The Inscribed Boxes Approach to Reserve Deliverability in Balancing Capacity Markets: Base Results

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***Abstract*— Cross-border balancing capacity markets are currently being explored in Europe, with multiple options under consideration for determining how much of the total cross-zonal capacity should be allocated to balancing capacity exchanges. In this context, since the specific amount of balancing capacity that will be activated in real time is unknown beforehand, a key question arises: how can we ensure that any pattern of real-time activations remains feasible given the network model used in the day-ahead time frame? Drawing from a publicly available professional report on co-optimization in Europe, in which some authors of the present paper participated, we demonstrate that this "(deterministic) reserve deliverability requirement" can be effectively approximated through an "inscribed boxes approach." This approach takes inspiration from methods used to extract ATC domains from flow-based domains, or to enforce "intuitiveness" in flow-based market coupling.**

***Index Terms*— Balancing Capacity Markets, Reserve Deliverability, Deterministic Requirement.**

I. INTRODUCTION

We consider the problem of balancing capacity auctions with flow-based network models, where we require that any activation pattern of the balancing capacity reserved by TSOs can be activated in real time while still respecting the flow-based network constraints.

After the definition of the problem, a stochastic formulation is shown to be exact.

We then demonstrate that a computationally efficient inner approximation of the problem can be formulated, significantly reducing its size without substantially affecting the total system costs..

II. RESERVE DELIVERABILITY

A. *Position of the problem*

Transmission System Operators have the key responsibility to ensure safe and reliable operations of the transmission grid, which in turn requires to balance generation and consumption in real time. To secure operations, European TSOs rely on balancing capacity, which corresponds to commitments of generators or consumers to either generate, or to consume, more or less. Balancing capacity is currently auctioned nationally in most European countries, with the exception of Nordic countries which have organized in recent years the cross-border procurement of balancing capacity for automatic Frequency Restoration Reserve (aFRR) and more recently, for a subset of these countries, manual Frequency Restoration Reserve (mFRR).

Reserve deliverability specifically refers to the requirement that TSOs must be able to freely activate reserves that are auctioned in the day-ahead or forward markets without violating network constraints in real time. In the sequel, violations of the network constraints will mean violations of the mathematical constraints representing the network at the day-ahead stage, regardless of whether or not this mathematical representation accurately models the underlying physical reality. Reserve deliverability is examined in [13], which proposes guaranteeing deliverability for specific imbalance scenarios by explicitly imposing 'Post Zonal Reserve Deployment Transmission Constraints' and earlier in [12] which proposes a zonal reserve model derived from simulating each contingency event in the predefined reserve zones. These approaches are somewhat analogous to the stochastic formulation described below, with a restricted set of second-stage scenarios. The reference [12] proposes enhancements to the approach in [13] and an analysis of market implications. References therein provide further details on (zonal) reserve deliverability enforcement in U.S. markets.

When the network constraints are represented by an ATC network model – which would be described in the operational research community as a "network flow model" in the spirit of models considered in [2, 3] - reserve deliverability would be granted for free: this will be made clear by Lemma 1 and the proof of Proposition 2 below. This does not hold anymore in DC models approximating meshed AC networks. The present paper however shows that appropriately leveraging the reserve deliverability property holding in an ATC context, combined with a computational geometry result leveraged to describe all possible ATC domains compatible with a given DC

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approximation of a meshed AC network, allows to provide a computationally efficient inner approximation of this reserve deliverability requirement.

B. Stochastic Formulation

We now consider the following linear welfare maximization program and only upward reserve requirements to simplify notation. The variables $p, r,\ pr$ respectively denote generator's day-ahead power, day-ahead balancing capacity, and real-time balancing energy, while the variables $dr,\ pdr$ respectively denote the TSO demand for balancing capacity and the balancing energy activated in real-time. The parameters $I_l^+(s)$ in constraints (6) take a value of 0 or 1, indicating whether the corresponding reserve is activated in real time for that scenario. Constraints (5a)-(5b) are day-ahead balancing capacity and energy balance conditions, while constraints (5c)-(5e) enforce balance conditions and network constraints considering both energy and balancing energy (activated reserves) for each reserve activation scenario.

$$
\begin{array}{lll}
(1) & \max\limits_{p,r,dr} \ \sum\limits_{l} VR^l dr_l^{up} - \sum\limits_{g} C^g p_g & \\
(2) & P_g^0 + p_g + r_g^{up} \leq Q_g^+, & g \in US \\
(3) & P_g^0 + p_g \geq Q_g^-, & g \in US \\
(4) & dr_l^{up} \leq QR_l, & l \in RL \\
(5a) & \sum\limits_{l \in RL} d\,r_l^{up} - \sum\limits_{g \in US} r_g^{up} = 0 & \\
(5b) & \sum\limits_{g \in US: B_g = n} p_g + P_n - D_n = ne_n & \mathrm{n} \in \mathrm{N} \\
(5c) & \sum\limits_{g \in US: B_g = n} pr_{g,s}^{up} - \sum\limits_{l: B_g = n} pdr_{l,s}^{up} = nre_{n,s}^{up} & \mathrm{n} \in \mathrm{N}\ \mathrm{s} \in \mathrm{S} \\
(5d) & \sum\limits_{n \in N} PTDF_{kn} \cdot (ne_n + nre_{n,s}^{up}) \leq F_k^{\max}, & k \in K,\ s \in S \\
(5e) & \sum\limits_{n \in N} (ne_n + nre_{n,s}^{up}) = 0, & s \in S \\
(6): & pdr_{l,s}^{up} = I_l^+(s)\, \mathrm{dr}_\mathrm{l}^{\mathrm{up}} & \mathrm{s} \in \mathrm{S}, \mathrm{l} \in \mathrm{RL} \\
(7a) & pr_{g,s0}^{up} = 0 & \mathrm{g} \in \mathrm{US} \\
(7b) & pr_{g,s}^{up} \leq r_g^{up} & \mathrm{s} \in \mathrm{S}, \mathrm{g} \in \mathrm{US} \\
(8) & p, dr, r, pr, pdr \geq 0 &
\end{array}
$$

The set K includes all *directed* lines, comprising both (i,j) for modeling the flow from node i to node j, and (j,i) for modeling the flow from node j to node i, as well as any other relevant 'critical network element'.

The stochastic formulation model proposed in [7] and partly related to [5] in the context of TSO-DSO coordination considers a discrete set of extreme scenarios and ensures "complete recourse": for any first-stage decisions (p, dr, r) in a solution of (1)-(8), all extreme scenarios of activation patterns, described by $I_l^+(s)$, are feasible.

We show below that this exactly models the deterministic requirement: constraints imposed by the extreme scenarios on the first-stage decisions (p, r, dr) are necessary and sufficient to ensure that any activation pattern – extreme or not – of the matched balancing capacity demands is feasible for the network (and by only relying on the balancing capacity supply bids that have been matched in the day-ahead). This is formalized in the following proposition.

Below, $X_{|S|}$ denotes the vector $(X_1, \ldots, X_{|S|})$. *To simplify notation, in the vector notation below, we drop the indices 'n' for nodes, 'g' for generators and 'l' for loads.*

Proposition 1:

Suppose $(p, r, dr,\ ne,\ pr_{|S|}, pdr_{|S|}\ , nre_{|S|}^{up})$ satisfy (1)-(8) with the set of scenarios S corresponding to all the possible combinations of extreme activation scenarios $I_l^+(s) = 1$ or $I_l^+(s) = 0$.

(a) Then, for any first stage decisions $(p, r, dr,\ ne)$, and for any set of values $I_l^+(s) \in [0;1], l \in\ RL$ – corresponding to an infinite set of scenarios – the constraints (5c)-(8) above admit a feasible solution $(pr_s, pdr_s\ , ne_s)$.

(b) Conversely, if (p, r, dr) is such that (5c)-(8) admit a solution for any $I_l^+(s)\ \in [0;1]$, then we can directly determine $pr_{|S|}, pdr_{|S|}\ ,\ ne_{|S|}$ such that $(p, r, dr, pr_{|S|}, pdr_{|S|}\ ,\ ne_{|S|})$ satisfy (1)-(8) for the finite set of scenarios where the $I_l^+(s)$ take a value of 0 or 1.

Proof.

(a) is a consequence of the fact that the set $Y := \{y\ |\ Ax \leq y$ admits a solution $'x'\}$ is a convex set, see Lemma 1 in [4]. Let y correspond to the RHS of (5c)-(8) seen as a set of inequalities in the variables $pr_{|S|}, pdr_{|S|}\ ,\ nre_{|S|}$, i.e. where the RHS corresponds to (the vector components follow the order (5c) to (8)):

$$
\text{(RHS)} \quad (0,\ F_k^{\max} - \textstyle\sum_{n \in N} PTDF_{kn} ne_n\ ,\ -\textstyle\sum_{n \in N} ne_n,\ I_l^+(s)\, dr_\mathrm{l}^{\mathrm{up}}\ ,\ 0,\ r^{up}).
$$

Then, since the system is feasible for all RHS points obtained for the different scenarios $I_l^+(s) = 1$ or $I_l^+(s) = 0$, it will also be feasible for any convex combination of these points, which exactly corresponds to (RHS) but with $I_l^+(s)$ that can now take any value in [0;1].

(b) is direct: if the constraints (5c)-(8) are feasible for all scenarios where $I_l^+(s)$ can take any value in [0;1], they are also feasible for the special cases where they take the values 0 or 1.

C. The Inscribed Boxes Approach

The major drawback of the stochastic programming formulation in [7] is the combinatorial explosion in terms of scenarios to consider as the number of nodes – and hence possible reserve demand activations – is growing.

We now describe how this combinatorial explosion could be avoided at the expense of being reasonably more conservative, by approximating the deterministic requirement by an approach initially suggested in [6]. We provide here additional information as well as a comprehensive proof of the validity of the inner approximation.

The approach uses tools which have initially been developed in Europe to (a) enforce "intuitiveness in flow-based network models", namely the existence of a decomposition of cross-border trades into bilateral flows such that no flow goes from a more expensive area to a cheaper one., and (b) to extract so-called "ATC network models" inscribed within flow-based polytopes. In both of these applications, the constraints describe the union of all ATC domains containing the origin (zero flows are allowed) such that the reachable net positions lie within the initial flow-based domain. One way to obtain these constraints is via a direct application of a result in [1] for describing the union of all boxes parallel to the axes and containing the origin which lie in a given polytope.

Let us consider a flow-based polytope given by:

$$(R^{up}FB)\text{: } \{(nre_{|N|}^{up}) | \sum_{n\in[N]} PTDF_{kn} nre_{n,s}^{up} \leq F_k^{max} - \sum_{n\in[N]} PTDF_{kn} ne_n \}$$

An ATC model – which corresponds to a network flow model in the OR literature – is instead of the form:

$$(R^{up}ATC)\text{:}$$

$$\{nre_{|N|}^{up} \;|$$

$$(A1) \qquad nre_n^{up} = \sum_{l\neq n} f_{nl}^{rup} - f_{ln}^{rup} \quad n \in [N]$$

$$(A2) \quad f_{nl}^{rup} \leq ATC_{nl}^{rup} \; n,l \in [N] \;\}$$

Leveraging [1], we can describe the union of all domains of the form $(R^{up}ATC)$ such that $Proj_{\left(nre_{|N|}^{up}\right)}(R^{up}ATC) \subseteq R^{up}FB$, i.e. the union of all ATC domains with reachable net positions in the leftover flow-based domain $R^{up}FB$ allocated to upward reserve. This union is obtained in two steps:

(1.) Rewrite $R^{up}FB$ in terms of the cross-border trades f_{nl}^{rup} using equalities identical to (A1) to replace nre_n^{up} by the sum of inward and outward cross-border trades f_{nl}^{rup}:

$$(R^{up}FB\ ZtoZ)\text{:}$$

$$\{(nre_{|N|}^{up}) \;|$$

$$(ZtoZ\ 1)\text{:} \qquad nre_n^{up} = \sum_{l\neq n} f_{nl}^{rup} - f_{ln}^{rup} \quad n \in [N],$$

$$(ZtoZ\ 2)\text{:} \sum_{n,l\in[N]} (PTDF_{kn} - PTDF_{kl}) f_{nl}^{rup} \leq F_k^{max} - \sum_{n\in[N]} PTDF_{kn} ne_n \}$$

(2.) Leveraging [1], replace the coefficients $(PTDF_{kn} - PTDF_{kl})$ by $(PTDF_{kn} - PTDF_{kl})^+ := \max[(PTDF_{kn} - PTDF_{kl}),\ 0]$ to describe the union of all boxes in the space of the f_{nl}^{rup} that contain (0, …,0) and within the polytope described by $(ZtoZ\ 2)$:

$$(ZtoZ\quad 2b) \sum_{n,l\in[N]} (PTDF_{kn} - PTDF_{kl})^+ f_{nl}^{rup} \leq F_k^{max} - \sum_{n\in[N]} PTDF_{kn} ne_n \}$$

This leads to the formulation (IB1)-(IB7) below which essentially consists in allowing to allocate part of the cross-zonal capacity F_k^{max} to energy (see the ne_n in the right-hand side of (IB5e)), while an ATC domain is optimally extracted for cross-border capacity exchanges of balancing capacity from the leftover domain after the subtraction of the cross-zonal capacity allocated to energy, see (IB5d)- (IB5e).

The ATC values of the implicitly extracted ATC domain correspond to the optimal values $(f_{nl}^{rup})^*$ obtained from the optimization of (IB1)-(IB7): indeed, it can easily be seen that flow values lower than $(f_{nl}^{rup})^*$ and the implied import/export values nre_n^{up} will continue to satisfy (IB5d)-(IB5e).

In the next section, we show that this inscribed boxes model (IB1)-(IB7) given below is an inner approximation of the stochastic programming formulation.

$$\text{(IB1)} \quad \max_{p,r,dr} \sum_l RupValue^l dr_l^{up} - \sum_g C^g p_g$$

$$(IB2) \quad P_g^0 + p_g + r_g^{up} \leq Q_g^+, \quad g \in US$$

$$(IB3) \quad P_g^0 + p_g \geq Q_g^-, \quad g \in US$$

$$(IB4) \quad dr_l^{up} \leq QR_l, \quad l \in RL$$

$$(IB5a) \quad \sum_{l\in RL} dr_l^{up} - \sum_{g\in US} r_g^{up} = 0$$

$$(IB5b) \quad \sum_{g\in US: B_g = n} p_g + P_n - D_n = ne_n \quad n \in N$$

$$(IB5c) \quad \sum_{g\in US: B_g = n} r_g^{up} - \sum_l dr_l^{up} = nre_n^{up} \quad n \in N$$

$$(IB5d) \quad nre_n^{up} = \sum_{l\neq n} f_{nl}^{rup} - f_{ln}^{rup} \quad n \in [N],$$

$$(IB5e) \quad \sum_{n,l\in[N]} (PTDF_{kn} - PTDF_{kl})^+ f_{nl}^{rup} \leq F_k^{max} - \sum_{n\in[N]} PTDF_{kn} ne_n, \quad k \in K$$

$$(IB6) \quad \sum_{n\in N} (ne_n + nre_n^{up}) = 0$$

$$(IB7) \quad p, dr, r, f_{nl}^{rup} \geq 0$$

III. Correctness of the Inscribed Boxes Approach

Given that the decision variables of interest in the objective functions are (p, r, dr, ne), our focus is on the projection of the feasible sets onto the space of these variables.

Proposition 2:

Let IBFS denotes the set of points satisfying (IB1)-(IB7) and SPFS denotes the set of points satisfying (1)-(8). For any DC network model (PTDF parameters, thermal limits F_k^{max}), if $(p, r, dr, ne, nr^{up}, f^{rup}) \in IBFS$ then there exists $(pr_{|S|}, pdr_{|S|}, nre_{|S|})$ such that $(p, r, dr, ne, pr_{|S|}, pdr_{|S|}, nre_{|S|}) \in SPFS$

Stated otherwise, Proposition 2 asserts that $Proj_{(p,r,dr,ne)}(IBFS) \subseteq Proj_{(p,r,dr,ne)}(SPFS)$. We first give some intuition before proving the result formally.

The reserve activation scenarios $I_l^+(s)$ model that in real time, only part of the demand for reserve dr_l^{up} may be activated. If the demand for reserve is lower than the dr_l^{up} booked in the day-ahead time frame in some given node, two cases could occur: the demand reduction can be absorbed locally by a reduction of the matched upward reserve supplies r_g^{up} located at the same node, or one needs to reduce reserve imports. The key intuitive argument is that solving (IB1)-(IB7) optimally extract an ATC domain for the exchange of upward reserve, given the discussion above of the constraints $(IB5d) - (IB5e)$, and, as shown in Lemma 1 below, in a network flow model like the ATC model $(R^{up}ATC)$, it is always possible to reduce imports in importing nodes by reducing exports at exporting nodes, and accordingly reduce flow values between nodes, while still satisfying the flow capacity constraints. Since the real-time reserve net positions remain feasible for the ATC domain, which is itself extracted from the leftover flow-based domain allocated to the exchange of balancing capacity, we can then define in each activation scenario 's' new values $nre_{n,s}^{up}$ and $pr_{g,s}^{up} \leq pr_{g}^{up}$ such that (1)-(8) hold.

Lemma 1 *In a network flow problem, it is always possible to reduce imports of importing nodes by reducing exports of exporting nodes and the arc flows while still satisfying the flow capacity constraints and the balance (or Kirckhoff) conditions.*

More formally, using the notation of (A1)-(A2): Consider a feasible point (A1)-(A2). Without loss of generality, let $(1,\dots,\mathrm{m})$ correspond to exporting nodes, i.e. $\mathrm{nre}_1^{\mathrm{up}} > 0,\dots,$ $\mathrm{nre}_{\mathrm{m}}^{\mathrm{up}} > 0$, and $(\mathrm{q}+1,\dots,\mathrm{N})$ correspond to importing nodes, i.e. $\mathrm{nre}_{\mathrm{q}+1}^{\mathrm{up}} < 0$, …, $\mathrm{nre}_{\mathrm{N}}^{\mathrm{up}} < 0$ (the nodes from m+1 to q being nodes with a zero balance). Then for any reduced imports $\mathrm{nre}_{\mathrm{q}+1}^{\mathrm{up}} \leq \widehat{\mathrm{nre}_{\mathrm{q}+1}^{\mathrm{up}}} \leq 0$, …, $\mathrm{nre}_{\mathrm{N}}^{\mathrm{up}} \leq \widehat{\mathrm{nre}_{\mathrm{N}}^{\mathrm{up}}} \leq 0$, there exist reduced exports $\mathrm{nre}_1^{\mathrm{up}} \geq \widehat{\mathrm{nre}_1^{\mathrm{up}}} \geq 0$, …, $\mathrm{nre}_{\mathrm{m}}^{\mathrm{up}} \geq \widehat{\mathrm{nre}_{\mathrm{m}}^{\mathrm{up}}} \geq 0$ and reduced flows $\widehat{\mathrm{f}_{\mathrm{nl}}^{\mathrm{rup}}} \leq \mathrm{f}_{\mathrm{nl}}^{\mathrm{rup}}$ such that $(\widehat{\mathrm{nre}_{\mathrm{m}}^{\mathrm{up}}}, \widehat{\mathrm{f}_{\mathrm{nl}}^{\mathrm{rup}}})$ is also feasible for (A1)-(A2).

Proof of Lemma 1.

This can be shown as a straightforward consequence of the classical flow decomposition theorem for network flows, see for instance Proposition 1.1 (Conformal Realization Theorem) [2] and the preceding definition of simple path flow, or Theorem 3.5 (Flow Decomposition Theorem) in [3]. Indeed, the network flow can be decomposed into *simple path flows* $\phi_{i,j}^k \geq 0$ connecting exporting nodes $i \in \{1,\dots m\}$ to importing nodes $j \in \{q+1,\dots N\}$, such that

- $nre_i^{up} = \sum_{j,k} \phi_{i,j}^k, \quad i \in \{1,\dots m\}$
- $nre_j^{up} = \sum_{i,k} -\phi_{i,j}^k, \quad j \in \{q+1,\dots N\}$,
- $f_{nl}^{rup} = \sum_{i,j,k \,|\, (n,l)\in Path_{(i,j,k)}} \phi_{i,j}^k$,

From this decomposition, we can see that for any node $j \in \{q+1,\dots,N\}$ in which there would be reduced imports $\widehat{nre_j^{up}}(i.e. \, such\ that\ nre_j^{up} \leq \widehat{nre_j^{up}} \leq 0)$, we can easily reduce the values of some simple path flows $\phi_{i,j}^k$ accordingly, which will lead to reducing accordingly the exports from the nodes $i \in \{1,\dots m\}$, and the arc capacity constraints will remain satisfied. ■

Proof of Proposition 2.

Let $(p, r, dr,\ ne,\ nr^{up},\ f^{rup})$ be a feasible point of (IB1)-(IB7). We need to show that for every scenario 's' and binary parameter values for the $I_l^+(s)$ uniquely determining $pdr_{l,s}^{up}$ via (6), we can define values for $pr_{g,s}^{up}$ and $nre_{n,s}^{up}$ such that all the scenario-dependent constraints (5c)-(8) are satisfied.

Let $\Delta_{n,s} \coloneqq \sum_{l:B_g=n} dr_l^{up} - \sum_{l:B_g=n} I_l^+(s) dr_l^{up}$ correspond to the difference between the demand for reserves matched in the day-ahead and the reserve to activate in real-time in scenario s. We determine the values of $pr_{g,s}^{up}$ and $nre_{n,s}^{up}$ in four steps. Note that the solution we build is just a feasible solution, not necessarily an optimal recourse solution for that scenario 's'.

Step 1) Preprocessing step absorbing as much as possible of the decrease $\Delta_{n,d}$ by locally reducing the r_g^{up} (we keep denoting them as r_g^{up} instead of $r_{g,s}^{up}$ to avoid heavy notation):

[Nodes of Type 1] For all nodes n such that $\Delta_{n,d} \leq \sum_{g:B_g=n} r_g^{up}$ (this includes all exporting nodes): reduce the values of r_g^{up} at that node (e.g. following the inverse price merit order) such that balance conditions (IB5c) remain satisfied, with the value of nre_n^{up} unchanged:

$$(*) \quad \sum_{g:B_g=n} r_g^{up} - \sum_{l:B_l=n} I_l^+(s) dr_l^{up} = nre_n^{up}$$

[Nodes of Type 2] For importing nodes 'n' such that

$(**)\ \Delta_{n,d} > \sum_{g:B_g=n} r_g^{up}$, define

$\Delta nre_n^{up} := \Delta_{n,d} - \sum_{g:B_g=n} r_g^{up}$ representing the portion of the difference between the day-ahead booked capacity and the actual real-time activation that cannot be offset by a *local* real-time reduction in the generator's reserves r_g^{up}. After fully reducing all generator's reserves in that node, i.e. setting all $r_g^{up} = 0$, the balance equation of type $(IB5c)$ becomes for that scenario:

$$-\sum_{l:B_l=n} I_l^+(s) dr_l^{up} = \widehat{nre_n^{up}}$$

with $nre_n^{up} \leq \quad \widehat{nre_{q+1}^{up}} \coloneqq (nre_n^{up} + \Delta nre_n^{up}) \quad \leq 0$

Step 2) Conditions (IB5d) are not satisfied anymore for nodes of Type 2 for which we defined the reduced imports $\widehat{nre_n^{up}}$. A direct application of Lemma 1 to $(IB5d)-(IB5e)$ however ensures that one can define reduced flows $\widehat{f_{nl}^{rup}}$ and reduced exports $nre_1^{up} \geq \widehat{nre_1^{up}} \geq 0$ at nodes of Type 1 which were exporting such that conditions (IB5d)-(IB5e) are all satisfied. The fact that the reduced flows $\widehat{f_{nl}^{rup}}$ remain feasible for (IB5e) directly follows from the non-negativity of the coefficients $(PTDF_{kn} - PTDF_{kl})^+$, or alternatively because they remain in the box $\left[0;\ f_{nl}^{rup}\right]_{n,l\in[N]}$ inscribed in the flow-based domain $(R^{up}FB\ ZtoZ)$.

Step 3) Finally, for an exporting node seeing its exports reduced, the balance condition $(*)$ of type $(IB5c)$ is restored by reducing accordingly the part of the $r_g^{up} > 0$ acceptances at that node that are not needed to match the local demand $\sum_{l:B_l=n} I_l^+(s) dr_l^{up}$.

Step 4) Let us denote by $\widehat{r_g^{up}}$ the new values for r_g^{up} obtained after steps 1 to 3 above. It is then straightforward to check that the following values of $pdr_{l,s}^{up}, pr_{g,s}^{up}, nre_{n,s}^{up}$ satisfy all the scenario-dependent constraints (5c)-(8):

$$pdr_{l,s}^{up} \coloneqq I_l^+(s) dr_l^{up}; \quad r_{g,s}^{up} \coloneqq \widehat{r_g^{up}}; \quad nre_{n,s}^{up} \coloneqq \widehat{nre_n^{up}} \qquad ■$$

N.B. The proof could be simplified by observing that for any solution of the model (IB1)-(IB7), there is an alternative solution in which only direct flows f_{nl}^{rup} from exporting nodes n to importing nodes l are used.

IV. Numerical Experiments

In this section we demonstrate in a real-world system the performance of the two aforementioned models.

A. Setup of case study

Transmission network. The system we use is a realistic representation of the Belgian transmission network, based on a commercial dataset. The network consists of 628 buses and 725 edges, including lines and transformers. The voltage range varies from 380kV to 12kV.

Loads. The dataset includes load values as a snapshot, which are modelled as price-inelastic power withdrawals.

High-voltage power units. There are 615 generators in the system. It is assumed that each generator is committed for the given snapshot of the grid and reserves can be activated fully, without ramping constraints for every generator. Additionally, the marginal costs of each unit are derived from [8].

Imports. The commercial dataset also includes the imports of Belgium for the certain snapshot that we are examining. Concretely, imports are modelled as price-inelastic power injections.

ORDCs. We assume a single reserve product in this case study, and we analyze only upward reserve. The ORDCs are sourced from the work of [9], where the authors have precisely calibrated ORDCs for the Belgian system.

The market models are implemented in Julia v.1.10.4 using JuMP v1.22.2, on an Asus Zenbook 14 with a Intel Core Ultra 7 155H 3.80 GHz processor with Windows 11 (64-bit). The chosen linear programming solver is Gurobi 11.

B. Results

To implement reserve requirement for various locations, we assumed that locations with load have an accordingly scaled ORDC in relation to the original Belgian curve. Moreover, we introduced congestion on a line with both reserve and energy flow to generate a more complex problem and challenge the system.

The metrics of performance used to evaluate each model include the number of reserve requirements, run-time, used solver, iterations needed, number of equations and social welfare and are shown in Table I and II.

The results demonstrate that the inscribed boxes approach is computationally tractable in contrast to the stochastic formulation, where the increase in number of reserve requirement increases exponentially the number of scenarios and consequently the number of constraints of the underlying linear program. The run-time of the inscribed boxes approach remains relatively stable, while the stochastic formulation exhibits a significant increase. Lastly, the welfare increase is in these simulations independent of the method used to enforce reserve deliverability.

TABLE I. PERFORMANCE OF STOCHASTIC FORMULATION

#dr	Run-time (s)	Simplex solver	Iterations	#equations	Welfare
1	4.10	primal	1658	2900	-10539
2	4.75	dual	1860	5800	-10537
3	9.45	primal	5177	11600	-9755
4	23.76	primal	5830	23200	-6573
5	65.63	primal	12110	46400	-5093
6	213.4	primal	23648	92800	-3625
7	650.19	primal	45552	185600	-2498

TABLE II. PERFORMANCE OF INSCRIBED BOXES APPROACH

#dr	Run-time (s)	Simplex solver	Iterations	#equations	Welfare
1	2.74	dual	788	1450	-10539
2	1.10	dual	1293	1450	-10537
3	1.04	dual	1294	1450	-9755
4	1.04	dual	1295	1450	-6573
5	1.51	dual	1295	1450	-5093
6	1.06	dual	1296	1450	-3625
7	1.05	dual	1298	1450	-2498

V. Conclusions and Future Work

We have examined how an approach based on inscribed boxes in polytopes can provide a computationally efficient inner approximation of the reserve deliverability requirement, significantly enhancing performance. This approach is inspired by the ATC extraction process and the enforcement of 'intuitive flows'—from low-cost to high-cost areas—in flow-based market coupling. However, these performance improvements may, in some cases, come at the expense of a slight reduction in welfare or an increase in system costs. Ongoing work aims to further explore the properties of the proposed approach and investigate alternative models that may achieve an even better balance between performance and system costs.

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References

[1] Bemporad, Alberto, Carlo Filippi, and Fabio D. Torrisi. "Inner and outer approximations of polytopes using boxes." *Computational Geometry* 27.2 (2004): 151-178.

[2] Bertsekas, Dimitri P. Linear network optimization: algorithms and codes. MIT press, 1991. Available online: http://web.mit.edu/people/dimitrib/LNets_Full_Book.pdf

[3] Ahyja, Ravindra K., James B. Orlin, and Thomas L. Magnanti. *Network flows: theory, algorithms, and applications*. Prentice-Hall, 1993.

[4] Geoffrion, Arthur M. Duality in nonlinear programming: a simplified applications-oriented development. *SIAM review* 13.1 (1971): 1-37

[5] Caramanis, Michael, et al. "Co-optimization of power and reserves in dynamic T&D power markets with nondispatchable renewable generation and distributed energy resources." *Proceedings of the IEEE* 104.4 (2016): 807-836.

[6] N-SIDE and SDAC MSD, SDAC MSD Co-optimization Roadmap Study, available online: https://www.nemo-committee.eu/assets/files/co-optimization-roadmap-study-.pdf

[7] N-SIDE, AFRY, "CZC allocation with co-optimisation", November 2020

[8] I. Aravena and A. Papavasiliou, "Renewable energy integration in zonal markets,"IEEE Transactions on Power Systems, vol. 32, no. 2, pp.1334-1349, 2017.

[9] J. Cartuyvels and A. Papavasiliou, "Calibration of operating reserve demand curves using a system operation simulator," IEEE Transactions on Power Systems, vol. 38, no. 4, pp. 3043–3055, 2023.

[10] Wang, F., & Chen, Y. (2020). Market implications of short-term reserve deliverability enhancement. *IEEE Transactions on Power Systems*, *36*(2), 1504-1514.

[11] Cobos, N. G., Arroyo, J. M., & Street, A. (2016). Least-cost reserve offer deliverability in day-ahead generation scheduling under wind uncertainty and generation and network outages. *IEEE Transactions on Smart Grid*, *9*(4), 3430-3442.

[12] Zheng, T., & Litvinov, E. (2008). Contingency-based zonal reserve modeling and pricing in a co-optimized energy and reserve market. *IEEE transactions on Power Systems*, *23*(2), 277-286.

[13] Chen, Y., Gribik, P. and Gardner, J., 2013. Incorporating post zonal reserve deployment transmission constraints into energy and ancillary service co-optimization. *IEEE Transactions on Power Systems*, *29*(2), pp.537-549.