Beyond Electricity

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Source: chapter 12, Papavasiliou [1]

Outline

- Hydrocarbons and biofuels
  - Hydrocarbons
  - Biofuels
- Short-term and long-term equilibrium
- Monopoly, cartel, and the dominant firm model
- Ta incidence
- One-way substitutability
- Hotelling's rule

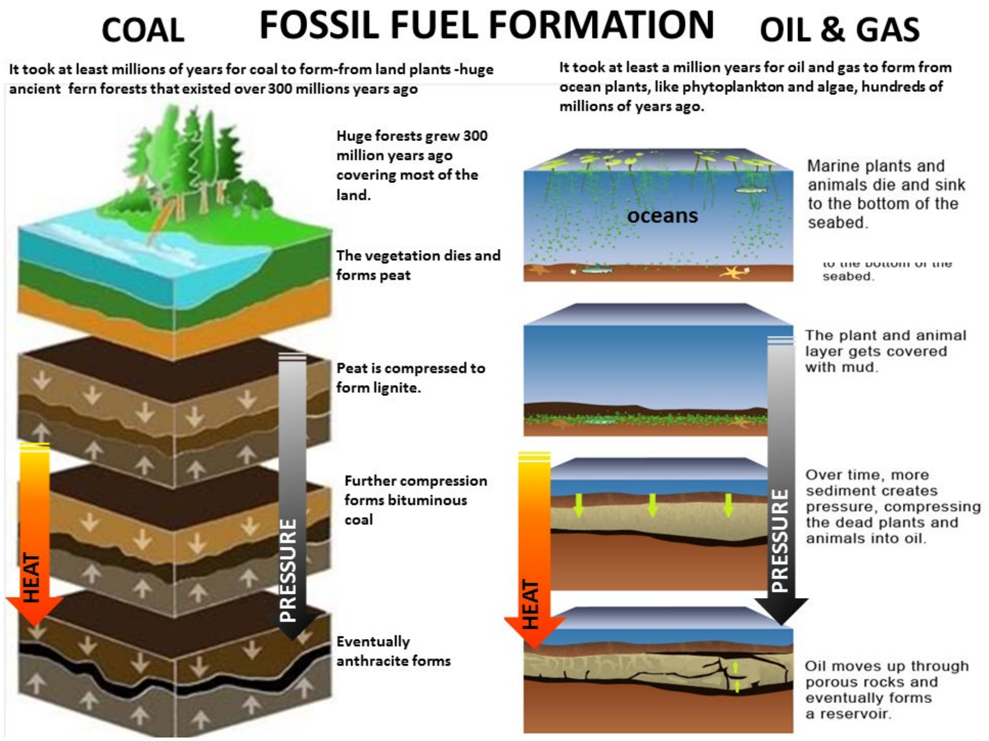
Hydrocarbons and biofuels

Hydrocarbons

Biofuels

Hydrocarbons: non-renewable resources

- Oil and natural gas are formed by organic matter, i.e. deceased plants and animals
- Formation of hydrocarbons requires millions of years under specific pressure and temperature conditions
- Non-renewable energy resources from practical standpoint



Oil: a global market

- The oil market is essentially global, since oil is stored and transported easily
- Largely impossible to split the market, which means that worldwide oil prices are approximately equal

Oil reserves

- The amount of available oil under the surface of the earth is unknown, and so is the amount that we will be able to extract in the future
- Reserves are typically classified as follows:

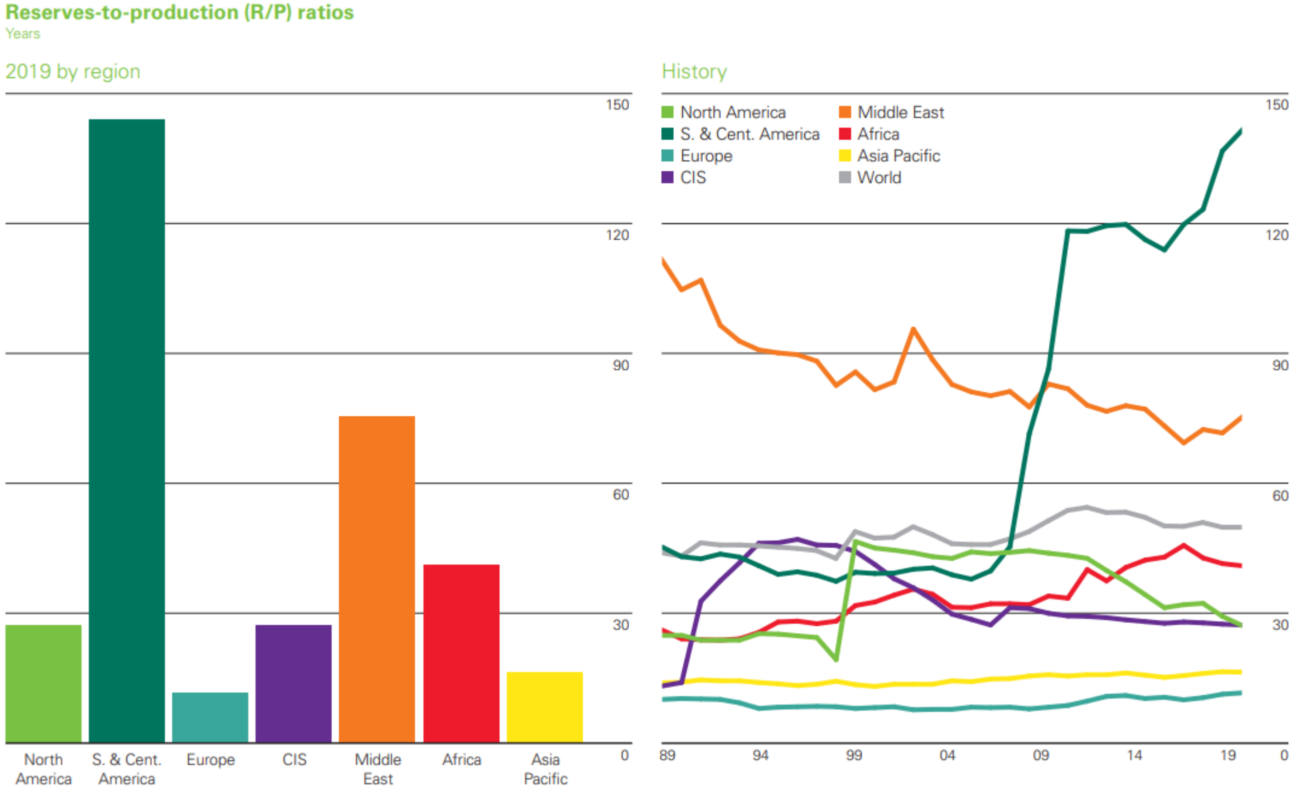
  (i) *Proven reserves*: 90-95% probability that commercially recoverable oil exists

  (ii) *Probable reserves*: sites with probability 50-89%

  (iii) *Possible reserves*: sites with probability 10-49%

Reserves-to-production (R/P) ratio

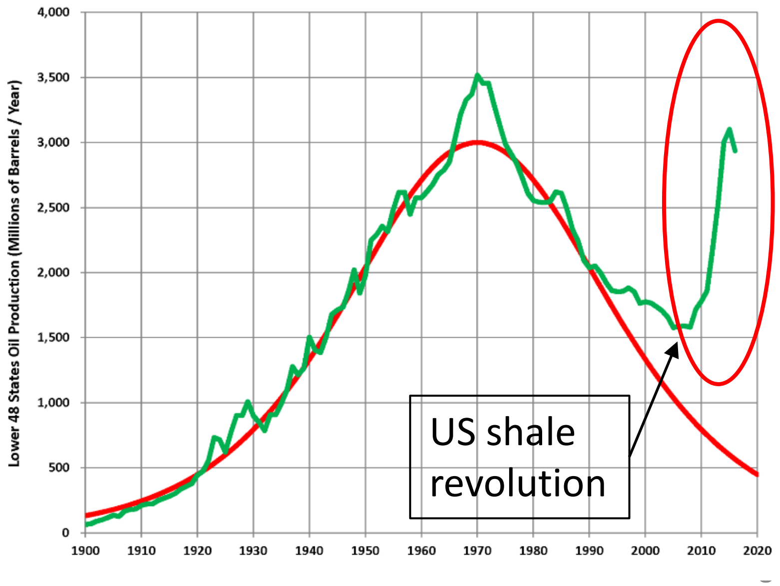
The reserves-to-production ratio or R/P ratio) refers to the remaining duration of extraction in years, if current production continues at the same pace



Global proved oil reserves were 1734 billion barrels at the end of 2019, down 2 billion barrels versus 2018. The global R/P ratio shows that oil reserves in 2019 accounted for 50 years of current production. Regionally, South & Central America has the highest R/P ratio (144 years) while Europe has the lowest (12 years). OPEC holds 70.1% of global reserves. The top countries in terms of reserves are Venezuela (17.5% of global reserves), closely followed by Saudi Arabia (17.2%) and Canada (9.8%).

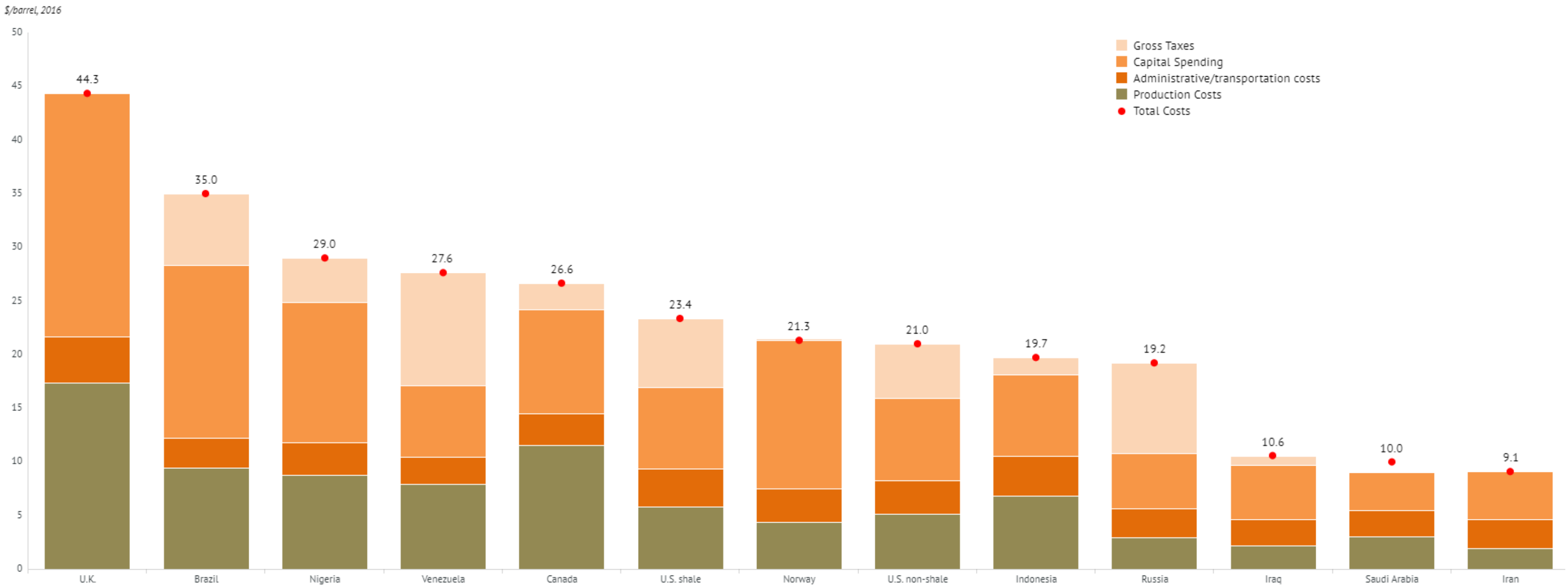
Peak oil

- **Peak oil**: moment in time when half of the global recoverable oil has been extracted
- Hubbert's curve (1956): predicted evolution of US crude oil production
- Until 2014: highly accurate prediction
  - Discovery of shale reserves in US overturned the prediction, important increase in US oil production after 2014
- Not clear if peak oil has already been reached or not



Global oil consumption

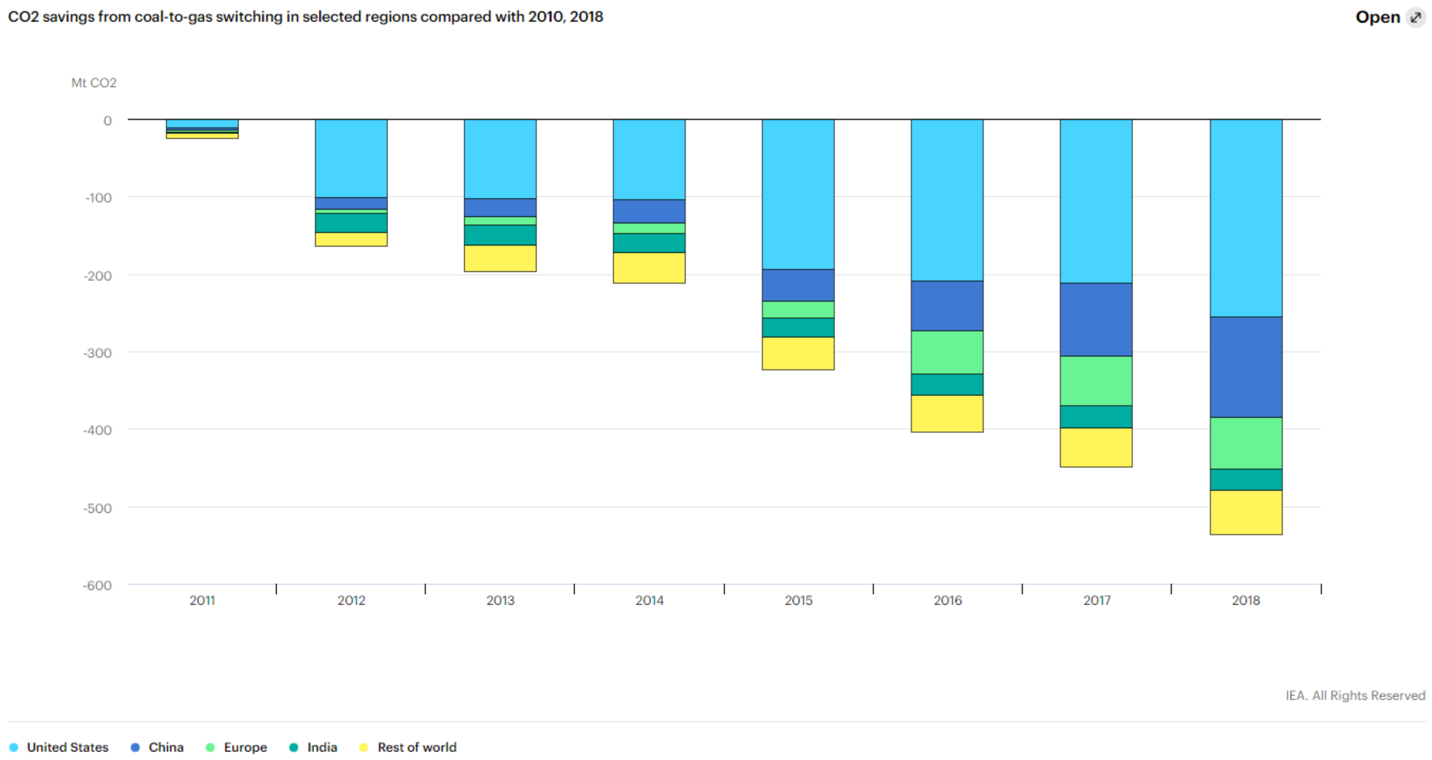
- Oil production and consumption typically measured in barrels
- Global annual trade of oil (2021): 35 billion barrels
- Marginal cost of producers (2016): $9 - 45 per barrel
- Each barrel corresponds to αντιστοιχεί σε 159 litres



Natural gas as transition fuel

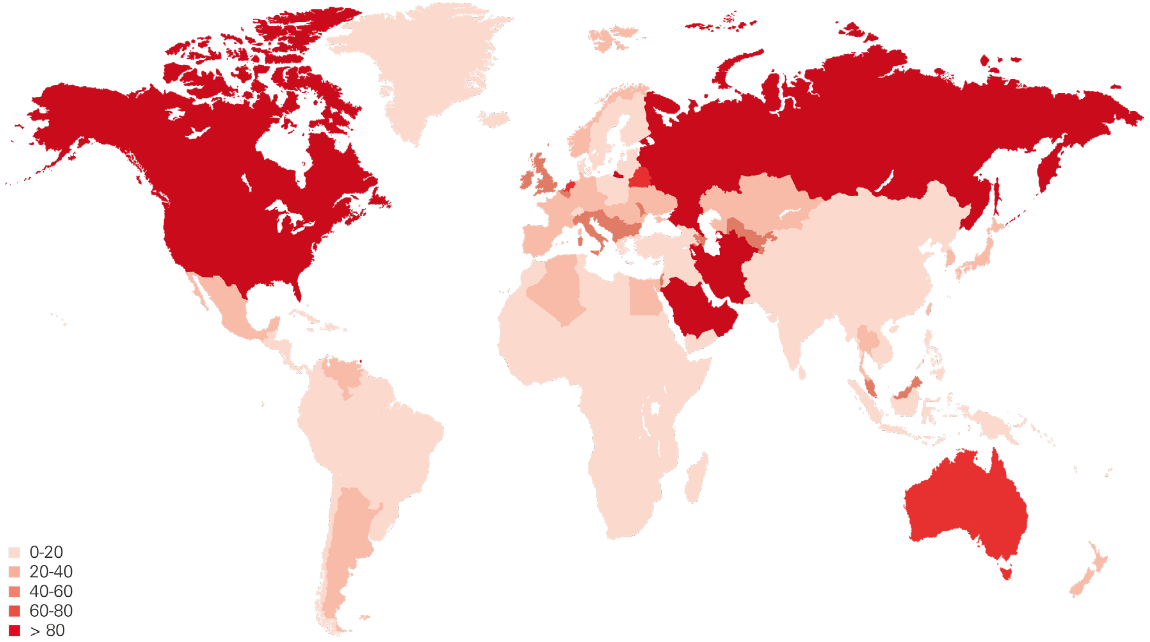
Important transition fuel

- Flexible power generation units, renewable energy integration (+)
- Improves air quality (+)
- Limits emissions of CO2 (+)
- Energy security problems (-)



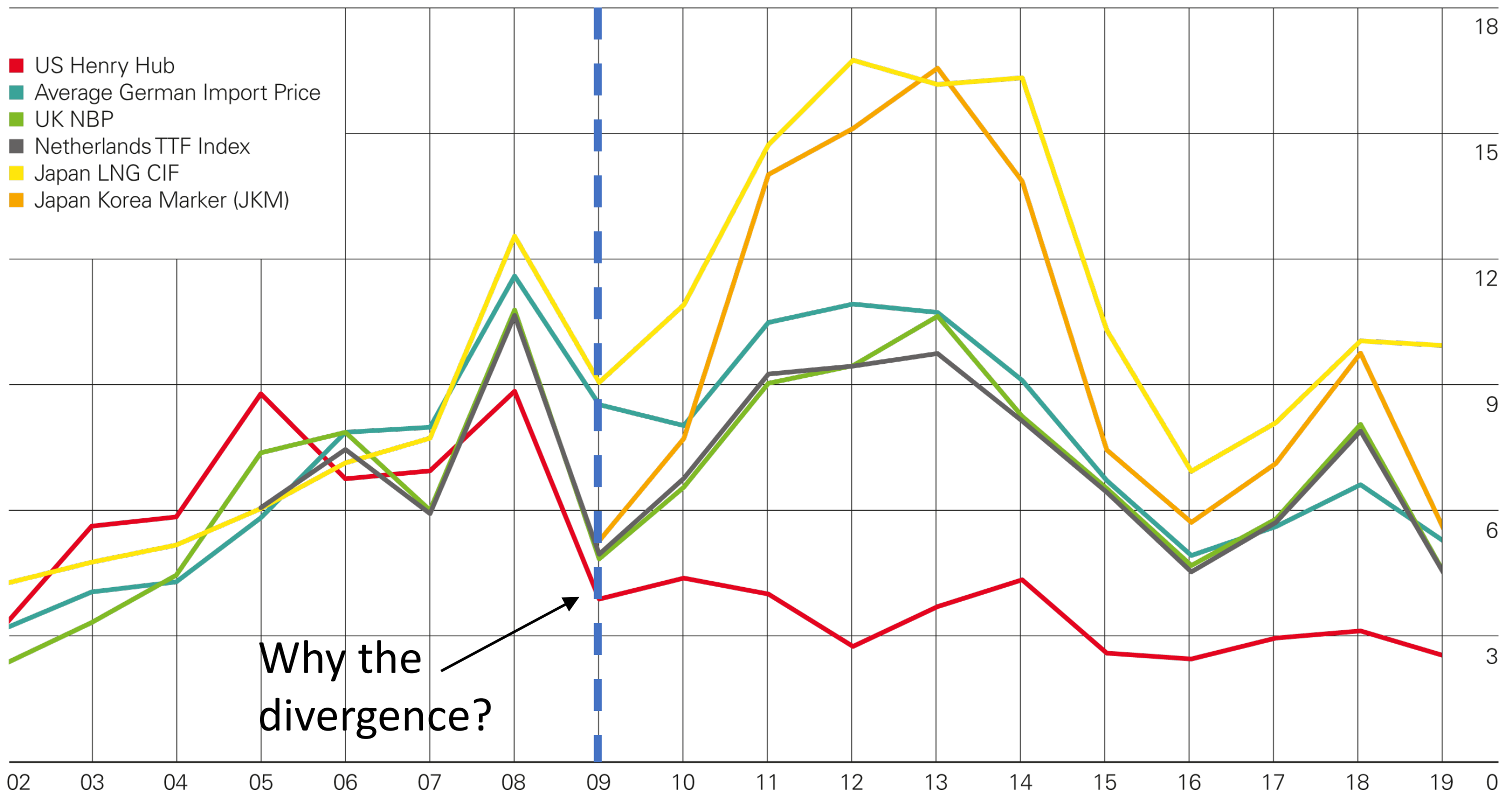
Global oil market

- Transportation of natural gas requires network, import/export infrastructure, compression equipment … (≠ oil)
- Three major natural gas markets:
  - (i) North America (USA, Canada)
  - (ii) East Asia, with *liquefied natural gas (LNG)* shipments
  - (iii) Europe, imports from Russia (until recently) and north Africa, via pipelines



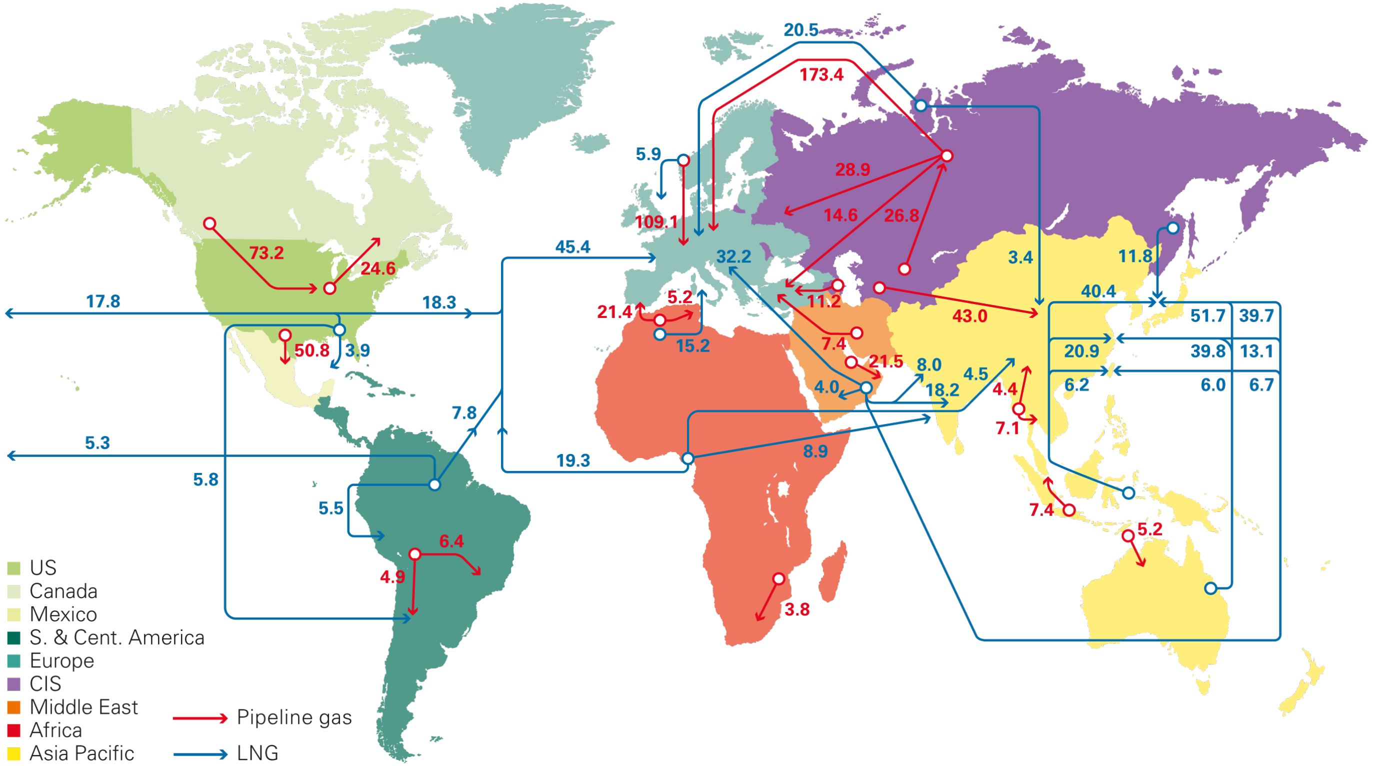
Per capita consumption of natural gas in GJ
Source: BP Statistical Review of World Energy 2020

Natural gas prices



Units: $/MMBtu

Transportation of natural gas (2019)



Units: billion cubic meters

Source: BP Statistical Review of World Energy 2020

The European and Greek natural gas network

entsog
european network of transmission system operators for gas

Since its foundation, ENTSOG member TSOs have provided wide coverage of the European gas market. In addition, according to ENTSOG's articles of association TSOs from EU countries currently derogated from the Third Energy Package, such as two of the Baltic States, are associated partners and are able to participate in its activities.

Since 2011, TSOs from Third Party countries (candidates for EU accession, members of the Energy Community or EFTA) interested in following development of the network codes were also admitted to the association as observers.

AUSTRIA, GERMANY AND SWITZERLAND

Units of measurement of natural gas

1 megajoule	238.8 kilocalories 947.8 Btu 0.278 kilowatt hours
1 kilocalorie	3.968 Btu
1 kilowatt hour (kWh)	359.8 kilocalories 3411 Btu
1 megawatt hour (MWh)	3.411 εκατομμύρια Btu 3.411 thousand cubic feet (mcf) natural gas 0.097 thousand cubic meters natural gas
1 million Btu (MMBtu)	1055 megajoules 2520 megacalories 293.1 kilowatt hours 1000 cubic feet natural gas
1 cubic meter natural gas	35.315 cubic feet natural gas

Hydrocarbons and biofuels

Hydrocarbons

Biofuels

Biofuels

Fuels that are produced from organic substances such as corn

Advantages	Disadvantages
Renewable and “sustainable” energy source	Production can be quite inefficient
Low greenhouse gas emissions	Not so low over the entire supply chain
“Cheaper” per unit of energy	Use of chemical pesticides
Large amount of biomass “available”	Loss of biodiversity
Increased energy security	Higher demand for water
Reduced transportation distance	**Competition between food and energy**
Job creation	

Short-term and long-term equilibrium

Global oil market

- Easy transport ⇒ (almost) uniform global oil price
- Aggregate marginal cost curve: $MC_G(p)$
- Aggregate marginal benefit curve: $MB_L(d)$
- Competitive market ⇒ the market equilibrium is the intersection fo the two curves

Estimating linear marginal cost and marginal benefit curves

- Suppose that the aggregate marginal cost curve is linear
- Measurable quantities:
  - Supply elasticity ϵ_S
  - Market clearing price λ_0 and market clearing quantity $P_0 = D_0$
- Enough information to estimate the aggregate marginal cost curve
- Identical argument for estimating linear marginal benefit curve

Estimating linear curves

- Linear marginal cost curve:

$$MC_G(p) = a_S + b_S \cdot p$$

- Inverse (supply function):

$$P_G(\lambda) = \frac{\lambda - a_S}{b_S}$$

- Elasticity of supply function:

$$\epsilon_S = \frac{dP_G(\lambda)/d\lambda}{P_0/\lambda_0} \Rightarrow b_S = \frac{\lambda_0}{P_0 \cdot \epsilon_S}$$

- Past market equilibrium:

$$\lambda_0 = a_S + b_S \cdot P_0$$

- Substituting out b_S, we estimate a_S:

$$a_S = \lambda_0 - b_S \cdot P_0 = \lambda_0 - \frac{\lambda_0}{\epsilon_S} = \lambda_0 \frac{\epsilon_S - 1}{\epsilon_S}$$

Estimating linear curves

- Marginal cost curve (elastic for $\epsilon_S > 1$, inelastic for $0 < \epsilon_S < 1$):

$$MC_G(p) = \lambda_0 \frac{\epsilon_S - 1}{\epsilon_S} + \frac{\lambda_0}{\epsilon_S} \cdot \frac{p}{P_0}$$

- Supply curve:

$$P_G(\lambda) = P_0 + \epsilon_S \frac{P_0}{\lambda_0}(\lambda - \lambda_0)$$

- Aggregate marginal benefit curve:

$$MB_L(d) = \lambda_0 \frac{\epsilon_D - 1}{\epsilon_D} + \frac{\lambda_0}{\epsilon_D} \cdot \frac{d}{D_0}$$

- Aggregate demand curve (elastic for $\epsilon_D < -1$, inelastic for $-1 < \epsilon_D < 0$):

$$D_L(\lambda) = D_0 + \epsilon_D \frac{D_0}{\lambda_0}(\lambda - \lambda_0)$$

Example 12.1 (Pindyck & Rubinfeld, 2014)

- Oil price: $50 per barrel
- Past annual trade: 35 billion barrels
- Production by OPEC (inelastic): 12 billion barrels per year
- Production from the rest of the industry (competitive): 23 billion barrels per year
- Production from Saudi Arabia: 3.6 billion barrels per year
- Saudi Arabia is a member of OPEC
- Short-term and long-term demand elasticity: -0.05 and -0.3 respectively
- Short-term and long-term supply elasticity: 0.05 and 0.3 respectively
- Problem: compute the short-term and long-term market equiliibrium

Example 12.1: short-term supply and demand functions

- Short-term aggregate demand curve:

$$D_L^{SR}(\lambda) = 36.75 \; - \; 0.035 \cdot \lambda$$

- Short-term competitive supply curve:

$$P_G^{SR,C}(\lambda) = 21.85 + 0.023 \cdot \lambda$$

  - When estimating the parameters of the supply curve, we ignore the supply of OPEC from the computations (thus the equilibrium supply is 23 billion barrels per year)

- Since OPEC is inelastic, the total supply curve of the industry is:

$$P_G^{SR}(\lambda) = 21.85 + 0.023 \cdot \lambda + 12 = 33.85 + 0.023 \cdot \lambda$$

Example 12.1: long-term supply and demand curves

- Long-term demand:

$$D_L^{LR}(\lambda) = 45.5 - 0.21 \cdot \lambda$$

- Long-term demand is more elastic (consumers find alternative ways to substitute oil in the long term, e.g. electric vehicles)
- Long-term supply curve:

$$P_G^{LR,C}(\lambda) = 16.1 + 0.138 \cdot \lambda$$

- For the total supply curve of the industry, we add OPEC production:

$$P_G^{LR}(\lambda) = 28.1 + 0.138 \cdot \lambda$$

- Long-term supply is more elastic (producers adapt over time, e.g. by adapting investments, refinery capacity, etc.)

Example 12.1: validating the historical observation

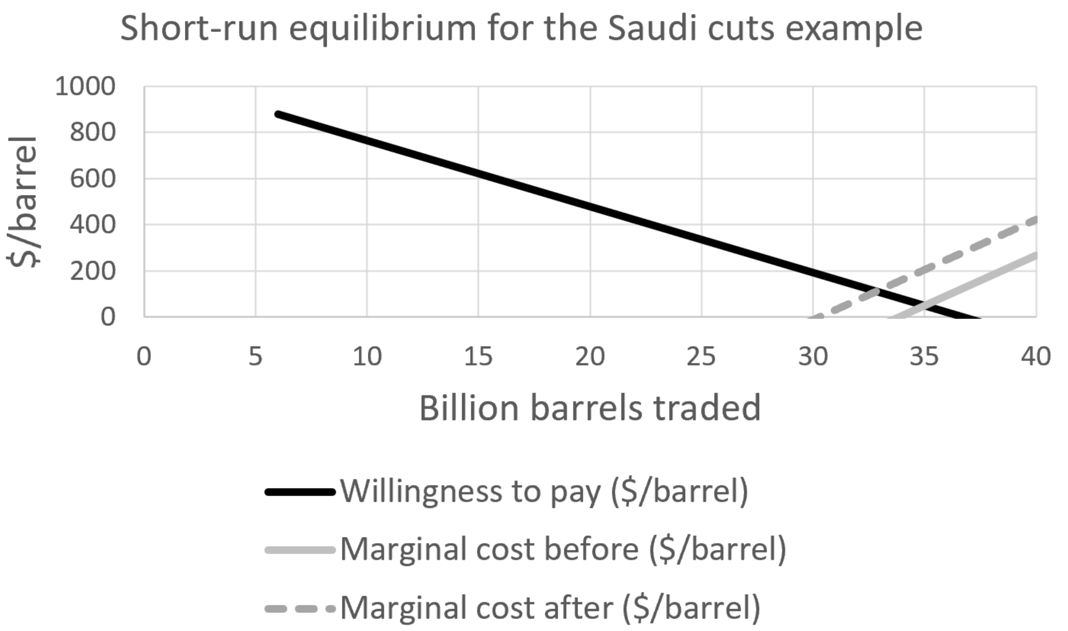
Substituting $\lambda_0 = \$50$ per barrel, we confirm that

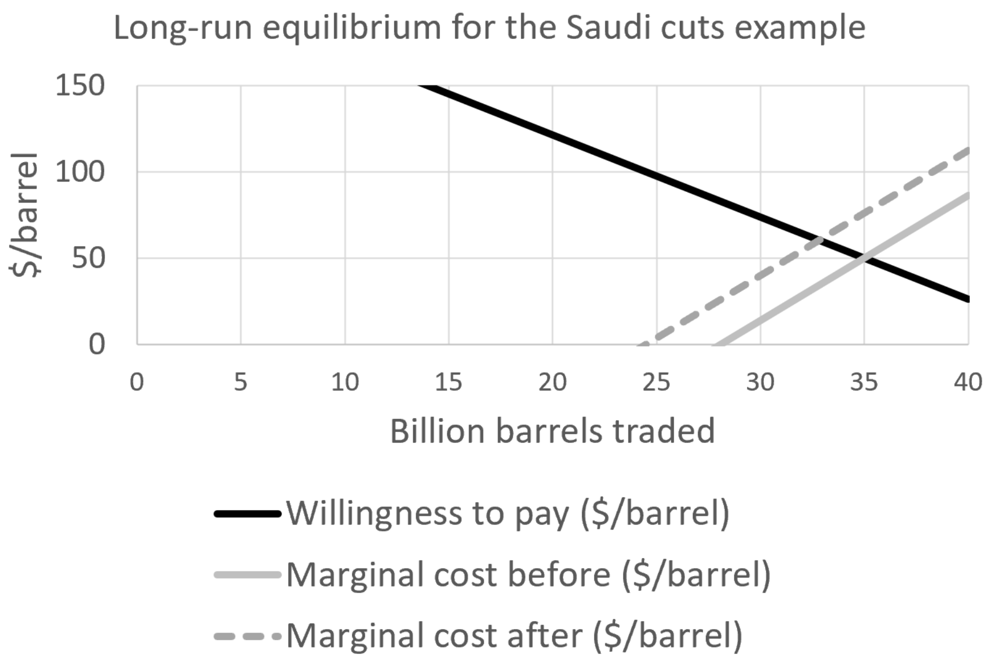
$$P_G^{SR}(50) = D_L^{SR}(50) = 35 \text{ billion barrels per year}$$

$$P_G^{LR}(50) = D_L^{LR}(50) = 35 \text{ billion barrels per year}$$

So the short-term and long-term equilibrium coincide

Example 12.1: graphical illustration of equilibrium





Example 12.1: Saudi cuts

- Interruption of Saudi production ⇒ -3.6 billion barrels per year
- The short-term and long-term demand curves of the market remain identical
- The short-term and long-term *competitive* supply curve of the market remain identical

Short-term *total* supply curve:

$$P_G^{SR}(\lambda) = 33.85 + 0.023 \cdot \lambda - 3.6 = 30.25 + 0.023 \cdot \lambda$$

Long-term total supply curve:

$$P_G^{LR}(\lambda) = 28.1 + 0.138 \cdot \lambda - 3.6 = 24.5 + 0.138 \cdot \lambda$$

Example 12.1: the new equilibrium

- New short-term equilibrium price: $\lambda = \$112.07$
- Long-term equilibrium price: $\lambda = \$60.34$

The market absorbs the initial increase in prices over the long term

  - Higher than the initial equilibrium market price of $50
  - But much lower than the short-term equilibrium price

Monopoly, cartel, and the dominant firm model

OPEC

- Organization of the Petroleum Exporting Countries (OPEC)
- OPEC was founded in Iraq in 1960 by five main producers: Iran, Iraq, Kuwait, Saudi Arabia, and Venezuela
- Evolution: 13 members for now
  - Joined: Qatar (1961), Indonesia (1962), Libya (1962), United Arab Emirates (1967), Algeria (1969), Nigeria (1971), Ecuador (1973), Gabon (1975), Angola (2007), Equatorial Guinea (2017), Congo (2018)
  - Left: Ecuador (2020), Indonesia (suspended membership in 2016), Qatar (2019)
- OPEC share:
  - 30 - 40% of global oil production
  - 50% of oil transactions
  - 80% of proven oil reserves
- Very low production cost

Cartel

- OPEC is an example of a stable cartel
- Cartel: collusion aiming at reducing output and increasing prices above competitive levels
- Cartel structure:
  - Members cannot affect market individually
  - Collective can influence market when coordinating actions
  - Cartel members need to agree on strategy for sharing the market
    - OPEC: production of each member is a fixed fraction of total production, decided during OPEC meetings

The dominant firm model

- **Dominant firm model**: how a monopoly determines output when confronted with a population of **fringe competitors**
- Fringe competitors: perfectly competitive firms
- The dominant firm is a monopoly that trades off loss of market share with increase in prices
- In the classic monopoly model, the firm analyzes market elasticity when deciding on output
- In the presence of fringe competition, the competitors affect the *net demand* elasticity of the market

Net demand

- $D(\lambda)$: demand function
- $P_F(\lambda)$: supply function of perfect competitors
- Net demand: $D_N(\lambda) = D(\lambda) - P_F(\lambda)$
  - Amount of demand that is left over for the monopoly to serve
- $MB_N(d)$: inverse net demand function
  - Inverse of $D_N(\lambda)$

Production from the dominant firm

- Behavior of monopoly:

$$\max_{p \geq 0} MB_N(p) \cdot p - TC(p)$$

- At an interior solution ($p > 0$):

$$MB_N'(p) \cdot p + MB_N(p) = MC(p)$$

Example 12.2: net demand

- We return to example 12.1
- Inverse demand function (inverse of $D_L^{SR}(\lambda)$ in example 12.1):

$$MB_L^{SR}(d) = 1050 - 28.751 \cdot d$$

- Marginal cost function of competitive producers (inverse of $P_G^{SR,C}(\lambda)$ in example 12.1):

$$MC_G^{SR,C}(p) = -950 + 43.478 \cdot p$$

$$D_L^{SR}(\lambda) - P_G^{SR,C}(\lambda) = 36.75 - 0.035 \cdot \lambda - (21.85 + 0.023 \cdot \lambda) = 14.9 - 0.058 \cdot \lambda$$

- Inverse net demand function (inverse of $D_L^{SR}(\lambda) - P_G^{SR,C}(\lambda)$):

$$MB_N(d) = 256.897 - 17.241 \cdot d$$

Example 12.2: market equilibrium

- Marginal cost of cartel: \$10 per barrel
- Monopoly first-order condition:

$$-17.241 \cdot p + (256.897 - 17.241 \cdot p) = 10$$

- Solving for p: $p = 7.16$ billion barrels
- Total quantity supplied to the market: 7.16 billion barrels (OPEC) + fringe production
- So the following hold:

$$\lambda = MB_L^{SR}(d) = 1050 - 28.571 \cdot d$$

$$d = 7.16 + P_L^{SR,C}(\lambda) = 7.16 + 21.85 + 0.023 \cdot \lambda$$

- Solution:
  - $d = 32.080$ billion barrels
  - Market price: $\lambda = \$133.46$ per barrel

Example 12.2: profit of Saudi Arabia

- Saudi Arabia controls 3.6/12=30% of OPEC production
- Based on the rule of slide 32, Saudi Arabia offers $0.3 \cdot 7.16 = 2.148$ billion barrels
- Profit of Saudi Arabia: $(133.46 - 10) \cdot 2.148 = \$265.192$ billion

Mathematical programs subject to equilibrium constraints

- The dominant firm model is a **Stackelberg game**
- **Leader** of the game: moves first
  - In our model this is the dominant firm
- Follower of the game: moves second
  - In our model these are the fringe producers
- The Stackelberg game is a **mathematical program subject to equilibrium constraints (MPEC)**

Tax incidence

Tax means two prices

- Tax models: a different market price for each side of the market
- Supply side faces price λ_s
- Demand side faces same price plus tax: $\lambda_b = \lambda_s + t$
- MC_G: aggregate marginal cost function
- MB_L: aggregate marginal benefit function

Taxation model

- Producer quantity adjustment:

$$\max_{p \geq 0} \lambda_s \cdot p - \int_{x=0}^{p} MC_G(x)dx$$

- Consumer quantity adjustment:

$$\max_{d \geq 0} \int_{x=0}^{d} MB_L(x)dx - \lambda_b \cdot d$$

- Price adjustment:

$$d - p = 0$$

- Definition of tax:

$$\lambda_b = \lambda_s + t$$

Taxation model as an equilibrium problem

- Equilibrium conditions:

$$0 \leq p \perp -\lambda_s + MC_G(p) \geq 0$$
$$0 \leq d \perp -MB_L(d) + \lambda_b \geq 0$$
$$d - p = 0$$
$$\lambda_b = \lambda_s + t$$

- This is a **complementarity problem**
  - Computationally challenging
- Equivalent to a linear program

The taxation model as an optimization problem: option 1

- Replacing λ_b into the equilibrium system:

$$0 \leq p \perp -\lambda_s + MC_G(p) \geq 0$$
$$0 \leq d \perp -MB_L(d) + (\lambda_s + t) \geq 0$$
$$d - p = 0$$

- Equivalent linear program:

$$\max_{p \geq 0, d \geq 0} \int_{x=0}^{d} (MB_L(x) - t)dx - \int_{x=0}^{p} MC_G(x)dx$$
$$(\lambda_s): d - p = 0$$

- Interpretation: the tax corresponds to a uniform decrease in consumer marginal benefit by t
- The dual variable λ_s is the price paid by producers

The taxation model as an optimization problem: option 2

- Replacing λ_s into the equilibrium system, we have the following equivalent linear program:

$$\max_{p\geq 0, d\geq 0} \int_{x=0}^{d} MB_L(x)dx - \int_{x=0}^{p} (MC_G(x) + t)dx$$

$$(\lambda_b): d - p = 0$$

- Interpretation: the marginal cost of producers increases uniformly by t
- The dual variable λ_b is the price paid by consumers

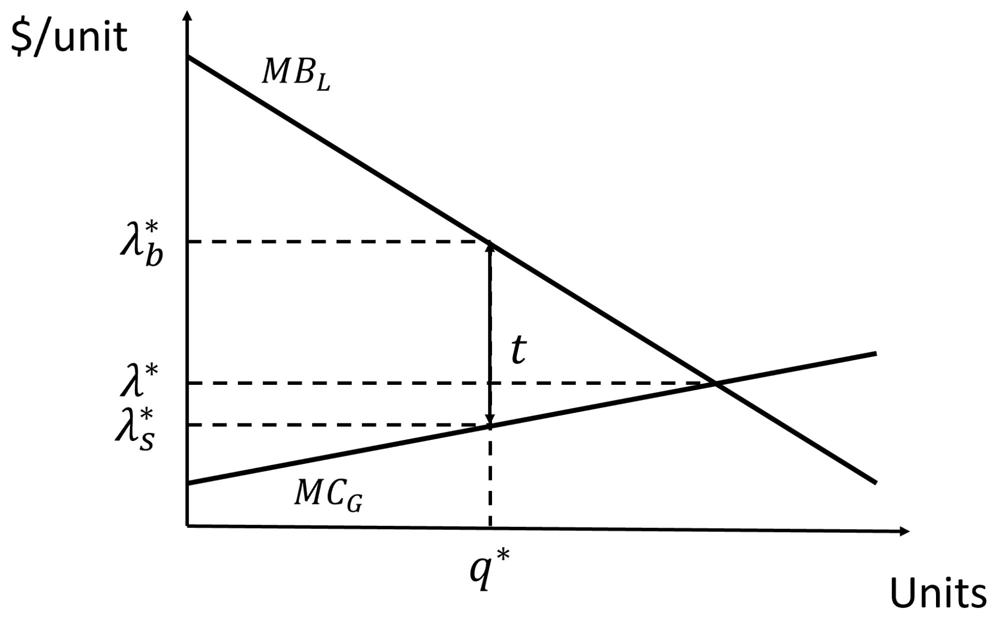
Subsidies

- Subsidies: exactly like taxes, but the buy prices are equal the sell prices *minus* a non-negative subsidy s:

$$\lambda_b = \lambda_s - s$$

- And the equilibrium models with subsidies are equivalent to optimization models

Graphical illustration of equilibrium: clearing prices and quantity

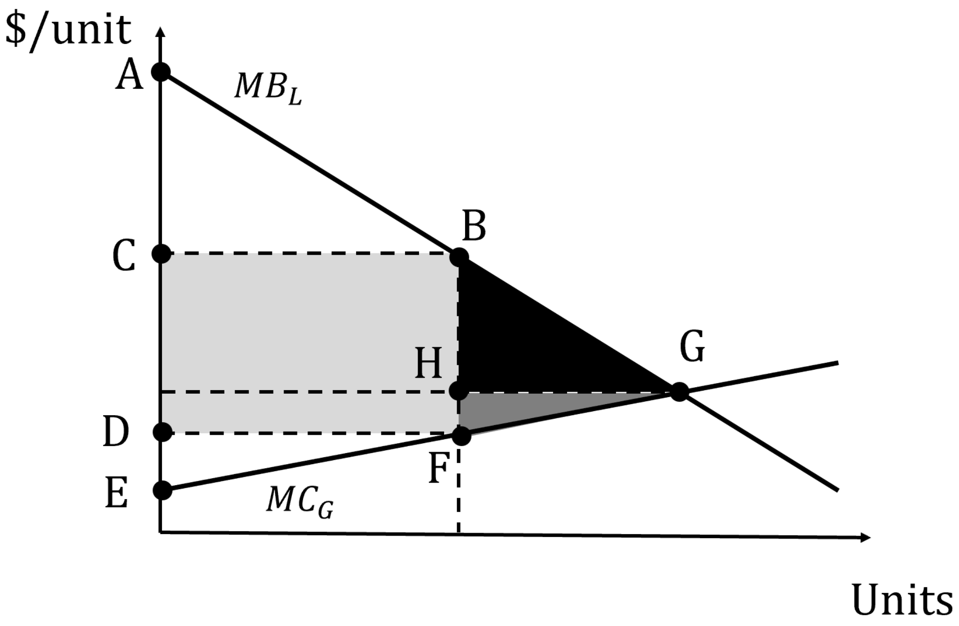


- Tax incidence: how the payment of the tax is split between buyers/sellers
- Quantitatively: how the prices λ_b^* and λ_s^* compare to λ^*
  - Increase in buy price: $\lambda_b^* - \lambda^*$
  - Decrease in sales price: $\lambda^* - \lambda_s^*$

Tax incidence

- The majority of the tax is absorbed from the side of the market that is less elastic
- Intuition: the less each side of the market can adapt to the introduction of the tax, the less able it is to avoid the tax at equilibrium
- Fully inelastic demand (vertical marginal benefit curve): the tax is fully absorbed by buyers
- Fully inelastic supply (vertical marginal cost curve): the tax is fully absorbed by sellers

Graphical illustration of equilibrium: welfare



- Consumer surplus: ABC
- Producer surplus: DEF
- Tax collected by state: BCDF
- Total social welfare: ABFE
- Social welfare before introduction of tax: AEG
- **Deadweight loss**: BFG
- Loss is due to excluding possibly profitable trades

Example 12.3: European natural gas market

- European natural gas consumption (2021): 412 bcm ($D_0 = 14549.78$ bcf)
- Natural gas price: approximately 78 \$/MWh (λ_0 = 78/3.3122 = 22.86 \$/mcf)
- Short-term demand elasticity: $\epsilon_D = -0.05$

$$D_T(\lambda) = D_0 + \epsilon_D \frac{D_0}{\lambda_0}(\lambda - \lambda_0) = 15277.3 - 31.82 \cdot \lambda$$

Example 12.3: European demand for Russian natural gas

- Suppose inelastic import of 155 bcm from Russia (therefore $155 \cdot 35.315 = 5473.825$ bcf)
- The rest ($P_0 = 9075.955$ bcf) is imported from the rest of the world
- Consider elasticity of supply for the rest of the world $\epsilon_S = 1.1$
- Supply curve $P_{NR}(\lambda)$ for the rest of the world except Russia:

$$P_{NR}(\lambda) = P_0 + \epsilon_S \frac{P_0}{\lambda_0}(\lambda - \lambda_0) = -907.60 + 436.73 \cdot \lambda$$

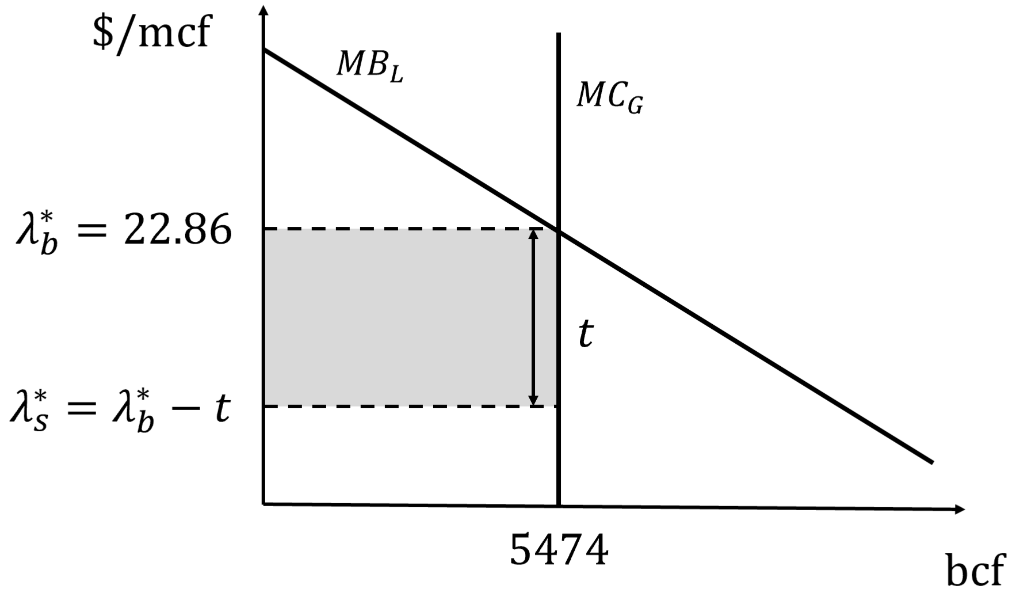
- Demand for Russian gas:

$$\begin{aligned} D_R(\lambda) &= D_T(\lambda) - P_{NR}(\lambda) \\ &= (15277.3 - 31.82 \cdot \lambda) - (-907.60 + 436.73 \cdot \lambda) \\ &= 16184.87 - 468.55 \cdot \lambda \end{aligned}$$

Example 12.3: taxing Russian natural gas

- Suppose that the European Union introduces a tax on Russian gas
- Suppose that the supply of Russian gas is inelastic (5473.825 bcf)
- Intuition: due to pipeline infrastructure, Russia can only sell its natural gas to the European market
- Tax incidence model: inelastic Russian demand faces a demand curve $D_R(\lambda)$

Example 12.3: graphical solution



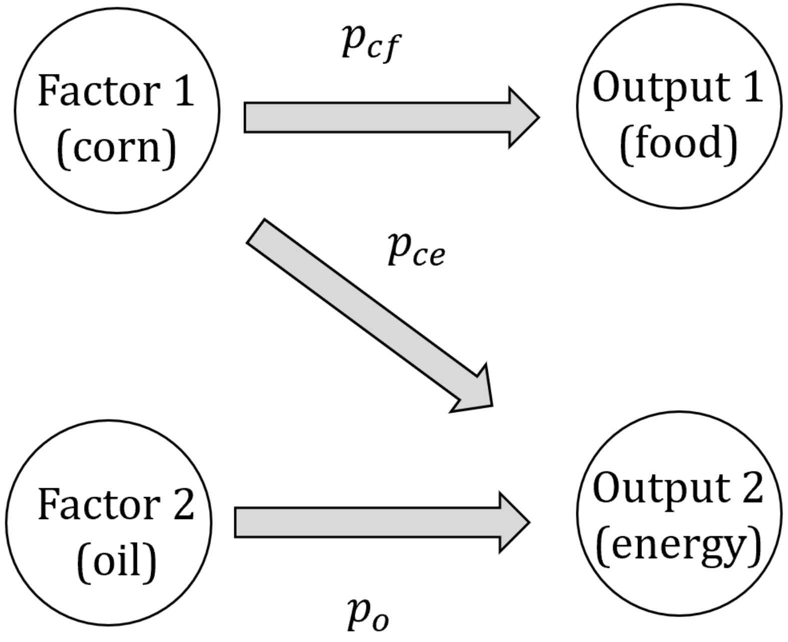
- Inelastic producer: fully absorbs tax
- European equilibrium price: $\lambda_b^* = 22.86$ \$/mcf (same as pre-tax)
- 10 \$/mcf tax ⇒ European Union collects $10 \cdot 5473.825 \cdot 10^6 \simeq$ \$54 billion in taxes annually from Russia

Example 12.3: comments

- The example is over-simplified: the monopoly will react to the introduction of tax by reducing sales
- We can compute the market equilibrium, assuming that the monopoly adjusts its output to the imposed tax
- An interesting question is whether the resulting loss of surplus of European consumers is compensated by the tax revenues paid by the monopolist

One-way substitutability

One-way substitutability



- **One-way substitutability**: a production factor can be used for covering the needs of two markets, while the same is not true for other production factors
- Examples:
  - Reserves in electricity
  - Biofuels

The tortilla crisis

The **tortilla crisis**: one-way substitutability in the food and energy market leads to tight coupling of food and energy prices

⇒ significant backlash against use of corn as biofuel

The New York Times

Cost of Corn Soars, Forcing Mexico to Set Price Limits

By James C. McKinley Jr.

Jan. 19, 2007

MEXICO CITY, Jan. 18 — Facing public outrage over the soaring price of tortillas, President Felipe Calderón abandoned his free-trade principles on Thursday and forced producers to sign an agreement fixing prices for corn products.

Tortilla crisis: the model

Simplified market model that quantifies the phenomenon:

$$\max_{p\geq 0} - C_c \cdot \left(p_{cf} + p_{ce}\right) - C_o \cdot p_o$$

$$\left(\lambda_f\right): D_f - p_{cf} = 0$$

$$(\lambda_e): D_e - p_{ce} - p_o = 0$$

$$(\mu_c): p_{cf} + p_{ce} \leq P_c^+$$

$$(\mu_o): p_o \leq P_o^+$$

Tortilla crisis model

- Decisions:
  - p_{cf}: quantity of corn used for producing food
  - p_{ce}: quantity of corn used for producing energy
  - p_o: quantity of oil used for producing energy
- C_c: marginal cost of corn
- C_o: marginal cost of oil

- First constraint (clearing of food market): only corn can be used for covering (inelastic) demand for food D_f
  - λ_f: food market clearing price
- Second constraint: (inelastic) demand for energy D_e can be covered by both corn as well as oil
- Third constraint: use of corn cannot exceed P_c^+
- Fourth constraint: use of oil cannot exceed P_o^+

Tortilla crisis: KKT conditions

$$D_f - p_{cf} = 0$$

$$D_e - p_{ce} - p_o = 0$$

$$0 \leq \mu_c \perp P_c^+ - p_{cf} - p_{ce} \geq 0$$

$$0 \leq \mu_o \perp P_o^+ - p_o \geq 0$$

$$0 \leq p_{cf} \perp C_c - \lambda_f + \mu_c \geq 0$$

$$0 \leq p_{ce} \perp C_c - \lambda_e + \mu_c \geq 0$$

$$0 \leq p_o \perp C_o - \lambda_e + \mu_o \geq 0$$

KKT analysis: oil price

- Suppose that we use oil, but not fully: $0 < p_o < P_o^+$
- And suppose that there is not enough corn: $p_{cf} + p_{ce} = P_c^+$
- Since there is enough oil: $\mu_o = 0$ (4th condition)
- Since oil is used: $\lambda_e = C_o + \mu_o = C_o$ (last condition)
- Interpretation of μ_o: profit margin of oil because of scarcity
  - Since oil is available at a surplus, the profit margin of oil equals zero
  - The price of energy is determined by oil

KKT analysis: price of food

- Since $p_{cf} > 0$: $\mu_c = \lambda_f - C_c$ (fifth KKT condition)
- Interpretation of μ_c: profit margin due to scarcity of corn
  - The condition $\mu_c = \lambda_f - C_c$ states that the profit margin of corn equals the difference between the revenue of corn from the food market and its marginal cost
- Since $p_{ce} > 0$: $\mu_c = \lambda_e - C_c$ (sixth KKT condition)
  - The profit margin of corn equals the difference between the revenue from the energy market and its marginal cost

- Substituting out μ_c from these two equalities: $\lambda_f - C_c = \lambda_e - C_c$
- Thus, the profit margin of corn should be equal in both the energy market and the food market

- So the prices in both market should be equal: $\lambda_f = \lambda_e$
- This is the essence of the tortilla crisis: tortillas (which are produced by corn) follow the price of energy, due to the one-way substitutability of corn

Example 12.5: data

- $C_c = 10$ \$/unit
- $C_o = 20$ \$/unit (corn is cheaper than oil)

- $D_f = 150$ units
- $D_e = 150$ units

- $P_c^+ = 200$ units
  - The availability of corn can cover the demand for food
  - But it cannot also fully cover the demand for energy
- $P_o^+ = 200$ units

Example 12.5: market equilibrium

- Optimal solution:
  - $p_{cf} = 150$
  - $p_{ce} = 50$
  - $p_o = 100$

- Demand is fully covered in both markets
- Coupling between the price of energy and food:
  - Although the marginal cost of corn is only \$10 per units, the equilibrium price of food becomes \$20, which is also the price of energy
  - In other words: $\lambda_f = \lambda_e = C_o = \$20$ per unit

Hotelling’s rule

Non-renewable resources and Hotelling’s rule

Non-renewable resources (oil, natural gas) will run out within a given time horizon

Hotelling’s rule: the profit that can be achieved from the price of a non-renewable resources increases according to the interest rate of the economy

- In particular, the price does not follow the marginal cost of extraction, even in a perfectly competitive economy
- Essentially a no-arbitrage condition

Market mode for non-renewable resource

$$\max_{p\geq 0, d\geq 0} \sum_{t=1}^{H} (1+r)^{-(t-1)} \cdot \left(\frac{\epsilon}{\epsilon-1} d_t^{\frac{\epsilon-1}{\epsilon}} - C \cdot p_t \right)$$

$$(\lambda_t): (1+r)^{-(t-1)} \cdot (d_t - p_t) = 0$$

$$(\mu): \sum_{t=1}^{H} p_t \leq S$$

Isoelastic demand

- Consumer benefit is expressed as an iso-elastic marginal benefit function
- ϵ: demand elasticity
- Marginal benefit of consumer:

$$MB(d) = d^{-\frac{1}{\epsilon}}$$

- The elasticity of the inverse demand function is equal to ϵ for all demand levels d

Model explanation

- Goal of the economy: maximize difference between consumer benefit and extraction cost in a horizon of H periods

- Parameters
  - r: interest rate of the economy
  - C: marginal cost of extraction (constant over time)
  - S: amount of non-renewable resource that is available

- Decision variables:
  - p_t: quantity of non-renewable resource that is extracted at time period t
  - d_t: quantity of non-renewable resource that is consumed at every time period t

- Fist constraint: market clearing condition at each time period
  - λ_t: market price at period t
- Second constraint: the resource is non-renewable

Hotelling’s rule

For a non-renewable resource, profit increases according to the interest rate of the economy:

$$\lambda_{t+1} - C = (1 + r) \cdot (\lambda_t - C)$$

for all $t = 1, \dots, H - 1$

Proof of Hotelling's rule

- Consider a time period t and the following time period $t+1$
- Among all KKT conditions, we find:

$$0 \le p_t \perp \frac{1}{(1+r)^{t-1}}(C - \lambda_t + \mu) \ge 0$$

$$0 \le p_{t+1} \perp \frac{1}{(1+r)^{t}}(C - \lambda_{t+1} + \mu) \ge 0$$

- Suppose we extract non-zero quantities during the entire horizon:

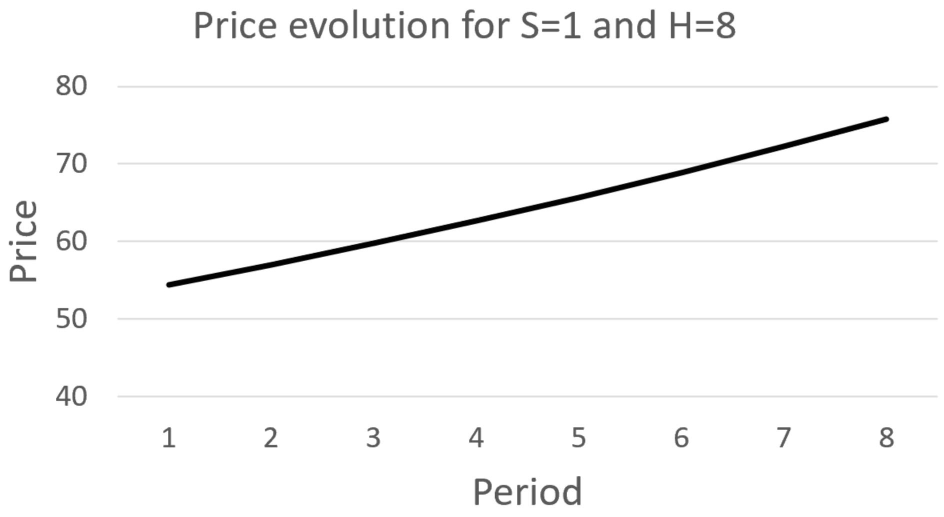
$$p_t > 0 \Rightarrow \mu = \frac{1}{(1+r)^{t-1}}(\lambda_t - C)$$

$$p_{t+1} > 0 \Rightarrow \mu = \frac{1}{(1+r)^{t}}(\lambda_{t+1} - C)$$

- Substituting out μ (interest-rate adjusted producer profit):

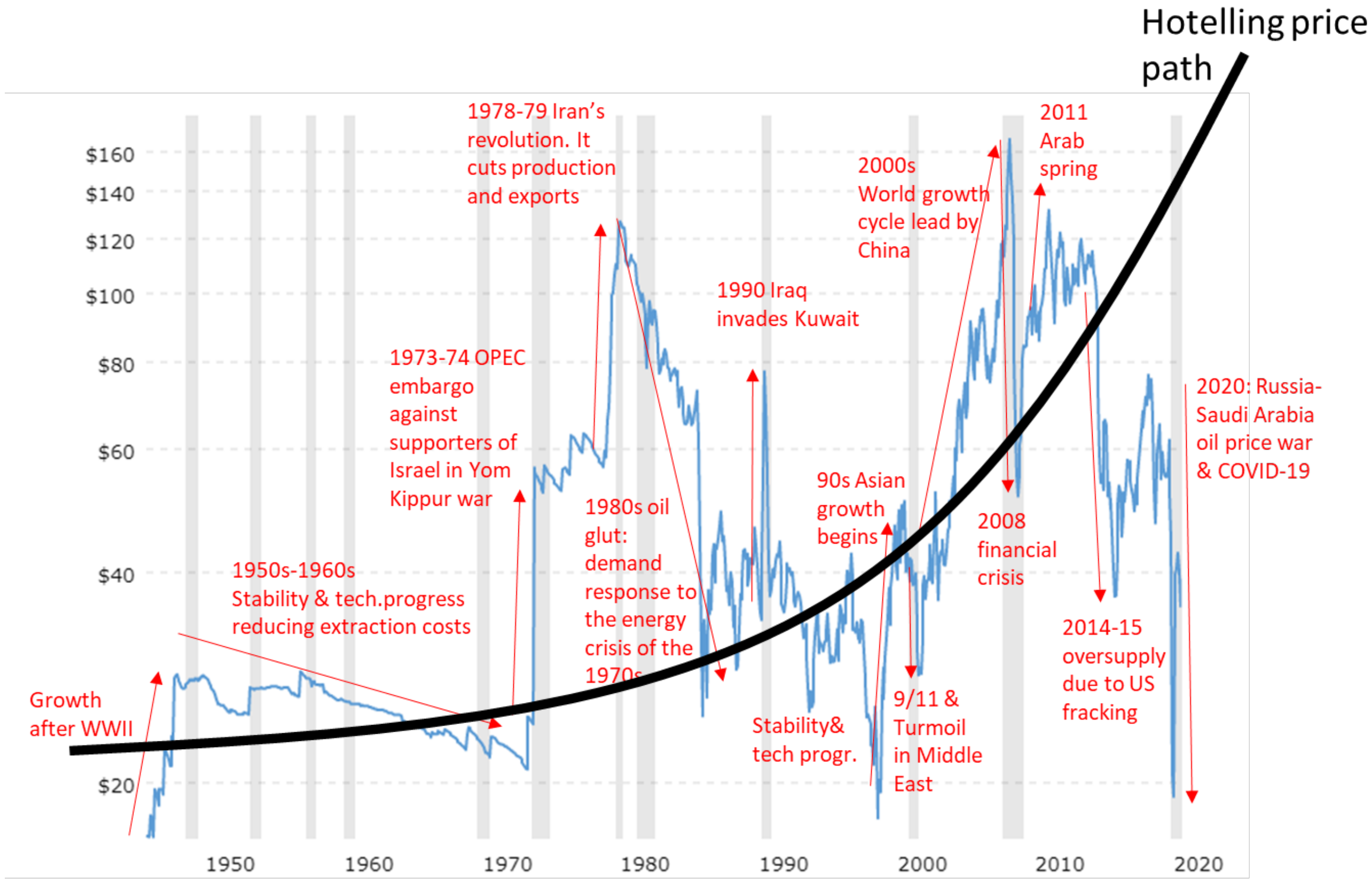
$$\frac{1}{(1+r)^{t-1}}(C - \lambda_t) = \frac{1}{(1+r)^{t}}(C - \lambda_{t+1}) \Rightarrow \lambda_{t+1} - C = (1+r)\cdot(\lambda_t - C)$$

Example 12.5



- Consider the following parameters: $H = 8, S = 1, r = 5\%, \epsilon = 0.5$, and $C = 2$
- Increase of price over time
- The exact solution of the model is determined by the demand side
- Reasoning backwards, since prices increase: $d_1 > d_2 > \cdots > d_H$
- These quantities cannot evolve arbitrarily, because they have to add up to the quantity S of non-renewable resources

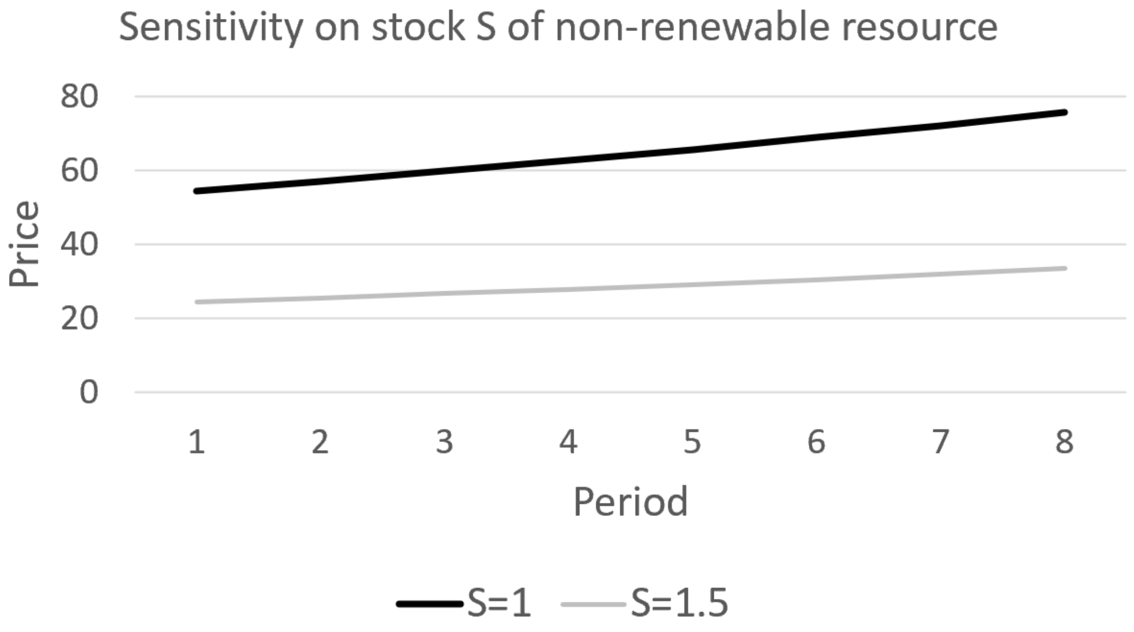
Theory and practice



18 https://www.macrotrends.net/1369/crude-oil-price-history-chart

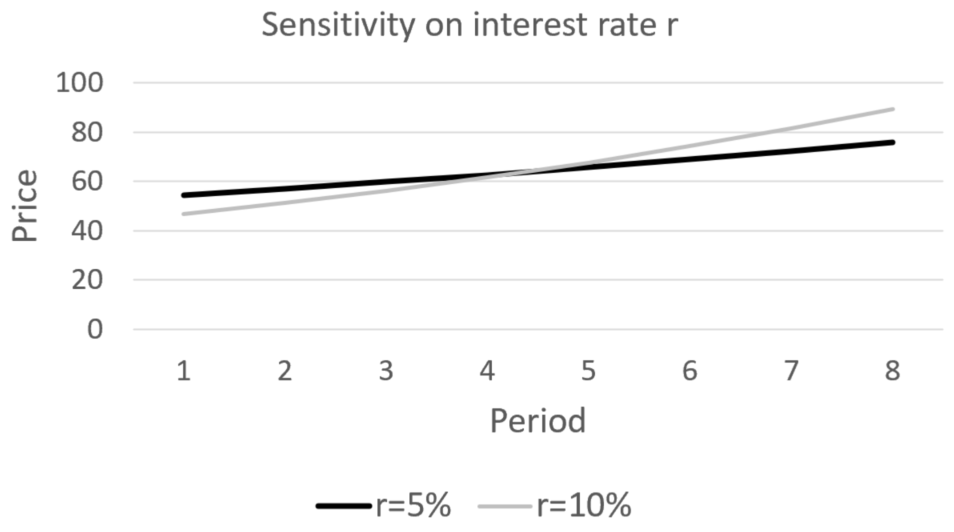
- Hotelling's rule not entirely confirmed empirically
- Makes sense: most parameters change, often unpredictably:
  - Interest rate r
  - Marginal cost of extraction C (e.g. scientific discovery reducing extraction cost)
  - Better estimate of available reserve S

Sensitivity on amount of reserve S



Intuition: increase in available reserve results in higher price trajectory

Sensitivity on interest rate r



Intuition: acceleration in price increase in the case of an economy with higher interest rate r (foreseen by Hotelling's rule)

Generalizations

- One can introduce a backstop technology that can contribute from a certain point onwards at a (potentially high) marginal cost
- One can introduce temporal variation in marginal cost
- One can introduce capacity constraints
- The supply side can correspond to a monopoly, instead of a collection of competitive producers

References

[1] A. Papavasiliou, Optimization Models in Electricity Markets, Cambridge University Press

https://www.cambridge.org/highereducation/books/optimization-models-in-electricity-markets/0D2D36891FB5EB6AAC3A4EFC78A8F1D3#overview