Exact Mixed-Integer Programming Approach for Chance-Constrained Multi-Area Reserve Sizing

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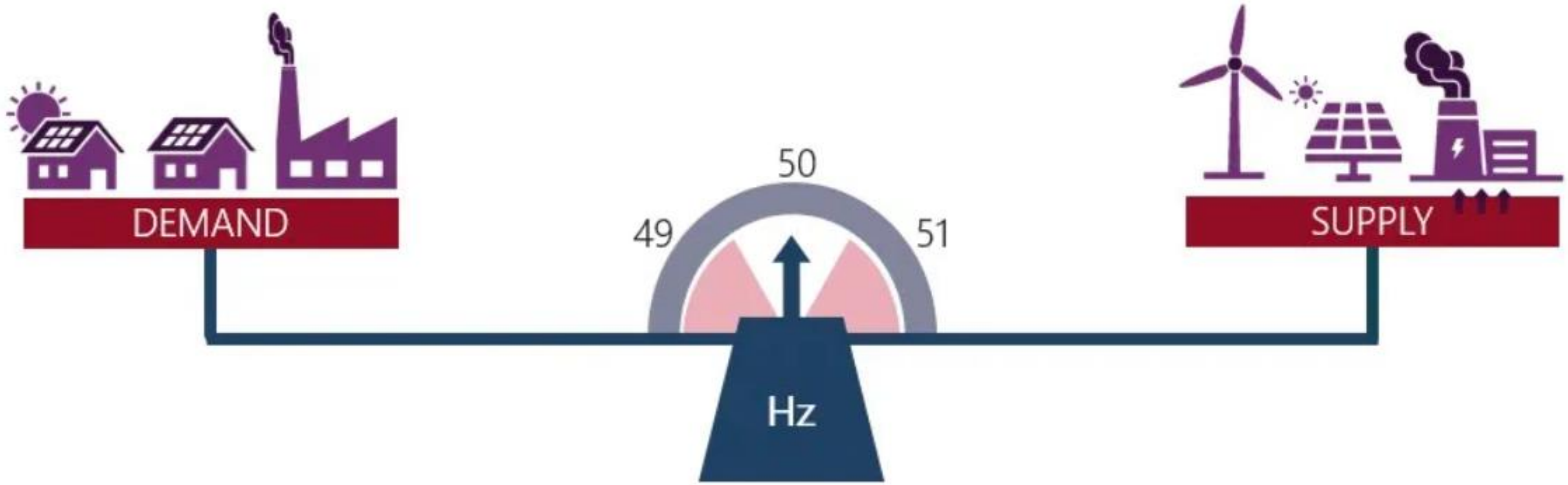
Motivation

Balancing



Continuous maintenance of system frequency by balancing supply and demand

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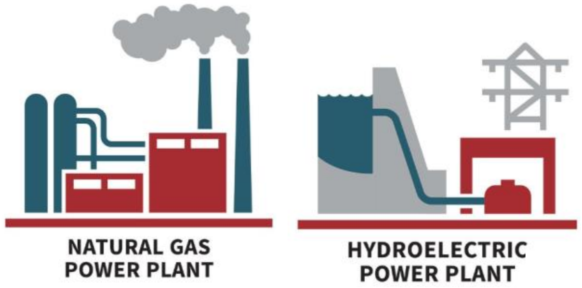
Motivation

Power Generation

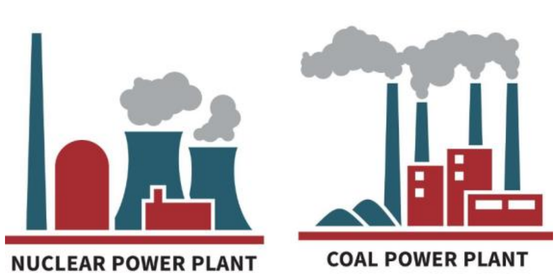
Flexible Generators

Examples: Gas turbines, Hydroelectric Power Plants



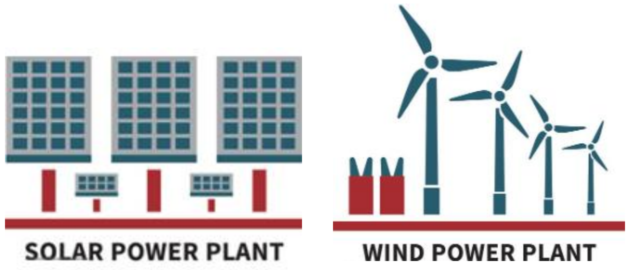
Inflexible Generators

Examples: Coal-Fired Power Plants, Nuclear Power Plants



Uncontrollable Generators

Examples: Solar Panels, Wind Turbines



Motivation

Imbalances

Before Real-Time

Generators are scheduled in advance according to forecasted power: supply (solar and wind energy production) and demand.

Real-Time Operations

Flexibility providers (flexible generator, battery storage system, demand response) adjust the mismatch between supply and demand in real-time.

"Imbalances" refer to the discrepancy between electricity supply and demand at any given moment, caused by forecasting errors or unpredicted system disturbances.

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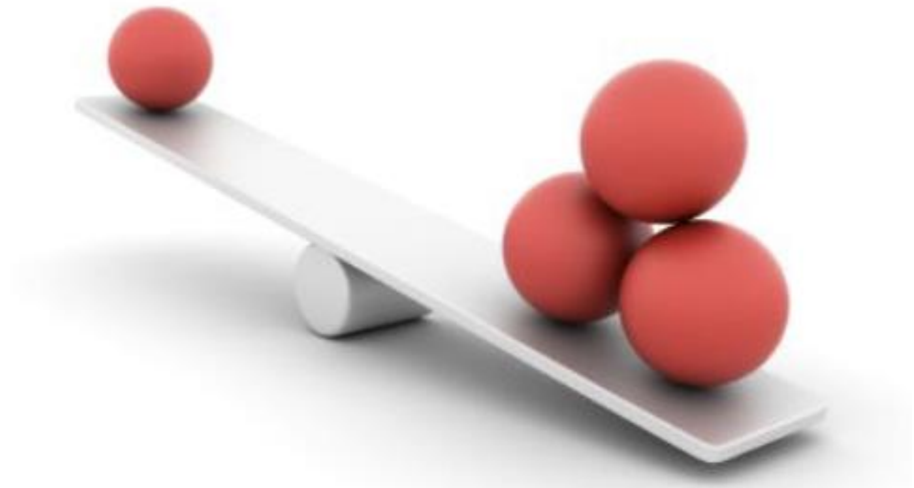
Motivation

Reserves

"**Reserves**" refer to the capacity set aside by the system operator for covering imbalances.

Costs for Reserve Capacity **System Reliability**



Trade off between costs for reserves and system reliability

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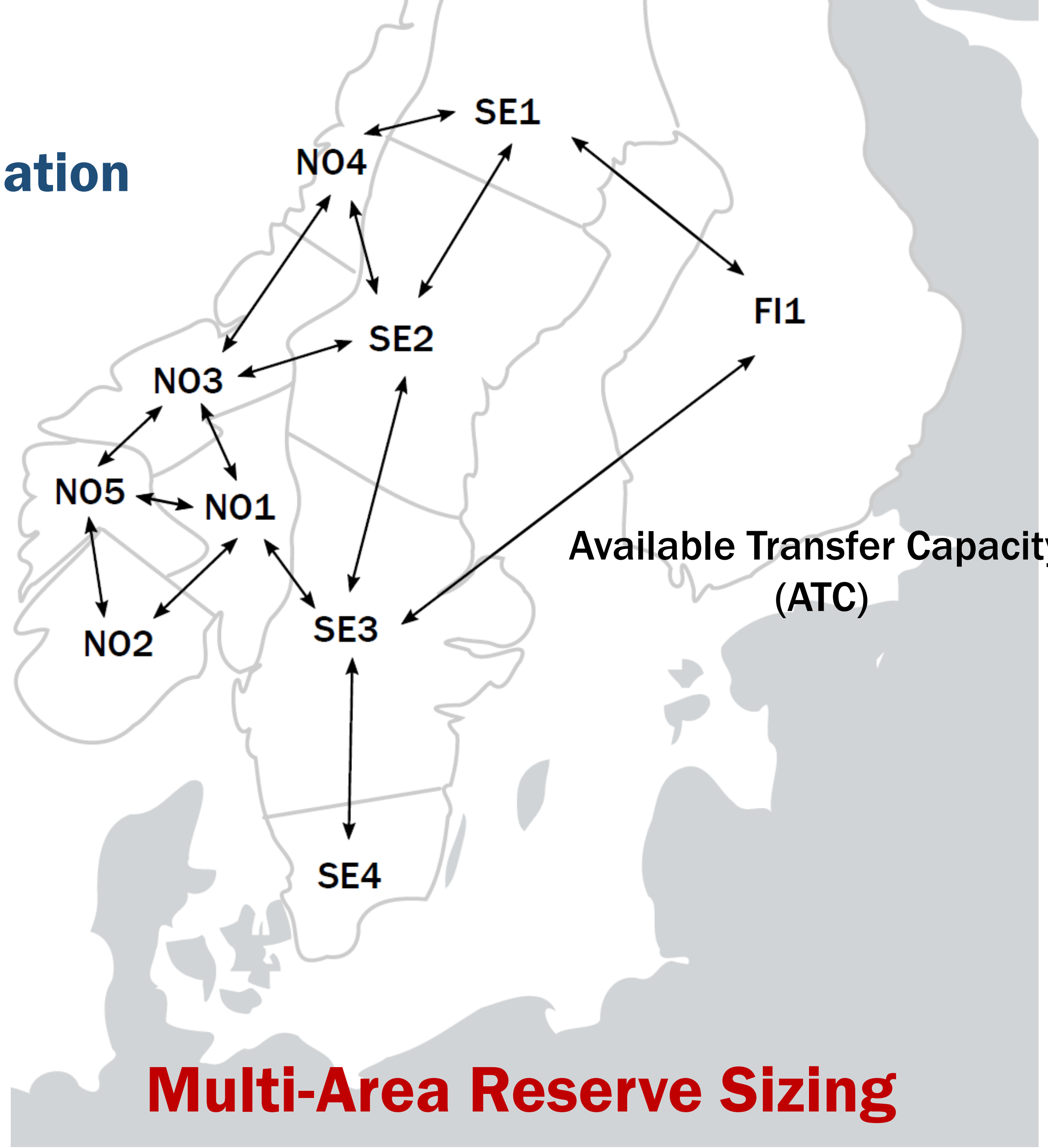
Motivation

Cross-Zonal Coordination



Available Transfer Capacity (ATC)

Multi-Area Reserve Sizing

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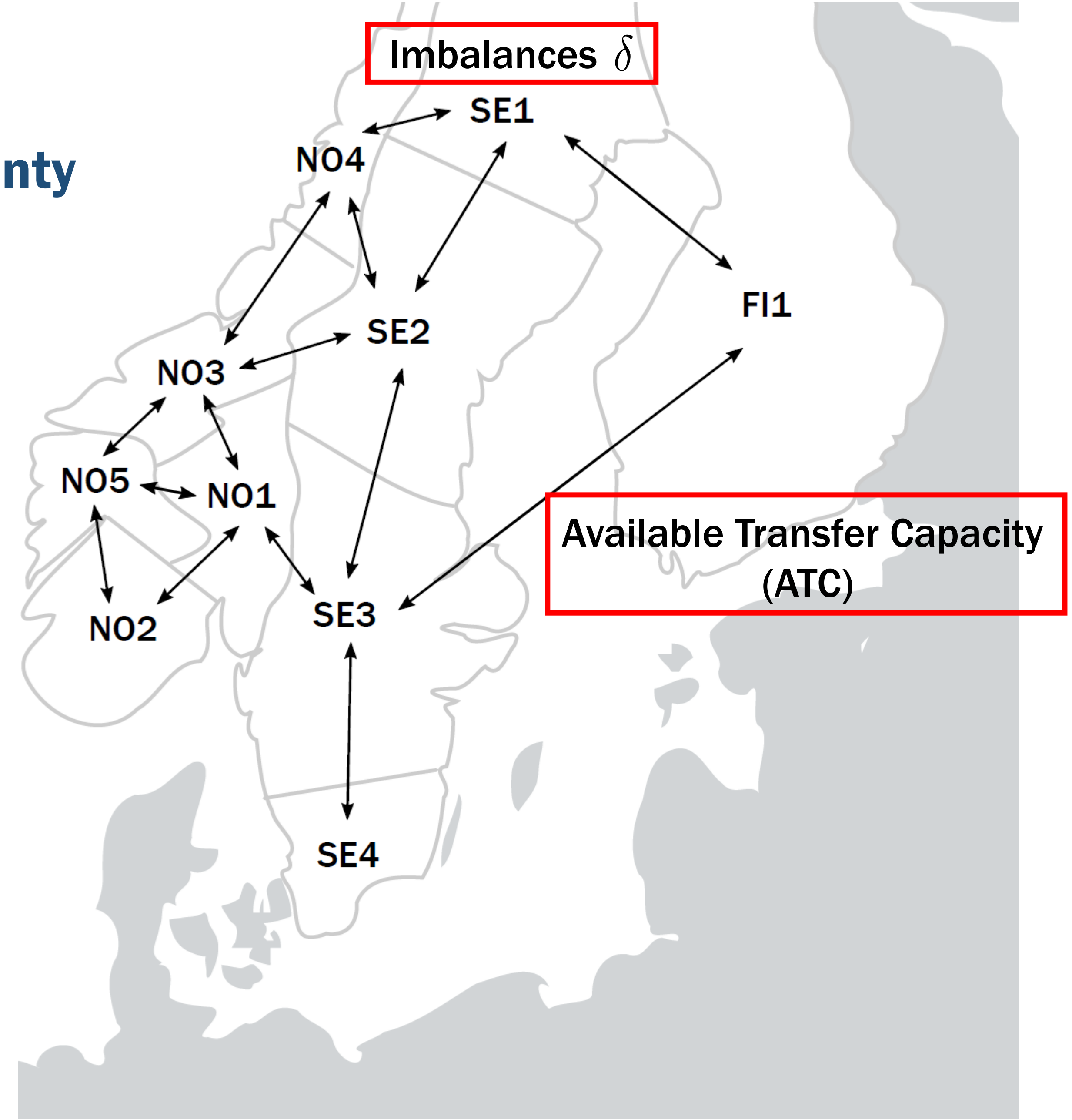
Motivation

Sources of Uncertainty



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Motivation

Probabilistic Requirements

Paragraphs (h,i) in article 157
of the System Operation Guideline of the EU

All system operators of each zone shall ensure that the positive [resp. negative] reserve capacity is sufficient to cover the positive [resp. negative] imbalances for at least 99 % of the time, based on the historical records.

TITLE 6

FREQUENCY RESTORATION RESERVES

Article 157

FRR dimensioning

1. All TSOs of a LFC Block shall set out FRR dimensioning rules in the LFC Block operational agreement.

2. The FRR dimensioning rules shall include at least the following:

(a) all TSOs of a LFC block in the CE and Nordic synchronous areas shall determine the required reserve capacity of FRR of the LFC block based on consecutive historical records comprising at least the historical LFC block imbalance values. The sampling of those historical records shall cover at least the time to restore frequency. The time period considered for those records shall be representative and include at least one full year period ending not earlier than 6 months before the calculation date;

(b) all TSOs of a LFC block in the CE and Nordic synchronous areas shall determine the reserve capacity on FRR of the LFC block sufficient to respect the current FRCE target parameters in Article 128 for the time period referred to in point (a) based at least on a probabilistic methodology. In using that probabilistic methodology, the TSOs shall take into account the restrictions defined in the agreements for the sharing or exchange of reserves due to possible violations of operational security and the FRR availability requirements. All TSOs of a LFC block shall take into account any expected significant changes to the distribution of LFC block imbalances or take into account other relevant influencing factors relative to the time period considered;

(c) all TSOs of a LFC block shall determine the ratio of automatic FRR, manual FRR, the automatic FRR full activation time and manual FRR full activation time in order to comply with the requirement of paragraph (b). For that purpose, the automatic FRR full activation time of a LFC block and the manual FRR full activation time of the LFC block shall not be more than the time to restore frequency;

(d) the TSOs of a LFC block shall determine the size of the reference incident which shall be the largest imbalance that may result from an instantaneous change of active power of a single power generating module, single demand facility, or single HVDC interconnector or from a tripping of an AC line within the LFC block;

(e) all TSOs of a LFC block shall determine the positive reserve capacity on FRR, which shall not be less than the positive dimensioning incident of the LFC block;

(f) all TSOs of a LFC block shall determine the negative reserve capacity on FRR, which shall not be less than the negative dimensioning incident of the LFC block;

(g) all TSOs of a LFC block shall determine the reserve capacity on FRR of a LFC block, any possible geographical limitations for its distribution within the LFC block and any possible geographical limitations for any exchange of reserves or sharing of reserves with other LFC blocks to comply with the operational security limits;

(h) all TSOs of a LFC block shall ensure that the positive reserve capacity on FRR or a combination of reserve capacity on FRR and RR is sufficient to cover the positive LFC block imbalances for at least 99 % of the time, based on the historical records referred to in point (a);

(i) all TSOs of a LFC block shall ensure that the negative reserve capacity on FRR or a combination of reserve capacity on FRR and RR is sufficient to cover the negative LFC block imbalances for at least 99 % of the time, based on the historical record referred to in point (a);

(j) all TSOs of a LFC block may reduce the positive reserve capacity on FRR of the LFC block resulting from the FRR dimensioning process by concluding a FRR sharing agreement with other LFC blocks in accordance with provisions in Title 8. The following requirements shall apply to that sharing agreement:

(i) for the CE and Nordic synchronous areas, the reduction of the positive reserve capacity on FRR of a LFC block

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All system operators of each zone shall ensure that the positive [resp. negative] reserve capacity is sufficient to cover the positive [resp. negative] imbalances for at least 99 % of the time, based on the historical records.

Chance-Constrained Multi-Area Reserve Sizing

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Motivation

Complexity of Chance-Constrained Multi-Area Reserve Sizing Problem

Two-Zone Case with Unlimited Capacity

Imbalance Distribution

$$\delta_1, \delta_2 \sim N(\mu, \sigma^2)$$

$$\mu = 0, \sigma = 100$$

What is the size of the (positive) reserve which meets the reliability target of 99%?

Motivation

Complexity of Chance-Constrained Multi-Area Reserve Sizing Problem

Two-Zone Case with Unlimited Capacity

Imbalance Distribution

$$\delta_1, \delta_2 \sim N(\mu, \sigma^2)$$

$$\mu = 0, \sigma = 100$$

What is the size of the (positive) reserve which meets the reliability target of 99%?

$$\Pr[-(\delta_1 + \delta_2) \ \leq r_1^+ + r_2^+] \geq 0.99$$

$$r_1^+ + r_2^+ = 2\mu + z_{0.01} \cdot \sqrt{2}\sigma \approx 329.5$$

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Motivation

Complexity of Chance-Constrained Multi-Area Reserve Sizing Problem

Two-Zone Case with Capacity Limit

$$T^{+/-} = 100$$

Imbalance Distribution

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What is the size of the (positive) reserve which meets the reliability target of 99%?

What if the imbalances are not Gaussian-distributed? Not independent?

What if the Capacity Limit (ATC) is also uncertain?

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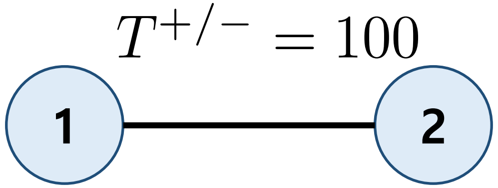
Complexity of Chance-Constrained Multi-Area Reserve Sizing Problem

Two-Zone Case with Capacity Limit



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Chance-Constrained Optimization Problem

Basic Formulations

Two-Stage Chance-Constrained Formulation

$$
\begin{aligned}
\min \sum_{z \in Z} & (r_z^+ + r_z^-) \\
\text{s.t.}\ & \Pr\{r^{+/-} \in F^{+/-}\} \geq 1 - \epsilon^{+/-} \\
& r^{+/-} \geq 0
\end{aligned}
$$

Basic Formulations

Two-Stage Chance-Constrained Formulation

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\min \; & \sum_{z\in Z}(r_z^+ + r_z^-) \\
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& r^{+/-} \geq 0
\end{aligned}
$$

$$
\begin{aligned}
F^+ = \{r^+ \in \mathbb{R}_+^{|Z|} : \exists(p, f) \text{ s.t.} \\
& p_z + \delta_z = \sum_{e=(z,\cdot)\in E} f_e - \sum_{e=(\cdot,z)\in E} f_e, && \forall z \in Z \\
& p_z \leq r_z^+, && \forall z \in Z \\
& -T_e^- \leq f_e \leq T_e^+, && \forall e \in E\}
\end{aligned}
$$

$$
\begin{aligned}
F^- = \{r^- \in \mathbb{R}_+^{|Z|} : \exists(p, f) \text{ s.t.} \\
& p_z + \delta_z = \sum_{e=(z,\cdot)\in E} f_e - \sum_{e=(\cdot,z)\in E} f_e, && \forall z \in Z \\
& -r_z^- \leq p_z, && \forall z \in Z \\
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Basic Formulations

Two-Stage Chance-Constrained Formulation

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$$r^{+/-} \geq 0$$

Sample Approximation

Generate scenarios for uncertainty realizations

$$F^+ = \{r^+ \in \mathbb{R}_+^{|Z|} : \exists (p,f) \text{ s.t.}$$
$$p_z + \delta_z = \sum_{e=(z,\cdot)\in E} f_e - \sum_{e=(\cdot,z)\in E} f_e, \quad \forall z \in Z$$
$$p_z \leq r_z^+, \quad \forall z \in Z$$
$$-T_e^- \leq f_e \leq T_e^+, \quad \forall e \in E\}$$

$$F^- = \{r^- \in \mathbb{R}_+^{|Z|} : \exists (p,f) \text{ s.t.}$$
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Basic Formulations

Big-M Style Reformulation

$$
\begin{aligned}
&\min \sum_{z\in Z}(r_z^+ + r_z^-) \\
&\text{s.t. } p_{zi} + l_{zi}^+ - l_{zi}^- + \delta_{zi} = \sum_{e=(z,\cdot)\in E} f_{ei} - \sum_{e=(\cdot,z)\in E} f_{ei}, \quad \forall z\in Z, i\in[N] \\
&\quad -r_z^- \le p_{zi} \le r_z^+, \quad \forall z\in Z, i\in[N] \\
&\quad l_{zi}^+ \le \max\{0, -\delta_{zi}\}\cdot u_i^+, \quad \forall z\in Z, i\in[N] \\
&\quad l_{zi}^- \le \max\{0, \delta_{zi}\}\cdot u_i^-, \quad \forall z\in Z, i\in[N] \\
&\quad -T_{ei}^- \le f_{ei} \le T_{ei}^+, \quad \forall e\in E, i\in[N] \\
&\quad \sum_{i\in N} u_i^{+/-} \le \left\lfloor \epsilon^{+/-} N \right\rfloor \\
&\quad r^{+/-} \ge 0, l^{+/-} \ge 0, u^{+/-} \in \{0,1\}^N
\end{aligned}
$$

$u_i^{+/-}$ **: binary variables representing**

$u_i^{+/-} = 0 \implies r^{+/-} \in F_i^{+/-}$

$l_{zi}^{+/-}$ **: slack variables that are non-zero when** $u_i^{+/-} = 1$

Basic Formulations

Big-M Style Reformulation – Heuristic Method

LP relaxation

$$
\begin{aligned}
&\min \sum_{z \in Z} (r_z^+ + r_z^-) \\
&\text{s.t. } p_{zi} + l_{zi}^+ - l_{zi}^- + \delta_{zi} = \sum_{e=(z,\cdot)\in E} f_{ei} - \sum_{e=(\cdot,z)\in E} f_{ei}, \quad \forall z \in Z, i \in [N] \\
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&\quad \sum_{i \in N} u_i^{+/-} \le \left\lfloor \epsilon^{+/-} N \right\rfloor \\
&\quad r^{+/-} \ge 0, l^{+/-} \ge 0, u^{+/-} \in [0,1]^N
\end{aligned}
$$

Heuristic Method

Step 1 : Solve the LP relaxation problem (left)

Step 2 : Pick $\lfloor \epsilon^{+/-} N \rfloor$ largest values from $\{u_i^{*+/-}, i \in [N]\}$

Step 3 : Fix them as 1 and the rest as 0

Step 4 : Solve the optimization problem with fixed $u_i^{+/-}$

Exact Mixed-Integer Programming Approach

Step 1 : Minimal Projection Formulation

$$\min \sum_{z\in Z}(r_z^+ + r_z^-)$$
$$\text{s.t. } \Pr\{r^{+/-} \in F^{+/-}\} \geq 1-\epsilon^{+/-}$$
$$r^{+/-} \geq 0$$

Projection on (r^+, r^-) space →

$$\min \sum_{z\in Z}(r_z^+ + r_z^-)$$
$$\text{s.t. } \Pr\{T^{+/-}r^{+/-} \geq \xi^{+/-}\} \geq 1-\epsilon^{+/-}$$
$$r^{+/-} \geq 0$$

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$$r^{+/-} \geq 0$$

Integer Programming Techniques

Mixing Inequalities

Strong Extended Formulation

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Exact Mixed-Integer Programming Approach

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$$r^{+/-} \geq 0$$

Projection on (r^+, r^-) space

$$\min \sum_{z\in Z} (r_z^+ + r_z^-)$$
$$\text{s.t. } \Pr\{T^{+/-} r^{+/-} \geq \xi^{+/-}\} \geq 1-\epsilon^{+/-}$$
$$r^{+/-} \geq 0$$

Step 2 : Strengthened Minimal Projection Formulation

$$\min \sum_{z\in Z} (r_z^+ + r_z^-)$$
$$\text{s.t. } \sum_{z\in S} r_z^{+/-} + \sum_{i=1}^{q^{+/-}} (h_{S,i}^{+/-} - h_{S,i+1}^{+/-}) w_{S,i}^{+/-} \geq h_{S,1}^{+/-}, S \in \mathcal{W}(\mathcal{G})$$
$$w_{S,i}^{+/-} - w_{S,i+1}^{+/-} \geq 0, \quad \forall i \in [q^{+/-}-1], S \in \mathcal{W}(\mathcal{G})$$
$$u_{\sigma_{S,i}^{+/-}}^{+/-} - w_{S,i}^{+/-} \geq 0, \quad \forall i \in [q^{+/-}], S \in \mathcal{W}(\mathcal{G})$$
$$\sum_{i=1}^{N} u_i^{+/-} \leq q^{+/-}$$
$$r^{+/-} \geq 0, u^{+/-} \in \{0,1\}^N, w^{+/-} \in \{0,1\}^{q^{+/-}\cdot|\mathcal{W}(\mathcal{G})|}$$

Integer Programming Techniques

Mixing Inequalities

Strong Extended Formulation

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Exact Mixed-Integer Programming Approach

Step 1 : Minimal Projection Formulation

$Theorem\ 2.1 : Proj_{(r^+,r^-)}(F) = F_r$

$Theorem\ 2.3 : F_r$ is a minimal representation on the space of (r^+, r^-)

$$F = \{(r^+, r^-, p, f) \in \mathbb{R}_+^{|Z|} \times \mathbb{R}_+^{|Z|} \times \mathbb{R}^{|Z|} \times \mathbb{R}^{|E|} : \begin{array}{ll} p_z + \delta_z = \sum_{e=(z,\cdot)\in E} f_e - \sum_{e=(\cdot,z)\in E} f_e, & z \in Z \\ -r_z^- \leq p_z \leq r_z^+, & z \in Z \\ -T_e^- \leq f_e \leq T_e^+, & e \in E \end{array} \}$$

$$F_r = \{(r^+, r^-) \in \mathbb{R}_+^{|Z|} \times \mathbb{R}_+^{|Z|} : \begin{array}{ll} \sum_{z\in S} r_z^- \geq \sum_{z\in S} \delta_z - O(S|E), & S \in \mathcal{W}(\mathcal{G}) \\ \sum_{z\in S} r_z^+ \geq -\sum_{z\in S} \delta_z - I(S|E), & S \in \mathcal{W}(\mathcal{G}) \end{array} \}$$

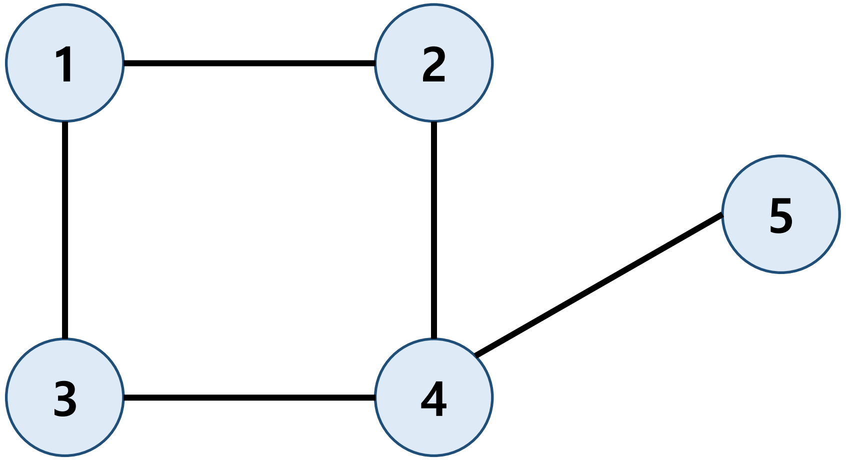
Exact Mixed-Integer Programming Approach

Step 1 : Minimal Projection Formulation

Definition 2.1 (Conneted Vertex Set) : For a graph $\mathcal{G}(V, E)$, the connected vertex set $\mathcal{W}(\mathcal{G})$ is defined as follows:

$$\mathcal{W}(\mathcal{G}) = \{S \subseteq V : \forall v, w \in S, \exists \text{ a path } P \text{ on } \mathcal{G} \text{ s.t. } v, w \in V(P) \subseteq S\},$$

where $V(P)$ denotes the set of vertices in the path P.



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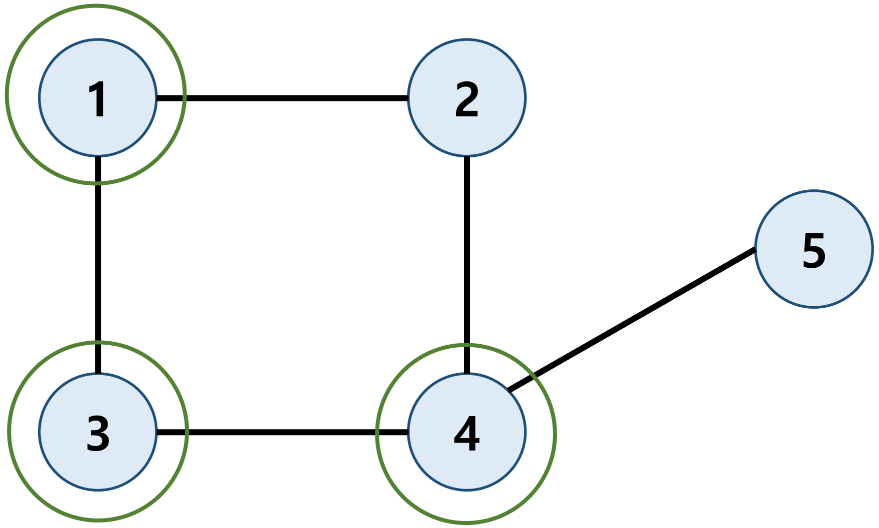
Exact Mixed-Integer Programming Approach

Step 1 : Minimal Projection Formulation

Definition 2.1 (Conneted Vertex Set) : For a graph $\mathcal{G}(V, E)$, the connected vertex set $\mathcal{W}(\mathcal{G})$ is defined as follows:

$$\mathcal{W}(\mathcal{G}) = \{S \subseteq V : \forall v, w \in S, \exists \text{ a path } P \text{ on } \mathcal{G} \text{ s.t. } v, w \in V(P) \subseteq S\},$$

where $V(P)$ denotes the set of vertices in the path P.



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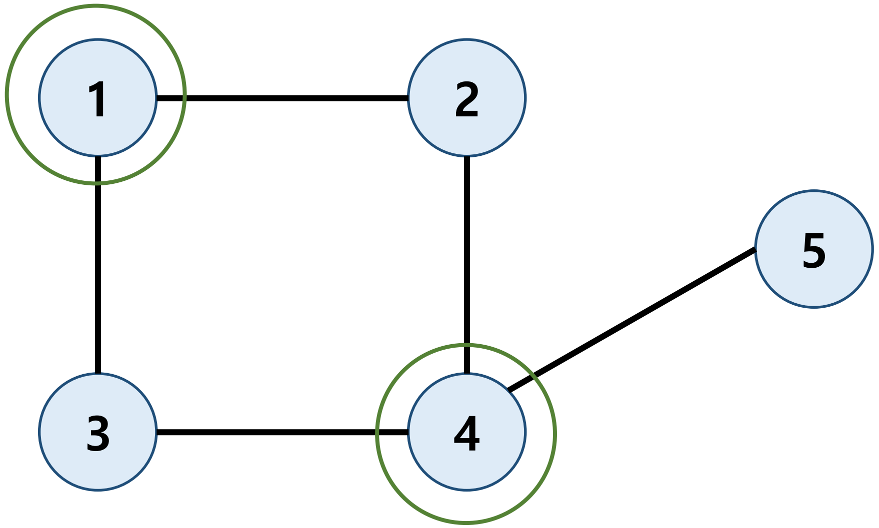

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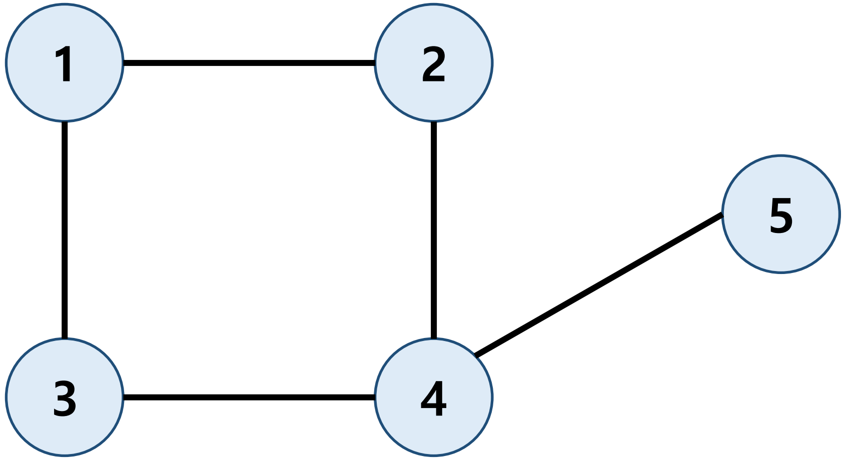

Total : 21

{1},{2},{3},{4},{5}
{1,2},{1,3},{2,4},{3,4},{4,5}
{1,2,3},{1,2,4},{1,3,4},{2,3,4},{2,4,5},{3,4,5}
{1,2,3,4},{1,2,4,5},{1,3,4,5},{2,3,4,5}
{1,2,3,4,5}

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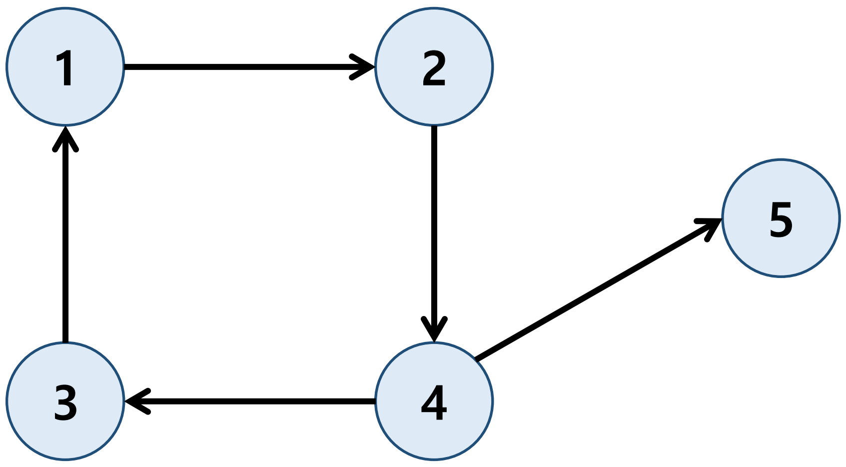
Exact Mixed-Integer Programming Approach

Step 1 : Minimal Projection Formulation

Definition 2.2 (Maximum Input/Output Flow): For a directed graph $\mathcal{G}(V, E)$ where $\forall e \in E, f(e)$ denotes the flow in e and $-T_e^- \leq f(e) \leq T_e^+$, for all $S \subseteq V, E' \subseteq E$,

$$I(S|E') = \sum_{v \in S, w \in S^c : (v,w) \in E'} T^-_{(v,w)} + \sum_{v \in S, w \in S^c : (w,v) \in E'} T^+_{(w,v)},$$

$$O(S|E') = \sum_{v \in S, w \in S^c : (v,w) \in E'} T^+_{(v,w)} + \sum_{v \in S, w \in S^c : (w,v) \in E'} T^-_{(w,v)}.$$



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Exact Mixed-Integer Programming Approach

Step 1 : Minimal Projection Formulation

Definition 2.2 (Maximum Input/Output Flow): For a directed graph $\mathcal{G}(V,E)$ where $\forall e \in E, f(e)$ denotes the flow in e and $-T_e^- \le f(e) \le T_e^+$, for all $S \subseteq V, E' \subseteq E$,

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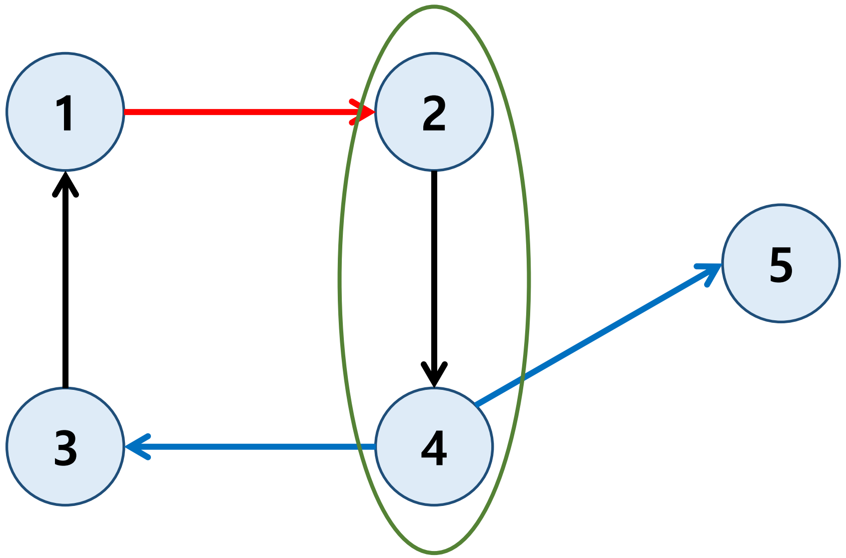

$$I(\{2,4\}|E) = T^+_{(1,2)} + T^-_{(4,3)} + T^-_{(4,5)}$$

$$O(\{2,4\}|E) = T^-_{(1,2)} + T^+_{(4,3)} + T^+_{(4,5)}$$

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Exact Mixed-Integer Programming Approach

Step 1 : Minimal Projection Formulation

$$F_r = \{(r^+, r^-) \in \mathbb{R}_+^{|Z|} \times \mathbb{R}_+^{|Z|} : \begin{array}{ll} \sum_{z\in S} r_z^- \geq \sum_{z \in S} \delta_z - O(S|E), & S \in \mathcal{W}(\mathcal{G}) \\ \sum_{z\in S} r_z^+ \geq -\sum_{z \in S} \delta_z - I(S|E), & S \in \mathcal{W}(\mathcal{G}) \end{array} \}$$

Two Zones Example

$$\begin{array}{ll} r_1^+ + r_2^+ \geq -\delta_1 - \delta_2 & r_1^- + r_2^- \geq \delta_1 + \delta_2 \\ r_1^+ \geq -\delta_1 - T^- & r_1^- \geq \delta_1 - T^+ \\ r_2^+ \geq -\delta_2 - T^+ & r_2^- \geq \delta_2 - T^- \end{array}$$

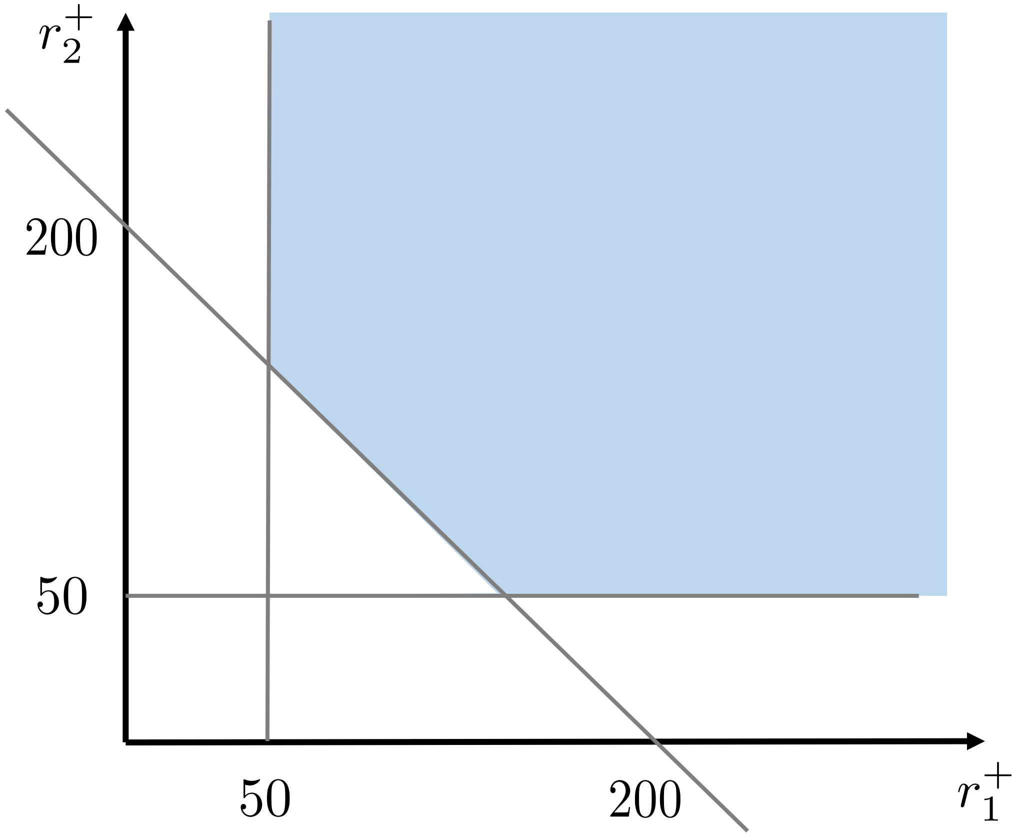
Exact Mixed-Integer Programming Approach

Step 1 : Minimal Projection Formulation

Two Zones Example (Positive Reserve)

$$r_1^+ + r_2^+ \geq -\delta_1 - \delta_2$$
$$r_1^+ \geq -\delta_1 - T^-$$
$$r_2^+ \geq -\delta_2 - T^+$$

Ex) $\delta_1, \delta_2 = -100$ **&** $T^+, T^- = 50$



Exact Mixed-Integer Programming Approach

Step 2 : Strengthened Minimal Projection Formulation

$$\min \sum_{z \in Z} (r_z^+ + r_z^-)$$
$$\text{s.t. } \Pr\{r^{+/-} \in F^{+/-}\} \geq 1 - \epsilon^{+/-}$$
$$r^{+/-} \geq 0$$

Projection on (r^+, r^-) space →

$$\min \sum_{z \in Z} (r_z^+ + r_z^-)$$
$$\text{s.t. } \Pr\{T^{+/-} r^{+/-} \geq \xi_i^{+/-}\}$$
$$r^{+/-} \geq 0$$

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Exact Mixed-Integer Programming Approach

Step 2 : Strengthened Minimal Projection Formulation

$$\begin{aligned} \min \ & \sum_{z \in Z} (r_z^+ + r_z^-) \\ \text{s.t.} \ & \Pr\{r^{+/-} \in F^{+/-}\} \geq 1 - \epsilon^{+/-} \\ & r^{+/-} \geq 0 \end{aligned}$$

Projection on (r^+, r^-) space

$$\begin{aligned} \min \ & \sum_{z \in Z} (r_z^+ + r_z^-) \\ \text{s.t.} \ & \Pr\{T^{+/-} r^{+/-} \geq \xi_i^{+/-}\} \\ & r^{+/-} \geq 0 \end{aligned}$$

Sample Approximation

$$\begin{aligned} \min \ & \sum_{z \in Z} (r_z^+ + r_z^-) \\ \text{s.t.} \ & u_i^{+/-} = 0 \implies T^{+/-} r^{+/-} \geq \xi_i^{+/-}, \quad \forall i \in [N] \\ & \sum_{i \in N} u_i^{+/-} \leq \lfloor \epsilon N \rfloor \\ & r^{+/-} \geq 0, u^{+/-} \in \{0,1\}^N \end{aligned}$$

Exact Mixed-Integer Programming Approach

Step 2 : Strengthened Minimal Projection Formulation

$$\begin{aligned}\min\ & \sum_{z\in Z}(r_z^+ + r_z^-)\\ \text{s.t. } & \Pr\{r^{+/-} \in F^{+/-}\} \geq 1-\epsilon^{+/-}\\ & r^{+/-} \geq 0\end{aligned}$$

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$$\begin{aligned}\min\ & \sum_{z\in Z}(r_z^+ + r_z^-)\\ \text{s.t. } & u_i^{+/-} = 0 \implies T^{+/-}r^{+/-} \geq \xi_i^{+/-}, \quad \forall i \in [N]\\ & \sum_{i\in N} u_i^{+/-} \leq \lfloor \epsilon N \rfloor\\ & r^{+/-} \geq 0, u^{+/-} \in \{0,1\}^N\end{aligned}$$

Big-M Reformulation ←

$$T^{+/-}r^{+/-} + \xi_i^{+/-}u_i^{+/-} \geq \xi_i^{+/-}$$

Mixing Set

Exact Mixed-Integer Programming Approach

Step 2 : Strengthened Minimal Projection Formulation

Mixing Set

$$P = \{(y,u) \in \mathbb{R}_+ \times \{0,1\}^N : y + h_i u_i \geq h_i, i \in [N]\}$$

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Exact Mixed-Integer Programming Approach

Step 2 : Strengthened Minimal Projection Formulation

Mixing Set

$$P = \{(y, u) \in \mathbb{R}_+ \times \{0,1\}^N : y + h_i u_i \geq h_i, i \in [N]\}$$

$$h_1 \geq h_2 \geq \cdots \geq h_N$$

Mixing Inequalities

$$y + \sum_{j=1}^{l} (h_{t_j} - h_{t_{j+1}}) u_{t_j} \geq h_{t_1}, \forall \{t_1, \ldots, t_l\} \subset [N],$$

Convex hull defining inequalities

Exact Mixed-Integer Programming Approach

Step 2 : Strengthened Minimal Projection Formulation

Mixing Set

$$P = \{(y,u) \in \mathbb{R}_+ \times \{0,1\}^N : y + h_i u_i \geq h_i, i \in [N]\}$$

$$h_1 \geq h_2 \geq \cdots \geq h_N$$

Mixing Inequalities

$$y + \sum_{j=1}^{l} (h_{t_j} - h_{t_{j+1}}) u_{t_j} \geq h_{t_1}, \forall \{t_1, \ldots, t_l\} \subset [N]$$

Convex hull defining inequalities

Mixing Set with a Cardinality Constraint

$$G = \{(y,u) \in \mathbb{R}_+ \times \{0,1\}^N : \sum_{i=1}^{N} u_i \leq q, y + h_i u_i \geq h_i, i \in [N]\}$$

$$q = \lfloor \epsilon N \rfloor$$

Strengthened Mixing Inequalities

$$y + \sum_{j=1}^{l} (h_{t_j} - h_{t_{j+1}}) u_{t_j} \geq h_{t_1}, \forall \{t_1, \ldots, t_l\} \subset [q]$$

Facet defining inequalities

Exact Mixed-Integer Programming Approach

Step 2 : Strengthened Minimal Projection Formulation

Strong Extended Formulation

$$EG := \{(y,u,w) \in \mathbb{R}_+ \times \{0,1\}^{n+q} : \sum_{i=1}^{N} u_i \leq q,\ y + \sum_{i=1}^{q} (h_i - h_{i+1}) w_i \geq h_1 \quad \}$$

$$w_i - w_{i+1} \geq 0, \qquad \forall i \in [q-1]$$

$$u_i - w_i \geq 0, \qquad \forall i \in [q]$$

Theorem 6 from (Luedtke 2010): $Proj_{(y,u)}(EG) = G$. Moreover, the projection of the linear relaxation of EG is the linear relaxation of G with all the strengthened mixing inequalities added.

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Exact Mixed-Integer Programming Approach

Step 2 : Strengthened Minimal Projection Formulation

$$\min \sum_{z \in Z} (r_z^+ + r_z^-)$$

$$\text{s.t.} \sum_{z \in S} r_z^{+/-} + \sum_{i=1}^{q^{+/-}} (h_{S,i}^{+/-} - h_{S,i+1}^{+/-}) w_{S,i}^{+/-} \geq h_{S,1}^{+/-}, S \in \mathcal{W}(\mathcal{G})$$

$$w_{S,i}^{+/-} - w_{S,i+1}^{+/-} \geq 0, \quad \forall i \in [q^{+/-} - 1], S \in \mathcal{W}(\mathcal{G})$$

$$u_{\sigma_{S,i}^{+/-}}^{+/-} - w_{S,i}^{+/-} \geq 0, \quad \forall i \in [q^{+/-}], S \in \mathcal{W}(\mathcal{G})$$

$$\sum_{i=1}^{N} u_i^{+/-} \leq q^{+/-}$$

$$r^{+/-} \geq 0, u^{+/-} \in \{0,1\}^N, w^{+/-} \in \{0,1\}^{q^{+/-} \cdot |\mathcal{W}(\mathcal{G})|}$$

$$q^{+/-} = \lfloor \epsilon^{+/-} N \rfloor$$

$$h_{S,\sigma_{S,i}^+}^+ = -\sum_{v \in S} \delta_{v,i} - I_i(S|E)$$

$$h_{S,\sigma_{S,i}^-}^- = \sum_{v \in S} \delta_{v,i} - O_i(S|E)$$

$\sigma_{S,i}^{+/-}$ are the permutations that rearrange the indices as

$$h_{S,1}^+ \geq h_{S,2}^+ \geq \cdots \geq h_{S,N}^+$$

$$h_{S,1}^- \geq h_{S,2}^- \geq \cdots \geq h_{S,N}^-$$

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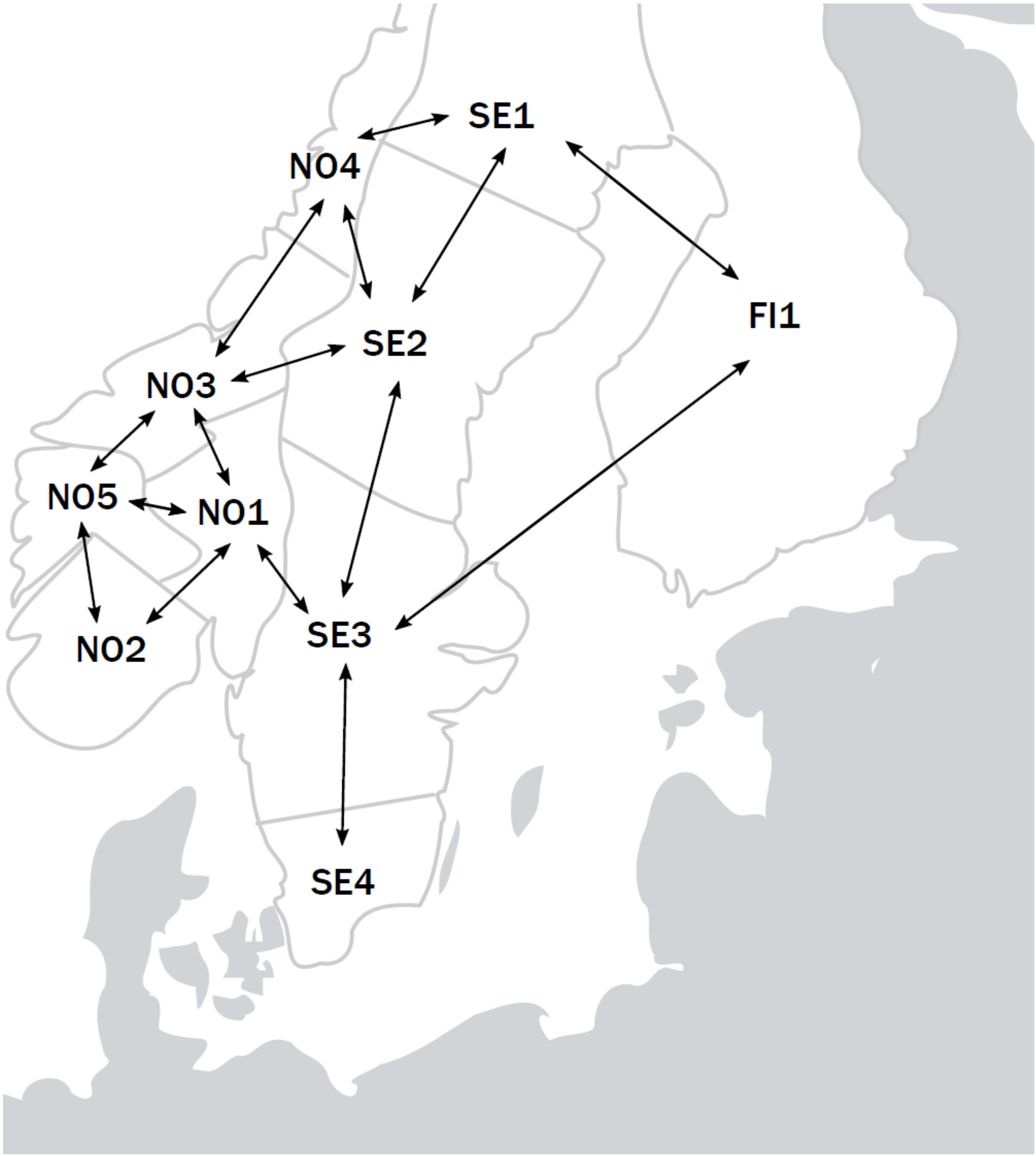
Computational Results

Nordic System Case Study

Reference data from (Boe 2017)

Imbalances data is Gaussian distributed with zero mean and standard deviation equal to the reference data

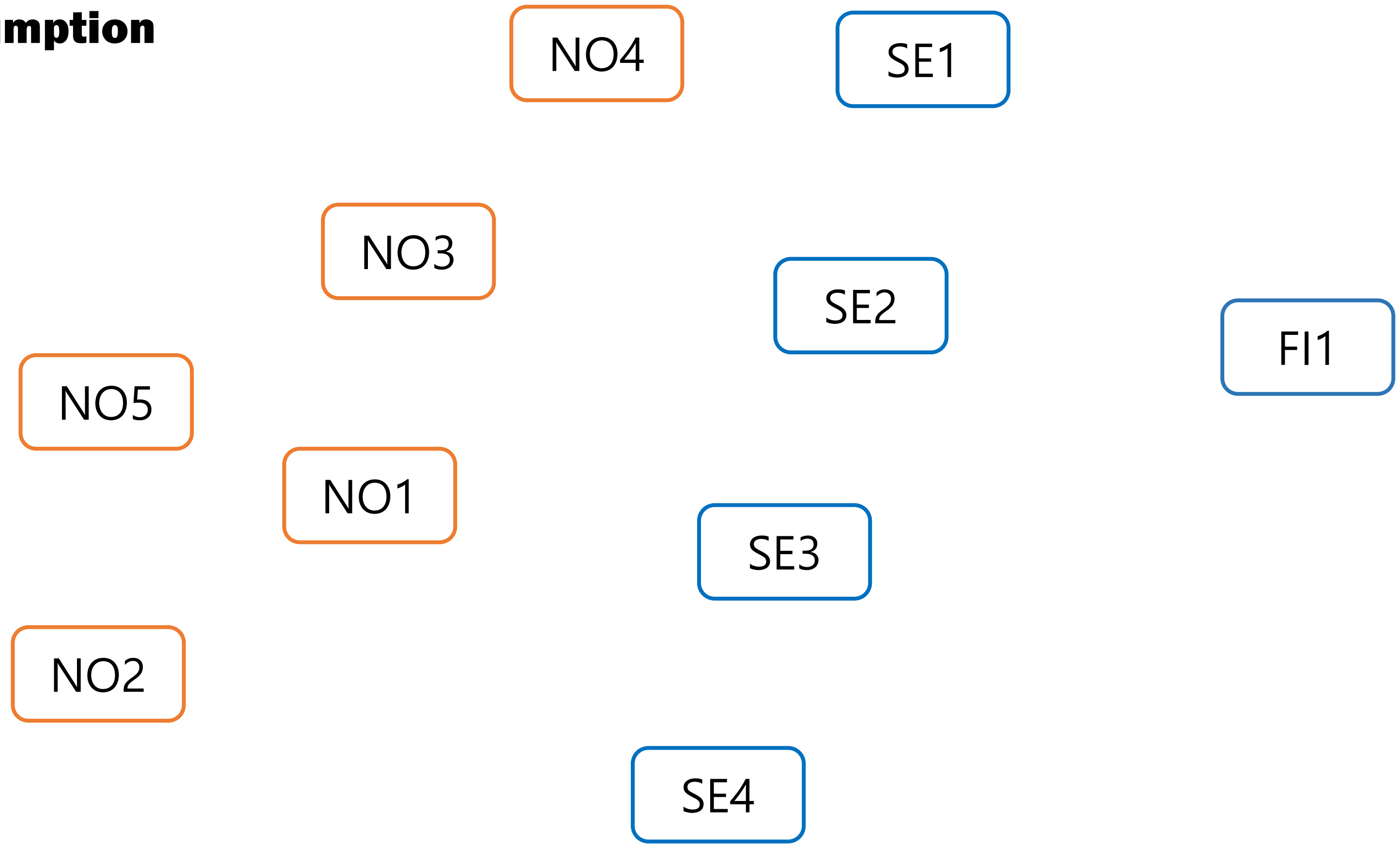
Network capacity (ATC) data is perturbed with Gaussian noise.



Computational Results

Nordic System Case Study – Effect of Coordination

Zero ATC Assumption



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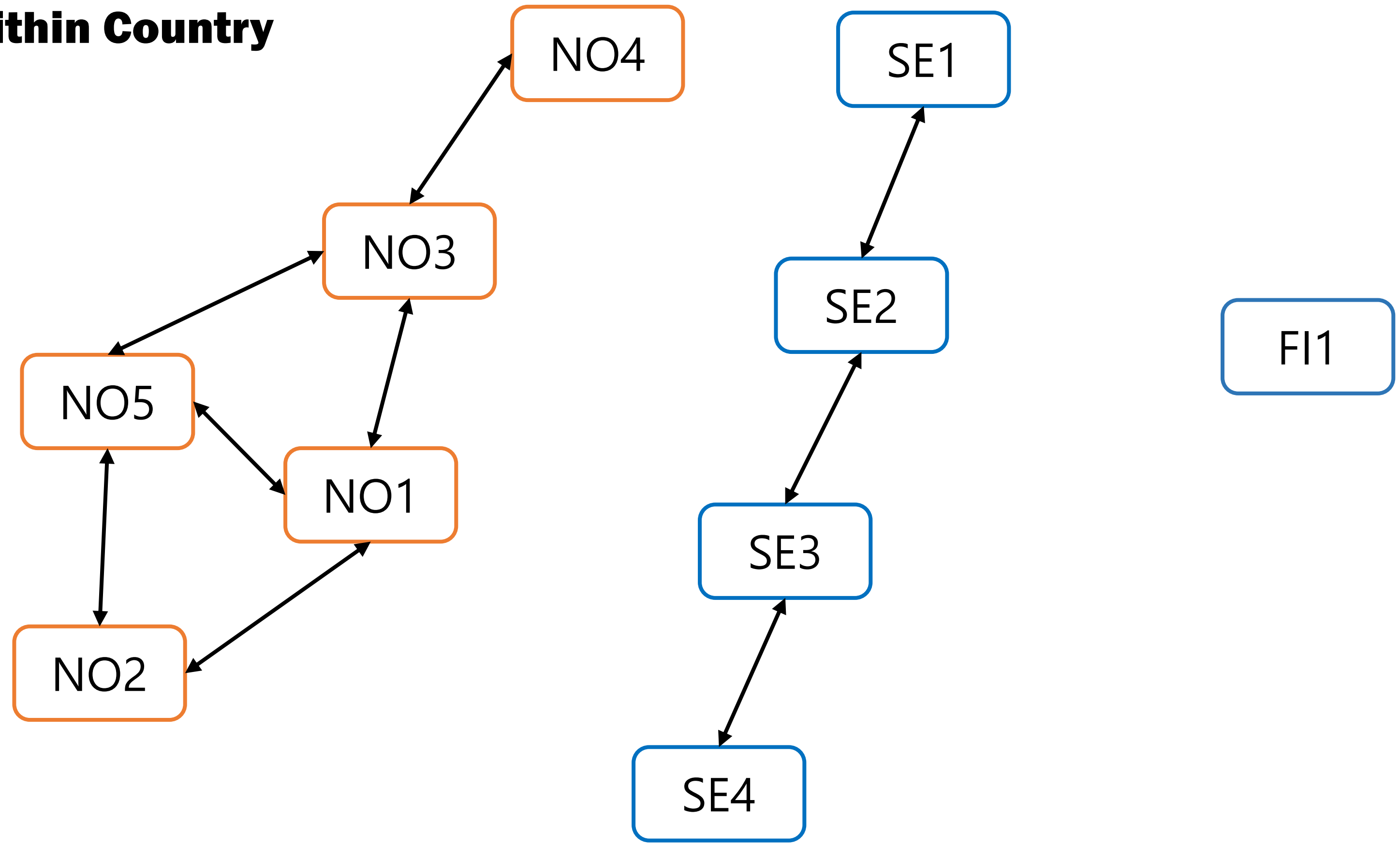
Computational Results

Nordic System Case Study – Effect of Coordination

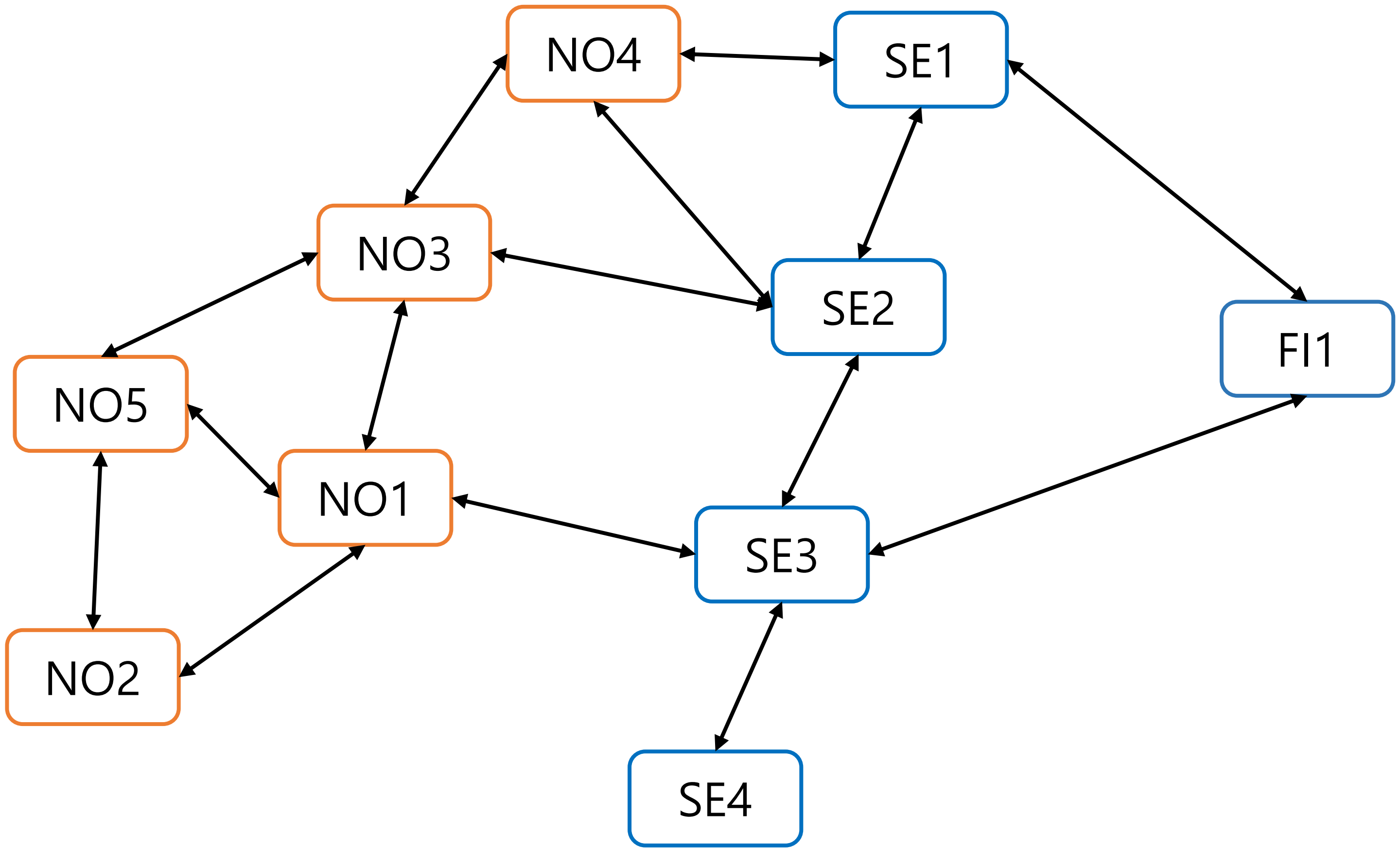
Connection within Country



Computational Results

Nordic System Case Study – Effect of Coordination

All Connected



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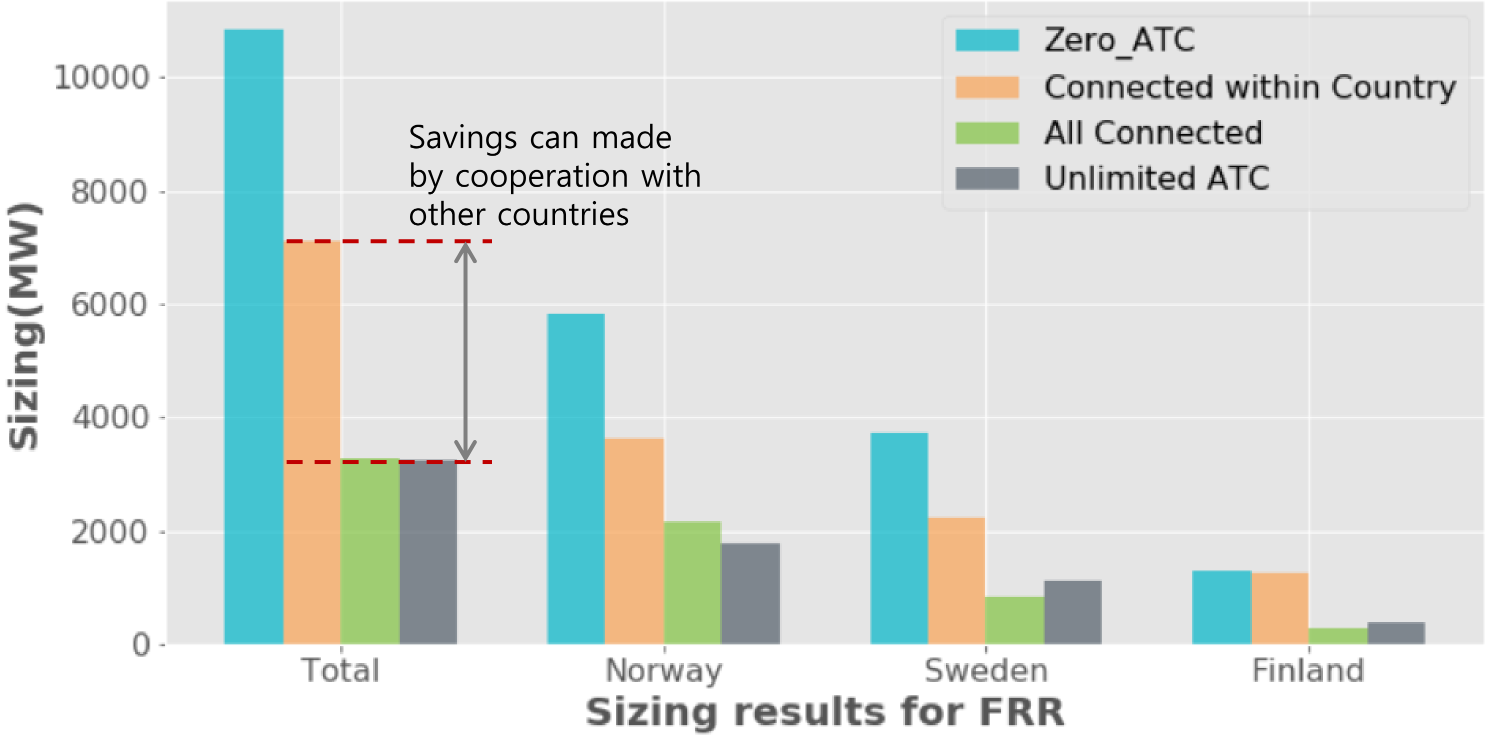
Computational Results

Nordic System Case Study – Effect of Coordination

Simulation with 50,000 Samples



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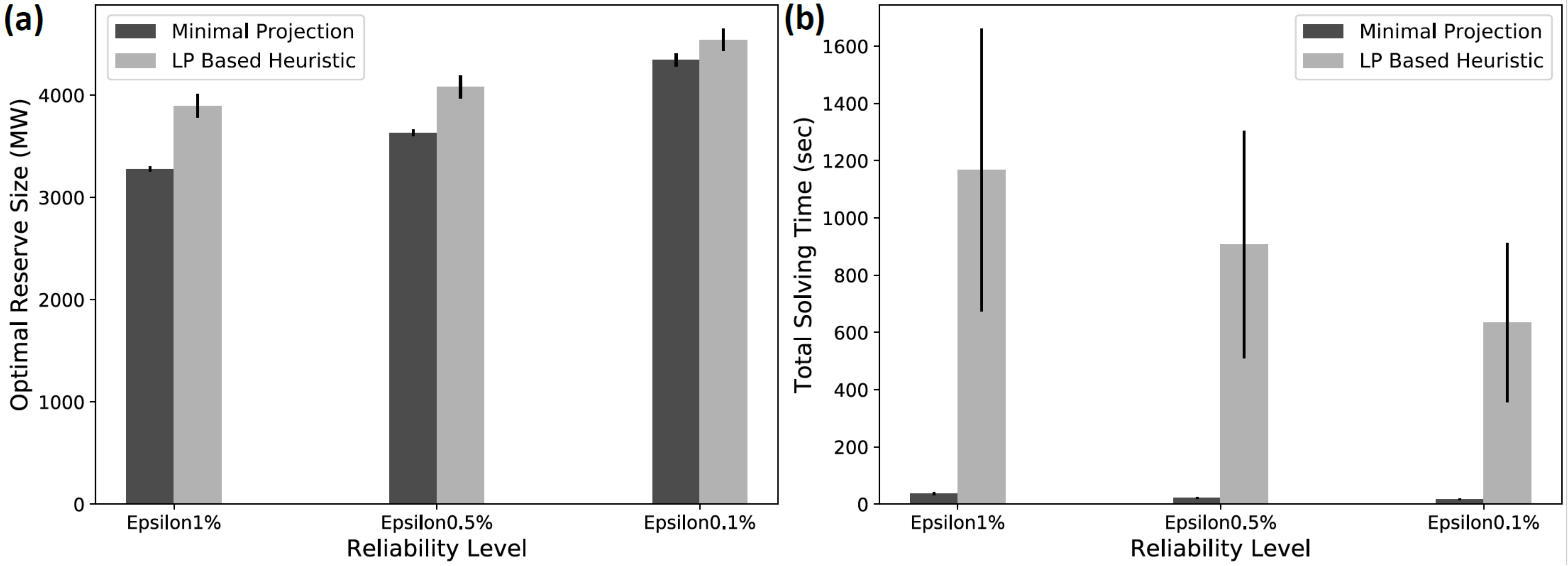
Computational Results

Nordic System Case Study – Result Comparison with the Heuristic Method



100 Times Simulation with 25,000 Samples

Computational Results

Nordic System Case Study – Result Comparison with the Heuristic Method

Heuristic Method Formulation (LP relaxation)

$$
\begin{aligned}
\min \ & \sum_{z\in Z}(r_z^+ + r_z^-) \\
\text{s.t. } & p_{zi} + l_{zi}^+ - l_{zi}^- + \delta_{zi} = \sum_{e=(z,\cdot)\in E} f_{ei} - \sum_{e=(\cdot,z)\in E} f_{ei}, \quad \forall z \in Z, i \in [N] \\
& -r_z^- \le p_{zi} \le r_z^+, \quad \forall z \in Z, i \in [N] \\
& l_{zi}^+ \le \max\{0, -\delta_{zi}\} \cdot u_i^+, \quad \forall z \in Z, i \in [N] \\
& l_{zi}^- \le \max\{0, \delta_{zi}\} \cdot u_i^-, \quad \forall z \in Z, i \in [N] \\
& -T_{ei}^- \le f_{ei} \le T_{ei}^+, \quad \forall e \in E, i \in [N] \\
& \sum_{i\in N} u_i^{+/-} \le \left\lfloor \epsilon^{+/-} N \right\rfloor \\
& r^{+/-} \ge 0, l^{+/-} \ge 0, u^{+/-} \in [0,1]^N
\end{aligned}
$$

Strengthened Minimal Projection Formulation

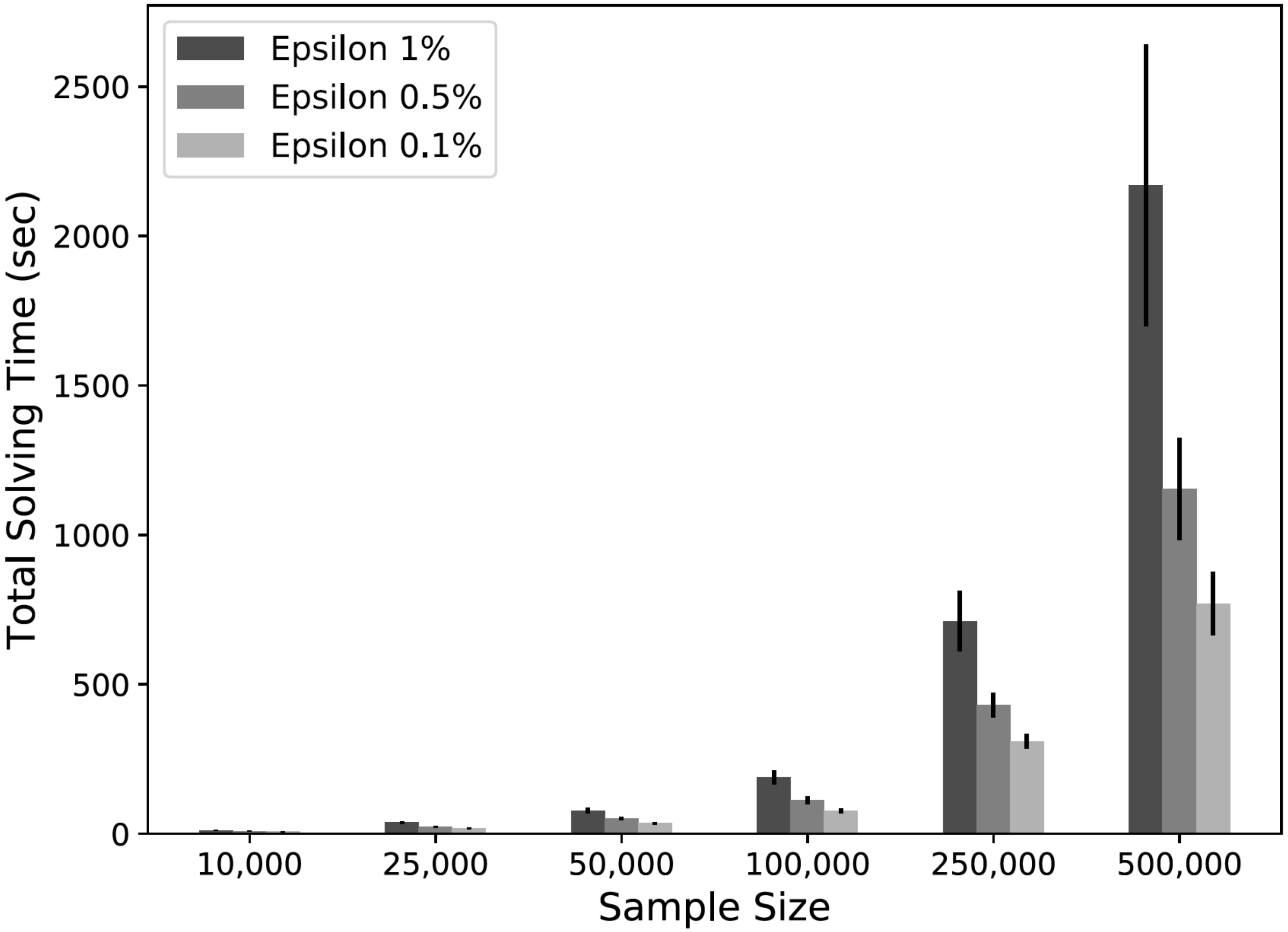
$$
\begin{aligned}
\min \ & \sum_{z\in Z}(r_z^+ + r_z^-) \\
\text{s.t. } & \sum_{z\in S} r_z^{+/-} + \sum_{i=1}^{q^{+/-}} (h_{S,i}^{+/-} - h_{S,i+1}^{+/-}) w_{S,i}^{+/-} \ge h_{S,1}^{+/-}, S \in \mathcal{W}(\mathcal{G}) \\
& w_{S,i}^{+/-} - w_{S,i+1}^{+/-} \ge 0, \quad \forall i \in [q^{+/-} - 1], S \in \mathcal{W}(\mathcal{G}) \\
& u_{\sigma_{S,i}^{+/-}}^{+/-} - w_{S,i}^{+/-} \ge 0, \quad \forall i \in [q^{+/-}], S \in \mathcal{W}(\mathcal{G}) \\
& \sum_{i=1}^{N} u_i^{+/-} \le q^{+/-} \\
& r^{+/-} \ge 0, u^{+/-} \in \{0,1\}^N, w^{+/-} \in \{0,1\}^{q^{+/-}\cdot|\mathcal{W}(\mathcal{G})|}
\end{aligned}
$$

$$
h_{S,\sigma_{S,i}^+}^+ = -\sum_{v\in S} \delta_{v,i} - I_i(S|E)
$$

$$
h_{S,\sigma_{S,i}^-}^- = \sum_{v\in S} \delta_{v,i} - O_i(S|E)
$$

Computational Results

Nordic System Case Study – Solving Time



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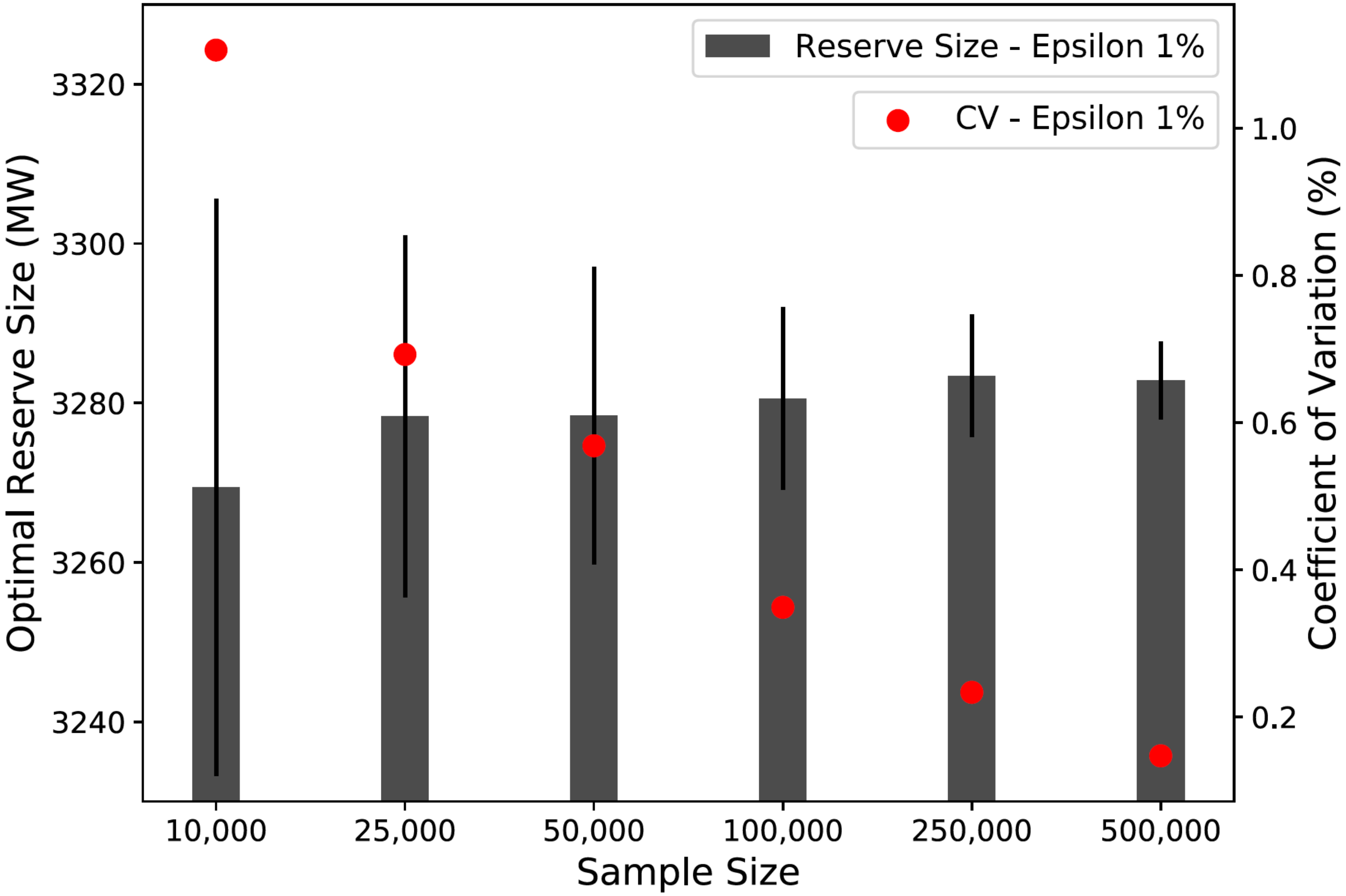
Computational Results

Nordic System Case Study – Sensitivity Analysis



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Conclusion

New Method for Chance-Constrained Multi-Area Reserve Sizing Problem

Minimal Projection Formulation

Strengthened Minimal Projection Formulation

Practical Usage

Tractable for Realistic Scale of Instances

Easy Implementation

Applicability

Transportation-Based Network

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Thank you

Appendix

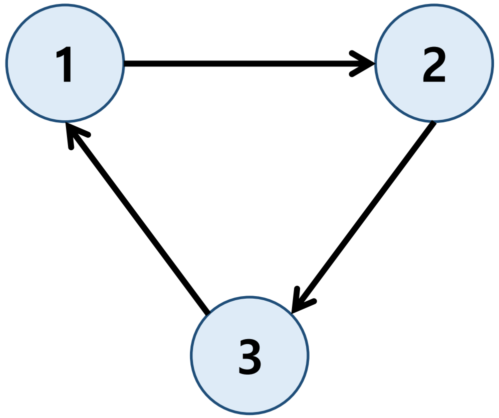
Exact Mixed-Integer Programming Approach

Step 1 : Minimal Projection Formulation

$$F_r = \{(r^+, r^-) \in \mathbb{R}_+^{|Z|} \times \mathbb{R}_+^{|Z|} : \begin{array}{ll} \sum_{z \in S} r_z^- \geq \sum_{z \in S} \delta_z - O(S|E), & S \in \mathcal{W}(\mathcal{G}) \\ \sum_{z \in S} r_z^+ \geq - \sum_{z \in S} \delta_z - I(S|E), & S \in \mathcal{W}(\mathcal{G}) \end{array} \}$$

Three Zones Example

$$r_1^+ + r_2^+ + r_3^+ \geq -\delta_1 - \delta_2 - \delta_3$$
$$r_1^+ + r_2^+ \geq -\delta_1 - \delta_2 - T_{(2,3)}^- - T_{(3,1)}^+$$
$$r_2^+ + r_3^+ \geq -\delta_2 - \delta_3 - T_{(1,2)}^+ - T_{(3,1)}^-$$
$$r_1^+ + r_3^+ \geq -\delta_1 - \delta_3 - T_{(1,2)}^- - T_{(2,3)}^+$$
$$r_1^+ \geq -\delta_1 - T_{(1,2)}^- - T_{(3,1)}^+$$
$$r_2^+ \geq -\delta_2 - T_{(1,2)}^+ - T_{(2,3)}^-$$
$$r_3^+ \geq -\delta_2 - T_{(2,3)}^+ - T_{(3,1)}^-$$



$$r_1^- + r_2^- + r_3^- \geq \delta_1 + \delta_2 + \delta_3$$
$$r_1^- + r_2^- \geq \delta_1 + \delta_2 - T_{(2,3)}^+ - T_{(3,1)}^-$$
$$r_2^- + r_3^- \geq \delta_2 + \delta_3 - T_{(1,2)}^- - T_{(3,1)}^+$$
$$r_1^- + r_3^- \geq \delta_1 + \delta_3 - T_{(1,2)}^+ - T_{(2,3)}^-$$
$$r_1^- \geq \delta_1 - T_{(1,2)}^+ - T_{(3,1)}^-$$
$$r_2^- \geq \delta_2 - T_{(1,2)}^- - T_{(2,3)}^+$$
$$r_3^- \geq \delta_2 - T_{(2,3)}^- - T_{(3,1)}^+$$

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Exact Mixed-Integer Programming Approach

Step 2 : Strengthened Minimal Projection Formulation (Implementation)

Connected Vertex Set Generation

Algorithm 1 $\mathcal{W}(\mathcal{G})$ Generation

Input: $\mathcal{G} = (V, E)$
Output: $\mathcal{W}$
Select a start node $v_0 \in V$
Initialize $\mathcal{W} = \{\{v_0\}\}, V_{sel} = \{v_0\}, E_{sel} = \emptyset$
while $E_{sel} \neq E$ **do**
  Choose $e = (v, w) \in E(V_{sel}) = \{e' \in E : \exists v' \in V_{sel}$ s.t. $e' = (v', \cdot)$ or $e' = (\cdot, v')\}$
  $E_{sel} \leftarrow E_{sel} \cup \{e\}$
  if $v, w \in V_{sel}$ **then**
    $\mathcal{W}^v \leftarrow \{S \in \mathcal{W} : v \in S\}$
    $\mathcal{W}^w \leftarrow \{S \in \mathcal{W} : w \in S\}$
    for $S_1 \in \mathcal{W}^v, S_2 \in \mathcal{W}^w$ **do**
      $\mathcal{W} \leftarrow \mathcal{W} \cup \{S_1 \cup S_2\}$
    end for
  else
    (WLOG assume $v \in V_{sel}$ and $w \notin V_{sel}$)
    $\mathcal{W} \leftarrow \mathcal{W} \cup \{\{w\}\}$
    $V_{sel} \leftarrow V_{sel} \cup \{w\}$
    $\mathcal{W}^v \leftarrow \{S \in \mathcal{W} : v \in S\}$
    for $S \in \mathcal{W}^v$ **do**
      $\mathcal{W} \leftarrow \mathcal{W} \cup \{S \cup \{w\}\}$
    end for
  end if
end while

Worst Case Complexity $\mathcal{O}(|E| \cdot 2^{|V|})$

Coefficient Generation

$$h^+_{S,\sigma^+_{S,i}} = -\sum_{v \in S} \delta_{v,i} - I_i(S|E), \forall S \in \mathcal{W}(\mathcal{G})$$

$$h^-_{S,\sigma^-_{S,i}} = \sum_{v \in S} \delta_{v,i} - O_i(S|E), \quad \forall S \in \mathcal{W}(\mathcal{G})$$

Sorting $\sigma^{+/-}_{S,i} \quad \forall S \in \mathcal{W}(\mathcal{G})$

Complexity $\mathcal{O}(N\log N \cdot |\mathcal{W}(\mathcal{G})|)$

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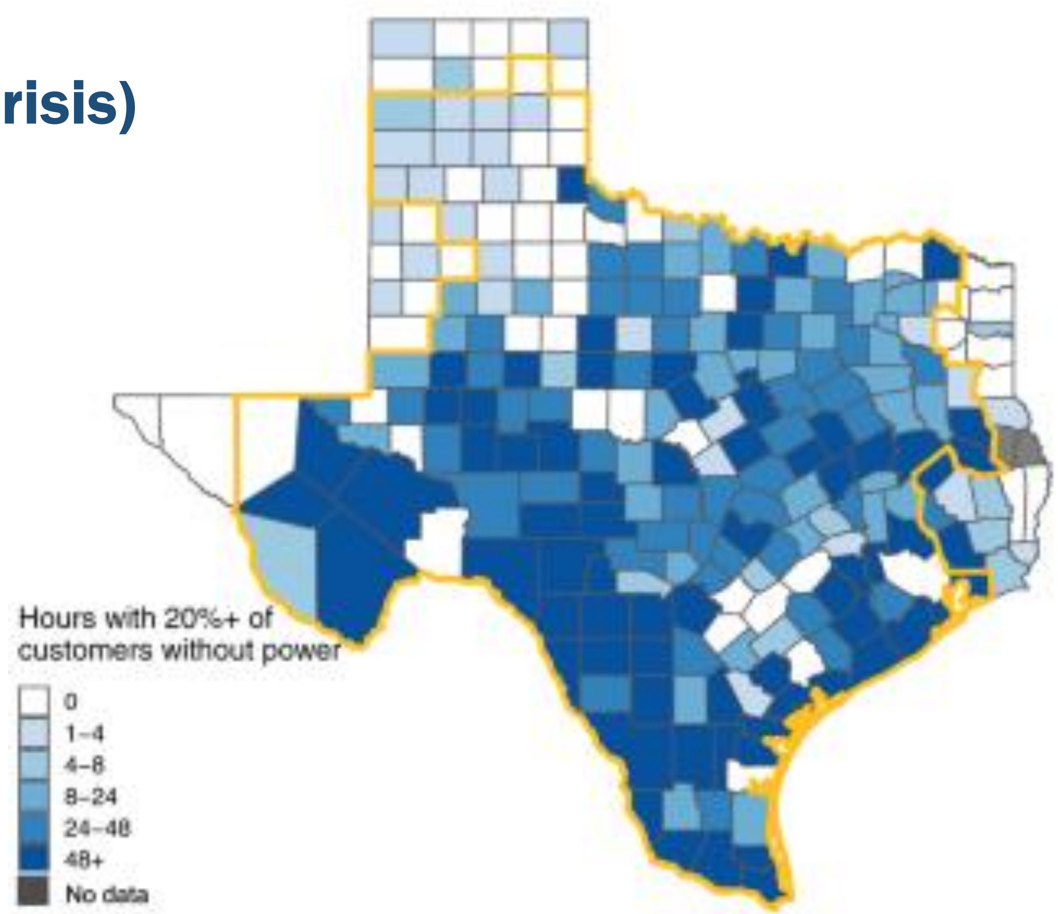
Motivation

System Failure
(2021 Texas Power Crisis)



over 246 people were killed / over 200 billion US dollars of property damage

Source: ERCOT