Direct Current Optimal Power Flow

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Source: section 5.1, Papavasiliou [1]

Outline

- The OPF using PTDFs
- The OPF using reactance

Transmission constraints

Lines can carry a limited amount of power

- Thermal limits
- Stability limits
- Voltage drop limits

Power flow equations

- Non-linear mapping: power injection in buses → power flow on lines
- We will linearize these

Optimal power flow (OPF): Maximize welfare (minimize cost) subject to power flow equations + transmission limits

Network representation

Transmission system is represented as a directed graph

- N: set of nodes
- K: set of lines (denoted by $k = (m, n)$)
- G_n: set of generators located in node n, $G = \cup_{n \in N} G_n$
- L_n: set of loads located in node n, $L = \cup_{n \in N} L_n$

Two equivalent models

Decisions:

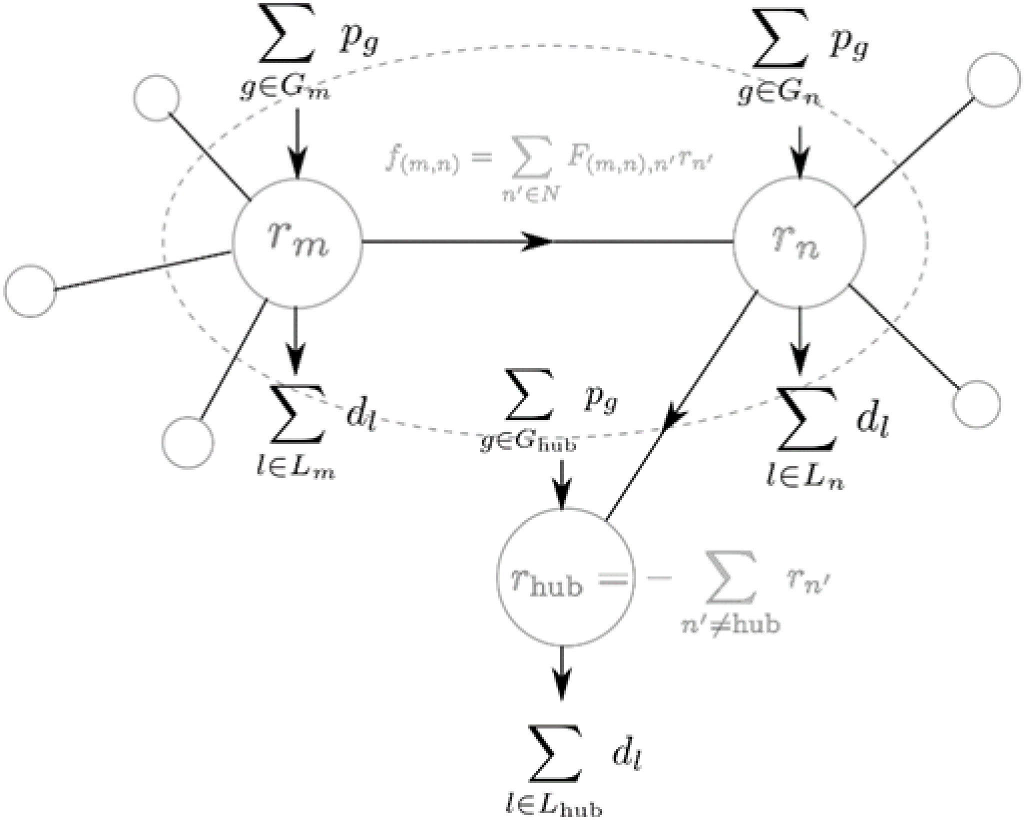
- p_g: amount of power produced by generator g
- d_l: amount of power consumed by load l

Two *equivalent* models, depending on system state and input data

- Model 1
  - System state: nodal injections
  - Input data: power transfer distribution factors (depend on physical characteristics of lines)
- Model 2
  - System state: nodal phase angles
  - Input data: reactance (depend on physical characteristics of lines)

The OPF using PTDFs

Model 1: power transfer distribution factors



Net injection

Hub node: reference node that "absorbs" all injections

Injection r_n: amount of power shipped from node n to the hub

$$r_n = \sum_{g \in G_n} p_g - \sum_{l \in L_n} d_l \, , n \in N$$

Not amount of power flowing over line connecting n and hub

Conservation of energy:

$$\sum_{n \in N} r_n = 0$$

Power flows

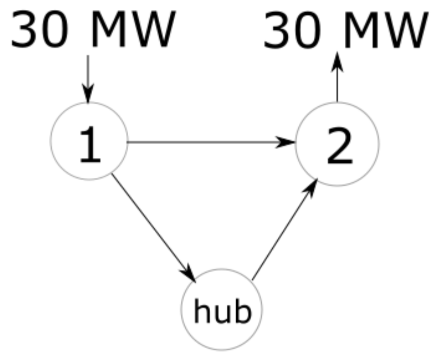
- **Power transfer distribution factor (PTDF)** F_{kn}: amount of power flowing on line k ως συνέπεια αποστολής 1 MW as a result of shipping 1 MW from n to hub
  - $F_{k,\mathrm{hub}} = 0$
  - PTDF: input data, depend on physical characteristics of lines
  - PTDFs depend on choice of hub
  - Flow f_k is

$$f_k = \sum_{n \in N} F_{kn} \cdot r_n \, , k \in K$$

  - Flow can be positive or negative (interpretation?)
  - T_k : limit on power that each line can carry

$$-T_k \leq f_k \leq T_k$$

Example



All lines have identical electrical characteristics

1. $F_{1-2,1} =? , F_{1-2,2} =?$
2. Express shipment of 30 MW from 1 to 2 as transaction through hub
3. Compute flow f_{1-2} from steps 1, 2
4. Note: r_1 and $f_{1-\text{hub}}$ are *different*

The OPF using PTDFs

(DCOPF):

$$\max_{p,d,f,r} \sum_{l \in L} \int_0^{d_l} MB_l(x)\, dx - \sum_{g \in G} \int_0^{p_g} MC_g(x) dx$$

(λ_k^+):

$$f_k \leq T_k, k \in K$$

(λ_k^-):

$$-f_k \leq T_k, k \in K$$

(ψ_k):

$$f_k - \sum_{n \in N} F_{kn} \cdot r_n = 0, k \in K$$

(ρ_n):

$$r_n - \sum_{g \in G_n} p_g + \sum_{l \in L_n} d_l = 0, n \in N$$

$(-\varphi)$:

$$\sum_{n \in N} r_n = 0$$

$$p_g \geq 0, g \in G$$

$$d_l \geq 0, l \in L$$

Optimal solution

- Denote P_g, D_l as maximum production/consumption of generators/loads (imposed through domain of objective function)

There exists a threshold ρ_n for all n such that:

- If $0 < p_g < P_g$, then $\rho_n = MC_g(p_g)$. If $0 < d_l < D_l$, then $\rho_n = MB_l(d_l)$.
- If $p_g = P_g$, then $\rho_n \geq MC_g(P_g)$. If $d_l = D_l$, then $\rho_n \leq MB_l(D_l)$.
- If $p_g = 0$, then $\rho_n \leq MC_g(0)$. If $d_l = 0$, then $\rho_n \geq MB_l(0)$.

Proof

Use KKT conditions

$$0 \le p_g \perp MC_g(p_g) - \rho_{n(g)} + \mu_g \ge 0$$
$$0 \le \mu_g \perp P_g - p_g \ge 0$$

$$0 \le d_l \perp -MB_l(d_l) + \rho_{n(l)} + \nu_l \ge 0$$
$$0 \le \nu_l \perp D_l - d_l \ge 0$$

- $n(g)$: node where generator g is located
- $n(l)$: node where load l is located

Sensitivity

Helpful in understanding transmission pricing

- φ: marginal change in welfare from marginal increase in production/marginal decrease in consumption
- λ_k^+ and λ_k^-: marginal impact of increasing line capacity
- ρ_n: marginal impact of marginal increase of consumption/decrease of generation in node n (what if demand is inelastic?)

What sign do we expect for these dual variables?

Components of ρ_n

Useful identity for computing prices:

$$\rho_n = \varphi + \sum_{k \in K} F_{kn} \cdot \lambda_k^- - \sum_{k \in K} F_{kn} \cdot \lambda_k^+$$

Proof: KKT conditions

Example

Case 1

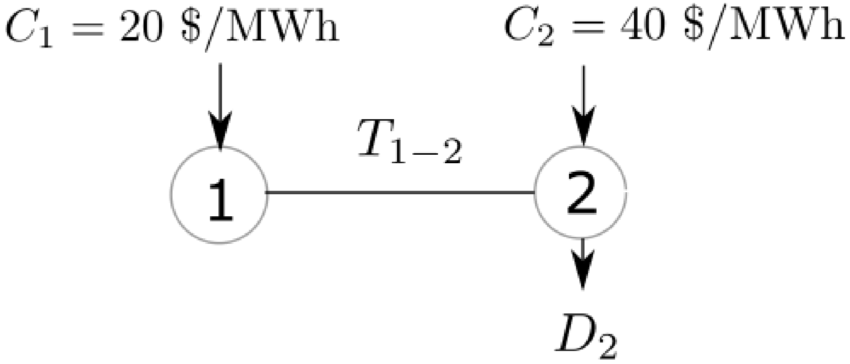
- $D_2 = 50\ \mathrm{MW}, T_{1-2}$ unlimited
- $\rho_1 = \rho_2 = 20\,\frac{\$}{\mathrm{MWh}}$

Case 2

- $D_2 = 50\ \mathrm{MW}, T_{1-2} = 50\ \mathrm{MW}$
- $\rho_1 = 20\,\frac{\$}{\mathrm{MWh}}, 20\,\frac{\$}{\mathrm{MWh}} \leq \rho_2 \leq 40\,\frac{\$}{\mathrm{MWh}}$

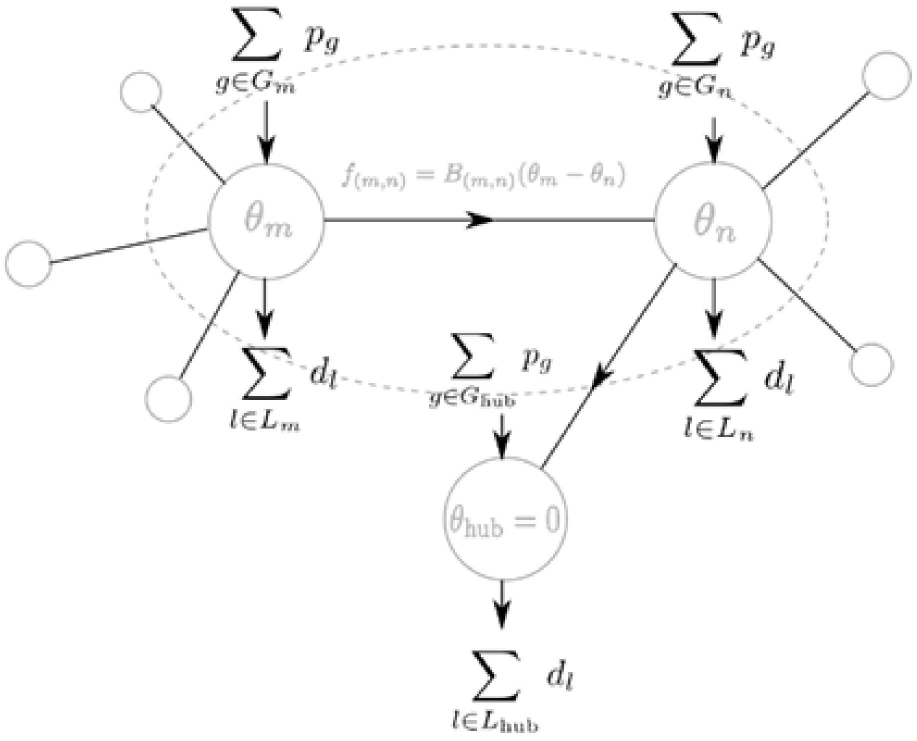
Case 3

- $D_2 = 60\ \mathrm{MW}, T_{1-2} = 50\ \mathrm{MW}$
- $\rho_1 = 20\,\frac{\$}{\mathrm{MWh}}, \rho_2 = 40\,\frac{\$}{\mathrm{MWh}}$



The OPF using reactance

Model 2: reactance



Power flows

- Reactance: input data, depends on physical characteristics of lines
- Independent of choice of hub
- Flow f_k is

$$f_{(m,n)} = B_{(m,n)} \cdot (\theta_m - \theta_n)$$

- Translation of θ results in identical flows, fix $\theta_{\text{hub}} = 0$
- Conservation of energy:

$$\sum_{g \in G_n} p_g + \sum_{k=(\cdot,n)} f_k = \sum_{k=(n,\cdot)} f_k + \sum_{l \in L_n} d_l \, , n \in N$$

- Input data is independent of network topology: transmission line investment, transmission line outages

The OPF using reactance

(DCOPF2):

$$\max_{p,d,f,\theta} \sum_{l \in L} \int_0^{d_l} MB_l(x)\, dx - \sum_{g \in G} \int_0^{p_g} MC_g(x) dx$$

(λ_k^+):

$$f_k \leq T_k, k \in K$$

(λ_k^-):

$$-f_k \leq T_k, k \in K$$

(γ_k):

$$f_k - B_k \cdot (\theta_m - \theta_n) = 0, k = (m,n) \in K$$

(ρ_n):

$$-\sum_{g \in G_n} p_g - \sum_{k=(\cdot,n)} f_k + \sum_{k=(n,\cdot)} f_k + \sum_{l \in L_n} d_l = 0, n \in N$$

$$p_g \geq 0, g \in G$$

$$d_l \geq 0, l \in L$$

References

[1] A. Papavasiliou, Optimization Models in Electricity Markets, Cambridge University Press

https://www.cambridge.org/highereducation/books/optimization-models-in-electricity-markets/0D2D36891FB5EB6AAC3A4EFC78A8F1D3#overview