Pricing with Non-Convexities

Anthony Papavasiliou, National Technical University of Athens (NTUA)

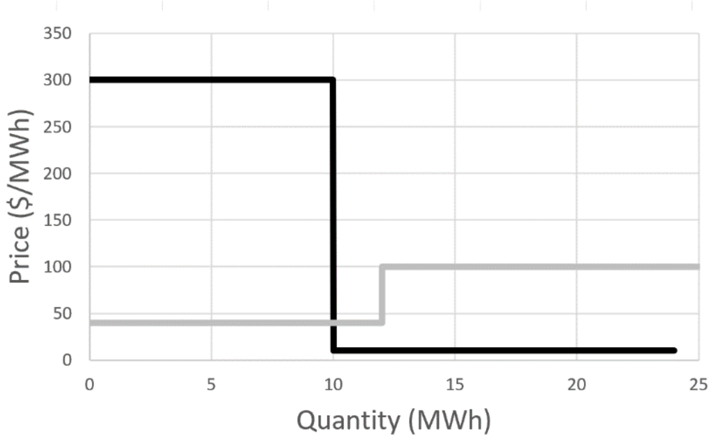
Source: section 7.3, Papavasiliou [1]

Outline

- Inexistence of a clearing price
- Measuring deviation from equilibrium
- Alternative pricing proposals
- The European exchange and EUPHEMIA

Inexistence of a clearing price

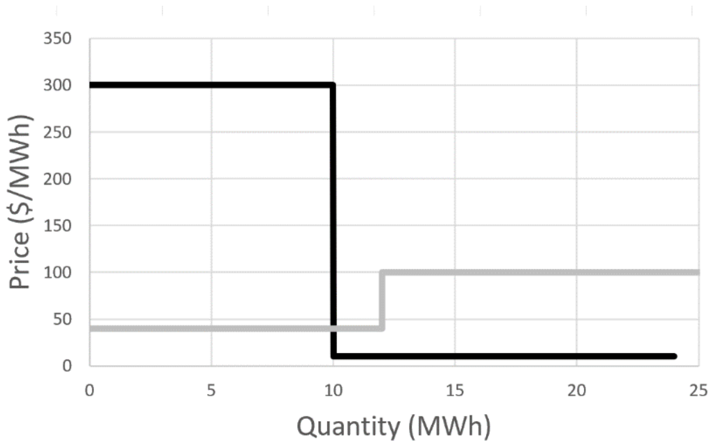
Example 7.6: a uniform price **always** exists in convex markets



Bid	Quantity (MWh)	Price ($/MWh)
A (buy)	10	300
B (buy)	14	10
C (sell)	12	40
D (sell)	13	100

- Definition of economic equilibrium:
  - Surplus maximization (quantity adjustment)
  - Market clearing (price adjustment)
- For this example: 40 $/MWh
- A market clearing price can be proven to exist in convex models (proposition 4.11)
- Corresponds to welfare maximization

Example 7.6: in markets with "complicated" products uniform clearing prices may **not** exist



Bid	Quantity (MWh)	Price ($/MWh)	Minimum acceptance (MWh)
A (buy)	10	300	0
B (buy)	14	10	0
C (sell)	12	40	11
D (sell)	13	100	0

"Complicated" orders: non-convex

In practice:

- Block orders
- MICs
- PUNs

Uniform price does not exist for this example

- At 40 $/MWh, supply ≠ demand because of min acceptance
- Below 40 $/MWh: supply < demand
- Above 40 $/MWh: supply > demand

The model of example 7.6

$$\max_{p,d,u} 300 \cdot d_A + 10 \cdot d_B - 40 \cdot p_C - 100 \cdot p_D$$
$$d_A + d_B - p_C - p_D = 0$$
$$11 \cdot u_C \leq p_C \leq 12 \cdot u_C$$
$$d_A \leq 10, d_B \leq 14, p_D \leq 13$$
$$d_A, d_B, p_D \geq 0$$
$$u_C \in \{0,1\}$$

- Optimal solution:
  - Bid A: $d_A^* = 10$ MWh
  - Bid B: $d_B^* = 1$ MWh
  - Bid C: $p_C^* = 11$ MWh, $u_C^* = 1$
  - Bid D: $p_D^* = 0$ MWh
- Surplus: $2570

Two solutions to the existence problem

Uniform pricing and paradoxically rejected blocks (PRBs)

Same price for all, but allow some paradoxically rejected orders

Rationale:

- Paradoxically rejected orders: no losses incurred, more tolerable, no need for side payments
- Paradoxically accepted orders: losses are incurred, not acceptable in European design

Mathematically:

- Maximize welfare, subject to extra constraints (allow PRBs)
- Extra constraints: lower welfare

Practice in EUPHEMIA

Very complex problem, not solved to optimality

Uplifts

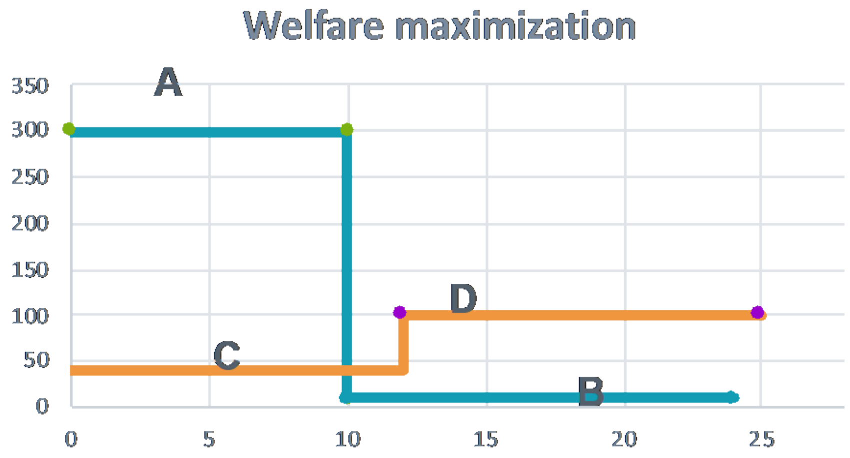
Rationale: maximize welfare, uses uplift payments to "make everybody happy"

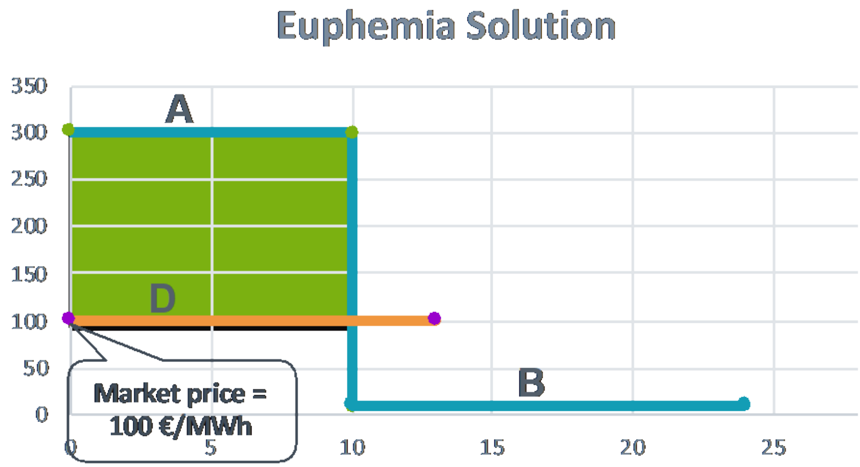
Mathematically:

- Solve for optimal selection of orders
- Solve for price
- Compute uplifts separately

Practice in the US

Approach 1: paradoxically rejected bids (example 7.7)





Bid	Quantity (MWh)	Price ($/MWh)	Min acceptance ratio (MWh)
A (buy)	10	300	0
B (buy)	14	10	0
C (sell)	12	40	11
D (sell)	13	100	0

- For this example, we showed that we cannot accept C
- So, if we reject C, the price becomes 100 $/MWh
- Welfare: $2000

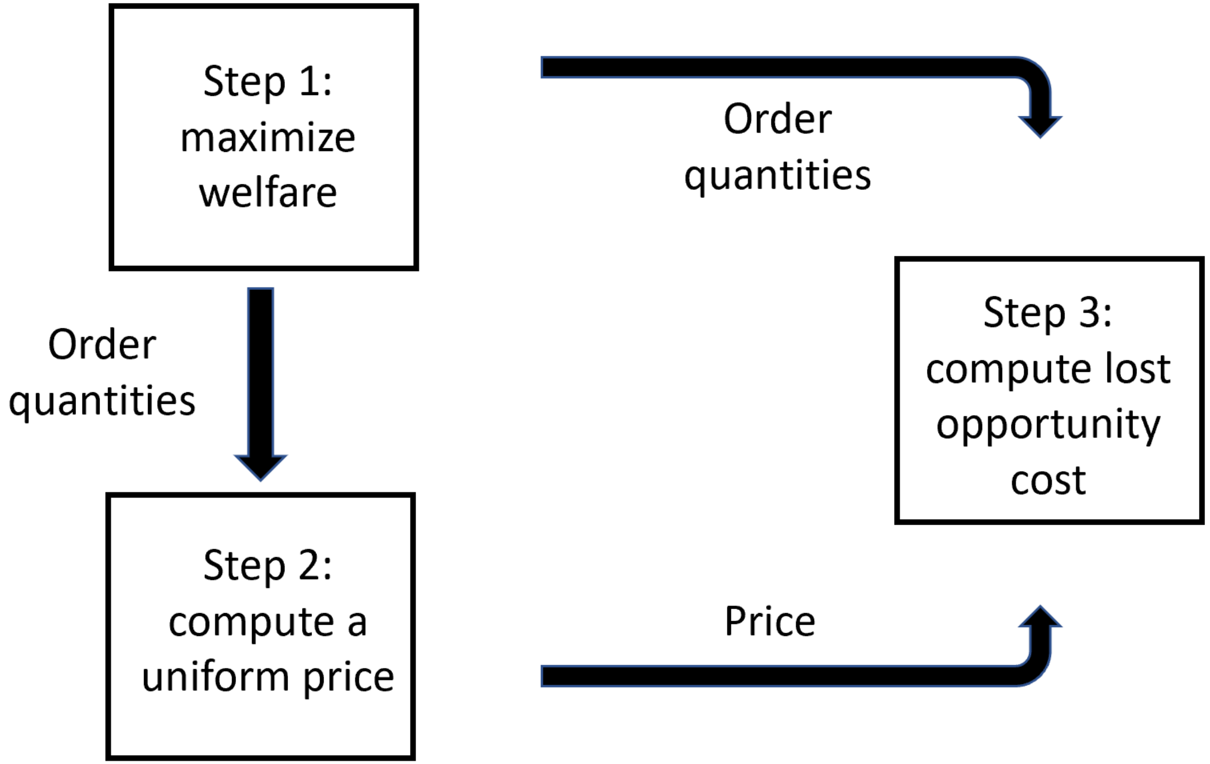
Mathematical model of example 7.7

$$\max_{p,d} 300 \cdot d_A + 10 \cdot d_B - 100 \cdot p_D$$
$$(\lambda): d_A + d_B - p_D = 0$$
$$d_A \leq 10, d_B \leq 14, p_D \leq 13$$
$$d_A, d_B, p_D \geq 0$$

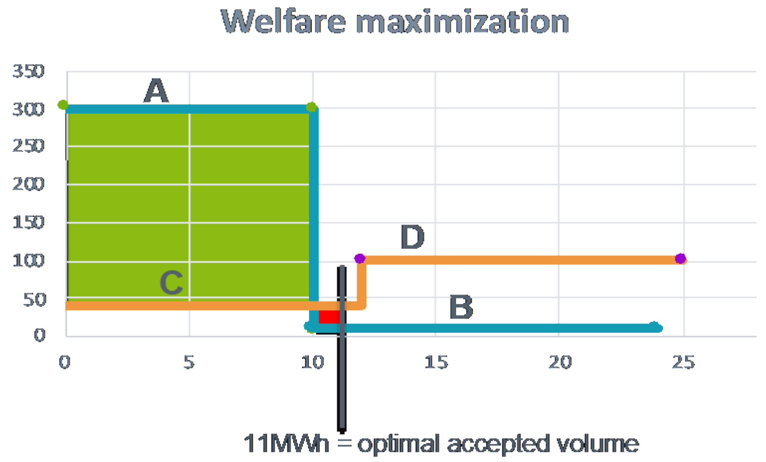
Solution:

- λ = 100 $/MWh
- Welfare: $2000 (loss of $570 of welfare)

Approach 2: separate computation of matches and prices



Approach 2: separate computation of matches and prices



Bid	Quantity (MWh)	Price ($/MWh)	Minimum quantity (MWh)
A (buy)	10	300	0
B (buy)	14	10	0
C (sell)	12	40	11
D (sell)	13	100	0

- Step 1: select quantities by matching welfare
  - Bid A: 10 MWh
  - Bid B: 1 MWh
  - Bid C: 11 MWh
  - Bid D: 0 MWh
- Step 2: find a uniform price, let's try 40 $/MWh
- Step 3: pay lost opportunity cost, if needed
  - Bids A, C, D: $0, no lost opportunity cost
  - Bid B: $30

Measuring deviation from equilibrium

Self-scheduling and lost opportunity cost

- Although the existence of an equilibrium price is not guaranteed, we want a price that "approximates" this goal
- How do we quantify "approximates"?
- Given a market price λ, the **lost opportunity cost** is

$$LOC(\lambda) = \Pi^*(\lambda) - \bar{\Pi}(\lambda)$$

- Where:
  - $\Pi^*(\lambda)$: profit of an agent that **self-schedules** its production
  - $\bar{\Pi}(\lambda)$: the profit of an agent if it follows the instruction of the auction

Mathematical definition of lost opportunity cost

- Consider a general agent which decides (x, q), a benefit function $f(x)$, and a set of constraints $g(x) \leq 0$ and $h(x) = q$
- Given a market clearing price λ^*, the surplus maximization problem of the agent is expressed as:

$$\max_{x,q}(f(x) - (\lambda^*)^T q)$$
$$g(x) \leq 0$$
$$h(x) = q$$

Denote:
- $(\bar{x}, \bar{q})$ the instruction of the market
- (x^*, q^*) the optimal self-scheduling decision

- Π^*: the surplus of the agent given (x^*, q^*)
- $\bar{\Pi}$: the surplus of the agent given $(\bar{x}, \bar{q})$

Computation of lost opportunity cost

- Suppose that we price at 40 \$/MWh (see slide 11)

- For λ^* = 40 \$/MWh we have $\bar{\Pi}$ of offer B:

$$(1)[\text{MWh}] \cdot (10 - 40) \left[\frac{\$}{\text{MWh}}\right] = -\$30$$

- For λ^* = 40 \$/MWh we have Π^* for offer B:

$$(0)[\text{MWh}] \cdot (10 - 40) \left[\frac{\$}{\text{MWh}}\right] = \$0$$

- So the lost opportunity cost is \$30

Make-whole payments

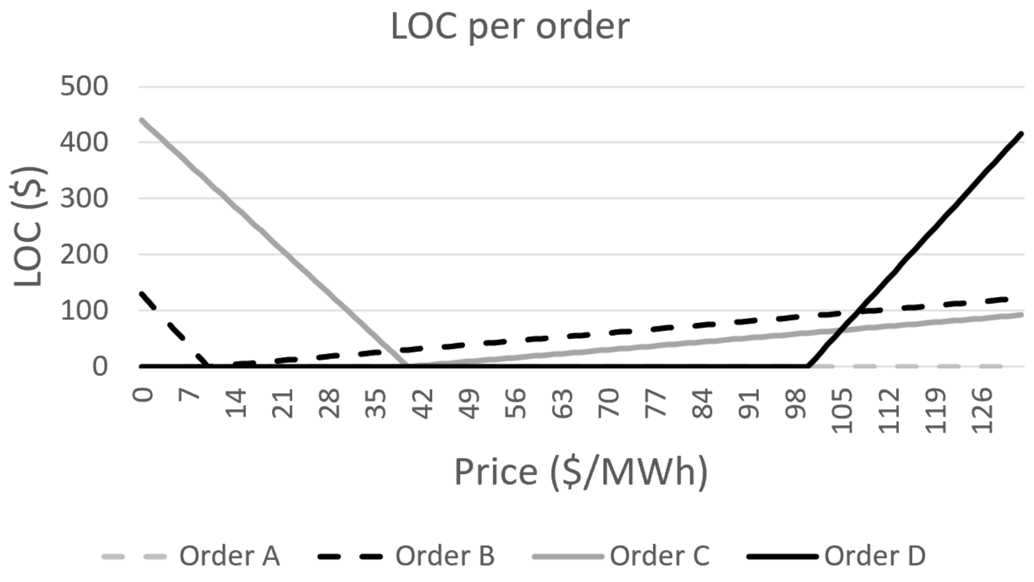
Make-whole payments: payment that compensates an agent if the agent is exposed to a negative surplus when following the market schedule $(\bar{x}, \bar{q})$, given market price λ^*

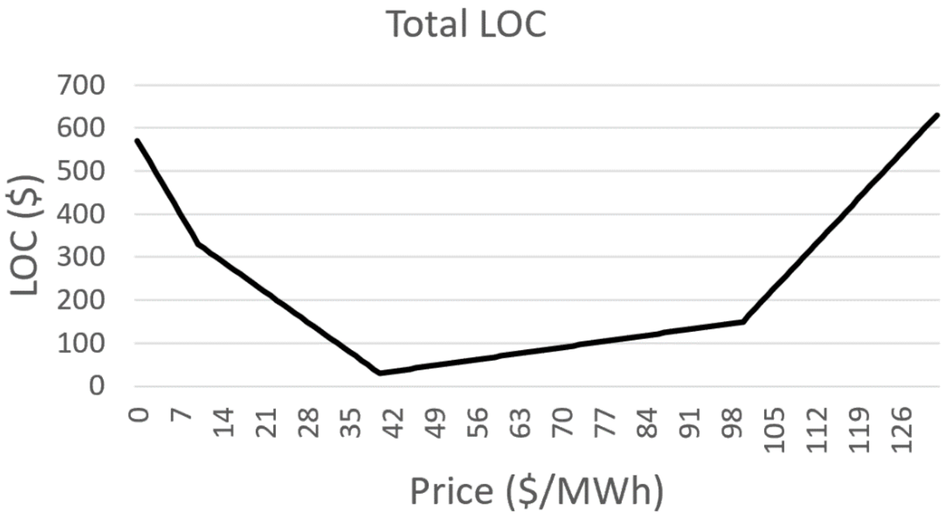
$$MWP = \max\left(0, -\left(f(\bar{x}) - (\lambda^*)^T \bar{q}\right)\right)$$

Clarifying terminology:

- Lost opportunity cost sometimes referred to as **uplift**
- Other publications reserve the term uplift for make-whole payments
- Difference in terminology reflects deeper divide regarding pricing in non-convex market models
  - Some experts argue that uplift payments should be minimized
  - Others argue in favor of minimizing lost opportunity cost

Example 7.8: lost opportunity cost depends on the market price



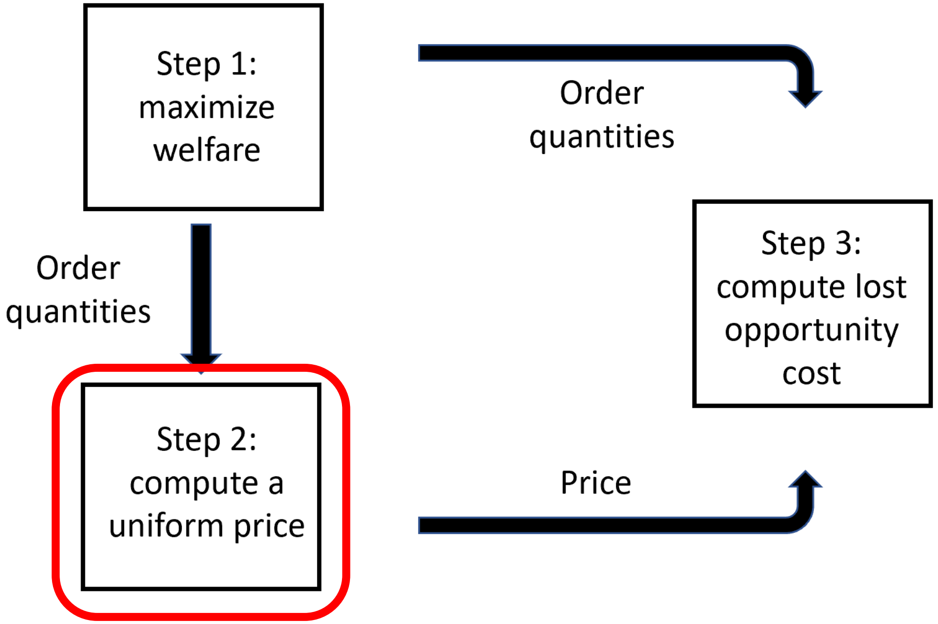


Alternative pricing proposals

Alternative pricing proposals

Alternatives for step 2:

- Convex hull pricing (MISO)
- Integer programming pricing (California)
- Linear programming relaxation (PJM)



Mathematical definition

- $conv(X)$: convex hull of set X
- f^{**}: the tightest convex approximation of f

Convex hull pricing

Energy and reserve prices (λ, λR) are computed from the solution of the following problem:

$$(CHP): \min_{p,u,r} \sum_{g \in G} TC_g^{**}(u_g, p_g)$$

$$(p_g, u_g, r_g) \in conv(X_g), g \in G$$

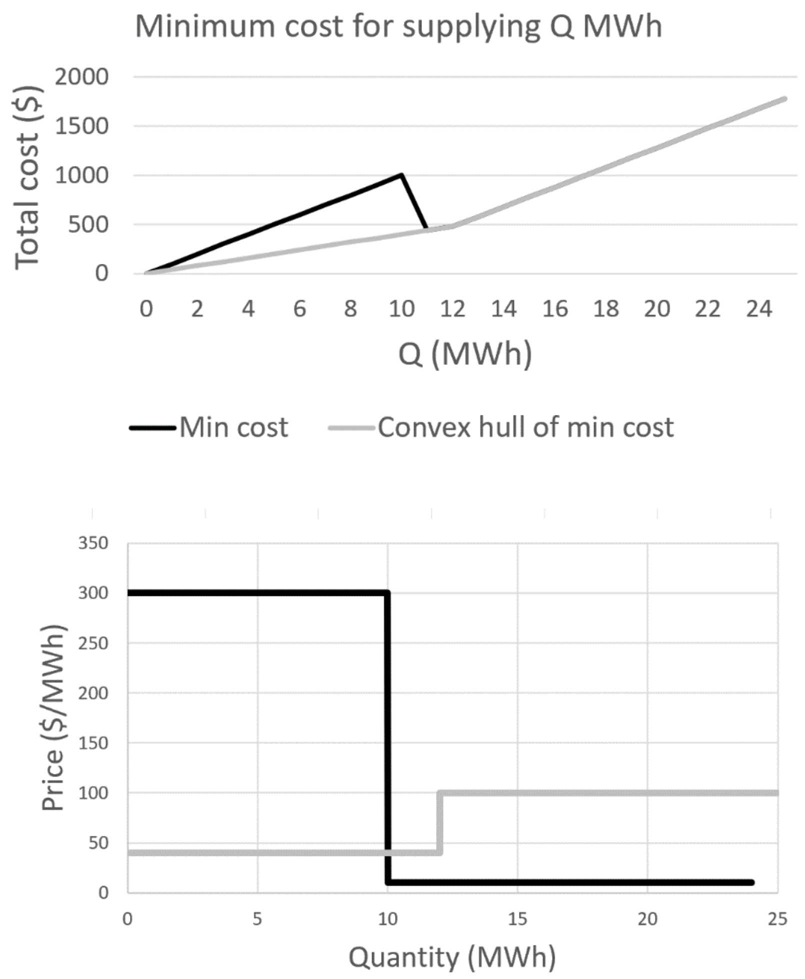
$$(\lambda): \sum_{g \in G} p_{gt} = D_t, t = 1, \ldots, T$$

$$(\lambda R): \sum_{g \in G} r_{gt} = R_t, t = 1, \ldots, T$$

where $X_g = \{(p_g, u_g, r_g): h_g(p_g, r_g, u_g) \leq 0, u_g \in \{0,1\}^T\}$

Intuition: the equilibrium price that would emerge from a convex economy that the "closest possible" to our non-convex economy

Example 7.11: convex hull pricing



- **Black curve**: what is the cheapest way to produce Q MWh?
- **Gray curve**: the convex hull is the largest convex function that sits under the black curve

- If offers were not "complicated" (i.e. if they were convex) we would get the orange total cost function
- Convex hull prices are the prices that would emerge in this convex model, which is closest convex approximation to the true setting

Convex hull pricing minimizes lost opportunity cost

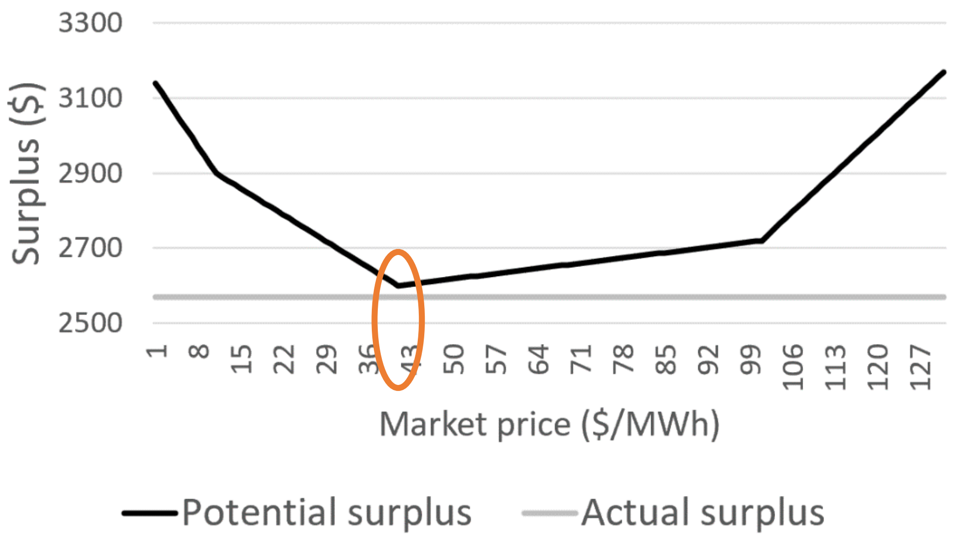
- Lost opportunity cost is the duality gap of the market model when we relax market clearing constraints

- The convex hull price minimizes lost opportunity cost

Mathematically:

$$p^* - d^* = \sum_{g \in G} \Pi_g^*(\lambda^*, \lambda R^*) - \sum_{g \in G} \bar{\Pi}_g(\lambda^*, \lambda R^*) = LOC(\lambda^*, \lambda R^*)$$

Convex hull pricing minimizes lost opportunity cost



Convex hull price (CHP): price that minimizes lost opportunity cost

For our example, the CHP is **40 $/MWh**

The minimum lost opportunity cost is **$30**

Integer programming pricing

Energy and reserve prices (λ, λR) are computed by solving the following problem:

$$(IP): \min_{p,r} \sum_{g \in G} TC_g(u_g^*, p_g)$$
$$h_g\left(p_g, r_g, u_g^*\right) \leq 0, g \in G$$
$$(\lambda): \sum_{g \in G} p_{gt} = D_t, t = 1, \ldots, T$$
$$(\lambda R): \sum_{g \in G} r_{gt} = R_t, t = 1, \ldots, T$$

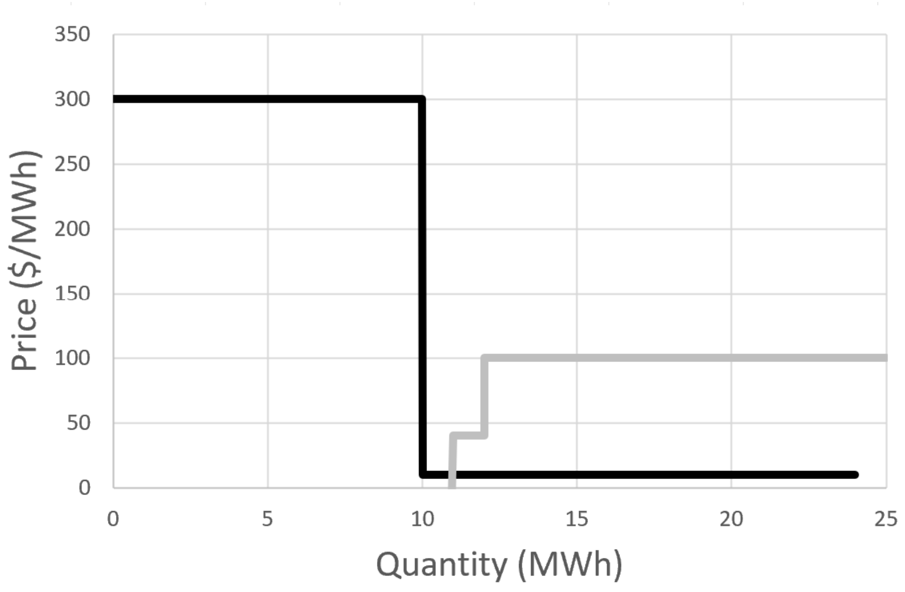
where u^* are the optimal commitments that are computed in step 1 of slide 19

Example 7.12: integer programming pricing

- From slide 6, we have that $u_C^* = 1$
- So the IP price is the dual multiplier λ of the following model:

$$\max_{p,d} 300 \cdot d_A + 10 \cdot d_B - 40 \cdot p_C - 100 \cdot p_D$$
$$(\lambda): d_A + d_B - p_C - p_D = 0$$
$$11 \leq p_C \leq 12$$
$$d_A \leq 10, d_B \leq 40, p_D \leq 13$$
$$d_A, d_B, p_D \geq 0$$

Graphical interpretation of integer programming pricing



- Due to the constraint $p_C \geq 11$, the supply curve of C and D essentially "starts" at 11 MWh
- In other words, at this point, the supply curve moves from minus infinity to 40 $/MWh
- The IP price is the intersection of this supply curve with the demand curve, thus equal to 10 $/MWh

Linear programming relaxation pricing

The prices of energy and reserve (λ, λR) are computed from the solution of the following problem:

$$(LPR): \min_{u,p,r} \sum_{g \in G} TC_g(u_g, p_g)$$

$$h_g(p_g, r_g, u_g) \leq 0, g \in G$$

$$(\lambda): \sum_{g \in G} p_{gt} = D_t, t = 1, \ldots, T$$

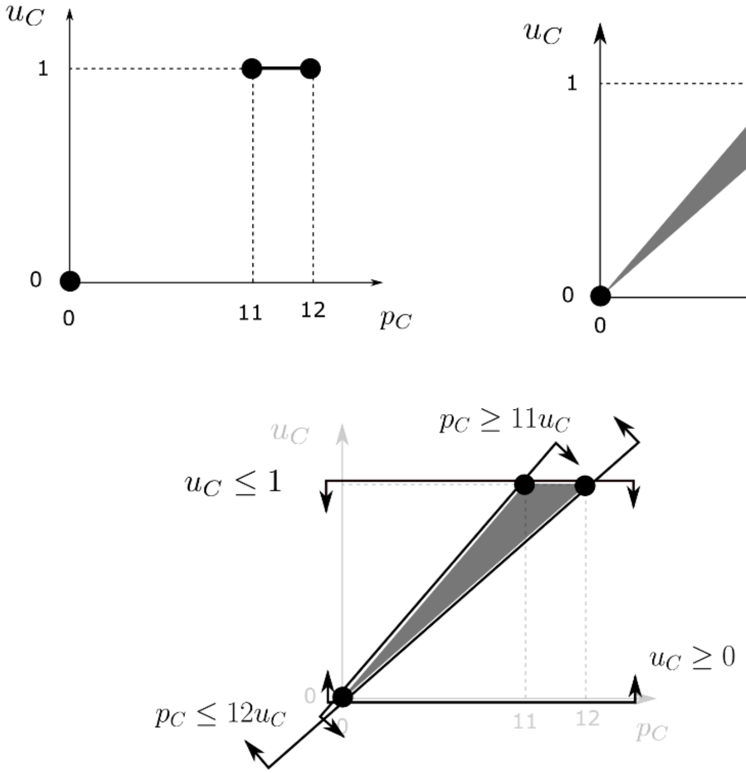
$$(\lambda R): \sum_{g \in G} r_{gt} = R_t, t = 1, \ldots, T$$

$$0 \leq u_{gt} \leq 1, g \in G, t = 1, \ldots, T$$

Some advantages of linear programming relaxation pricing

- Simple implementation (+)
- Computationally easy (+)
- The method leads to the exact same solution as convex hull pricing in certain cases (+)

Equivalence of linear programming relaxation and convex hull pricing (in simple cases)



- Upper left: the feasible set of bid C
- Upper right: the convex hull of the feasible set of bid C
- Lower: linear relaxation of bid C

Important observation: the upper right and lower sets are <u>equivalent</u>

Example 7:13: linear programming relaxation pricing

The linear programming relaxation price is the dual multiplier λ of the following model:

$$\max_{p,u,d} 300 \cdot d_A + 10 \cdot d_B - 40 \cdot p_C - 100 \cdot p_D$$
$$(\lambda): d_A + d_B - p_C - p_D = 0$$
$$11 \cdot u_C \leq p_C \leq 12 \cdot u_C$$
$$d_A \leq 10, d_B \leq 40, p_D \leq 13$$
$$d_A, d_B, p_D \geq 0$$
$$0 \leq u_C \leq 1$$

Clearing price: 40 $/MWh (same as CHP)

The European exchange and EUPHEMIA

Products in the European power exchange

- Aggregated hourly orders
  - Linear orders
  - Stepwise orders
- Complex orders
  - Load gradients
  - Minimum income conditions
- Block orders
- Merit orders and unique national price (Prezzo Unico Nazionale, PUN) orders

Το πρόβλημα εκκαθάρισης της ευρωπαϊκής αγοράς ηλεκτρισμού

- Integer quadratic program subject to complementarity constraints
- Quadratic objective function: surplus from trading energy
- Integer variables: accept or reject block orders and other products
- Complementarity constraints: pricing business rules in the European market

Formulation of the model (in words) for the running example

Maximize surplus, subject to the following constraints:

- Pricing rules for continuous offers (bids A, B, and D): maximize surplus given market prices
- Pricing rules for block orders (bid C): cannot be paradoxically accepted, but can be paradoxically rejected

Pricing rules of continuous offers

Surplus maximization problem of bid A is expressed as follows:

$$\max_{d_A} (300 - \lambda) \cdot d_A$$
$$(\mu_A): d_A \leq 10$$

Equivalent to the following KKT conditions:

$$0 \leq d_A \perp \lambda - 300 + \mu_A \geq 0$$
$$0 \leq \mu_A \perp 10 - d_A \geq 0$$

Confirming the pricing rules

- If the bid is fully accepted ($d_A = 10$), then it is in the money or at the money ($300 \geq \lambda$)
- If the bid is partially accepted ($0 < d_A < 10$) then it is on the money ($300 = \lambda$)
- If the bid is rejected ($d_A = 0$), then it is on the money or out of the money ($300 \leq \lambda$)

Block order pricing rules

We work with the *linear relaxation* of the block order profit maximization problem:

$$\max_{u_C, p_C} (\lambda - 40) \cdot p_C$$
$$(\mu_C): p_C - 12 \cdot u_C \leq 0$$
$$(\nu_C): 11 \cdot u_C - p_C \leq 0$$
$$(s_C): u_C \leq 1$$
$$p_C, u_C \geq 0$$

KKT conditions of linear relaxation

The KKT conditions of the previous slide are:

$$0 \leq p_C \perp 40 - \lambda + \mu_C - \nu_C \geq 0$$
$$0 \leq u_C \perp -12 \cdot \mu_C + 11 \cdot \nu_C + s_C \geq 0$$
$$0 \leq \mu_C \perp 12 \cdot u_C - p_C \geq 0$$
$$0 \leq \nu_C \perp p_C - 11 \cdot u_C \geq 0$$
$$0 \leq s_C \perp 1 - u_C \geq 0$$

In order to express the pricing rules, we remove the complementarity operator from the last condition:

$$0 \leq s_C \not\perp 1 - u_C \geq 0 \quad \rightarrow \quad s_C \geq 0, u_C \leq 1$$

What is the effect of removing the complementarity operator?

The complementarity operator prevents

- that the offer has positive surplus ($s_C > 0$) *and*
- that it is not fully accepted ($u_C < 1$)

Lifting complementarity is precisely the essence of paradoxically rejected orders

Full model for the running example

Maximize surplus $(EUDA): \max_{p,d,u,\lambda,\mu,\nu,s} 300 \cdot d_A + 10 \cdot d_B - 40 \cdot p_C - 100 \cdot p_D$

Pricing rules of continuous orders

$$0 \leq d_A \perp \lambda - 300 + \mu_A \geq 0, 0 \leq \mu_A \perp 10 - d_A \geq 0$$
$$0 \leq d_B \perp \lambda - 10 + \mu_B \geq 0, 0 \leq \mu_B \perp 14 - d_B \geq 0$$
$$0 \leq p_D \perp 100 - \lambda + \mu_D \geq 0, 0 \leq \mu_D \perp 13 - p_D \geq 0$$

Pricing rules of block orders

$$0 \leq p_C \perp 40 - \lambda + \mu_C - \nu_C \geq 0$$
$$0 \leq u_C \perp -12 \cdot \mu_C + 11 \cdot \nu_C + s_C \geq 0$$
$$0 \leq \mu_C \perp 12 \cdot u_C - p_C \geq 0$$
$$0 \leq \nu_C \perp p_C - 11 \cdot u_C \geq 0$$
$$s_C \geq 0, u_C \in \{0,1\}$$

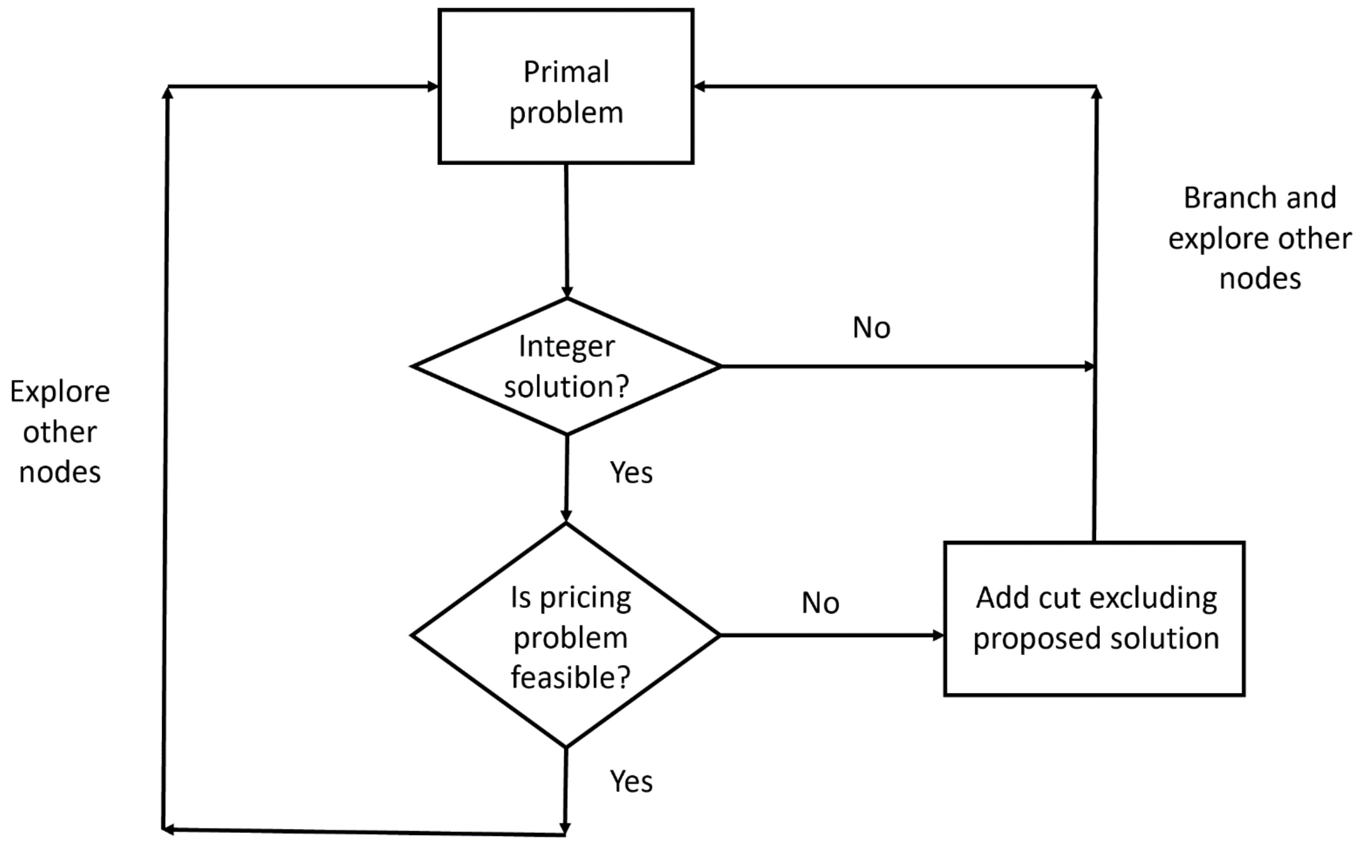
$$d_A + d_B - p_C - p_D = 0$$

Some observations

- The model is expressed in both primal and dual variables
- The problem is **mixed integer** (binary variable u_C)
- The problem is subject to **complementarity constraints** (pricing rules)

- Mixed integer ⇒ hard
- Complementarity conditions ⇒ extremely hard

EUPHEMIA


Primal problem
Integer solution?
No
Branch and explore other nodes
Yes
Is pricing problem feasible?
No
Add cut excluding proposed solution
Yes
Explore other nodes

EUPHEMIA

- EUPHEMIA is a branch and bound algorithm
- Developed by N-SIDE, spinoff of the Center for Operations Research and Econometrics (CORE) at the Université catholique de Louvain
- Clears the **pan-European** day-ahead electricity market

The EUPHEMIA algorithm

- *Primal problem* function*:* convex integer program (computationally easy)
  - Binary variables are replaced by their linear relaxations
- *Integer solution* check: (easy) check if the solution of the problem results in integer values for u
- *Feasible pricing problem* check: can be expressed as a linear program
- *Add cut excluding proposed solution* function: one of the strong points of the EUPHEMIA algorithm

Feasible pricing problem

- Corresponds to the complementarity system that implements the pricing rules
- We use *constant* values for primal variables (u, p, d) from the optimal solution of the *primal problem*
- Computationally easy step
- The only free variables in this computation are the dual variables, i.e. (μ, ν, λ, s)

Adding cuts

- Cuts exploit the structure of the problem
- They cut off not only the current candidate solution but also many other alternatives that will never need to be explored in the branch and bound tree
- Such “deep” cuts exploit the special structure of the problem that is tackled by EUPHEMIA

References

[1] A. Papavasiliou, Optimization Models in Electricity Markets, Cambridge University Press

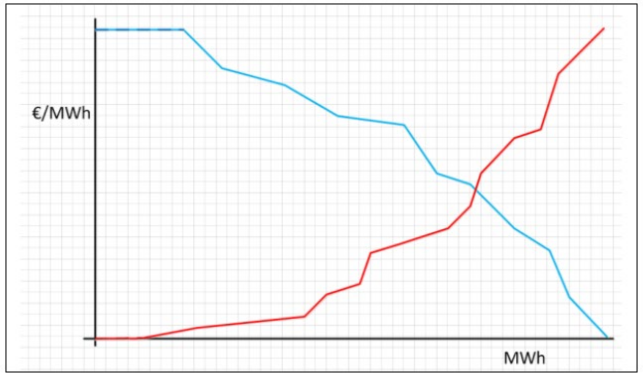
https://www.cambridge.org/highereducation/books/optimization-models-in-electricity-markets/0D2D36891FB5EB6AAC3A4EFC78A8F1D3#overview

[2] Madani, Mehdi, and Mathieu Van Vyve. "Computationally efficient MIP formulation and algorithms for European day-ahead electricity market auctions." European Journal of Operational Research 242.2 (2015): 580-593

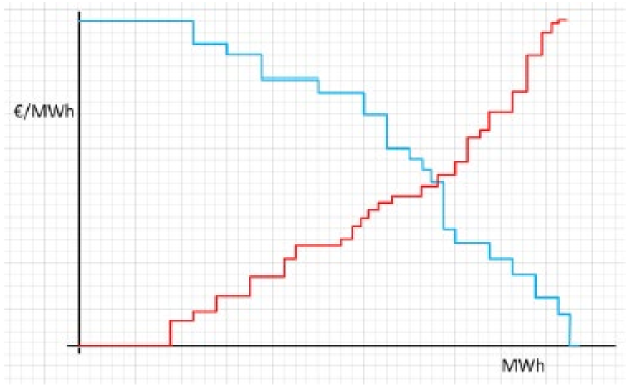
Appendix

Aggregated hourly orders

- Specific to each hour within a bidding area
- Supply orders are sorted in increasing price
- Demand orders are sorted in decreasing price
- Stepwise curves: two consecutive points always have same price or same quantity



Linear piecewise aggregated curve



Stepwise aggregated curve

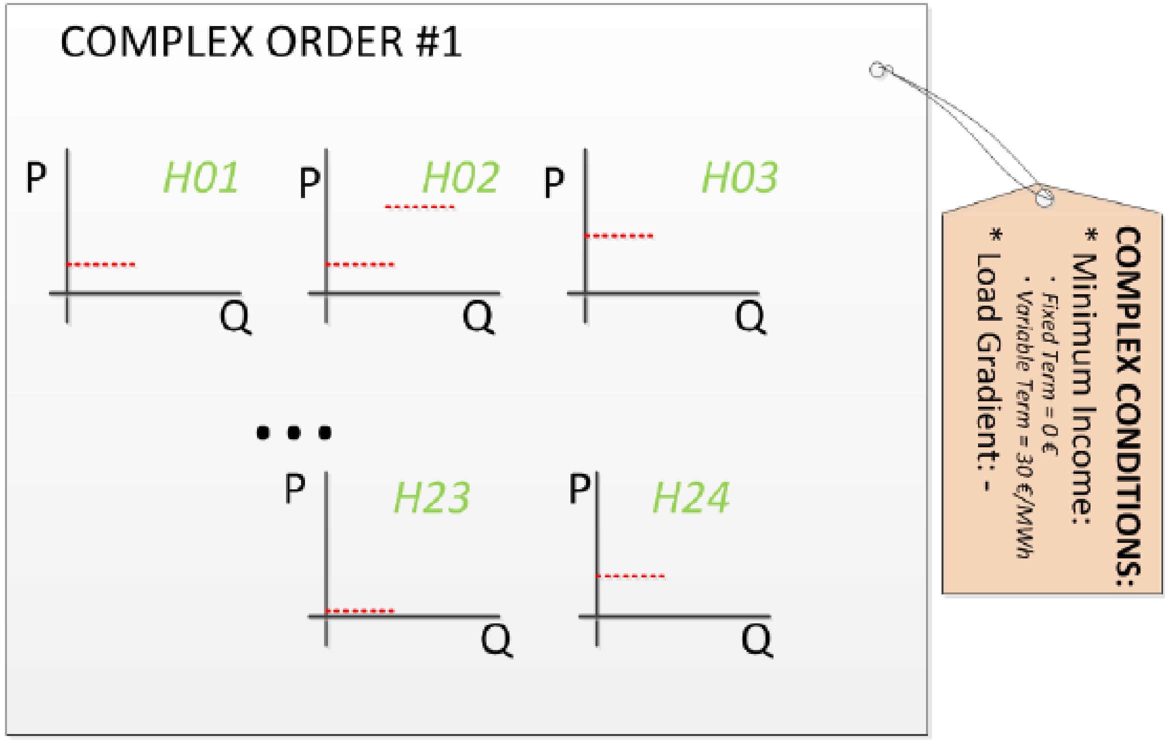
Acceptance of aggregated orders

- Definition of ITM/ATM/OTM:
  - A demand (resp. supply) hourly order is said to be ***in-the-money*** (ITM) when the market clearing price is lower (resp. higher) than the price of the hourly order
  - A demand or supply hourly order is said to be ***at-the-money*** (ATM) when the price of the hourly order is equal to the market clearing price
  - A demand (resp. supply) hourly order is said to be ***out-of-the-money*** (OTM) when the market clearing price is higher (resp. lower) than the price of the hourly order
- Acceptance rules:
  - Any in-the-money order must be fully accepted
  - Any out-of-the money order must be rejected
  - At-the-money orders can be either accepted (fully or partially) or rejected

Complex orders

- Used for representing dependencies across time periods

- Two types of complex conditions:
  - Minimum Income Condition (with or without scheduled stop)
  - Load Gradient

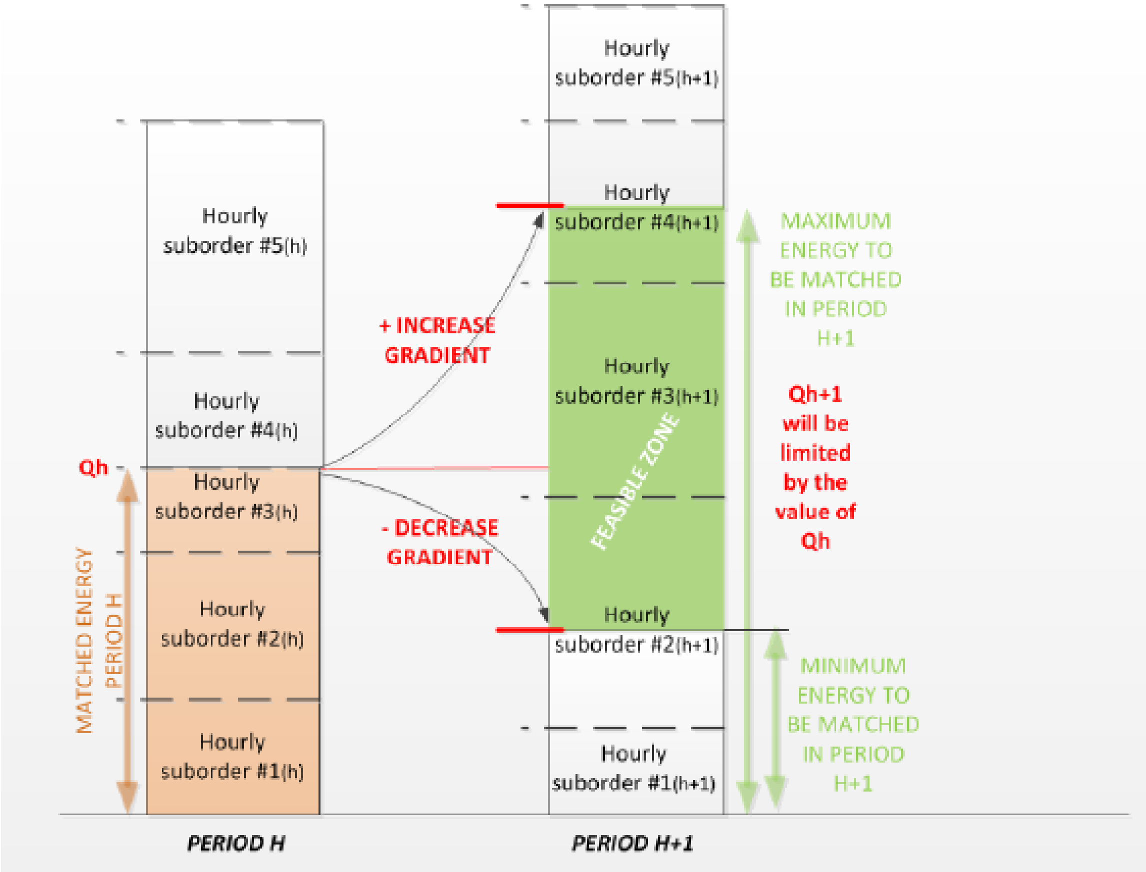


Minimum income conditions (MIC)

- Minimum income conditions mean that the order should cover
  - a fixed (startup) cost and
  - a variable (fuel) cost
- If MIC is activated, each of the hourly sub-orders is
  - accepted if in-the-money
  - rejected if out-of-the-money
  - can be either accepted (fully or partially) or rejected if at-the-money
- If a MIC order is deactivated, every sub-order is fully rejected (even if in-the-money)
- No paradoxically accepted MICs

Load gradient orders

- Used to represent ramp constraints
- Ramp limited by an increment/decrement limit (same value for all periods)



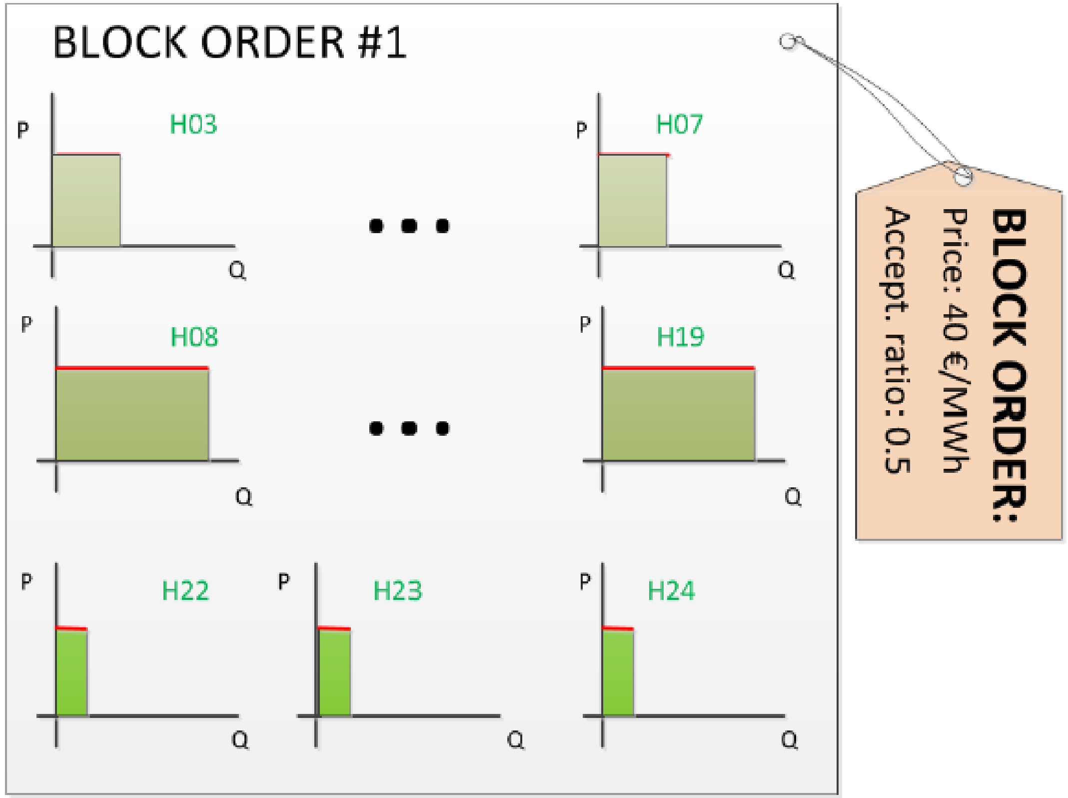
Block orders

- A block order is defined by:
  - Sense (supply or demand)
  - Price limit
  - Number of periods
  - Volume (can be different for every period)
  - Minimum acceptance ratio
- **Regular (fill-or-kill) block order**:
  - Block order defined for a consecutive set of periods
  - Same volume
  - A minimum acceptance ratio of 1
- The periods of the block order can be non-consecutive
- The volume can differ over the periods
- **Curtailable Block Orders**: the minimum acceptance ratio can be less than 1

Example block order

Example block order:

- Sense: supply
- Price: 40 €/MWh
- Minimum acceptance ratio: 0.5
- Intervals: hours (3-7), hours (8-19) and hours (22-24)
- Volume: 80 MWh in the first interval, 220 MWh in the second one, and 40 MWh in the third one

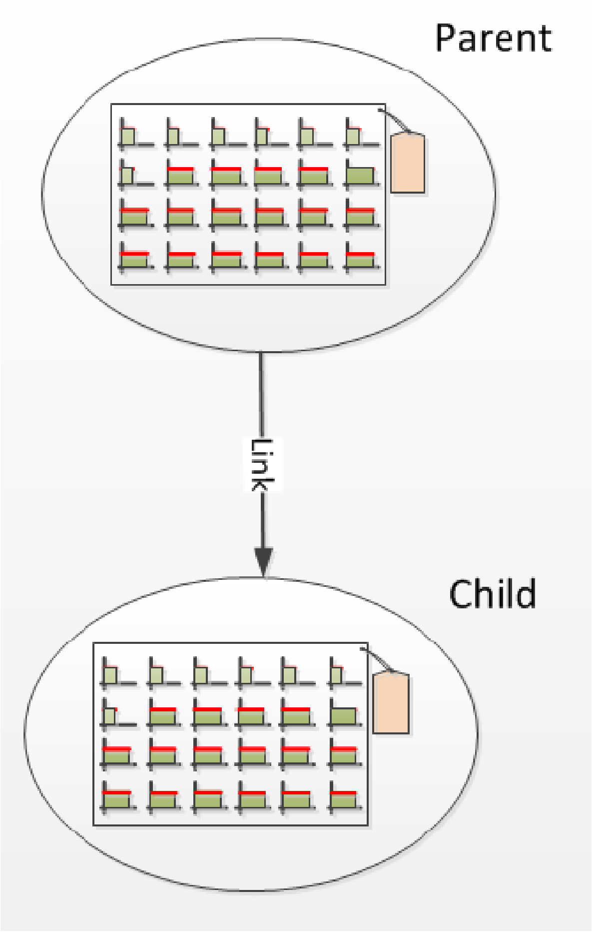


Acceptance of block orders

- Block orders that are ***out-of-the-money*** must be rejected
- If the block is ***in-the-money*** (or ***at-the-money***), then the block can be entirely accepted
- If the block is ***in-the-money*** (or ***at-the-money***), then the block can be fully rejected => Paradoxically Rejected Bid (PRB)
- If the block is ***in-the-money*** (or ***at-the-money***), then the block can be partially accepted => Partially Paradoxically Rejected Bid (PPRB)

Linked block orders

- The child can never be accepted “more” than the parent
- A child which is individually generating losses cannot be accepted, unless it is itself a parent of another order
- Rules in a single link:
  - The parent can be accepted alone
  - The child can “save” the parent with its surplus, but not the opposite



Block orders in an exclusive group

- An exclusive group is a set of block orders for which the sum of the accepted ratios cannot exceed 1

- When blocks have a minimum acceptance ratio of 1, <u>at most one</u> of the blocks in the exclusive group can be accepted

Flexible hourly block order

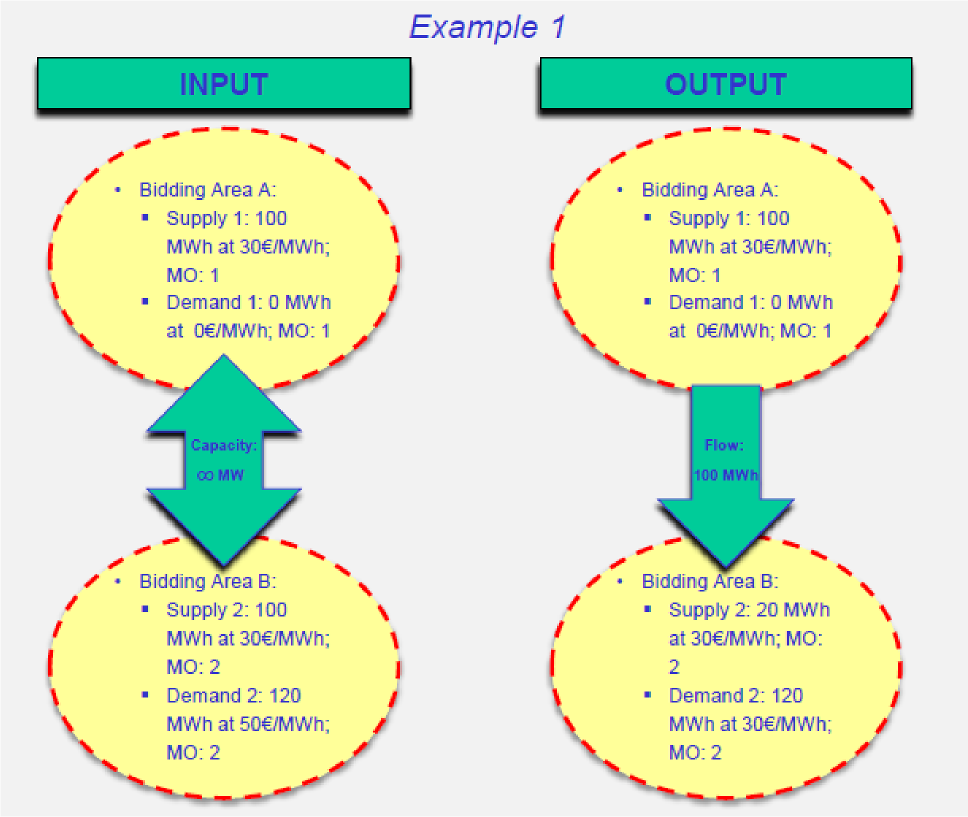
- A flexible "hourly" order is a block order with a fixed price limit, a fixed volume, minimum acceptance ratio of 1, with duration of 1 hour

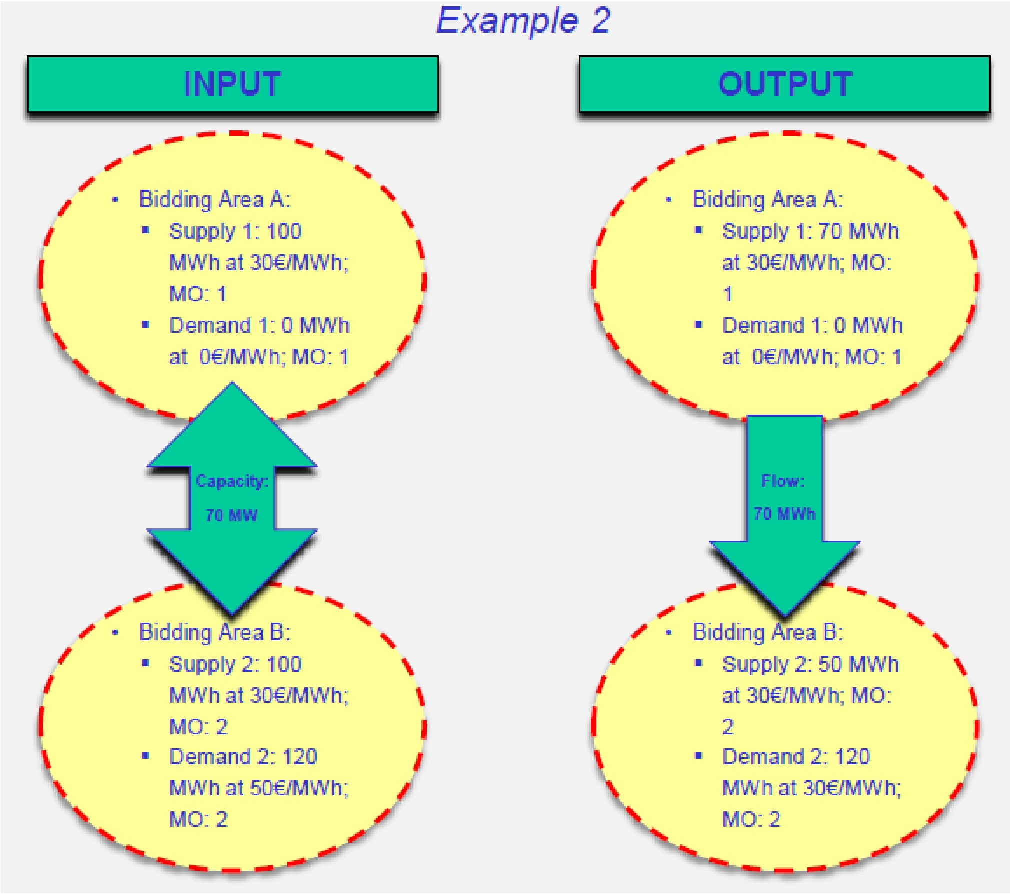
- The hour is not defined by the participant but is determined by the algorithm (hence the name "flexible")

Merit orders

- Merit orders are individual step orders defined at a given period that have an associated so-called **merit order number**
- A merit order number is unique per period and order type (demand, supply, PUN)
- The merit order number is used for ranking merit orders in the *bidding areas* containing this order type
- The lower the merit order number, the higher the priority for acceptance

Merit order examples





PUN orders

- PUN orders are a particular type of demand merit orders
- Cleared at the ***PUN price*** ("Prezzo Unico Nazionale") rather than the *bidding area* market clearing price
- For each period, financial balance needs to hold:

$$P_{PUN} \cdot \sum_{z} Q_z = \sum_{z} P_z \cdot Q_z \pm \Delta$$

- P_{PUN}: PUN price
- Q_z: volumes consumed in bidding area z
- P_z: price of bidding area z
- Δ: PUN imbalance (has to be near-zero)