A Comparison of Alternative Algorithms for Resolving Stochastic Expansion Planning Problems with Endogenous Adequacy Targets

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***Abstract*—The scope of this work is to test alternative algorithmic schemes that could tackle a large stochastic capacity expansion problem with specific constraints on expected energy not served. The challenge lies in decoupling scenario-specific variables, originally bundled by adequacy constraints, so that scenario-specific problems can be solved in parallel. The examined schemes include Benders' decomposition, the projected subgradient method, the level method and Dantzig-Wolfe decomposition. Numerical experiments are conducted on a small-scale problem of four countries and two scenarios, for which we are able to calculate the extended form solution and thus benchmark the effectiveness of the proposed decomposition algorithms.**

***Index Terms*—Capacity expansion planning, resource adequacy, stochastic optimization, parallel algorithms**

I. Introduction

Transmission System Operators (TSOs), as well as the European Network of Transmission System Operators for Electricity (ENTSOe), conduct resource adequacy studies on a periodic basis [1]–[3]. These studies use various methodologies that differ in the scope of system considerations and assumptions. However, they have a similar overarching structure; they start with a capacity expansion plan that is considered realistic for the future based on certain assumptions, and they evaluate its ability to meet certain adequacy criteria, more often in terms of loss of load expectation (LOLE), but also in terms of expected energy not served (EENS). Based on this evaluation, they identify areas of "adequacy risk" and formulate assessments on the necessity of capacity mechanisms.

The natural follow-up question is what is a cost-optimal expansion plan that meets the adequacy criteria. The aforementioned studies typically rely on heuristics to tackle this question. Another approach is to solve for a cost-optimal expansion plan with endogenous adequacy criteria, but this is computationally challenging if we consider a pan-European scale, and if we want to account systematically for uncertainty.

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Trying to address the problem in such a dimension is justifiable. Power markets in Europe are integrated. Moreover, the vast penetration of generation from renewable energy sources (RES) dictates that uncertainty be systematically accounted for. The European Resource Adequacy Assessment (ERAA) [3] is an example of this problem in practice. With 59 zones and 35 climatic scenarios, the respective economic dispatch problem is estimated to have approximately 181 million variables and 225 million constraints.

An overview of different approaches to solve a stochastic capacity expansion model is provided in [4]. When the size of the problem is manageable, a number of authors tackle the extended formulation using a commercial solver. To solve problems of higher dimensionality, two fronts have been explored; one front has focused on developing scenario selection techniques to reduce the size of the uncertainty space, with the hope of decreasing the size of the problem so that its extended form becomes manageable. The other front, which aligns with the scope of this paper to include a large number of scenarios, is employing decomposition algorithms, which split the problem into scenario-specific sub-problems.

However, the introduction of adequacy constraints renders this scenario decomposition challenging. Adequacy constraints are expressed as an expectation of load shedding across scenarios of uncertainty, and therefore couple all scenarios together. In the literature, the most common practice to address stochastic capacity expansion problems with adequacy constraints is to use methods based on Benders' decomposition [5]–[7]. Nevertheless, it is well known that Benders' decomposition, can encounter scalability issues. For a discussion on the drawbacks of cutting plane methods (which include Benders' decomposition) we refer the reader to [8], as well as [9]. For a specific case where Benders' decomposition encounters scalability issues when applied to a pan-European problem, we refer the reader to [4].

To the knowledge of the authors, the examination of decomposition methods other than Benders', specifically to address a stochastic capacity expansion problem with adequacy constraints, is thinly addressed in the literature. In order to counter scalability challenges, we therefore focus on comparing Benders' decomposition to alternatives based on dual decomposition, in particular the subgradient method and the level method [10], [11]. We also consider two cases based on the Dantzig-Wolfe decomposition [12]. While the ultimate objective is to work on a large-scale setting, we devote this paper to examining the alternative schemes on a small test case, for which we are able to solve the problem directly in its extended form and obtain a benchmark for their results.

The structure of the paper is as follows: in section II we present the mathematical formulation of a stochastic capacity expansion model with adequacy constraints. In section III we discuss three different decomposition schemes to tackle the model. In section IV we present the results from the implementation of the examined schemes on a test system, discuss their relative performance. Finally, we conclude with a summary of the main findings of this work.

II. Stochastic Capacity Expansion with Adequacy Constraints

In this section, we provide stylized formulations of stochastic capacity expansion planning problems (SCEP) with adequacy constraints, that are sufficient for demonstrating their key features and the decomposition methods that we develop in the subsequent sections. We begin with the SCEP formulation without adequacy constraints. We then introduce adequacy constraints, considering two cases, one with EENS limits and one with LOLE limits, and discuss their relative features. We use the following notation:

- Sets
  Ω : uncertainty realizations (climatic conditions that affect demand and RES generation)
  G: generators
  T: time horizon (we consider an annual time horizon with hourly time steps)
- Parameters
  IC_g: annualized investment cost (€/MWh)
  MC_g: variable and fuel cost (€/MWh)
  X_g: upper bound on investment (MW)
  $VOLL$: Value of lost load (€/MWh)
  $LOLE$: upper limit on LOLE (h)
  $EENS$: upper limit on EENS (MWh)
  $D_{t,\omega}$: demand (MW)
  P_ω: probability of scenario realization ω
- Variables
  x_g: invested capacity (MW)
  $p_{g,t,\omega}$: generation by capacities (MWh)
  $ls_{t,\omega}$: load shedding (MWh)
  $u_{t,\omega}$: occurrence of load shedding (binary)

A. Formulation without adequacy constraints

Let us begin by introducing the stochastic capacity expansion problem, ignoring the adequacy constraints:

$$\min_{x,p,ls\geq 0} \quad \sum_{g\in G} I_g \cdot x_g + \tag{1}$$

$$+\,\mathbb{E}_{\omega\in\Omega}\left[\sum_{g\in G}\sum_{t\in T} MC_g \cdot p_{t,g,\omega}\right] \tag{2}$$

$$+\,VOLL\cdot\mathbb{E}_{\omega\in\Omega}\left[\sum_{t\in T} ls_{t,\omega}\right] \tag{3}$$

$$\text{s.t.}\quad x_g \leq X_g \quad \forall g\in G \tag{4}$$

$$p_{g,t,\omega} \leq x_g \quad \forall g\in G, t\in T, \omega\in\Omega \tag{5}$$

$$D_{t,\omega} - \sum_{g\in G} p_{t,g,\omega} - ls_{t,\omega} = 0 \quad \forall t\in T, \omega\in\Omega \tag{6}$$

(SCEP)

The objective function is the minimization of the sum of investment cost (1), the expected cost of power generation (2) and the expected cost of load shedding (3). Constraint (4) imposes upper bounds on investment decisions. Constraint (5) limits power generation to the level of installed capacity. Constraint (6) requires the balance between supply and demand, for every scenario and time period. While there is no explicit adequacy constraint, load shedding is limited by the respective cost term (3) in the objective function.

This problem can be effectively solved using decomposition methods. To realize why this is possible, the reader may notice that, the scenario-dependent variables ($p_{g,t,\omega}$ and $ls_{t,\omega}$) and expressions ((2), (3), (5), (6)) are not bundled explicitly to each other.

B. Formulation with EENS constraints

Let us now rewrite the SCEP problem introducing EENS constraints:

$$\begin{aligned}\min_{x,p,ls\geq 0} \quad & (1)+(2)\\ \text{s.t.}\quad & (4),(5),(6),\\ & \mathbb{E}_{\omega\in\Omega}\left[\sum_{t\in T} ls_{t,\omega}\right] \leq EENS \qquad (7)\end{aligned}$$

(SCEP-EENS)

With the inclusion of EENS constraint (7) the cost of load shedding in the objective function (3) is no longer necessary, as the two can serve the same purpose of limiting the amount of load shedding. This intuition can be formalized with a strong duality argument.

It becomes obvious from (7) that the EENS constraint introduces an explicit bundling of all scenarios to the SCEP-EENS problem. Therefore, the decomposition by scenario that was possible for the SCEP problem is not a direct option. This bundling is the main difficulty in tackling the SCEP-EENS problem. The decomposition schemes presented in section III are designed to address this issue.

C. Formulation with LOLE constraints

In order to formulate the SCEP problem with LOLE constraints, we introduce the binary variables $u_{t,\omega}$ that reflect the occurrence (or not) of load shedding at a specific time segment. The model formulation is given by:

$$
\begin{aligned}
\min_{x,p,ls\geq 0,u\in\{0,1\}} \quad & (1)+(2)+(3) \\
\text{s.t.} \quad & (4),(5),(6), \\
& ls_{t,\omega} \leq u_{t,\omega}\cdot D_{t,\omega} \quad \forall t\in T,\omega\in\Omega \qquad (8) \\
& \mathbb{E}_{\omega\in\Omega}\left[\sum_{t\in T} u_{t,\omega}\right] \leq LOLE \qquad (9)
\end{aligned}
$$

(SCEP-LOLE)

Constraint (8) ensures that load shedding can take a non-zero value ($ls_{t,\omega} > 0$) only if we decide to shed load ($u_{t,\omega}$ = 1) in the same period and same scenario. Constraint (9) is the adequacy constraint and ensures that the expected total number of periods that load is shed is lower than the LOLE limit. The reader may notice that we have reintroduced the cost of load shedding in the objective function. This is necessary, because otherwise, load shedding in a period for which $u_{t,\omega} > 0$ could be arbitrarily high. In fact, the model can shed load entirely ($ls_{t,\omega} = D_{t,\omega}$) with no impact on the LOLE constraint.

The SCEP-LOLE model is far more complex computationally than the SCEP-EENS model, because the inclusion of the binary variables u renders the problem non-convex. The schemes presented in this paper focus on the challenge of decomposing the problem by scenario, where the resulting sub-problems would each be a pan-European capacity expansion problem with hourly resolution, thus a very large model. Solving such large sub-problems is possible with a strong computational system, if they are linear; on the other hand, if they include binary variables, they become intractable even for smaller cases (examples can be found in [13]), unless additional temporal or spatial decomposition is implemented. We thus focus the remainder of the paper on tackling the SCEP-EENS problem.

III. Decomposition Schemes

In this section we present a suite of decomposition schemes for tackling the SCEP-EENS problem. We begin with a formulation based on Benders' decomposition, which is a commonly used technique in the literature on similar problems. We then present two methods that both aim at solving the Lagrangian dual problem of SCEP-EENS, namely the subgradient method and the level method. Finally, we present two formulations based on Dantzig-Wolfe decomposition. The challenge in all cases is to break the scenario bundling of the EENS constraints so that the second-stage problems can be solved in parallel.

A. Benders' decomposition

The Benders' decomposition of the SCEP-EENS problem that we implement in this paper is based on the work of [13]. We introduce budget variables to the SCEP-EENS problem, $q_\omega \in \mathbb{R}_{\geq 0}, \forall\omega\in\Omega$, which effectively measure the aggregate load shedding in a given scenario over the time horizon. This allows us to replace the EENS constraints (7) with the following:

$$\sum_{t\in T} ls_{t,\omega} \leq q_\omega \quad \forall\omega\in\Omega \qquad (10)$$

$$\mathbb{E}_{\omega\in\Omega}\left[q_\omega\right] = EENS \qquad (11)$$

A proof of equivalence between (7) and (10, 11) is provided in [13] as Theorem 1.

What we have achieved with this formulation is for all ls variables to appear in constraints that are scenario-specific. We have, therefore, unbundled second-stage decisions and can decompose by scenario. This comes at the cost of introducing new variables q, which are also scenario-specific. They, nonetheless, can become meaningful first-stage decisions, and allow us to maintain a structure on which we can apply Benders' decomposition.

In particular, in the implementation of Benders' decomposition, constraint (11) becomes part of the master problem, therefore the variables q will be optimized along with x. Constraints (10) become part of the operation sub-problems (slaves) which are scenario-specific and unbundled, and can therefore be solved in parallel. The formulations of the master problem and the slave problems can be found in the Appendix.

B. Lagrange relaxation schemes

The subgradient and level method that we implement rely on dual decomposition [10], [11] and focus on solving the dual of the SCEP-EENS problem:

$$\max_{\lambda\geq 0} g(\lambda) \qquad (12)$$

where λ is the dual value of the EENS constraint (7) and $g(\lambda)$ is the Lagrange dual function. The reader is referred to [14], section III-A for the complete definition of the Lagrange dual function for the SCEP-EENS problem.

Both methods require computing $g(\lambda)$ repeatedly until they arrive at a λ^* that maximizes $g(\lambda)$. In practice, these methods involve two procedures; the first procedure is to update the values of λ at each iteration, and the second procedure is to calculate $g(\lambda)$ for the given λ. The two methods differ in regard to determining the λ of each iteration (as discussed in the following paragraphs), but employ the same method for calculating $g(\lambda)$.

It should be noted that, for a given λ, $g(\lambda)$ is an optimization problem equivalent to the SCEP problem, with the difference that in (3) we price load shedding at λ instead of $VOLL$. As such, it is a problem that can be tackled with a decomposition scheme. The selection of the decomposition scheme to use for $g(\lambda)$ should be based on its potential to be implemented on pan-European instances, otherwise it will not be aligned with the purpose of this paper. With this consideration in mind, we utilize the approach that is developed in [4], which has proven to be superior to other methods in solving the SCEP problem for a pan-European setting. A representation of this algorithm is provided in [14], section 3.3.

We proceed with the presentation of the subgradient and the level method.

1) The subgradient method: A detailed exposition of a subgradient method for solving the SCEP-EENS problem is presented in [14]. In particular, the reader is referred to Algorithm 1 of section 3.1.

2) The level method: The implementation of the level method is based on chapter 3.3.3 of [10]. The reader is further referred to [9] for uses of the level method in power system optimization problems.

According to the level method, the Lagrange dual function $g(\lambda)$ is outer approximated by a function $\hat{g}(\lambda) \geq g(\lambda)$:

$$\hat{g}(\lambda) = \max_{\theta, \lambda \geq 0} \theta \tag{13}$$

$$\text{s.t.} \quad \theta \leq g(\lambda^k) + \left(\rho^k\right)^T (\lambda - \lambda^k), \quad k = 1, ..., K \tag{14}$$

The upper bound $\hat{g}(\lambda)$ is defined by a set of cutting planes (14) that are being built by computing $g(\lambda)$ iteratively, for various values of $\lambda \in \left\{\lambda^k\right\}_{k=1}^K$. In particular, for any given λ^k, the cutting plane is built from the value of $g(\lambda^k)$ itself, as well as the value of the subgradient of g at λ, which is denoted as ρ^k. As we obtain iteratively more solutions and thus more cuts, $\hat{g}(\lambda)$ approaches g(λ).

It remains to explain how we determine the λ values for every iteration. If we are at iteration k, then λ^{k+1} is derived by solving the following problem:

$$\min_{\lambda \geq 0} \quad \|\lambda - \lambda^k\|_2^2 \tag{15}$$

$$\text{s.t.} \quad g(\lambda^i) + (\rho^i)^T (\lambda - \lambda^i) \geq L^k, \quad i = 1, ..., k \tag{16}$$

The L^k in (16) is calculated at every iteration using the maximum $g(\lambda^k)$ and the minimum θ obtained so far (denoted as g_{best}^k and θ_{best}^k respectively), weighted by a parameter $\beta \in (0, 1)$, which we will refer to as the level parameter:

$$L^k = \beta \cdot g_{best}^k + (1 - \beta) \cdot \theta_{best}^k \tag{17}$$

C. Dantzig-Wolfe decomposition

The implementation of the Dantzig-Wolfe decomposition algorithm is primarily based on the textbook of [15], chapter 6.4. For implementing the Dantzig-Wolfe decomposition, we isolate the complicating constraint (7) and we represent the polyhedron defined by the remaining constraints as a convex combination of its extreme points. In particular, if we define the polyhedron $\mathcal{X} = \{(x, p, ls) \in \mathbb{R}^n : (4), (5), (6)\}$, and the set of extreme points of $\mathcal{X}$, $\mathcal{J}_{\mathcal{X}} = \left\{(\hat{x}, \hat{p}, \hat{ls})^j, j \in \mathcal{I}_{\mathcal{J}}\right\}$, then the SCEP-EENS problem is equivalent to:

$$\begin{aligned}
\min_{z \geq 0} \quad & \sum_{j \in \mathcal{I}_{\mathcal{J}}} z_j \cdot \sum_{g \in G} IC_g \cdot \hat{x}_g^j \\
& + \sum_{j \in \mathcal{I}_{\mathcal{J}}} z_j \cdot \mathbb{E}_{\omega \in \Omega} \left[\sum_{g \in G} \sum_{t \in T} MC_g \cdot \hat{p}_{t,g,\omega}^j \right] \\
\text{s.t.} \quad & \sum_{j \in \mathcal{I}_{\mathcal{J}}} z_j \cdot \mathbb{E}_{\omega \in \Omega} \left[\sum_{t \in T} \hat{ls}_{t,\omega}^j \right] \leq EENS \\
& \sum_{j \in \mathcal{I}_{\mathcal{J}}} z_j = 1
\end{aligned} \tag{SCEP-EENS-DW}$$

Since enumerating the full set of extreme points $\mathcal{J}_{\mathcal{X}}$ is impossible, the Dantzig-Wolfe decomposition considers the SCEP-EENS-DW problem for only a subset $\bar{\mathcal{I}_{\mathcal{J}}} \subseteq \mathcal{I}_{\mathcal{J}}$, namely, the Dantzig-Wolfe "restricted master problem", and then strategically adds new elements to $\bar{\mathcal{I}_{\mathcal{J}}}$ in an iterative manner, employing the methodology of delayed column generation (chapter 6.1 of [15]), until the optimal solution is reached.

The reader may notice that in SCEP-EENS-DW, variables z are not indexed by scenario, which is owing to the fact that the polyhedron $\mathcal{X}$ includes the first-stage constraints (4). Column generation will thus involve a single optimization sub-problem. This sub-problem is equivalent to the Lagrange dual function (see also [16]), which, as already argued in section III-B, can be solved effectively through decomposition. However, solving a single sub-problem means that we only obtain a single column at every iteration. Such an application of the Dantzig-Wolfe decomposition is a trivial option (see [15] in discussion of the "Applicability of the method"), as the method is generally intended to involve multiple sub-problems, that produce multiple columns per iteration. Relevant implementations in power system optimization problems can be found in [9], [17] and [18].

To attempt a reformulation of our problem that involves multiple sub-problems, we inspire ourselves from the work of [18]. In particular, we introduce scenario-specific variables $xw_{g,\omega}, \forall g \in G, \omega \in \Omega$ to the SCEP-EENS problem and we associate every $xw_{g,\omega}$ to the corresponding x_g though non-anticipativity constraints [19]. With this "split-variable" reformulation [18], we can maintain constraints (4) and variables x_g in the restricted master problem and introduce $xw_{g,\omega}$ to the column generation sub-problems, which can now be solved in parallel. We will refer to this alternative formulation as the Dantzig-Wolfe-NAC, where "NAC" stands for non-anticipativity constraints.

The detailed problem formulations of the Dantzig-Wolfe decomposition schemes can be found in the Appendix.

IV. Case Study

A. Description

The algorithms presented in section III are tested on a capacity expansion model that is similar to the model of the European Resource Adequacy Assessment (ERAA) analysis of ENTSO-E in 2021 [20]. The reader may find the detailed description of the test model in section II of [14]. Input data for the model have been obtained from the ERAA website [21]. We focus on an instance that includes 4 countries, namely Germany, France, the Netherlands and Belgium, and 2 climatic scenarios out of the 35 that are included in the ERAA 2021 analysis (those corresponding to climate years 1982 and 1983). We refer to this case as the "main test case".

The EENS limits that we impose correspond to figures reported in Table 8 of the ERAA 2021 report [22]. Additional cases of EENS limits have also been tested, but their results are not presented here, as the key observations regarding the relative performance of the algorithms remain consistent across cases.

The algorithms are implemented in Julia (version 1.11.1). Optimization is performed with Gurobi (version 11.0.3), using the optimization algorithm that demonstrated the best performance in preliminary test runs, namely primal simplex for the Benders' decomposition sub-problems, and dual simplex for the sub-problems of the Lagrange relaxation and Dantzig-Wolfe schemes. The parametrization of the algorithms includes the following:

- The level parameter β of the level method is set to 0.7, as the algorithm demonstrated the best performance at this value in preliminary runs.
- The initial value of λ for the subgradient and the level method is set to 50 EUR/MWh for all countries. The relative performance of the algorithms remained similar with alternative initialization points.
- The Dantzig-Wolfe restricted master problems were initialized with a feasible solution to the SCEP-EENS problem, obtained by fixing investments at their upper limit and load shedding at zero.

B. Results

1) Evolution of the algorithms: In Fig. 1 the reader can observe the evolution of the relative gap between the upper and the lower bound of each algorithm with time. Some key performance indicators are provided in Table II in the Appendix.

The Benders' and the level methods both demonstrate fast convergence. They both require a relatively small number of iterations to reach low gaps, and only a few seconds are required to complete each iteration in this small-scale instance.

The subgradient method proceeds at a slower, but mainly less steep pace (Fig. 1) relative to the other methods. The evolution of the algorithm gap plateaus after the 30th iteration ($\sim$500 seconds), improving only marginally in the remaining iterations, and not going below $\sim$0.15%.

The Dantzig-Wolfe decomposition algorithm required the most time to arrive to low gaps, as it involved the slowest iterations. This is a non-intuitive observation; the computationally heaviest process of the Dantzig-Wolfe, as well as the level and the subgradient methods, is the solution of the Lagrangian dual function ($g(\lambda)$ in (12)), and in all cases we are utilizing the same methodology. However, we have observed that the time required to solve the Lagrangian dual function varies with different values of λ, and every method adopts different strategies for selecting λ in each iteration.

The Dantzig-Wolfe decomposition algorithm with the non-anticipativity constraints (Dantzig-Wolfe-NAC) evolved slower than the other algorithms, and it will thus not be discussed further. The reader may find a depiction of its evolution in the Appendix.

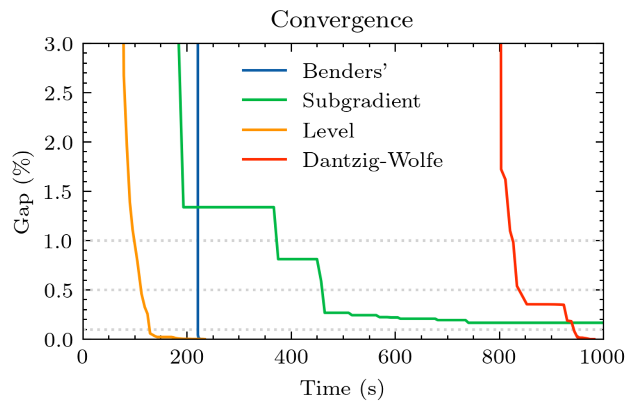

Fig. 1. Convergence of the examined methods, excluding the Dantzig-Wolfe-NAC method.

2) Quality of the solution: The deviation of the Benders' final solution to the extended form solution is practically zero, for both costs and retirement of capacities. The other cases also approach very closely the extended form solution (respective results are included in the Appendix). Some deviations are observed in the distribution of retirements by country. If we consider the aggregate results on retirements for all countries, the total deviation is of the order of 1-3%. Nevertheless, these deviations have a minor effect on total cost, with an impact of less than 0.1%.

V. Conclusion

This paper investigates a variety of algorithmic schemes to decompose a stochastic capacity expansion problem with endogenous adequacy targets. These schemes could be rigorous alternatives to the heuristics that are being adopted in real-world applications when the size of the problem does not allow to optimize it without decomposition. The proposed schemes are based on well-established methods, nevertheless their implementation to the particular problem is non-trivial.

In a small-scale setup, the proposed schemes that are based on Benders' decomposition and the level method are both able to solve the problem at a high speed. The proposed subgradient method has the slowest convergence, and it can not surpass a 0.1% optimality gap at a reasonable amount of time in the examined case study; nevertheless, the best solution obtained approximates very closely the extended form solution. The Dantzig-Wolfe decomposition also demonstrates relatively slow evolution. An alternative formulation of the Dantzig-Wolfe scheme using non-anticipativity constraints did not improve convergence time.

Immediate future work will involve the examination of the scalability of the analyzed methods, and their potential to be used in pan-European applications.

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Appendix

A1. Problem formulations

TABLE I
MATHEMATICAL NOTATION USED IN PROBLEM FORMULATIONS

Notation	**Meaning**
Hat over variable symbol ($\hat{\alpha}$)	Input from optimization at a previous step
V_ω	Set of iteration indices where the Benders' slave problem for scenario ω is feasible
R_ω	Set of iteration indices where the Benders' slave problem for scenario ω is infeasible
ϕ, σ, τ	Extreme ray components corresponding to the first, second, and last constraint respectively of the Benders' slave problem for scenario ω, in case it is infeasible

Benders' master problem:

$$\begin{aligned}
\min_{\theta,x,q\geq 0} \quad & \sum_{g\in G} I_g \cdot x_g + \mathbb{E}_{\omega\in\Omega}\left[\theta_\omega\right] \\
\text{s.t.} \quad & \theta_\omega \geq \sum_{t\in T} \hat{\xi}^i_{t,\omega} \cdot D_{t,\omega} + \sum_{g\in G}\sum_{t\in T} \hat{\mu}^i_{t,g,\omega} \cdot x_g + \\
& \quad + \hat{\nu}^i_\omega \cdot q_\omega, \quad \forall i \in V_\omega, \omega \in \Omega \\
& 0 \geq \sum_{t\in T} \hat{\phi}^i_{t,\omega} \cdot D_{t,\omega} + \sum_{g\in G}\sum_{t\in T} \hat{\sigma}^i_{t,g,\omega} \cdot x_g + \\
& \quad + \hat{\tau}^i_\omega \cdot q_\omega, \quad \forall i \in R_\omega, \omega \in \Omega \\
& x_g \leq X_g, \quad \forall g \in G \\
& \mathbb{E}_{\omega\in\Omega}\left[q_\omega\right] = EENS
\end{aligned}$$

Benders' slave problems, $\forall\omega \in \Omega$:

$$\begin{aligned}
\min_{p,ls\geq 0} \quad & \sum_{g\in G}\sum_{t\in T} MC_g \cdot p_{t,g,\omega} \\
\text{s.t.} \quad & \sum_{g\in G} p_{t,g,\omega} + ls_{t,\omega} = D_{t,\omega}, \quad \forall t \in T && [\xi_{t,\omega}] \\
& p_{t,g,\omega} \leq \hat{x}_g, \quad \forall g \in G, t \in T && [\mu_{t,g,\omega}] \\
& \sum_{t\in T} ls_{t,\omega} \leq \hat{q}_\omega && [\nu_\omega]
\end{aligned}$$

Dantzig-Wolfe restricted master problem:

$$\begin{aligned}
\min_{z\geq 0} \quad & \sum_{j\in\bar{\mathcal{I}}_{\mathcal{J}}} z_j \cdot \sum_{g\in G} IC_g \cdot \hat{x}^j_g + \\
& + \sum_{j\in\bar{\mathcal{I}}_{\mathcal{J}}} z_j \cdot \mathbb{E}_{\omega\in\Omega}\left[\sum_{g\in G}\sum_{t\in T} MC_g \cdot \hat{p}^j_{t,g,\omega}\right] \\
\text{s.t.} \quad & \sum_{j\in\bar{\mathcal{I}}_{\mathcal{J}}} z_j \cdot \mathbb{E}_{\omega\in\Omega}\left[\sum_{t\in T} \hat{ls}^j_{t,\omega}\right] \leq EENS && [\lambda]
\end{aligned}$$

$$\sum_{j \in \bar{\mathcal{I}}_{\mathcal{J}}} z_j = 1 \qquad [\pi]$$

Dantzig-Wolfe pricing problem:

$$\begin{aligned}
\min_{x,p,ls \geq 0} \quad & \sum_{g \in G} IC_g \cdot x_g + \mathbb{E}_\omega \left[\sum_{t \in T} \sum_{g \in G} MC_g \cdot p_{t,g,\omega} \right] - \\
& - \hat{\lambda} \cdot \mathbb{E}_\omega \left[\sum_{t \in T} ls_{t,\omega} \right] - \hat{\pi} \\
\text{s.t.} \quad & \sum_{g \in G} p_{t,g,\omega} + ls_{t,\omega} = D_{t,\omega}, \quad \forall t \in T, \omega \in \Omega \\
& p_{t,g,\omega} \leq x_g, \quad \forall t \in T, g \in G, \omega \in \Omega \\
& x_g \leq X_g, \quad \forall g \in G
\end{aligned}$$

Dantzig-Wolfe-NAC restricted master problem:

$$\begin{aligned}
\min_{x,z \geq 0} & \sum_{g \in G} IC_g \cdot x_g + \\
& + \sum_{\omega \in \Omega} \sum_{j \in \bar{\mathcal{I}}_{\mathcal{J}\omega}} z_\omega^j \cdot P_\omega \cdot \sum_{t \in T} \sum_{g \in G} MC_g \cdot \hat{p}_{t,g,\omega}^j \\
\text{s.t.} & \sum_{j \in \bar{\mathcal{I}}_{\mathcal{J}\omega}} z_\omega^j \cdot \hat{xw}_{g,\omega}^j - x_g = 0, \quad \forall g \in G, \omega \in \Omega \qquad [\kappa_{g,\omega}] \\
& \sum_{\omega \in \Omega} \sum_{j \in \bar{\mathcal{I}}_{\mathcal{J}\omega}} z_\omega^j \cdot P_\omega \cdot \sum_{t \in T} \hat{ls}_{t,\omega}^j \leq EENS \qquad [\lambda] \\
& \sum_{j \in \bar{\mathcal{I}}_{\mathcal{J}\omega}} z_\omega^j = 1, \quad \forall \omega \in \Omega \qquad [\pi_\omega] \\
& x_g \leq X_g, \quad \forall g \in G
\end{aligned}$$

Dantzig-Wolfe-NAC pricing problem, $\forall \omega \in \Omega$:

$$\begin{aligned}
\min_{xw,p,ls \geq 0} & P_\omega \cdot \sum_{t \in T} \sum_{g \in G} MC_g \cdot p_{t,g,\omega} - \sum_{g \in G} \hat{\kappa}_{g,\omega} \cdot xw_{g,\omega} - \\
& - \hat{\lambda} \cdot P_\omega \cdot \sum_{t \in T} ls_{t,\omega} - \hat{\pi_\omega} \\
\text{s.t.} \quad & \sum_{g \in G} p_{t,g,\omega} + ls_{t,\omega} = D_{t,\omega}, \quad \forall t \in T \\
& p_{t,g,\omega} \leq xw_{g,\omega}, \quad \forall t \in T, g \in G \\
& xw_{g,\omega} \leq X_g, \quad \forall g \in G
\end{aligned}$$

A2. Additional Graphs and Tables

TABLE II
PERFORMANCE INDICATORS

Indicators	Algorithms			
	Benders'	***Subgr.***	***Level***	***D-W***
Time for 1% gap[a] (s)	221	375	105	716
Time for 0.01% gap[a] (s)	223	>1000	182	891
Average time per iteration (s)	5	10	6	20
Iterations for 1% gap[a]	37	22	9	22
Iterations for 0.01% gap[a]	39	>100	24	42

a. Percentage difference of the lower bound relative to the upper bound of each algorithm.

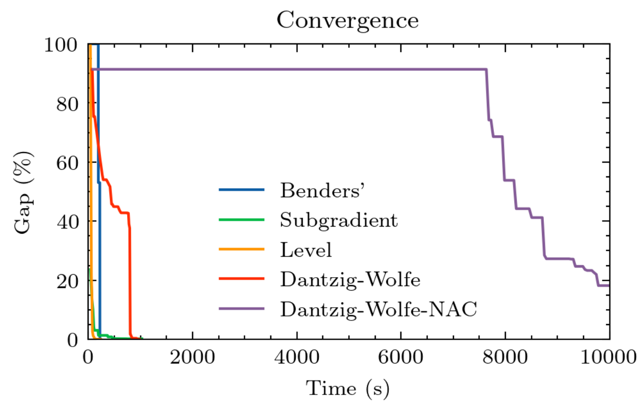

Fig. 2. Convergence of the examined methods.

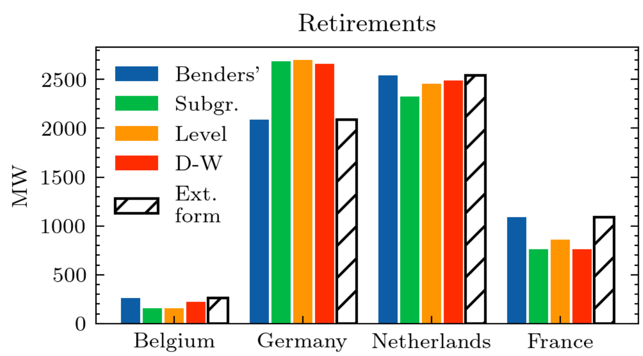

Fig. 3. Retirements in the solution of the examined methods. Investments were zero in all cases.

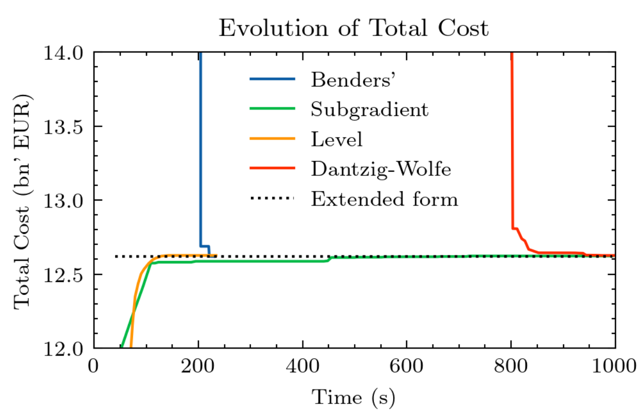

Fig. 4. Evolution of the total cost in the examined methods.