Efficient Dispatch in Cross-Border Balancing Platforms: Elastic Demand through Parametric Cost Function Approximation

Jacques Cartuyvels
CORE/LIDAM, UCLouvain
Louvain-la-Neuve, Belgium
jacques.cartuyvels@uclouvain.be

Gilles Bertrand
ACER
Ljubljana, Slovenia

Anthony Papavasiliou
NTUA
Athens, Greece

***Abstract*—We propose a method for rationalizing the determination of elastic demand curves for MARI, the mFRR cross-border balancing paltform, in the integrated European balancing process. Elastic demand curve for mFRR balancing energy has been proposed due to their ability to reduce the overall balancing cost compared to single-point inelastic demand. The problem is modelled as a non-convex stochastic program, with the variables of interest being the parametrization of the demand curve used by the system operator for acquiring mFRR. The structure of the model allows us to derive its gradient with respect to the demand curve parameters. The problem is solved by applying a randomized stochastic gradient scheme. A price parametrization is shown to outperform a quantity parametrization.**

***Index Terms*—MARI, elastic demand curve, cross-border balancing market, manual frequency restoration reserve**

I. Introduction

The integrated European frequency restoration process is composed of two sequential *cross-border balancing platforms* : MARI for the trading of *manual frequency restoration reserve* (mFRR) and PICASSO for the trading of *automatic frequency restoration reserve* (aFRR). A few minutes before the start of an *imbalance settlement period* (ISP), the MARI platform clears mFRR *balancing energy* on a pan-European level based on the demand curve of *transmission system operators* (TSOs), the supply curve from the *balancing service providers* (BSPs), and the available cross zonal capacities. As the ISP unfolds over the next fifteen minutes, the PICASSO platform optimizes every four seconds the dispatch of *aFRR balancing energy*. The objective of this balancing process is to cover the instantaneous *system imbalance* through the combined activation of mFRR and aFRR balancing energy. This can be conceptualized as a two-stage stochastic optimization problem, where the activated mFRR balancing energy is a first-stage decision and the activated aFRR balancing energy is a recourse decision.

The cross-border balancing platforms went live in 2022 and are operational in Germany, the Czech Republic, and Austria. Other European TSOs are expected to join in 2024 after having delayed their connection. This study is motivated by recurring instances of price spikes (periods when prices exceed €7500/MWh) in Austria after their connection to PICASSO. We explore how improved activation strategies for mFRR can serve as mitigation measure, as recommended by the European Union Agency for the Cooperation of Energy Regulators (ACER) [1]. These price spikes may have resulted from a saturation of the aFRR merit order, which could be alleviated by the activation of slower reserve such as mFRR.

The potential benefit of proactively dispatching mFRR to reduce balancing cost has been demonstrated in [2] . This work determine a level of mFRR balancing energy that minimizes the expected cost of activation, accounting for the realization of wind uncertainty during the ISP. This method is not suitable for an integrated European setting due to the interaction between the different TSOs' mFRR activation strategies. In addition to real-time uncertainty, TSOs connected to the platforms have to consider the uncertainty caused by the other TSOs. Their balancing cost will be impacted by the demand for mFRR balancing energy in other zones and inelastic demands for mFRR balancing energy render TSOs vulnerable to increased mFRR balancing cost. mFRR demand curves increase the control of TSOs over this cost, compared to inelastic demands.

Another component of this analysis concerns the *rationalization* of the method to determine the mFRR demand curve. The method applied here is part of the class of *cost function approximation* methods for tackling sequential decision-making problems as defined by Powell in [3]. Cost function approximation is a *reinforcement learning* method that involves finding a *policy* (i.e. a decision rule for determining the action to take given the state of the system) by minimizing a cost function based on a parametrically modified version of the initial problem. This approach formalizes common industry practices such as the introduction of reserve margin in unit commitment problems [4]. Instead of solving a stochastic unit commitment, which can be potentially computationally expensive, an alternative method would consist of setting a reserve margin in order to ensure the reliability of the system.

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The cost function approximation method solves this type of problem by endogenizing the characterization of the reserve margin. In our case, the variable of interest that is endogenized by the TSO is the parametrization of the mFRR demand curve. The authors in [5], [6] have demonstrated the validity of this method in a renewable energy storage problem. Covic [7] has applied a similar method for the control of a microgrid.

The main contribution of this work is to propose a framework for rationalizing the determination of demand curves for mFRR balancing energy on the cross-border balancing platform MARI.

The remainder of the paper is structured as follows: section II describes the modelling of the parametrized cost function approximation, section III describes the randomized stochastic gradient algorithm that is used for finding the optimal parametrization, section IV illustrates our method on a toy model, and section V concludes.

II. Minimizing Balancing Activation Cost

This section describes the cost-minimization problem of a system operator as a function of the parametrization of the mFRR demand curve. Two parametrizations are discussed: the *price parametrization* in which the variable of interest is the TSO's valuation of mFRR in MARI, and the *quantity parametrization* whose demand curve is composed of demand segments with fixed values and variable quantities.

A. TSO's Cost Minimization

The objective of a *benevolent* system operator is to minimize the expected cost of balancing activation, $F(\cdot)$, as a function of the parametrization of the mFRR demand curve, θ. The objective of a benevolent system operator is to minimize the activation cost of balancing the entire integrated European system. It can be contrasted with a *patriotic* system operator whose goal is to minimize the procurement cost of balancing energy in its own zone. This cost is composed of stochastic mFRR and aFRR balancing energy components, that are indexed by the scenario for the demand of mFRR balancing energy in other zones, ω, and the scenario for the actual realization of system imbalance in real time, ϕ. The aggregated cost for balancing energy as a function of the TSO's demand for mFRR balancing energy cleared by MARI, $x_\omega(\theta)$, is denoted as $C_\omega^{mFRR}(\cdot)$ and $C_{\omega,\phi}^{aFRR}(\cdot)$ for mFRR and aFRR respectively. The problem of the system operator is described in (1):

$$\min_\theta \{F(\theta) = \mathbb{E}_{\omega,\phi}[\bar{F}(\theta,\omega,\phi)]\}, \tag{1}$$

where the balancing cost is expressed in (2):

$$\bar{F}(\theta,\omega,\phi) = C_\omega^{mFRR}(x_\omega(\theta)) + C_{\omega,\phi}^{aFRR}(x_\omega(\theta)). \tag{2}$$

The stochastic cost functions for aFRR and mFRR balancing energy are determined based on initial aggregated aFRR and mFRR merit order curve, $C^{aFRR}(\cdot)$ and $C^{mFRR}(\cdot)$ respectively. The stochastic demand for mFRR balancing energy in other zones and the realized system imbalance are then accounted for, in conjunction with the grid topology. The simplest case without congestion, and with an inelastic demand for mFRR balancing energy in other zones, X_ω, and an aggregated realized system imbalance, X_ϕ^{SI}, can be described as follows:

$$C_\omega^{mFRR}(x) = C^{mFRR}(X_\omega + x), \tag{3}$$

$$C_{\omega,\phi}^{aFRR}(x) = C^{aFRR}(X_\phi^{SI} - X_\omega - x). \tag{4}$$

As there is no congestion, the initial merit order curve pools the aFRR and mFRR *balancing energy bids* from both zones.

The optimization variable of the model is the demand curve's parametrization, θ. It defines a policy for determining the stochastic demand for mFRR balancing energy cleared in MARI. The expected cost of this policy is then computed by estimating F. The next two subsections will describe a price-based and a quantity-based policy.

B. Price Parametrization

The price parametrization is a function of the valuation of the TSO for mFRR balancing energy, θ. The TSO's demand for mFRR balancing energy cleared by MARI is obtained by solving the parametrized economic dispatch is presented in (5) - (7) in function of the variables x and d representing the activated mFRR balancing energy and the cleared demand for balancing energy:

$$x_\omega(\theta) = \arg\min \quad C_\omega^{mFRR}(x) - \theta d \tag{5}$$

$$s.t. \quad 0 \leq d \tag{6}$$

$$x = d \tag{7}$$

Under this parametrization, the total demand for mFRR balancing energy cleared by MARI in an uncongested system with strictly monotonic increasing marginal cost function for mFRR is invariant to the scenario ω. Solving the economic dispatch (5) - (7) states that for every θ there exists a unique total demand for mFRR balancing energy cleared by MARI, $x(\theta)$, defined as the sum of the stochastic demand for mFRR by the other TSOs and the TSO's demand cleared by MARI,

$$x(\theta) = X_\omega + x_\omega(\theta) \quad \forall\omega, \tag{8}$$

such that the marginal cost of the total demand for mFRR balancing energy is equal to θ[1],

$$\theta = (C^{mFRR})'(x(\theta)). \tag{9}$$

If, additionally, the scenarios ϕ for the aggregated system imbalance are independent from the scenarios ω for the demand for mFRR balancing energy by the other TSOs, the system operator cost-minization problem in (1) can be reduced to

$$\min_\theta C^{mFRR}(x(\theta)) + \mathbb{E}_\phi[C^{aFRR}(X_\phi^{SI} - x(\theta))]. \tag{10}$$

The bijection function from θ to $x(\theta)$ in (9) follows from the strict monotonicity of the marginal cost function of mFRR and allows us to state the existence of an optimal θ^* for the reduced

[1]This assumes that the marginal cost of the stochastic demand for mFRR balancing by the other TSOs does not exceed θ.

problem in (10). The first order condition for characterizing θ^* is characterized in (11).

$$(C^{mFRR})'(x(\theta^*)) = \mathbb{E}_\phi[(C^{aFRR})'(X_\phi^{SI} - x(\theta^*))]. \quad (11)$$

An implication of (11) is the equivalence of the system operator problem parametrized through the price and the *foresighted system operator problem*. A foresighted system operator sets its demand for mFRR balancing energy given the other TSOs' demand for mFRR balancing energy. In practice, this can be formulated as minimizing the expectation of the balancing costs as functions of the foresighted TSO's demands for mFRR balancing energy, x_ω, as displayed in (12).

$$\min_x \mathbb{E}_{\omega,\phi}[C_\omega^{mFRR}(x_\omega) + C_{\omega,\phi}^{aFRR}(x_\omega)] \quad (12)$$

In that formulation, every scenario ω is independent and this problem can be reformulated as the expectation of the minimum over the balancing cost in scenario ω.

$$\mathbb{E}_\omega[\min_{x_\omega} C_\omega^{mFRR}(x_\omega) + \mathbb{E}_\phi[C_{\omega,\phi}^{aFRR}(x_\omega)]] \quad (13)$$

The first order conditions of (13) are characterized in (14) and are equivalent to the one of the price parametrization (11) for scenarios ϕ independent from ω.

$$(C^{mFRR})'(X_\omega + x_\omega) = \mathbb{E}_\phi[(C^{aFRR})'(X_\phi^{SI} - X_\omega - x_\omega)] \quad \forall\omega \quad (14)$$

To summarize the price parametrization, the system operator aims to find the valuation of mFRR balancing energy that, when cleared by MARI, minimizes the expected cost of balancing the system. If the marginal cost function of mFRR is strictly monotonic increasing, and the scenarios ω and ϕ are independent, the TSO's valuation ensures that the price of mFRR is equal to the expected price of aFRR, irrespective of the other TSOs demand for mFRR balancing energy. The outcome of that price parametrization is then equivalent to the one resulting from a foresighted TSO.

C. Quantity Parametrization

The quantity parametrization is an alternative to the price parametrization. It is characterized by N segments of fixed value V_i and variable quantity θ_i. It is obtained by solving the parametrized economic dispatch in (15) to (17):

$$x_\omega(\theta) = \arg\min \quad C_\omega^{mFRR}(x) - \sum_{i=1\dots N} V_i d_i \quad (15)$$

$$s.t. \quad 0 \le d_i \le \theta_i \quad \forall i = 1\dots N \quad (16)$$

$$x = \sum_{i=1\dots N} d_i \quad (17)$$

Inelastic demand for mFRR is a special case of this formulation. It consists of one segment with a high value V and a variable quantity θ corresponding to the requested demand for mFRR.

Other parametrization methods include a mix of quantity and price. Linear demand curve can be mentioned.

III. Randomized Stochastic Gradient

Problem (1) is a non-convex stochastic program, the stochastic gradient of which can be computed. This allows us to use a *randomized stochastic gradient* (RSG) scheme to solve it [8]. The chain rule allows us to decompose the gradient of cost as follows for given scenarios ω and ϕ, and for a given optimal clearing of the parametrized economic dispatch, x^* and d^*:

$$\frac{\partial \bar{F}(\theta,\omega,\phi)}{\partial \theta_i} = \frac{\partial \bar{F}}{\partial x^*}\frac{\partial x^*}{\partial \theta_i}. \quad (18)$$

The first component is identical for both parametrizations, and can be formulated as follows:

$$\frac{\partial \bar{F}}{\partial x^*} = (C_\omega^{mFRR})'(x^*) + (C_{\omega,\phi}^{aFRR})'(x^*), \quad (19)$$

The second component can exploit the structure of the problem. The price parametrization is derived in (20) based on (5)-(7):

$$\frac{\partial x^*}{\partial \theta_i} = (C_\omega^{mFRR})''(x^*) \quad (20)$$

The quantity parametrization depends on whether the demand for mFRR balancing energy in segment i is binding and is shown (21):

$$\frac{\partial x^*}{\partial \theta_i} = \begin{cases} 1 & \text{if } d_i^* = \theta_i \\ 0 & \text{else.} \end{cases} \quad (21)$$

Combining (19) with (20) or (21) allows us to compute the gradient.

The randomized stochastic gradient method for solving the problem can be divided into two phases.

1) **Optimization phase:** S independent randomized stochastic gradient runs are launched independently. For each run s, a random stopping criterion, R_s, is drawn, and the following iteration is performed for $k = 1\dots R_s$:

$$\theta_{k+1} = \theta_k - \gamma\nabla_\theta \bar{F}(\theta_k, \omega_k, \phi_k), \quad (22)$$

with γ, ω_k and ϕ_k being respectively the fixed descent stepsize, a random realisation of ω, and a random realization of ϕ. The last iteration is the output of the randomized stochastic gradient run and is denoted as $\tilde{\theta}_s$.

2) **Post-optimization phase:** The optimal solution θ^* is chosen from the candidates $\{\tilde{\theta}_1 \dots \tilde{\theta}_S\}$ by minimizing the stochastic balancing cost over T random realizations:

$$\theta^* = \arg\min_{s=1\dots S} \frac{1}{T}\sum_{i=1\dots T} \bar{F}(\tilde{\theta}_s, \omega_i, \phi_i) \quad (23)$$

The two main components of this method are the random stopping criterion and the multiple independent run. They compensate for the inability to find the minimum of $F(\theta_k)$ in an independent stochastic gradient run as only $\bar{F}$ is available. Further details on the algorithm are given in [8]

IV. RESULTS

The cost function approximation method is applied to a toy model in this section. Throughout this section, a piecewise linear approximation of the aFRR and mFRR merit orders of Austria on the fourth of February 2022 is used for representing the initial aggregated aFRR and mFRR merit orders. They are illustrated in Fig. 1.

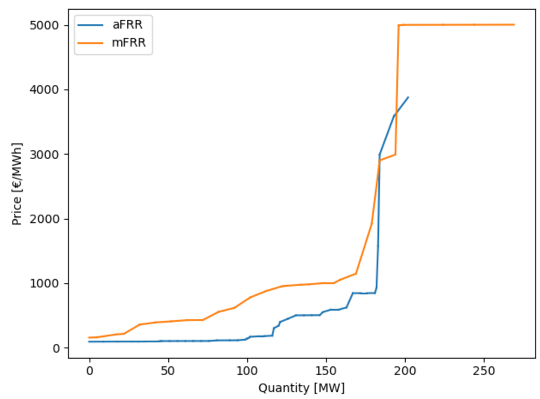

Fig. 1. aFRR and mFRR initial merit orders, $C^{aFRR}(\cdot)$ and $C^{mFRR}(\cdot)$.

This section shows the convergence of the algorithm, compares the price and quantity parametrization and tests the sensitivity of the method to the cost of saturating the aFRR merit order.

A. Convergence of the Randomized Stochastic Gradient

We assume that there is no congestion, the demand for mFRR balancing energy is fixed, and the aggregated realized system imbalance is the sum of two uniform random variables between 0 MWh and 200 MWh, $X_{\phi}^{SI} = U[0,200]+U[0,200]$. This represents the aggregation of two zones that are exposed to uniformly distributed system imbalances. Without uncertainty on the demand for mFRR balancing energy in other zones, there exist an optimal demand for mFRR balancing energy that minimizes the balancing cost. Figure 2 shows the evolution of the mFRR demand for balancing energy in independent RSG runs for a one-segment quantity parametrization. Figure 3 displays the balancing cost induced by the cleared mFRR balancing energy at the stopping criterion in the independent RSG runs.

B. Price versus Quantity Parametrization

Let now assume that the inelastic demand for mFRR balancing energy in other zones is distributed according to a uniform distribution between 0 MWh and 100 MWh, $X_{\omega} = U[0,100]$, We compare four possible parametrizations: (1) a price parametrization, (2) a quantity parametrization with one segment of value 3000 €/MWh (essentially representing an inelastic demand for mFRR balancing energy), (3) a quantity parametrization with three segments of value 1000 €/MWh, 750 €/MWh and €/500 MWh, and, (4) a second quantity parametrization, with three segments, of value 2000 €/MW,

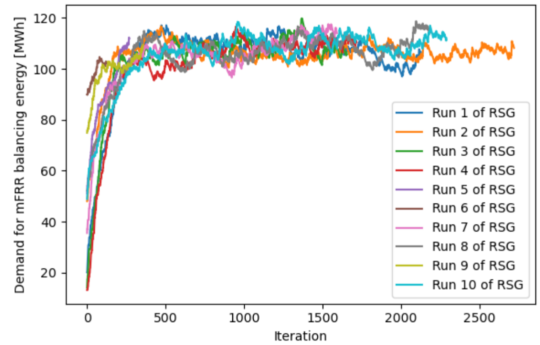

Fig. 2. Cleared mFRR demand for balancing energy for independent RSG runs.

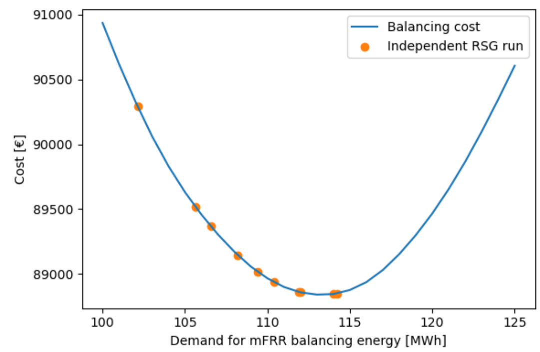

Fig. 3. Balancing cost for independent RSG runs.

1000 €/MW and 500 €/MW. The resulting parametrized mFRR demand curves are presented in Fig. 4.

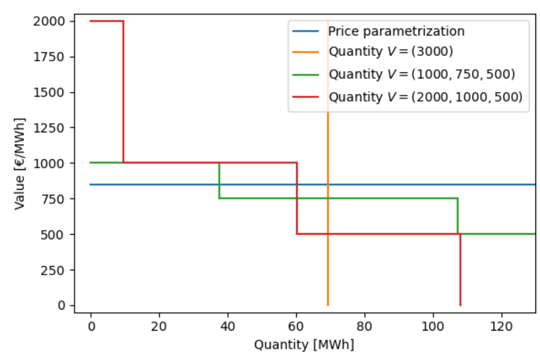

Fig. 4. Parametrized mFRR demand curves.

Fig. 5 presents the demand for mFRR balancing energy due to the TSO demand curve as a function of the demand for balancing energy in other zones. The different parametrizations are compared with a foresighted TSO that can react to the demand for mFRR balancing energy in other zones. As shown in (8), the total cleared mFRR balancing energy is invariant with respect to the demand for mFRR balancing energy in other zones.

Fig. 6 presents the balancing cost from the parametrization

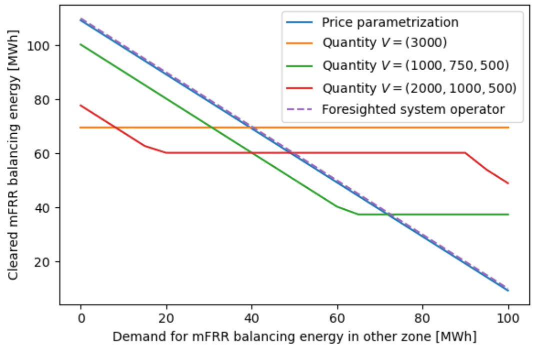

Fig. 5. Cleared mFRR balancing energy as a function of the demand for mFRR balancing energy in other zones.

again as a function of the demand for mFRR balancing energy in other zones. The TSO with foresight and the price parametrization have constant balancing cost, since they adapt to the demand for mFRR balancing energy in other zones. The other parametrization classes generate higher costs. In the case of low demand for mFRR balancing energy in other zones, expensive aFRR assets can be activated and this can even saturate the aFRR merit order, thereby affecting frequency quality. Exhausting the aFRR merit order does not automatically trigger loss of load due to the system's inertia and its final safeguard: *frequency containment reserve* (FCR). High demand for mFRR balancing energy in other zones results in an an over-activation of the mFRR assets.

The efficiency of the quantity parametrization with three demand segments depends heavily on the valuation of the demand segments, which is chosen ex-ante. Utilizing the quantity parametrization with $V = (1000; 750; 500)$ leads to a 2% increase in cost compared to the foresighted problem, while with $V = (2000; 1000; 500)$ results in a 9% cost increase. Inelastic demand exhibits an increase of 11%. The difference between the price parametrization and the foresighted system operator is negligible.

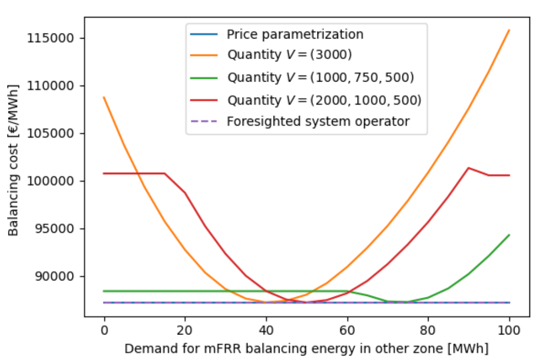

Fig. 6. Balancing cost as a function of the demand for mFRR balancing energy in other zones.

C. Sensitivity to the Most Expensive aFRR Bid

The impact of the price of the last aFRR bid in the merit order is now discussed. This corresponds to the price paid by the system operator when the aFRR merit order is depleted Figure 7 presents the sensitivity of the valuation of the mFRR balancing energy in the price parametrization with respect to the most expensive aFRR bid. It also shows the induced total mFRR balancing energy cleared by MARI. The higher the price of the highest bid in the aFRR merit order, the more the TSO should proactively activate mFRR. An increase from 5,000 to 20,000 €/MWh translates to a 20% increase in the valuation of mFRR balancing energy and a 33% increase in activated mFRR balancing energy. The non-smooth increase in figure 7 is due to the randomness of the RSG method that is used for finding the optimal valuation.

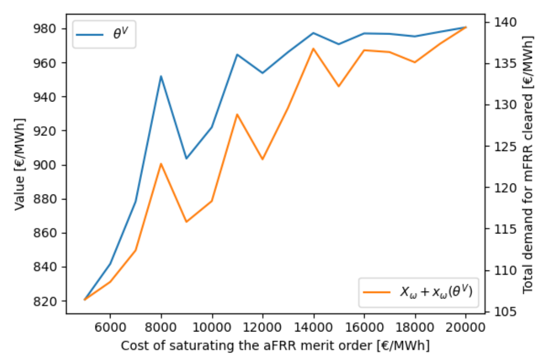

Fig. 7. Total cleared mFRR balancing energy and value of the price parametrization as a function of the cost of saturating the aFRR merit order.

V. Conclusion

This paper analyses the use of a parametrized mFRR demand curve in the context of the sequential clearing of cross-border balancing platforms in an integrated European balancing market. The system operator problem of finding the optimal parametrization is formulated as a cost function approximation problem. It is a non-convex stochastic program solved using a randomized stochastic gradient method. Parametrizations based on quantity and on price are investigated. The price parametrization is equivalent to the system operator problem with foresight for the simple case without congestion and with independent demand for mFRR balancing energy in other zones and aggreagated system imbalance. The method is validated on a toy example and the impact of the cost of saturating the aFRR merit order on the form of the demand curve is discussed.

Further research will consider realistic systems with congestion. Another question of interest concerns the benevolent assumption. Future research will also consider TSOs whose objective is to minimize the procurement cost of reserve in their own zone instead of the overall balancing activation cost in the integrated European balancing market.

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