33	-5866.67	3066.67	5666.67	0.00
Full tree	Zone B	59.74	3.71	-11.54	13.61	214.18	712.78	1369.85	657.07	-391.12	-51.77
	Zone D	59.74	0.00	-2.07	0.00	-3061.16	2134.20	4392.73	2258.52	-5319.68	0.00

Table 8: Example 3 – RT market for reserve

		$\mathbb{E}_\mu[\lambda_P]$	$\mathbb{E}_\mu[\lambda_R]$	$\mathbb{E}_\mu[x^{BSP}]$	α	Welfare	Activation cost	Producer payoff	Producer surplus	Consumer surplus	Capacity cost
Branch 1	Zone B	85.00	8.33	50.00	0.00	-3566.69	3766.67	6200.00	2433.33	-4777.80	-1222.22
	Zone D	85.00	8.33	200.00	0.00	-15266.69	15066.66	18133.32	3066.66	-18333.35	0.00
Branch 2	Zone B	45.00	0.14	-30.00	0.00	-1966.69	-1433.34	-1038.90	394.44	-2343.77	-17.36
	Zone D	45.00	0.14	-120.00	0.00	9133.31	-5733.34	-4266.68	1466.66	7666.65	0.00
Branch 3	Zone B	75.00	5.14	30.00	0.00	4033.31	2166.67	3561.11	1394.44	3899.28	-1260.41
	Zone D	75.00	5.14	120.00	0.00	-14866.69	8666.66	10133.32	1466.66	-16333.35	0.00
Branch 4	Zone B	35.00	0.00	-50.00	0.00	2433.32	-2233.34	-1466.67	766.67	1666.65	0.00
	Zone D	35.00	0.00	-200.00	0.00	8733.31	-8933.34	-5866.68	3066.66	5666.65	0.00
Full tree	Zone B	60.00	3.40	0.00	0.00	233.31	566.67	1813.88	1247.22	-388.91	-625.00
	Zone D	60.00	3.40	0.00	0.00	-3066.69	2266.66	4533.32	2266.66	-5333.35	0.00

Table 9: Example 4 – No adder

		$\mathbb{E}_\mu[\lambda_P]$	$\mathbb{E}_\mu[\lambda_R]$	$\mathbb{E}_\mu[x^{BSP}]$	α	Activation cost	Producer payoff	Producer surplus	Consumer surplus	Capacity cost	Congestion rent
Branch 1	Zone B	85.00	0.00	50.00	0.00	3754.80	4509.60	754.80	-4361.90	0.00	43.95
	Zone D	85.00	0.00	200.00	0.00	15085.12	18170.25	3085.12	-18361.90	0.00	
Branch 2	Zone B	61.63	0.00	3.26	0.00	368.04	540.77	172.73	-3442.10	0.00	1039.23
	Zone D	40.84	0.00	-153.26	0.00	-6945.56	-4695.80	2249.76	6557.90	0.00	
Branch 3	Zone B	58.37	0.00	-3.26	0.00	-22.58	150.15	172.73	2557.90	0.00	1039.23
	Zone D	79.16	0.00	153.26	0.00	11445.07	13694.82	2249.76	-17442.10	0.00	
Branch 4	Zone B	35.00	0.00	-50.00	0.00	-2245.20	-1490.40	754.80	1638.10	0.00	43.95
	Zone D	35.00	0.00	-200.00	0.00	-8914.88	-5829.75	3085.12	5638.10	0.00	
Full tree	Zone B	60.00	0.00	0.00	0.00	463.77	927.53	463.77	-902.00	0.00	541.59
	Zone D	60.00	0.00	0.00	0.00	2667.44	5334.88	2667.44	-5902.00	0.00	

Table 10: Example 4 – Adder on BRPs and BSPs

		$\mathbb{E}_\mu[\lambda_P]$	$\mathbb{E}_\mu[\lambda_R]$	$\mathbb{E}_\mu[x^{BSP}]$	α	Activation cost	Producer payoff	Producer surplus	Consumer surplus	Capacity cost	Congestion rent
Branch 1	Zone B	81.36	10.68	64.09	0.00	5091.37	6337.10	1245.73	-4785.53	-235.31	104.98
	Zone D	83.24	0.00	185.91	0.00	13838.71	16523.06	2684.35	-17944.30	0.00	
Branch 2	Zone B	59.54	2.12	3.32	0.00	372.20	545.19	172.98	-3443.57	105.23	935.47
	Zone D	40.83	0.00	-153.32	0.00	-6949.67	-4700.11	2249.55	6557.80	0.00	
Branch 3	Zone B	57.04	1.68	-2.56	0.00	27.76	209.17	181.41	2562.24	-79.93	1101.44
	Zone D	79.07	0.00	152.56	0.00	11396.17	13638.69	2242.52	-17431.61	0.00	
Branch 4	Zone B	35.00	0.00	-50.00	0.00	-2245.20	-1490.40	754.80	1638.10	0.00	43.95
	Zone D	35.00	0.00	-200.00	0.00	-8914.88	-5829.75	3085.12	5638.10	0.00	
Full tree	Zone B	58.24	3.62	3.71	0.00	811.53	1400.26	588.73	-1007.19	-52.50	546.46
	Zone D	59.54	0.00	-3.71	0.00	2342.58	4907.97	2565.39	-5795.00	0.00	

Table 11: Example 4 – Adder on BRPs

		$\mathbb{E}_\mu[\lambda_P]$	$\mathbb{E}_\mu[\lambda_R]$	$\mathbb{E}_\mu[x^{BSP}]$	α	Activation cost	Producer payoff	Producer surplus	Consumer surplus	Capacity cost	Congestion rent
Branch 1	Zone B	84.3	9.1	17.1	37.3	4074.4	5119.8	1045.4	-4811.6	140.5	37.6
	Zone D	84.5	0.0	195.7	0.0	14793.7	17847.4	3053.7	-18333.7	0.0	
Branch 2	Zone B	61.6	2.1	3.3	0.0	368.0	540.8	172.7	-3616.6	174.5	1039.2
	Zone D	40.8	0.0	-153.3	0.0	-6945.6	-4695.8	2249.8	6557.9	0.0	
Branch 3	Zone B	58.4	1.6	-3.3	0.0	-22.6	150.1	172.7	2585.0	-27.1	1039.2
	Zone D	79.2	0.0	153.3	0.0	11445.1	13694.8	2249.8	-17442.1	0.0	
Branch 4	Zone B	35.0	0.0	-50.0	0.0	-2245.2	-1490.4	754.8	1638.1	0.0	43.9
	Zone D	35.0	0.0	-200.0	0.0	-8914.9	-5829.8	3085.1	5638.1	0.0	
Full tree	Zone B	59.8	3.2	-8.2	9.3	543.7	1080.1	536.4	-1051.3	72.0	540.0
	Zone D	59.9	0.0	-1.1	0.0	2594.6	5254.2	2659.6	-5895.0	0.0	

Table 12: Example 4 – RT market for reserve

		$\mathbb{E}_\mu[\lambda_P]$	$\mathbb{E}_\mu[\lambda_R]$	$\mathbb{E}_\mu[x^{BSP}]$	α	Activation cost	Producer payoff	Producer surplus	Consumer surplus	Capacity cost	Congestion rent
Branch 1	Zone B	85.0	8.3	50.0	0.0	3754.8	6176.3	2421.5	-4815.9	-1212.7	43.9
	Zone D	85.0	0.0	200.0	0.0	15085.1	18170.2	3085.1	-18361.9	0.0	
Branch 2	Zone B	61.6	2.1	3.3	0.0	368.0	961.8	593.7	-3616.6	-246.5	1039.2
	Zone D	40.8	0.0	-153.3	0.0	-6945.6	-4695.8	2249.8	6557.9	0.0	
Branch 3	Zone B	58.4	1.6	-3.3	0.0	-22.6	462.6	485.2	2585.0	-339.6	1039.2
	Zone D	79.2	0.0	153.3	0.0	11445.1	13694.8	2249.8	-17442.1	0.0	
Branch 4	Zone B	35.0	0.0	-50.0	0.0	-2245.2	-1490.4	754.8	1638.1	0.0	43.9
	Zone D	35.0	0.0	-200.0	0.0	-8914.9	-5829.8	3085.1	5638.1	0.0	
Full tree	Zone B	60.0	3.0	0.0	0.0	463.8	1527.6	1063.8	-1052.3	-449.7	541.6
	Zone D	60.0	0.0	0.0	0.0	2667.4	5334.9	2667.4	-5902.0	0.0	

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