Unit Commitment

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Source: chapter 7.1, Papavasiliou [1]

Day-ahead and real-time operations

Day-ahead operations

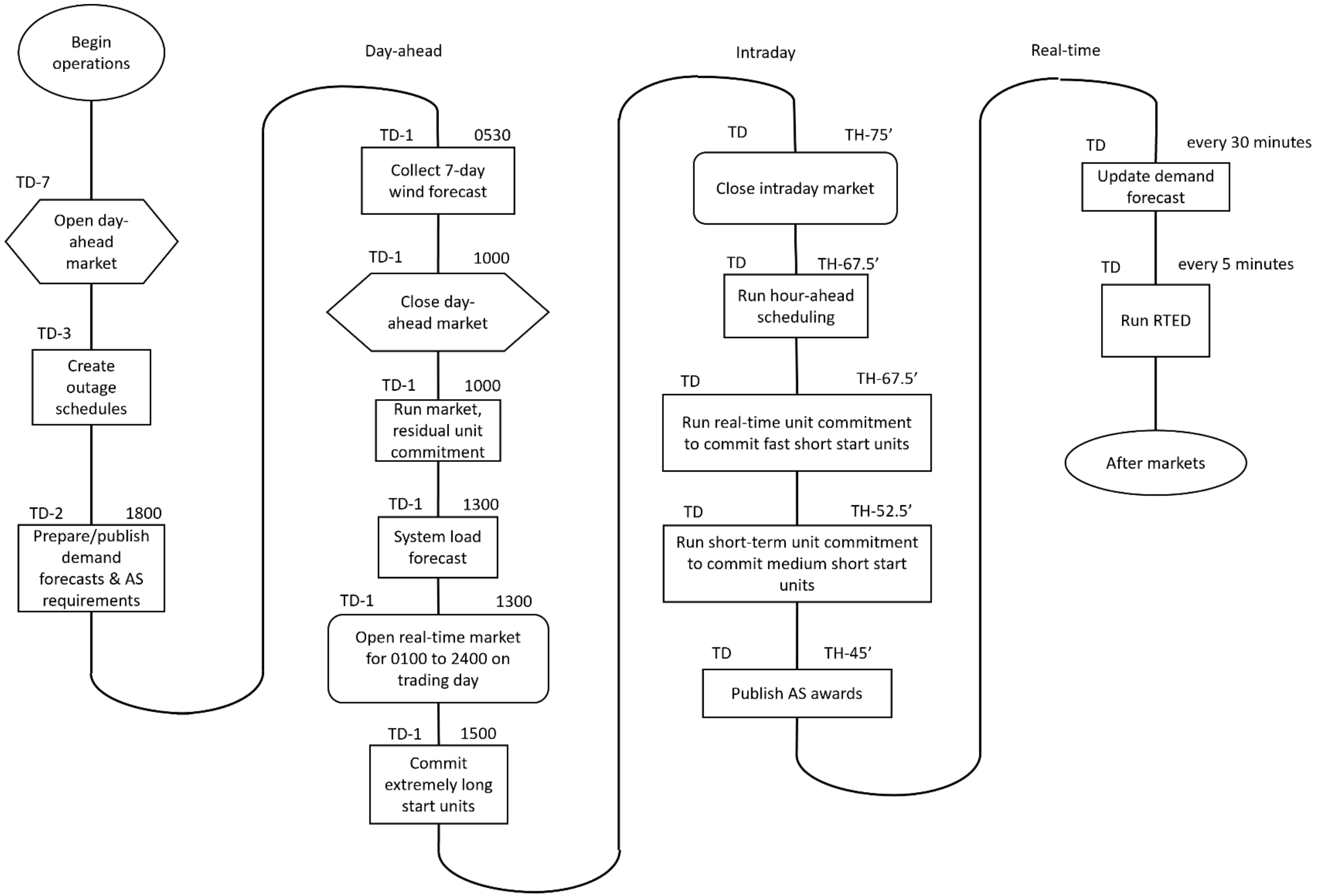
- Performed 24-36 hours in advance of real time
- Necessary because of delays in starting/moving units
- Based on forecasts (of demand, renewable energy, system state)
- Unit commitment

Real-time operations

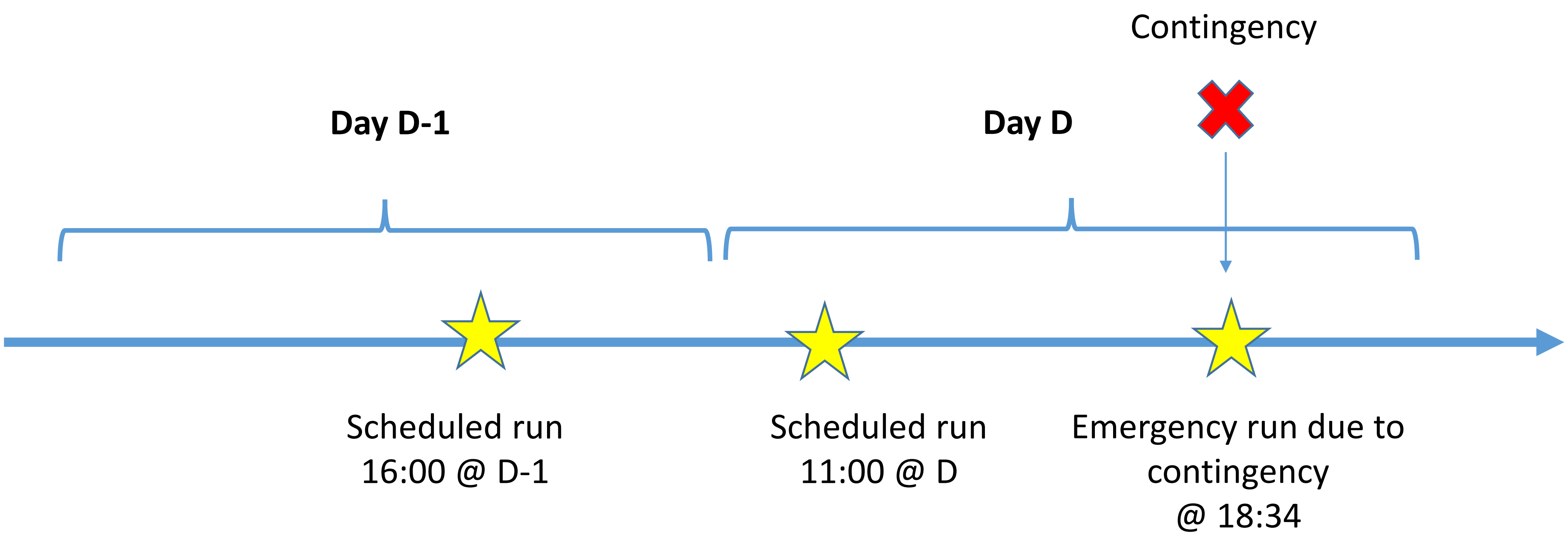
- Continuously
- Economic dispatch

Distinction between day-ahead scheduling and real-time dispatch is universal across systems

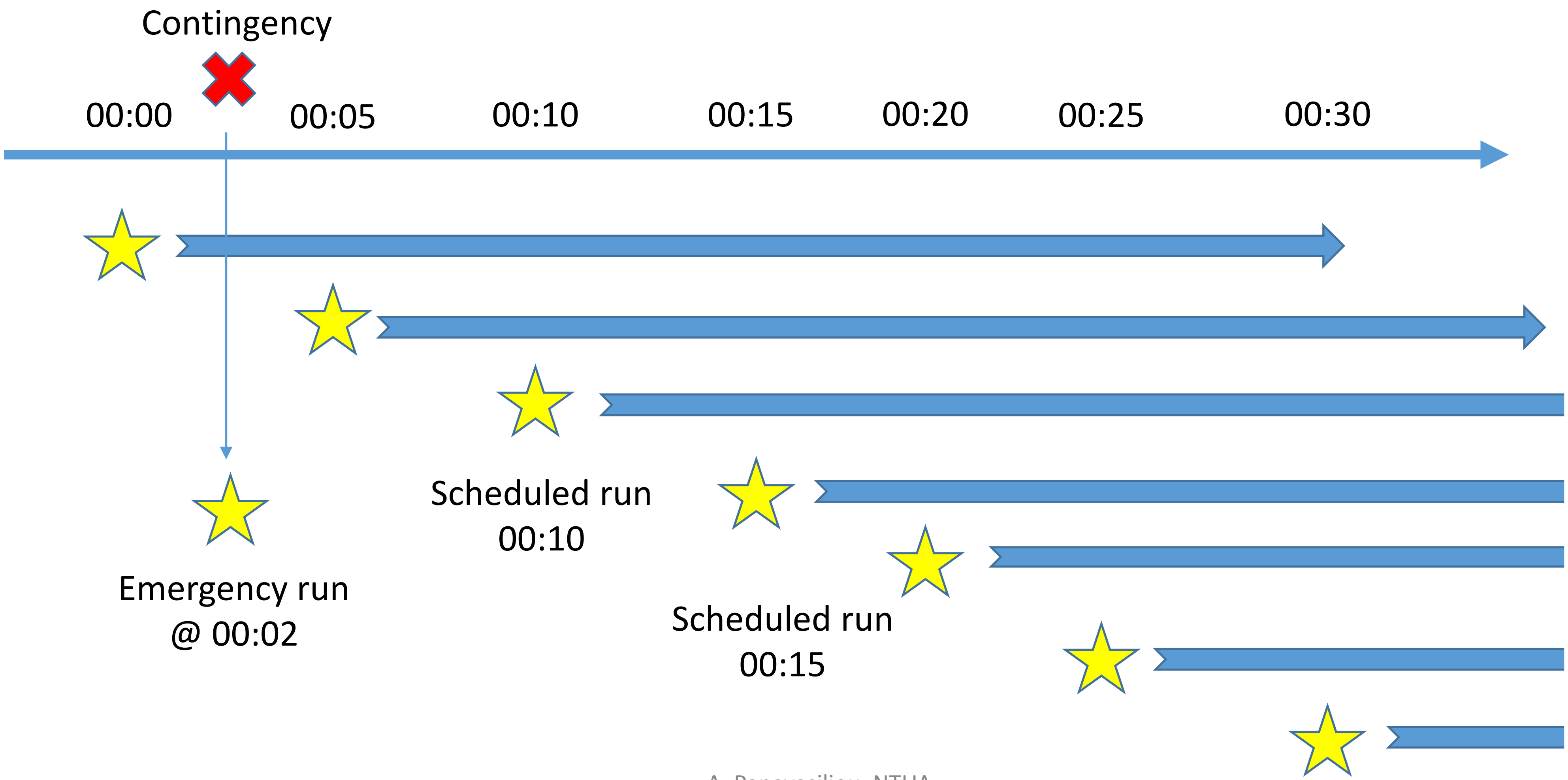
Flow chart of operations



Timeline of execution of integrated scheduling process (ISP)



Timeline of execution of real-time balancing mechanism (RTBM)



The real thing

Day-ahead Market – Average Daily Volumes

- 1,210 generators, 3 part offers (startup, no load, 10 segment incremental energy offer curve)
- 10,000 - Demand bids – fixed or price sensitive
- 50,000 - Virtual bids / offers
- 8,700 - eligible bid/offer nodes (pricing nodes)
- 6,125 - monitored transmission elements
- 10,000 - transmission contingencies modeled

Computational methods

Unit commitment is a large-scale mixed integer linear program

- Until 1960s: dispatch in order of increasing marginal cost
- 1970s, 1980s: dynamic programming with Lagrange relaxation
- Past decade: branch and bound solvers

Total cost

- Denote
  - $PC_g(p_{gt})$: production cost
  - $UC_g(u_g)$: commitment cost
  - $TC_g(u_g)$: total cost

$$TC_g(u_g, p_g) = UC_g(u_g) + \sum_{t=1}^{T} PC_g(p_{gt})$$

- T: scheduling horizon
- u_{gt}: indicates whether a unit is on or off, with $u_g = (u_{g1}, \dots, u_{gT}) \in \{0,1\}^T$
- p_{gt}: power production, with $p_g = (p_{g1}, \dots, p_{gT}) \in \mathbb{R}^T$

Example 7.1: cost function with startup and min load cost

Denote

- S_g: startup cost
- K_g: min load cost
- MC_g: marginal cost function

$$TC_g(u_g, p_g) = \sum_{t=1}^{T} \left(K_g \cdot u_{gt} + S_g \cdot v_{gt} + \int_0^{p_{gt}} MC_g(x)dx\right)$$

v_{gt}: indicator for startup in period t

$$v_{gt} = \begin{cases} 1, \text{if } u_{g,t-1} = 0, u_{g,t} = 1 \\ 0, \text{otherwise} \end{cases}$$

Generic unit commitment model

$$(UC): \min_{u,p,r} \sum_{g \in G} TC_g(u_g, p_g)$$

$$h_g(p_g, r_g, u_g) \leq 0, g \in G$$

$$\sum_{g \in G} p_{gt} = D_t \, , t = 1, \ldots, T$$

$$\sum_{g \in G} r_{gt} = R_t \, , t = 1, \ldots, T$$

$$u_{gt} \in \{0,1\}, g \in G, t = 1, \ldots, T$$

- h_g: private operating constraints of unit g
- D_t: energy demand
- R_t: reserve demand

Initial conditions

Denote

- $u0_g \in \{0,1\}^{T_0}$: initial commitment, T_0 periods prior to first period of scheduling horizon
- $p0_g \in \mathbb{R}^{T_0}$: initial production

How long should T_0 be?

Transitions

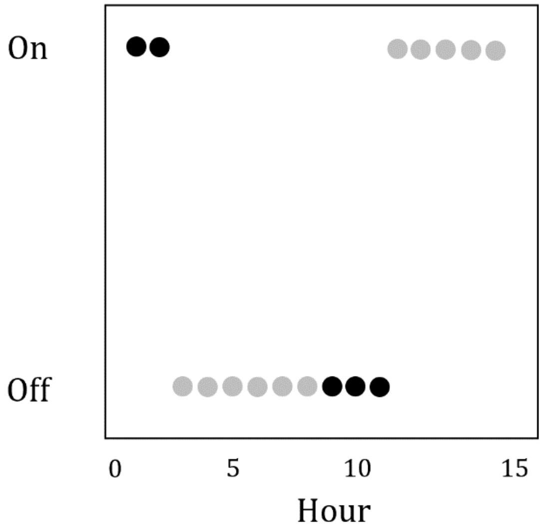
Notation:

- u indicates on status
- v indicates startup
- z indicates shutdown

$$u_{gt} = u_{g,t-1} + v_{gt} - z_{gt}, g \in G, t = 2, \ldots, T$$

Min up/down time

- Black dots: forced states
- Gray dots: free choices



What is the min up time? Down time?

Denote

- UT_g: min up time
- DT_g: min down time

$$\sum_{\tau=t-UT_g+1}^{t} v_{g\tau} \leq u_{gt}, g \in G, t = UT_g, \dots, T$$

$$\sum_{\tau=t-DT_g+1}^{t} z_{g\tau} \leq 1 - u_{gt}, g \in G, t = DT_g, \dots, T$$

Example 7.2: dependency of cost on temperature

Temperature of a generator determines how much fuel is required in order to start it up

Example:

- Hot: 200 GJ needed to start 1-16 hours after shut down
- Warm: 220 GJ needed to start 17-24 hours after shut down
- Cold: 250 GJ needed to start 25+ hours after shut down

$$\Theta = \{\text{Hot}, \text{Warm}, \text{Cold}\}$$

Temperature-dependent startup

- v_{glt}: indicator for startup in temperature state l at period t

- Generator can only start up from a single temperature state:

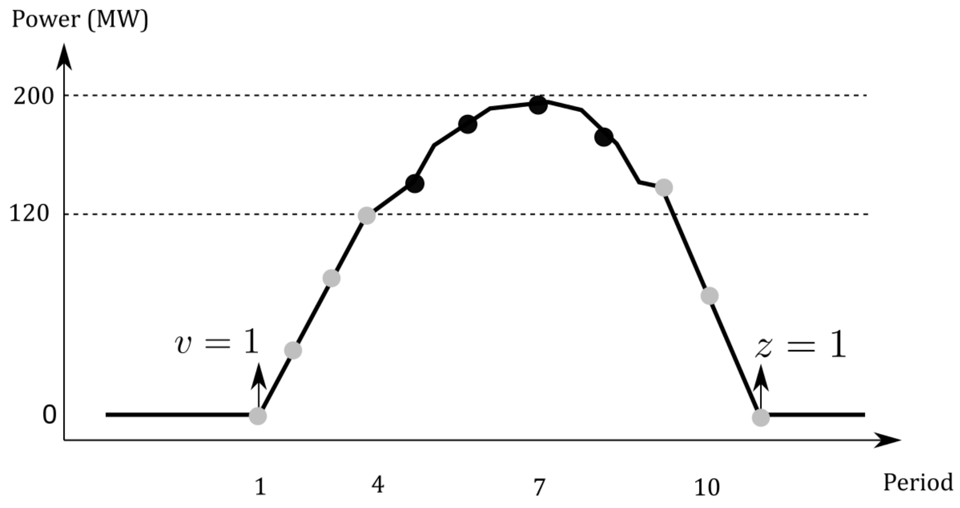
$$v_{gt} = \sum_{l \in \Theta} v_{glt}, g \in G, t = 1, \dots, T$$

- Temperature state l occurs within TA_{gl} to TB_{gl} periods after shutdown:

$$v_{glt} \leq \sum_{\tau = t - TA_{gl} + 1}^{t - TB_{gl}} z_{g\tau}, l \in \Theta, t = TA_{gl}, \dots, T$$

Startup/shutdown profiles

Startup/shutdown profiles: predefined sequence of production when generators are started up/shut down



- Gray dots: profile (restricted)
- Black dots: free dispatch

Example 7.3: startup profile with ramp limit

Consider a generator with

- Technical minimum: 120 MW (should be reached as soon as possible)
- Ramp rate: 1 MW/minute

Startup profile is (60 MW, 120 MW), why?

Temperature-dependent startup profiles

- u_{gt}^{SU}: indicator for startup
- u_{gt}^{SD}: indicator for shutdown
- u_{gt}^{DISP}: indicator for free dispatch

Generator must be in one of three states:

$$u_{gt} = u_{gt}^{SU} + u_{gt}^{DISP} + u_{gt}^{SD}, g \in G, t = 1, \ldots, T$$

- T_{gl}^{SU}: duration of startup profile (depends on temperature l)
- T_{gl}^{SD}: duration of shutdown profile

Determine whether generator is in startup/shutdown:

$$u_{gt}^{SU} = \sum_{l \in \Theta} \sum_{\tau = t - T_{gl}^{SU} + 1, \tau \geq 1}^{t} v_{gl\tau}, g \in G, t = \max_{l \in \Theta} T_{gl}^{SU}, \ldots, T$$

$$u_{gt}^{SD} = \sum_{\tau = t}^{t + T_{gl}^{SD} - 1} z_{g\tau}, g \in G, t = 1, \ldots, T - T_{g}^{SD} + 1$$

Startup/shutdown production

- $P_{gl\tau}^{SU}$: sequence of production levels for startup profile (note dependence on temperature l)
- $P_{g\tau}^{SD}$: sequence of production levels for shutdown profile

Production in startup/shutdown profile:

$$p_{gt}^{SU} = \sum_{l \in \Theta} \sum_{\tau = t - T_{gl}^{SU} + 1, \tau \geq 1}^{t} P_{gl,t-\tau+1}^{SU} \cdot v_{gl\tau}, g \in G, t = \max_{l \in \Theta} T_{gl}^{SU}, \ldots, T$$

$$p_{gt}^{SD} = \sum_{\tau = t+1}^{t + T_{gl}^{SD}} P_{g\tau}^{SD} \cdot z_{g\tau}, g \in G, t = 1, \ldots, T - T_g^{SD}$$

Dispatchable production

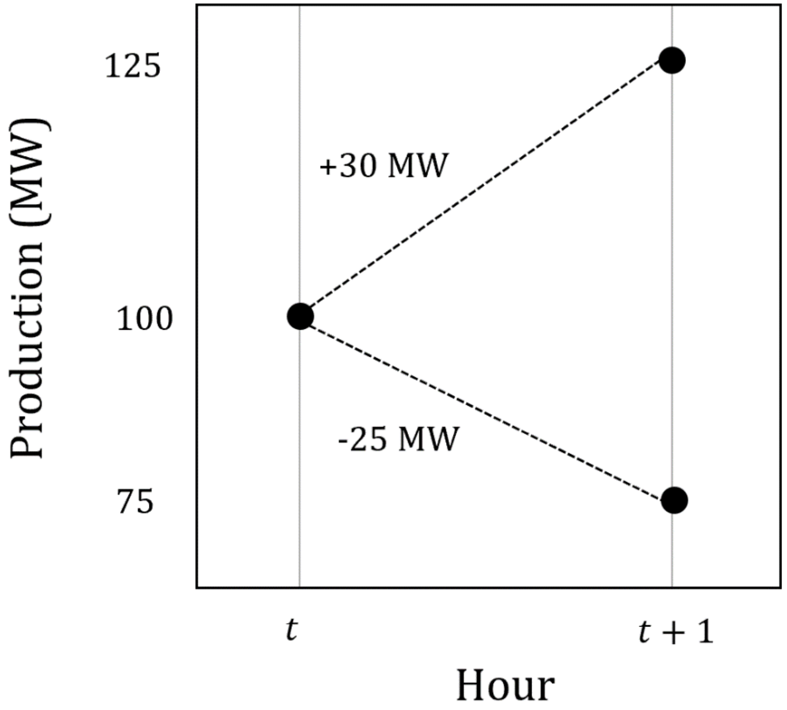
Denote P_g^- / P_g^+ as technical minimum/maximum

$$p_{gt} \geq p_{gt}^{SU} + p_{gt}^{SD} + P_g^- \cdot u_{gt}^{DISP}, g \in G, t = 1, \ldots, T$$

$$p_{gt} \leq p_{gt}^{SU} + p_{gt}^{SD} + P_g^+ \cdot u_{gt}^{DISP}, g \in G, t = 1, \ldots, T$$

What happens when $u_{gt}^{DISP} = 0$? $u_{gt}^{DISP} = 1$?

Ramp rates



Note: ramp rates may be violated by startup/shutdown profiles

Denote R_g^+ / R_g^- as ramp up/down rate limit

$$p_{gt} - p_{g,t-1} \leq R_g^+ + M \cdot u_{gt}^{SU}, g \in G, t = 2, \ldots, T$$

$$p_{g,t-1} - p_{gt} \leq R_g^- + M \cdot u_{gt}^{SD}, g \in G, t = 2, \ldots, T$$

What happens when $u_{gt}^{DISP} = 0$? $u_{gt}^{DISP} = 1$?

Commitment cost

Denote

- SUC_{gl}: startup cost for temperature l
- MLC_g: minimum load cost

$$UC_g(u_g) = \sum_{t=1}^{T} (\sum_{l \in \Theta} SUC_{gl} \cdot v_{glt} + MLC_g \cdot u_{gt}), g \in G$$

Note: Fuel cost from startup profiles *not* accounted here

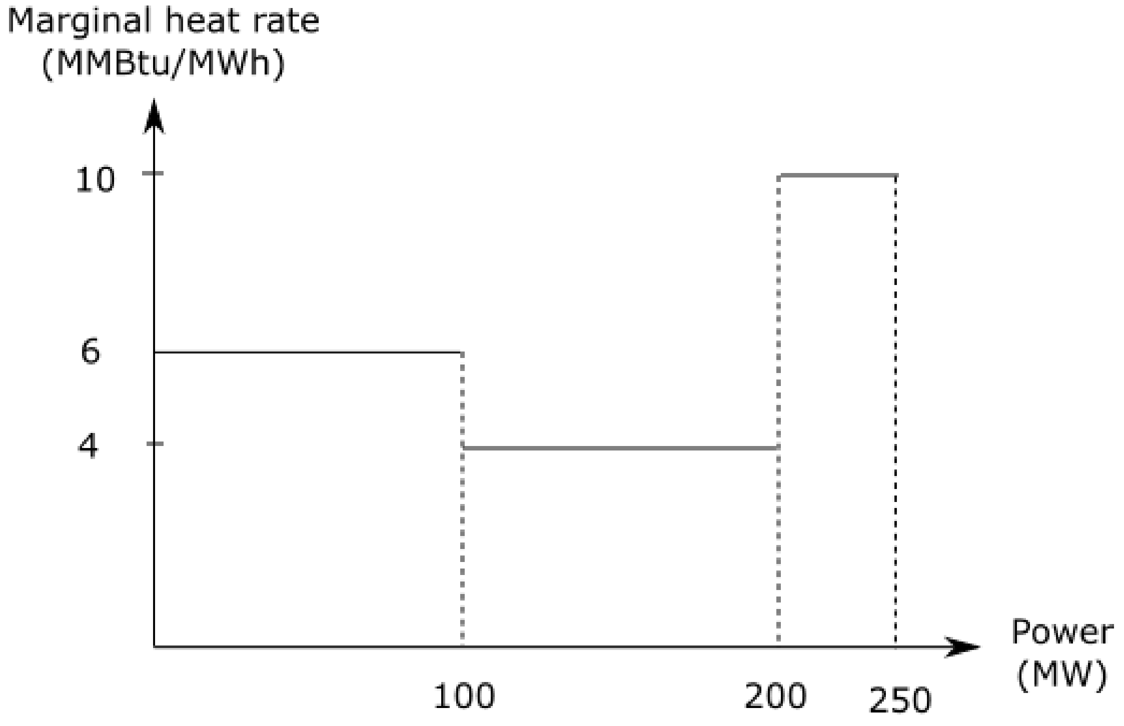
Fuel cost

- **Average heat rate** (MMBtu/MWh): ratio of *total* fuel consumption to *total* electric power production
- **Marginal heat rate** (MMBtu/MWh): derivative of fuel consumption with respect to electric power production

Denote $MHR_g(p)$ as marginal heat rate curve, FP as fuel price ($/MMBtu):

$$PC_{gt}(p_{gt}) = FP \cdot \int_0^{p_{gt}} MHR_g(x)dx \, , g \in G, t = 1, \ldots, T$$

Example 7.4: non-increasing marginal heat rate



Why does this heat rate curve cause modeling problems?

Modeling non-convex fuel cost

- Denote:
  - S: set of segments in heat rate curve
  - P_{gs}^{+}: width of each segment
  - MHR_{gs}: marginal heat rate of each segment

Activate first segment once generator is started up:

$$u_{g,s_1,t} = u_{gt}, g \in G, t = 1, \dots, T$$

Segment cannot be activated before previous segment is fully used:

$$u_{g,s+1,t} \leq \frac{p_{gst}}{P_{gs}^{+}}, g \in G, s = 1, \dots, S-1, t = 1, \dots, T$$

Production within each segment:

$$0 \leq p_{gst} \leq P_{gs}^{+} \cdot u_{gst}, g \in G, s = 1, \ldots, S-1, t = 1, \ldots, T$$

Total power production:

$$p_{gt} = \sum_{s \in S} p_{gst}, g \in G, t = 1, \ldots, T$$

Total production cost:

$$PC_{gt}(p_{gt}) = FC \cdot \sum_{s=1}^{S} MHR_{gs} \cdot p_{gst}, g \in G, t = 1, \ldots, T$$

Frequency restoration reserves

Denote upwards/downwards reserve as $r2^{+}_{gt}, r2^{-}_{gt} \geq 0$

Min/max capacity constraints:

$$p_{gt} - r2^{-}_{gt} \geq p^{SU}_{gt} + p^{SD}_{gt} + P^{-}_{g} \cdot u^{DISP}_{gt}, g \in G, t = 1, \dots, T$$
$$p_{gt} + r2^{+}_{gt} \leq p^{SU}_{gt} + p^{SD}_{gt} + P^{+}_{g} \cdot u^{DISP}_{gt}, g \in G, t = 1, \dots, T$$

Denote upward/downward reserve limits as $MR2^{+}_{g}$ / $MR2^{-}_{g}$:

$$r2^{-}_{gt} \leq MR2^{-}_{g} \cdot u^{DISP}_{gt}, g \in G, t = 1, \dots, T$$
$$r2^{+}_{gt} \leq MR2^{+}_{g} \cdot u^{DISP}_{gt}, g \in G, t = 1, \dots, T$$

Denote upward/downward requirements as $RR2^{+}_{t}$ / $RR2^{-}_{t}$:

$$\sum_{g \in G} r2^{-}_{gt} \geq RR2^{-}_{t}, t = 1, \dots, T$$
$$\sum_{g \in G} r2^{+}_{gt} \geq RR2^{+}_{t}, t = 1, \dots, T$$

Replacement reserves

Denote $r3_{gt}^{S} \geq 0$ as spinning replacement reserves

Max capacity:

$$p_{gt} + r2_{gt}^{+} + r3_{gt}^{S} \leq p_{gt}^{SU} + p_{gt}^{SD} + P_g^{+} \cdot u_{gt}^{DISP}, g \in G, t = 1, \ldots, T$$

Denote $r3_{gt}^{NS} \geq 0$ as non-spinning replacement reserve

Max capacity:

$$r3_{gt}^{NS} \leq P_g^{+} \cdot \left(1 - u_{gt}\right), \in G, t = 1, \ldots, T$$

Denote $MR3_g$ as replacement reserve limit:

$$r3_{gt}^{S} + r3_{gt}^{NS} \leq MR3_g, g \in G, t = 1, \dots, T$$

Denote $RR3_t$ as aggregate replacement reserves:

$$\sum_{g \in G} (r3_{gt}^{S} + r3_{gt}^{NS}) \geq RR3_t, t = 1, \dots, T$$

Reserve requirements for renewables

Unit commitment model can quantify

- Reserve requirements
- Operating cost
- Utilization of resources (thermal, renewable)

Policy support: we can quantify trade-offs of renewable energy

- Uncertainty (-)
- Free fuel cost (+)

An important question is: how many reserves do we need? Different models provide different answers...

Stochastic unit commitment

Two-stage formulation:

1. First stage: commitment
2. Revelation of uncertainty: component contingencies (generators, lines), forecast errors (renewables, demand)
3. Second stage: generator/load dispatch

Setup

- Thermal units: controllable, costly
- Renewable generators: zero cost, but unpredictable

Tradeoff:

- Too many reserves ⇒ high startup/min load costs, renewable energy curtailment
- Too few reserves ⇒ load shedding

Criticisms

- Model size
- Detailed model of uncertainty is needed
- Scenario selection is crucial and non-trivial

Security-constrained unit commitment

- Objective: minimize cost under normal conditions
- Each “scenario” corresponds to the outage of a single component
- All demand must be satisfied
- Renewable supply replaced by forecast

- In line with approach of system operator to unit commitment (+)
- Large-scale problem (-)
- Conservative (-)

References

[1] A. Papavasiliou, Optimization Models in Electricity Markets, Cambridge University Press

https://www.cambridge.org/highereducation/books/optimization-models-in-electricity-markets/0D2D36891FB5EB6AAC3A4EFC78A8F1D3#overview