Demand Response

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Source: chapter 10, Papavasiliou [1]

Απόκριση ζήτησης

Demand response: active participation of consumers in (i) efficient consumption of electricity and (ii) provision of ancillary services

Types of demand response:

1. Efficiency
2. Peak load shaving
3. Load shifting

Retail pricing

Mechanisms for retail pricing of electricity:

- Real-time pricing
- Time of use pricing (ToU)
- Critical peak pricing: ToU + critical peak events
- Interruptible service

Outline

- Time of use pricing
- Priority service pricing

Time of use pricing

Motivation of time of use pricing

- Electricity service consists of (i) fuel cost for producing power, and (ii) investment cost for building capacity
- If electricity were priced at marginal fuel cost, demand in peak periods would be too high
- ToU pricing breaks bill into two parts:
    1. Energy component: charge proportional to amount of power consumption, differs depending on the time of day
    2. Capacity component: applied to consumers who contribute to need of installing additional capacity to the system
- Goal is to flatten demand across time periods

Simple two-period model

- Consider the following system:
- Decreasing marginal benefit functions:
  - Peak: $MB_1(d)$, lasts fraction τ_1 of the time
  - Off-peak: $MB_2(d)$, lasts fraction τ_2 of the time
- Increasing marginal investment cost $MI(x)$, with $MI(x) > 0$ for all x
- Increasing marginal fuel cost $MC(p)$
- Suppose $MB_1(0) > MC(0) + \frac{MI(0)}{\tau_1}$

Welfare maximization model

- Denote
  - x: amount of constructed capacity
  - p_1/p_2: production in peak/off peak hours

$$\max_{p,x} \tau_1 \cdot \int_0^{p_1} MB_1(q)dq + \tau_2 \cdot \int_0^{p_2} MB_2(q)dq$$
$$- \int_0^{x} MI(q)dq - \tau_1 \cdot \int_0^{p_1} MC(q)dq - \tau_2 \cdot \int_0^{p_2} MC(q)dq$$
$$(\rho_1 \cdot \tau_1): p_1 \leq x$$
$$(\rho_2 \cdot \tau_2): p_2 \leq x$$
$$p_1, p_2, x \geq 0$$

Note: since $MI(x) > 0$, in the optimal solution $p_1 = x$, $p_2 = x$, or both

KKT conditions

$$0 \leq \rho_1 \perp x - p_1 \geq 0$$
$$0 \leq \rho_2 \perp x - p_2 \geq 0$$

$$0 \leq p_1 \perp -MB_1(p_1) + MC(p_1) + \rho_1 \geq 0$$
$$0 \leq p_2 \perp -MB_2(p_2) + MC(p_2) + \rho_2 \geq 0$$

$$0 \leq x \perp MI(x) - \rho_1 \cdot \tau_1 - \rho_2 \cdot \tau_2 \geq 0$$

Note: dual multipliers have been scaled by τ_i

Short-term marginal cost pricing is suboptimal

- **Proposition**: Suppose that electricity is priced at the marginal variable cost $MC(p_i)$ for each period i. This results in suboptimal investment.

Mathematically: Optimal solution cannot satisfy all of the following conditions

- $MC(p_1) = MB_1(p_1)$
- $MC(p_2) = MB_2(p_2)$
- $x = \max(p_1, p_2)$

Proof: by contradiction, using KKT conditions

We first show that $\rho_1 = \rho_2 = 0$:

- Since $MB_1(0) > MC(0) + MI(0)/\tau_1$, optimal investment must be such that $x > 0$
- Suppose that $\rho_i > 0$, then $p_i = x > 0$
- Since $p_i > 0$, $MB_i(p_i) = MC(p_i) + \rho_i > MC(p_i)$
- But short-term marginal cost pricing requires that $MB_i(p_i) = MC(p_i)$
- Therefore $\rho_1 = \rho_2 = 0$, otherwise there is a contradiction

We then show that $\rho_i > 0$ for some i:

- Since $x > 0$, by complementarity
$$MI(x) = \rho_1 \cdot \tau_1 + \rho_2 \cdot \tau_2$$

- Since $MI(x) > 0$ for all $x \geq 0$, $\rho_i > 0$ for $i = 1$, or $i = 2$, or both

Peak charges

Interpretation of multiplier ρ_i: charge above the marginal cost of the marginal technology, $MC(p_i)$

- For constant marginal investment cost, $MI(x) = MI$, additional charges are exactly equal to capital investment costs

Example: pricing on and off peak

Consider the following market:

- $MI(x) = 5$ \$/MWh
- $MC(p) = 80$ \$/MWh
- Peak demand $MB_1(d) = \max(1000 - d, 0)$ \$/MWh, with $\tau_1 = 20\%$
- Off-peak demand $MB_2(d) = \max(500 - d, 0)$ \$/MWh, with $\tau_2 = 80\%$

Problem: you are told that the optimal investment is $x = 895$ MW, what are the optimal ToU prices?

- Since optimal x is 895 MW, then either $p_1 = 895$ MW, or $p_2 = 895$ MW, or both
- Check that $MB_1(895) = 105\ \$/\mathrm{MWh}$ and $MB_2(895) = 0\ \$/\mathrm{MWh}$
- Obviously $p_2 < x$ (marginal benefit at 895 MW is zero, marginal cost is 80 \$/MWh)
- Therefore, $p_1 = 895$ MW
- Price in peak periods: 105 \$/MWh
- From KKT conditions,

$$MB_2(p_2) = MC(p_2)$$

- Price in off-peak periods: 80 \$/MWh

Graphical illustration of tariff

Consider the fixed retail tariff which is the average ToU tariff:

$$0.2 \cdot 105 + 0.8 \cdot 80 = 85\ \$/\mathrm{MWh}$$

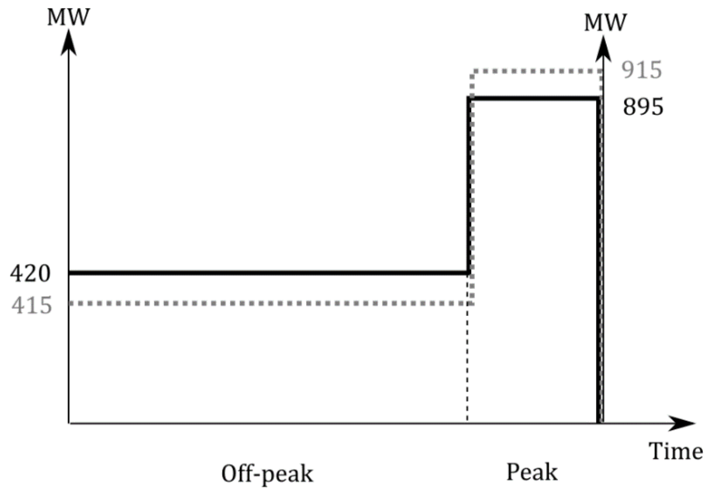


Figure: Demand under fixed retail pricing (black solid curve) and time of use pricing (gray dashed curve).
Effect of ToU pricing: depresses consumption in peak hours, increases consumption in off-peak hours.

Example: sharing peak charges

Consider the previous example, with $MB_2(d) = \max(980 - d, 0)$ $/MWh (and everything else as in slide 14)

Price of 80 $/MWh in off-peak hours results in demand that violates installed capacity

Optimal solution: $x = 899$ MW, $p_1 = p_2 = 899$ MW

Sharing of capital costs among peak and off-peak consumers:

- $\frac{\rho_1}{\tau_1} = 21$ $/MWh
- $\frac{\rho_2}{\tau_2} = 1$ $/MWh

Priority service pricing

System reliability

- We analyze the function $F\left(D(v)\right)$

where

  - $D(v)$: demand function (power demand from consumers who value power at v *or more*)
  - $F(L)$: probability of having L MW *or more* of available power

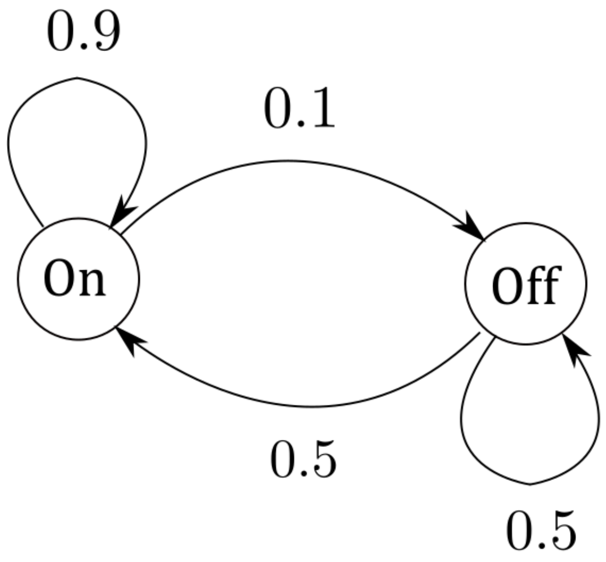
- Interpretation of $F(D(v))$: probability of being able to satisfy consumers with valuation v or higher

Example 10.3: computing $F(D(v))$

Consider the following system:

- Reliable technology: 295 MW
- Unreliable technology: 1880 MW
- Demand function: $D(v) = 1620 - 4v$

Unreliable technology described by Markov chain



Stationary distribution: $\pi_{\text{off}} = 0.167$ and $\pi_{\text{on}} = 0.833$

- Generator availability:

$$F(L) = \begin{cases} 1, & L \leq 295 \text{ MW} \\ 0.833, & 295 \text{ MW} < L \leq 2175 \text{ MW} \\ 0, & L > 2175 \text{ MW} \end{cases}$$

- Service reliability:

$$F\big(D(v)\big) = \begin{cases} 0.833, & 0 \frac{\$}{\text{MWh}} \leq v \leq 331.25 \frac{\$}{\text{MWh}} \\ 1, & 331.25 \frac{\$}{\text{MWh}} < v \leq 405 \frac{\$}{\text{MWh}} \end{cases}$$

Priority service contracts

Priority service contracts are defined as $p(r)$, where r is the reliability of the services, and $p(r)$ is the price paid for r

Note: $p(r)$ determines reliability chosen by customers

- Goal: design $p(r)$ so that customers with higher valuation receive more reliable service

Steering customer choice

Load with valuation v selects reliability by solving

$$\max_{0 \le r \le 1} r \cdot v - p(r)$$

First-order condition:

$$v - p'(r) = 0$$

Suppose $p(r)$ satisfies:

$$v - p'(r) = 0 \quad (1)$$
$$r \cdot v - p(r) \ge 0 \quad (2)$$

Load with valuation v

- Is willing to procure a reliability contract
- Chooses reliability level $F(D(v))$

Computing the price menu

Integrating equation (1):

$$\hat{p}(v) = p_0 + \int_{v_0}^{v} y \cdot dr(y) \quad (3)$$

where v_0 is **cutoff valuation**: valuation of consumer with lowest willingness to pay who chooses to subscribe

Parametrizing with respect to v, the menu $p(r)$ is

$$\{F(D(v)), \hat{p}(v), v \in [v_0, V]\}$$

where V is maximum valuation

Fixed charge

Fixed charge p_0 determines cutoff valuation v_0:

$$v_0 \cdot r(v_0) - p_0 = 0 \quad (4)$$

Customers with $v < v_0$ do not procure reliability contracts

Example 10.4: optimal pricing of a menu

$$F\big(D(v)\big) = \begin{cases} 0.833, & 0\,\frac{\$}{\text{MWh}} \le v \le 331.25\,\frac{\$}{\text{MWh}} \\ 1, & 331.25\,\frac{\$}{\text{MWh}} < v \le 405\,\frac{\$}{\text{MWh}} \end{cases}$$

Suppose $v_0 = 10\ \$/\text{MWh}$, then from equation (4):

$$p_0 = 10 \cdot 0.833 = 8.33\,\frac{\$}{\text{MWh}}$$

Example 10.4

From equation (3):

$$\hat{p}(v) = p_0 + \int_{v_0}^{v} u \cdot dr(u) =$$

$$= \begin{cases} 8.33 \dfrac{\$}{\text{MWh}}, & 10 \dfrac{\$}{\text{MWh}} \le v \le 331.25 \dfrac{\$}{\text{MWh}} \\ 8.33 + 331.25 \cdot 0.167 \dfrac{\$}{\text{MWh}}, & 331.25 \dfrac{\$}{\text{MWh}} < v \le 405 \dfrac{\$}{\text{MWh}} \end{cases}$$

$$= \begin{cases} 8.33 \dfrac{\$}{\text{MWh}}, & 10 \dfrac{\$}{\text{MWh}} \le v \le 331.25 \dfrac{\$}{\text{MWh}} \\ 63.65 \dfrac{\$}{\text{MWh}}, & 331.25 \dfrac{\$}{\text{MWh}} < v \le 405 \dfrac{\$}{\text{MWh}} \end{cases}$$

Example 10.4

Parametrizing with respect to v:

$$p(r) = \begin{cases} 8.33 \dfrac{\$}{\mathrm{MWh}}, & r = 0.833 \\ 63.65 \dfrac{\$}{\mathrm{MWh}}, & r = 1 \end{cases}$$

This is a menu with 2 options

Example 10.4: consumer self-selection

Consider the choice of a load with valuation v:

$$\max(0, 0.833 \cdot v - 8.33, v - 63.65)$$

- $r = 0$ is optimal if $0.833 \cdot v - 8.33 \leq 0$ and $v - 63.65 \leq 0$, i.e. $v \leq 10$
- $r = 0.833$ is optimal if $0 \leq 0.833 \cdot v - 8.33$ and $v - 63.65 \leq 0.833 \cdot v - 8.33$, i.e. $10 \leq v \leq 331.25$
- $r = 1$ is optimal if $0 \leq v - 63.65$ and $0.833 \cdot v - 8.33 \leq v - 63.65$, i.e. $v \geq 331.25$

Example 10.4: different choice of fixed charge

- If menu designer would like all customers to procure reliability contracts, i.e. $v_0 = 0$, then $p_0 = 0$ and

$$p(r) = \begin{cases} 0\,\dfrac{\$}{\text{MWh}}, & r = 0.833 \\ 55.32\,\dfrac{\$}{\text{MWh}}, & r = 1 \end{cases}$$

Service policy

In case of shortage, customers with higher r served first

Note: in order to design the menu, we used *aggregate* information ($F(L)$ and $D(v)$)

Menu selections allow us to dispatch *individual* customers efficiently!

References

[1] A. Papavasiliou, Optimization Models in Electricity Markets, Cambridge University Press

https://www.cambridge.org/highereducation/books/optimization-models-in-electricity-markets/0D2D36891FB5EB6AAC3A4EFC78A8F1D3#overview