Economic Dispatch

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Source: chapter 4, Papavasiliou [1]

Outline

- The economic dispatch model
- Competitive market equilibrium
- Modeling market equilibrium as an optimization problem

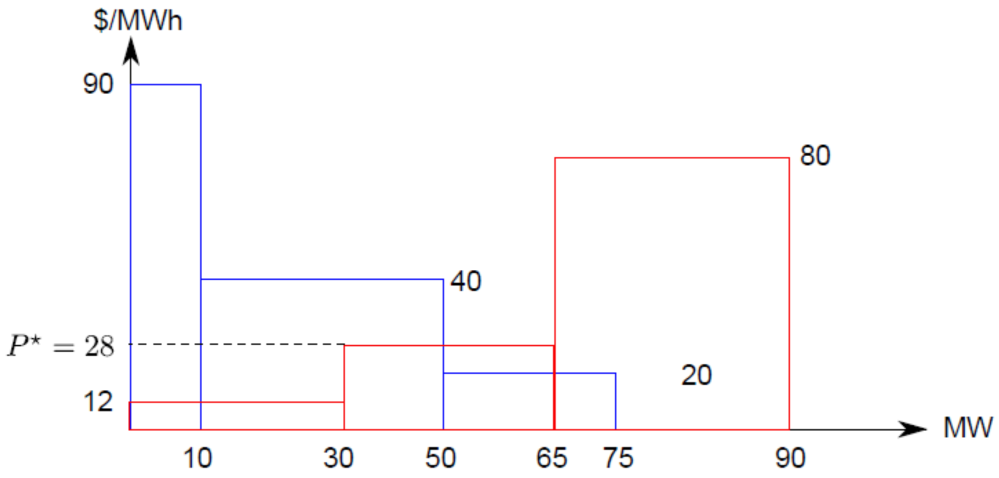
What is economic dispatch?

- Simplest resource allocation problem in electricity markets
- Model used in *real-time* electricity markets
  - Uniform price auctions
  - Repeated every five to fifteen minutes

An example

Consider the offers in the figure

1. Write the problem as a linear program
2. Write out the KKT conditions of the problem
3. Split the KKT conditions into three categories, depending on whether they correspond to
    1. A surplus maximization problem of buyers (quantity adjustment)
    2. A profit maximization problem of sellers (quantity adjustment)
    3. Market clearing conditions (price adjustment)
4. Propose a primal-dual optimal solution and confirm that it is optimal using the KKT conditions
5. Confirm that the market clearing price is indeed consistent with agent incentives



Question 1: linear program

The economic dispatch model is described as follows:

$$\max_{p,d} 90 \cdot d_1 + 40 \cdot d_2 + 20 \cdot d_3 - (12 \cdot p_1 + 28 \cdot p_2 + 80 \cdot p_3)$$
$$(\lambda): d_1 + d_2 + d_3 - p_1 - p_2 - p_3 = 0$$
$$(\mu_1): p_1 \leq 30$$
$$(\mu_2): p_2 \leq 35$$
$$(\mu_3): p_3 \leq 25$$
$$(\nu_1): d_1 \leq 10$$
$$(\nu_2): d_2 \leq 40$$
$$(\nu_3): d_3 \leq 25$$
$$p, d \geq 0$$

Questions 2, 3: KKT conditions and their decomposition

Market clearing:

$$d_1 + d_2 + d_3 - p_1 - p_2 - p_3 = 0$$

Questions 2, 3: KKT conditions and their decomposition

Profit maximization of sellers:

$$0 \leq \mu_1 \perp 30 - p_1 \geq 0$$
$$0 \leq \mu_2 \perp 35 - p_2 \geq 0$$
$$0 \leq \mu_3 \perp 25 - p_3 \geq 0$$
$$0 \leq p_1 \perp 12 + \mu_1 - \lambda \geq 0$$
$$0 \leq p_2 \perp 28 + \mu_2 - \lambda \geq 0$$
$$0 \leq p_3 \perp 80 + \mu_3 - \lambda \geq 0$$

Questions 2, 3: KKT conditions and their decomposition

Surplus maximization of buyers:

$$0 \leq \nu_1 \perp 10 - d_1 \geq 0$$
$$0 \leq \nu_2 \perp 40 - d_2 \geq 0$$
$$0 \leq \nu_3 \perp 25 - d_3 \geq 0$$
$$0 \leq d_1 \perp -90 + \nu_1 + \lambda \geq 0$$
$$0 \leq d_2 \perp -40 + \nu_2 + \lambda \geq 0$$
$$0 \leq d_3 \perp -20 + \nu_3 + \lambda \geq 0$$

Question 4: prima-dual optimal solution

Primal optimal solution:

$$p_1^* = 30, p_2^* = 20, p_3^* = 0$$
$$d_1^* = 10, d_2^* = 40, d_3^* = 0$$

Dual optimal solution:

$$\lambda^* = 28$$
$$\mu_1^* = 16, \mu_2^* = 0, \mu_3^* = 0$$
$$\nu_1^* = 62, \nu_2^* = 12, \nu_3^* = 0$$

We note that all KKT conditions are satisfied

Question 5: checking the incentives of agents

- From the point of view of producers:
  - Producer 1 is in the money and therefore wants to produce $p_1^* = 30$
  - Producer 2 is at the money and therefore indifferent about producing $p_2^* = 20$
  - Producer 3 is out of the money and therefore wants to produce $p_3^* = 0$

- Similarly for consumers

The economic dispatch model

Welfare maximizing economic dispatch

$$\max_{p,d} \sum_{l \in L} \int_0^{d_l} MB_l(x)dx - \sum_{g \in G} \int_0^{p_g} MC_g(x)dx$$

$$(\lambda): \sum_{l \in L} d_l - \sum_{g \in G} p_g \leq 0$$

$$(\nu_l): d_l \leq D_l, l \in L$$

$$(\mu_g): p_g \leq P_g, g \in G$$

$$p_g \geq 0, g \in G, d_l \geq 0, l \in L$$

Increasing marginal cost function $MC_g(\cdot)$, decreasing marginal benefit function $MB_l(\cdot)$

KKT conditions

$$0 \le p_g \perp -\lambda + MC_g(p_g) + \mu_g \ge 0, g \in G$$

$$0 \le d_l \perp \lambda - MB_l(d_l) + \nu_l \ge 0, l \in L$$

$$0 \le \mu_g \perp P_g - p_g \ge 0, g \in G$$

$$0 \le \nu_l \perp D_l - d_l \ge 0, l \in L$$

$$0 \le \lambda \perp \sum_{g \in G} p_g - \sum_{l \in L} d_l \ge 0$$

System lambda

There exists a threshold λ such that:

- If $0 < p_g < P_g$), then $MC_g(p_g) = \lambda$. If $0 < d_l < D_l$), then $MB_l(d_l) = \lambda$.
- If $p_g = 0$, then $MC_g(0) \geq \lambda$. If $d_l = 0$, then $MB_l(0) \leq \lambda$.
- If $p_g = P_g$, then $MC_g(P_g) \leq \lambda$. If $d_l = D_l$, then $MB_l(D_l) \geq \lambda$.

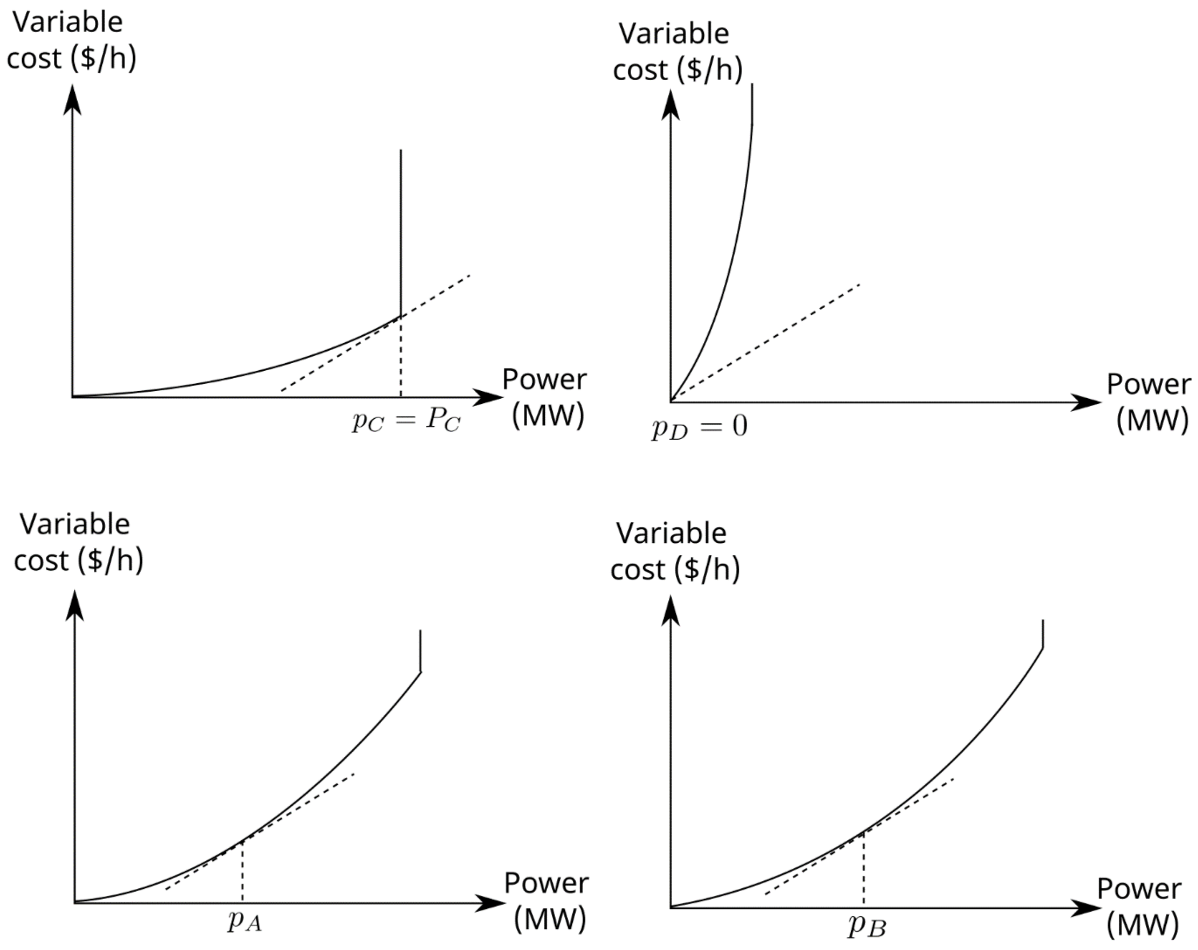
Proof: KKT conditions

- **System lambda**: marginal cost of the marginal generating unit (i.e. the generating unit which will supply the next unit of power at lowest cost)

Interpretation of KKT conditions

Optimal solution is matching cheapest generators with consumers who have greatest valuation (can you see why from the KKT conditions?)

Graphical illustration of KKT conditions



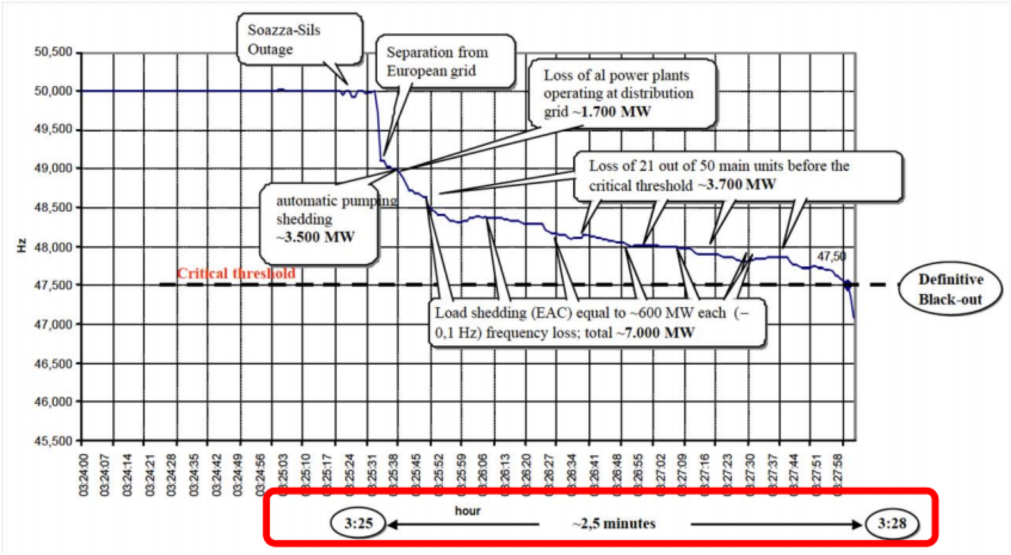
Competitive market equilibrium

Path to deregulation

- Late 1970s: power systems are operated as vertically integrated regulated monopolies
- Before 1980s: Premature markets (e.g. Norway)
- 1982: Chile introduces a sport market
- 1988: British government privatizes public power sector in England and Wales
- 1990: Nordic market expands to include Sweden, Finland and Denmark
- New Zealand and Australia introduced spot markets
- The United States follow with California (CAISO), Pennsylvania-New Jersey-Maryland (PJM), Texas (ERCOT), New York (NYISO) and the Midwest (MISO)

Trading in real time

- Real-time markets cannot rely on bilateral negotiations (only takes a few minutes of imbalance for a blackout
- ... but they can rely on a uniform price auction that charges system lambda for power
- But why is system lambda the “right” price?



Definition of competitive market

- A market is **competitive** if:
    - Agents are price-taking
    - Variable cost is convex and the benefit is concave (which implies that marginal cost is? marginal benefit is?)
    - Agents have access to public information (prices)

Aggregate and marginal cost

- **Aggregate cost** is the cheapest way to produce Q MW of power among a *collection* of producers

$$TC_G(Q) = \min_p \sum_{g \in G} \int_0^{p_g} MC_g(x)dx$$

$$\text{s.t.} \sum_{g \in G} p_g = Q$$

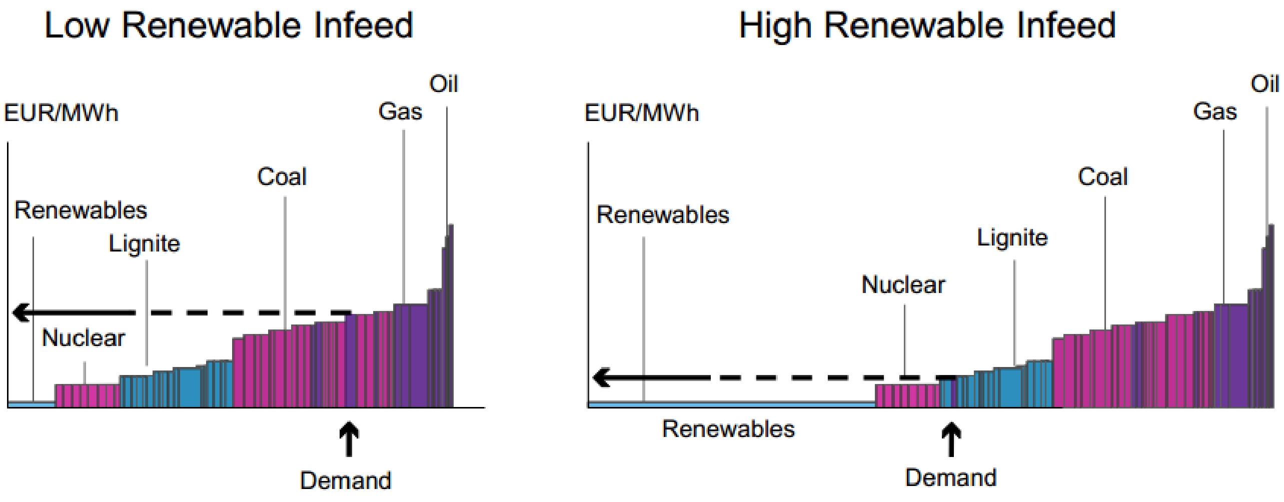
$$p_g \in \text{dom } MC_g, g \in G$$

Marginal cost: $MC_G(Q) = TC_G'(Q)$

- Constraints imposed through domain of objective function (last constraint)
- What do we know about MC in competitive markets?
- What is the unit of measurement of TC and MC?

Merit order curve

Merit order curve: (increasing) system marginal cost curve



Source: Agora Energiewende

Aggregate and marginal benefit

Aggregate benefit is most beneficial way to consume Q MW of power among a *collection* of consumers

$$TB_L(Q) = \max_d \sum_{l \in L} \int_0^{d_l} MB_l(x)dx$$

$$\text{s.t.} \sum_{l \in L} d_l = Q$$

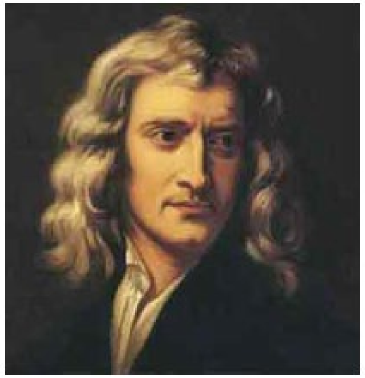
$$d_l \in \operatorname{dom} MB_l, l \in L$$

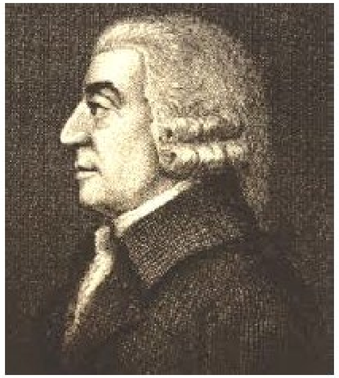
Marginal benefit: $MB_l(Q) = TB_L'(Q)$

Price and quantity adjustment

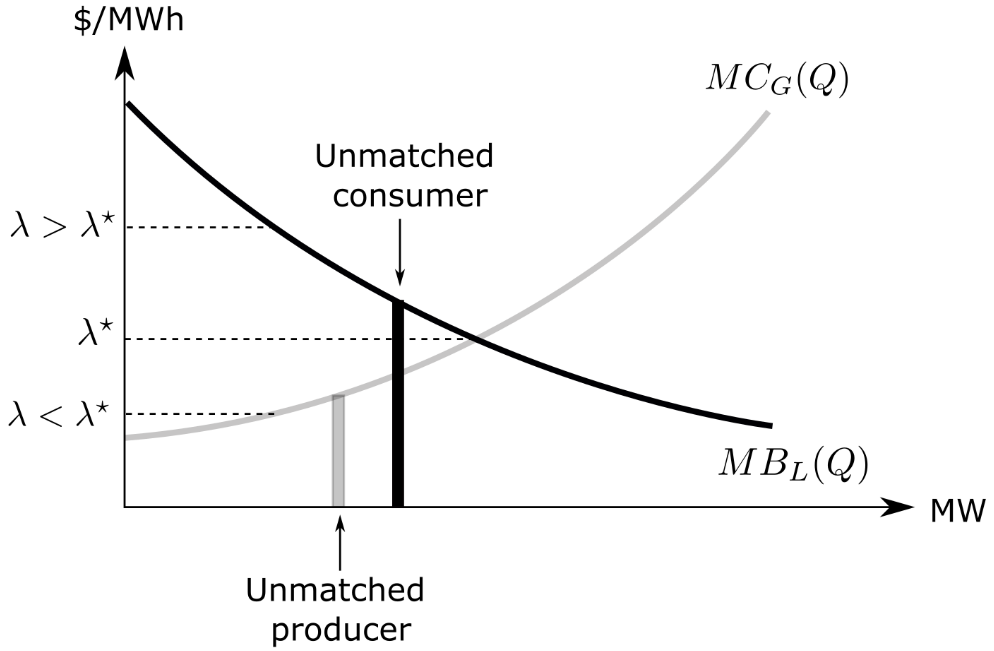
Mechanical system dynamics are governed by Newton's laws of motion

Price adjustment and **quantity adjustment** are the "laws of motion" for electricity markets





Price adjustment: graphical illustration



Any price different from λ^* creates opportunities for profitable trade

Price adjustment: mathematical description

When demand exceeds supply, upward pressure on *prices*

When supply exceeds demand, downward pressure on *prices*

Market clearing condition:

$$0 \leq \lambda \perp \sum_{g \in G} p_g - \sum_{l \in L} d_l \geq 0$$

Quantity adjustment

Price-taking supplier will increase *quantity* produced if marginal cost $\leq$ price, decrease output otherwise:

$$\max_p \lambda \cdot p_g - \int_0^{p_g} MC_g(x)dx$$

$$(\mu_g): p_g \leq P_g$$

$$p_g \geq 0$$

Price-taking consumer will decrease *quantity* consumed if marginal benefit $\leq$ price, increase consumption otherwise:

$$\max_d \int_0^{d_l} MB_l(x)dx - \lambda \cdot d_l$$

$$(\nu_l): d_l \leq D_l$$

$$d_l \geq 0$$

Equilibrium, market clearing price, competitive equilibrium, competitive price

- A market is in **equilibrium** when no profitable opportunities for trade exist
- The **market clearing price** is the price of a market in equilibrium
- An equilibrium in a competitive market is called a **competitive equilibrium**
- The price of a competitive market is the **competitive price**

Competitive markets are efficient

The competitive equilibrium results in an allocation which is optimal for the economic dispatch problem

Proof: Collect KKT conditions of quantity adjustment and market clearing condition of price adjustment:

Producers:

$$0 \le p_g \perp -\lambda + MC_g(p_g) + \mu_g \ge 0, g \in G$$

$$0 \le \mu_g \perp P_g - p_g \ge 0, g \in G$$

Consumers:

$$0 \le d_l \perp \lambda - MB_l(d_l) + \nu_l \ge 0, l \in L$$

$$0 \le \nu_l \perp D_l - d_l \ge 0, l \in L$$

Market clearing:

$$0 \le \lambda \perp \sum_{g \in G} p_g - \sum_{l \in L} d_l \ge 0$$

Identical to KKT conditions of economic dispatch

Producer and consumer surplus, welfare, efficiency

Suppose price is λ

- **Producer surplus/profit**: profit of producers who are willing to sell

$$\lambda q_G(\lambda) - \int_0^{q_G(\lambda)} MC_G(x)dx$$

where $q_G(\lambda)$ is quantity sold at price λ

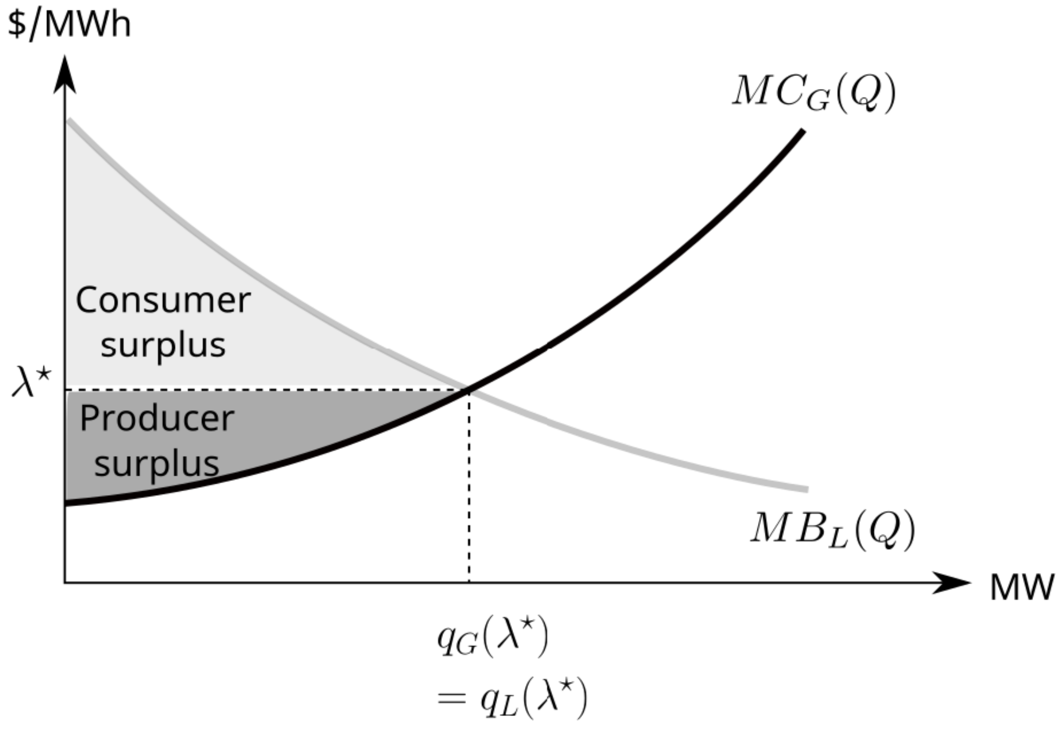
- **Consumer surplus**: surplus of consumers who are willing to buy

$$\int_0^{q_L(\lambda)} MB_L(x)dx - \lambda q_L(\lambda)$$

where $q_L(\lambda)$ is quantity bought at price λ

- **Welfare**: sum of producer and consumer surplus

Graphical illustration of surplus



Modeling market equilibrium as an optimization problem

Separable optimization

Consider the following problem

$$(\text{Sep}): \max_x \sum_{i=1}^{n} f_i(x_i)$$

$$(\rho_i): g_i(x_i) \leq 0, i = 1, \dots, n$$

$$(\lambda): \sum_{i=1}^{n} h_i(x_i) \leq 0$$

- $x_i \in \mathbb{R}^n$: private decisions
- $f_i: \mathbb{R}^{n_i} \to \mathbb{R}$: *concave* differentiable
- g_i: $\mathbb{R}^{n_i} \to \mathbb{R}^{a_i}$ and h_i: $\mathbb{R}^{n_i} \to \mathbb{R}^m$: convex differentiable

Interpretation

- m limited resources/**commodities**, n agents
- Each agent decides x_i, uses $h_i(x_i)$ of each of m resources
- For each resource, total consumption $\leq$ total production

KKT conditions

- Denote
  - $\nabla_{x_i} f_i(x_i) \in \mathbb{R}^{n_i}$: gradient of f_i
  - $\nabla_{x_i} g_i(x_i) \in \mathbb{R}^{a_i} \times \mathbb{R}^{n_i}$: Jacobian matrix of g_i (likewise for $\nabla_{x_i} h_i(x_i)$)

- KKT conditions of (Sep):

$$-\nabla_{x_i} f_i(x_i) + \left(\nabla_{x_i} g_i(x_i)\right)^T \rho_i - \left(\nabla_{x_i} h_i(x_i)\right)^T \lambda = 0, i = 1, \ldots, n$$

$$0 \leq \rho_i \perp -g_i(x_i) \geq 0, i = 1, \ldots, n$$

$$0 \leq \lambda \perp -\sum_{i=1}^{n} h_i(x_i) \geq 0$$

Market for multiple commodities

- Consider a competitive market for the m resources:
  - Producers are paid λ_j for selling commodity *j*
  - Consumers pay λ_j for buying commodity *j*
  - Each agent accepts price vector λ^* as *given* (not influenced by private decisions)
- Denote q_i as vector of resources procured (or sold, if negative) by agent i, then each agent solves:

$$
\begin{aligned}
&(\text{Profit-}i): && \max_{x_i, q_i} (f_i(x_i) - (\lambda^*)^T q_i) \\
&(\rho_i): && g_i(x_i) \leq 0 \\
&(\lambda_i): && h_i(x_i) = q_i
\end{aligned}
$$

Competitive equilibrium (for multiple products): combination of prices λ^*, agent decisions x_i^*, commodity procurements q_i^*, such that:

- (x_i^*, q_i^*) solve $(\text{Profit} - i)$ given λ^*, and
- Market clearing holds:

$$0 \leq \lambda^* \perp \sum_{i=1}^{n} q_i^* \leq 0$$

Modeling competitive market equilibrium via optimization

- Suppose KKT conditions are necessary and sufficient for the optimality of (Sep) and $(\text{Profit} - i)$:
  1. A competitive market equilibrium results in an optimal solution of (Sep), and
  2. a primal-dual solution to the KKT conditions of (Sep) is a competitive equilibrium

Proof

- Necessary and sufficient KKT conditions of $(\text{Profit} - i)$:

$$-\nabla_{x_i} f_i(x_i) + \left(\nabla_{x_i} g_i(x_i)\right)^T \rho_i - \left(\nabla_{x_i} h_i(x_i)\right)^T \lambda = 0$$

$$\lambda^* - \lambda = 0$$

$$0 \leq \rho_i \perp -g_i(x_i) \geq 0$$

$$h_i(x_i) = q_i$$

- Proceed by comparing KKT conditions of:
    - $(\text{Profit} - i)$ for all i and market clearing condition
    - (Sep)

Example: 2-agent oligopoly

Consider the following market:

- Linear marginal benefit function, $MB(Q) = a - b \cdot Q$
- Two agents, with identical cost functions TC_1 and TC_2

Competitive market equilibrium obtained by solving:

$$\max_{p_1,p_2,d} a \cdot d - 0.5 \cdot b \cdot d^2 - TC_1(p_1) - TC_2(p_2)$$

$$p_1 + p_2 = d$$

$$p_1, p_2, d \geq 0$$

If $p_1, p_2 > 0$ and $p_1 = p_2$ (since agents are symmetric), then

$$MC_1(p_1) = MC_2(p_2) = a - b \cdot (p_1 + p_2) \Rightarrow p_i = \frac{1}{2b}\big(a - MC_i(p_i)\big).$$

Example: Cournot duopoly

Suppose agent i realizes that it influences price, solves:

$$\max_{p_i}(a - b \cdot (p_1 + p_2)) \cdot p_i - TC_i(p_i)$$
$$p_i \geq 0$$

Denote p_{-i} as the decision of the competing agent, if $p_i > 0$ then:

$$p_i = \frac{1}{2b}\left(a - MC_i(p_i)\right) - \frac{1}{2}p_{-i}$$

And due to the symmetry of agents we have $p_i = p_{-i}$, and conclude that

$$p_i = \frac{1}{3b}\left(a - MC_i(p_i)\right)$$

We note that agents reduce their output below optimal in order to increase profitability

Market power

Market power: the strategic withholding of production from electricity markets by producers with the intention of *profitably* increasing prices

- Real problem in electricity markets
- Regulatory interventions (bid mitigation, price caps) can be used for mitigating market power ...
- ... but these interventions may create new problems (for example, the missing money problem)
- Strategic behavior of market agents typically analyzed using game theory (not optimization models)

References

[1] A. Papavasiliou, Optimization Models in Electricity Markets, Cambridge University Press

https://www.cambridge.org/highereducation/books/optimization-models-in-electricity-markets/0D2D36891FB5EB6AAC3A4EFC78A8F1D3#overview