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On some advantages of convex hull pricing for the European electricity auction ☆

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ABSTRACT

Since the liberalization of the power sector and the creation of wholesale electricity markets, the question of how to price the non-convexities that are present in the market has attracted the interest of both academics and practitioners. Over the years, US markets have studied and adopted different and evolving pricing rules. Since the "Trilateral Market Coupling" (2006), the European day-ahead market has opted for a notably different pricing rule. Recently, EU stakeholders have undertaken research to reform it, and have indicated an interest for some approaches that are discussed in the other side of the Atlantic. Our paper aims at contributing to the debate. We analyse six different pricing methods. We establish several mathematical properties for enabling their accurate comparison. Our findings are illustrated on stylized examples and numerical simulations that are performed on realistic datasets. Both theoretical and numerical evidences that are gathered in our paper point towards the advantages of convex hull pricing.

1. Introduction

Power auctions are notably characterized by the presence of non-convexities. In the US, these non-convexities emerge from the so-called unit commitment model, which has been run in control rooms since before the liberalization of the power sector took place. Although some economists have argued for simpler – convex – market models (cf. the arguments covered by Stoft (2002)), unit commitment has prevailed in many US auctions. In Europe, despite the fact that the market model is different, it also includes non-convex bids, the so-called "block orders" being the simplest example. Although the European market does not rely on *physical* unit commitment models, the non-convex orders also aim – indirectly at least – at providing the suppliers with the flexibility of representing the complex constraints of power generation into the auction. Non-convex multi-parts bids are a bet that the efficiency gained by a refined scheduling model are higher than the inefficiencies resulting from the increase in complexity. In particular, the main drawback of non-convexities is that they impede the existence of a competitive equilibrium. The "classical" marginal prices fail to support the efficient allocation of goods. The absence of equilibrium prices has resulted in various and evolving pricing practises among the US and EU markets.

The liberalization of the power sector in the US started in the 90s, encouraged by the government through the Energy Policy Act of 1992. The creation of the Independent System Operators (ISOs), that have assumed the role of operating the market, followed in the late 90s and early 2000s. Locational *marginal pricing* (LMP) has traditionally been adopted by many ISOs to clear the market, cf. Stoft (2002) and the historical account provided by EPRI (2019). Experience revealed several drawbacks of marginal pricing, especially the fact that short-term fixed costs are not reflected in the price signal which therefore does not provide adequate incentives to market participants. The inadequacy of marginal pricing has stimulated research about the right way to price non-convex power auctions. Convex hull pricing (CHP) (Hogan and Ring, 2003) has emerged as a promising – although contested (Schiro et al., 2015) – way to price energy in the presence of non-convex bids. Acknowledging these issues, several ISOs started moving away from marginal pricing. In 2014, the US Regulatory Commission launched a consultation about price formation in power auctions (FERC, 2014). In 2015, MISO implemented "Extended LMP" (ELMP, an approximation of convex hull pricing) and a similar proposal followed by PJM in 2017 (PJM, 2017). Other ISOs have implemented various "fast-start pricing" approaches (EPRI, 2019), which are variants of ELMP. They

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typically share the property of including, to some extent, fixed costs in the price and resorting to some sort of linear relaxation of the problem for computing market clearing prices. One example is the "hybrid pricing" approach, or "Fixed Block Unit Pricing", implemented by NYISO (2016) since the early 2000s. That being said, up to recently, some ISOs such as CAISO or SPP still rely on marginal pricing (CAISO, 2020; EPRI, 2019).

The restructuring of the power sector in Europe went down a similar path, notwithstanding its peculiarities, cf. the historical account by Meeus (2020). Following the creation of the European Single Market in 1993, the First Energy Package initiated the liberalization of the power sector in 1996. The actual unbundling of competitive (supply and retail) and regulated (TSO and DSO) segments effectively took place between 2003 and 2009 (the Second and Third Energy Packages), along with the creation of national Regulatory Authorities. The implementation of power markets followed, with a different institutional arrangement than in the US: instead of the US ISOs (private, *non*-profit), the EU market is operated by the Nominated Electricity Market Operator (NEMO, private and *for*-profit). The first centralized – and non-convex – auction, coupling parts of central-western European countries, went live in 2006 (the so-called "Trilateral Market Coupling"). This auction has been progressively extended to more member states and in 2014 it became the Single Day-Ahead Coupling (SDAC) that still prevails today. SDAC currently couples 27 countries (62 bidding zones, 30 TSOs and 16 NEMOs) with an average daily traded volume of 4.62 TWh for a market surplus of 9.9 B€ per session (NEMO Committee, 2023).

The pricing approach adopted early on by SDAC (NEMO Committee, 2020b), inherited from the design of the Trilateral Market Coupling (Belpex et al., 2006), significantly differs from those encountered across the US. A central difference in the design is the introduction of side-payments. Because an equilibrium does not exist with a uniform energy price, the US ISOs resort to discriminatory side-payments that complement the uniform price of energy. In contrast with this – so-called in EU parlance – "*non-uniform* pricing", the EU stakeholders have opted for a *uniform* pricing rule. This is anchored in the regulation: the Market Codes emphasize the importance for the payments to be non-discriminatory (CACM GL, Art. 38, 1.b, cf. Commission Regulation (EU) (2015)). According to Meeus (2020), this implies that the introduction of "non-uniform pricing" (i.e. the usage of side payments) would require to change the regulation. This has motivated market clearing rules that are notably different from those in the US. The general principle of the EU pricing approach can be described as follows. It is deemed unacceptable for a non-convex bid, such as block orders, to be cleared while it is out of the money (a so-called "paradoxically accepted block", or PAB). Since the market principles reject the usage of side payments, the market may not clear PABs. Thus, the auction first solves the dispatch problem by aiming at maximizing the welfare. Then, if no price can be found that respects the no-PAB requirement, some constraints are added to the dispatch problem which is solved again. This process repeats until the set of allocation and price satisfies all the requirements.[1]

There are three main issues with this pricing approach (Van Vyve, 2011). Firstly, as opposed to US auctions that clear the welfare-maximizing allocation, the EU market clearing rules can result in rejecting welfare-enhancing bids in order to satisfy the no-PAB requirement. From an economic viewpoint, this welfare loss is critical since *efficiency* (maximization of the total surplus) is the main justification for the market to exist.[2] From a regulatory standpoint, the CACM GL market codes (Art. 38, 1.a, cf. Commission Regulation (EU) (2015)) specifically emphasize that the EU pricing algorithm should "aim at maximizing economic surplus for single day-ahead coupling", which is, strictly speaking, currently not the case. Secondly, although the EU pricing rule ensures no PAB orders, the outcome is *not* a competitive equilibrium. There are market participants that are not cleared while they would be profitable: the so-called "paradoxically rejected blocks" (PRB). In 2022, there was an average volume of 12 GWh of PRBs per bidding zone per day, which amounted to a total profit loss of 129 thousand euros per day (NEMO Committee, 2023). From a regulatory viewpoint, using the previously cited Art. 38 1.b of CACM GL, one could argue that the current pricing rule *already* entails discrimination of market players through the PRBs. Thirdly, the complexity of the clearing rules creates computational challenges. This is problematic, since the current algorithm is granted 17 min to compute the market clearing allocation and price for the entire European continent. This limit increased from 12 to 17 min between 2019 and 2022 – and there are discussions to further extend it to 30 min or more (MCSC, 2023) – , reflecting the computational stress caused by this pricing requirement. The Market Codes also emphasize the importance of "scalability", cf. CACM GL, Art. 38, 1.e in Commission Regulation (EU) (2015).

For these reasons, SDAC is undertaking research to reform the current pricing rule (SDAC, 2023). Initial EU stakeholder discussions on "non-uniform prices" identified convex hull pricing as one possible option for the EU market (NEMO Committee, 2020a). More recent discussions have rather focused on marginal pricing (MCSC, 2022), although nothing is decided yet (SDAC, 2023). Our paper aims at contributing to these discussions relative to the reform of the European pricing rules, although our analysis also applies to US auctions. Our discussion focuses on possible alternatives to the current pricing rule, i.e. we discuss the advantages of these alternatives *between* them and not *over* the SDAC pricing rule. In particular, the contributions of this paper are threefold.

Firstly, we perform a cross-comparison of four different pricing approaches. Several properties are formalized mathematically on the same model, in order to allow for a rigorous comparison of the alternative prices. Our paper focuses on the *short-term* properties of the prices. The *long-term* properties – the effect of pricing on investment incentives – have notably been studied in other recent works (Mays et al., 2021; Byers and Hug, 2023). Our endeavor aims at addressing the urge for a better understanding of various pricing candidates, as called upon by EPRI (2019). To some extent, we follow up on the pioneering works of Schiro et al. (2015) and Liberopoulos and Andrianesis (2016). While Schiro et al. (2015) focuse solely on Convex Hull Pricing, we discuss it in comparison with other approaches to better grasp their relative benefits and drawbacks. We also critically review some of the arguments provided by Schiro et al. (2015). While Liberopoulos and Andrianesis (2016) study some properties on a "two-suppliers" model, we rather analyse other properties on a general market model.

Secondly, the theoretical properties are supported by numerical simulations on realistic systems. This is a novelty compared to both Schiro et al. (2015) and Liberopoulos and Andrianesis (2016). In particular, studying convex hull pricing on realistic instances is an effort that has not been widely undertaken in the literature. Thanks to recent algorithmic progresses (Stevens and Papavasiliou, 2022; Andrianesis et al., 2021) we are able to compute *exact* CHP on realistic instances. This enables an accurate numerical comparison. More specifically, we illustrate and study the properties of the four pricing approaches on

[1] To simplify the exposition, we only describe the PAB requirement. As a matter of fact, there are additional "primal–dual" constraints in the market rules, that an interested reader can find in NEMO Committee (2020b).

[2] Unfortunately, there is no public figure regarding the welfare loss, although it is a key indicator. ACER is the institution that defines the KPIs that are reported in the annual CACM reports. It would arguably make sense to include this additional KPI: the difference of welfare between the "root node" of the market clearing algorithm and the final solution.

two different datasets: the "FERC dataset" (public data, but without a network) and the "CWE dataset" (non-public data, but including a network).

Finally, we particularly include the pricing method proposed by Madani and Papavasiliou (2022) referred to as "Minimal Make-Whole Payment" (MMWP) pricing in our comparison. This novel approach is representative of various recent proposals that have appeared in the literature, which have not been critically assessed so far. We notably implement three alternative versions of MMWP, and we discuss their relative advantages.

The material of the paper is organized as follows. Sections 2 and 3 introduce the model, the main concepts and the four pricing schemes. Sections 4 to 8 then study their properties, and provide results from numerical simulations. To some extent, Sections 5, 6 and 7–8 focus respectively on the comparison between CHP vs. MMWP, CHP vs. ELMP and CHP vs. marginal pricing.

2. Market model and distance to equilibrium

Throughout this paper, we consider the following auction model, which can accommodate the settings of both the EU day-ahead market[3] as well as most US auctions.

$$z^* = \min_{c,q,x,f} \sum_{g\in\mathcal{G}} c_g \tag{1a}$$

$$\sum_{g\in\mathcal{G}_i} q_{g,t} - D_t^i = \sum_{\substack{l\in\\ from(i)}} f_{l,t} - \sum_{\substack{l\in\\ to(i)}} f_{l,t} \quad \forall i\in\mathcal{N}, t\in\mathcal{T} \tag{1b}$$

$$(c,q,x)_g \in \mathcal{X}_g \quad \forall g\in\mathcal{G} \tag{1c}$$

$$f\in\mathcal{F} \tag{1d}$$

The auction model (1) aims at minimizing the cost of satisfying the load D_t^i for each time period $t\in\mathcal{T}$ and each bidding zone $i\in\mathcal{N}$. To simplify the exposition of the paper, demand is assumed to be inelastic.[4] The market includes a set of $\mathcal{G}_i$ suppliers (or market offers) at each node i. Each offer is modelled with a total cost variable c_g, a power output $q_{g,t}$ at time t and a set of possibly non-convex constraints $\mathcal{X}_g$. The variables x_g stand for all the binary variables encountered in the supplier model. In a US auction, which typically relies on a unit commitment model, $\mathcal{X}_g$ should be understood as a detailed representation of the technical constraints of the power plant g. In the EU day-ahead auction, which relies on *portfolio* bidding instead of *unit* bidding, $\mathcal{X}_g$ should be understood as the constraints of the market order g (blocks, linked blocks, stepwise curves, etc.). Eq. (1b) represents the market clearing constraints. Finally, the auction model (1) also includes a network. The variable $f_{l,t}$ represents the flow on line l, while $from(i)$ is the set of lines originating from i and $to(i)$ the ones directed towards i. No assumption is made on the network constraints $\mathcal{F}$, except that it is a *convex* set. All suppliers are assumed to be *price-takers* and to act so as to maximize their private profit. We now proceed with some definitions.

Definition 1 (*Supplier Profit Maximization*)**.** The agent g is assumed to maximize its selfish profit function $\mathcal{P}_g$, under market price π, defined as follows:

$$\max_{(c,q,x)_g\in\mathcal{X}_g} \mathcal{P}_g(c,q,x,\pi) \equiv \sum_{t\in\mathcal{T}} q_{g,t}\pi_{i(g),t} - c_g. \tag{2}$$

[3] This has one exception: the so-called PUN orders (the "Prezzo Unico Nazionale" requirement in Italy, cf. NEMO Committee (2020b)) and complex orders are not compatible with the pricing approaches considered in this paper as they include primal–dual constraints. We point out that both the PUN and complex orders are planned to be discontinued (MCSC, 2023).

[4] All the pricing schemes and results of this paper can be extended straightforwardly to a model with *elastic* loads. With elastic load, the objective of the auction is welfare maximization.

Definition 2 (*Network Profit Maximization*)**.** The network is assumed to maximize its profit function $\mathcal{P}_N$ (the "congestion rent"), under market price π, defined as follows:

$$\max_{f\in\mathcal{F}} \mathcal{P}_N(f,\pi) \equiv \sum_{i\in\mathcal{N},t\in\mathcal{T}} -\pi_{i,t}\left(\sum_{\substack{l\in\\ from(i)}} f_{l,t} - \sum_{\substack{l\in\\ to(i)}} f_{l,t}\right). \tag{3}$$

Definition 3 (*Competitive Walrasian Equilibrium*)**.** The allocation (c^*, q^*, x^*, f^*) together with the market price π constitute a competitive Walrasian equilibrium if

(i) for each supplier g, $(c^*, q^*, x^*)_g$ optimizes the profit problem (2) under price π; f^* optimizes the network profit problem (3) under price π, and
(ii) the market clears (constraint (1b)).

A paramount desideratum for an auction is to reach *economic efficiency*: the allocation of goods resulting from the market should be welfare-maximizing (cost-minimizing under inelastic load). All the pricing schemes considered in this paper assume a welfare-maximizing allocation: they assume that the auctioneer solves problem (1) and selects the welfare-maximizing allocation. An example of a pricing scheme that departs from welfare maximization is the current European pricing rule (cf. Section 1). In the remainder of this paper, (c^*, q^*, x^*, f^*) refers to the optimal solution of problem (1). Since the market is non-convex, a competitive equilibrium is not guaranteed to *exist* (i.e. the concern is about the *existence* of an equilibrium rather than its *efficiency*: the First Theorem of Welfare Economics does not require convexity, so if an equilibrium exists in a non-convex market, it will be efficient, cf. Debreu (1959)). By assumption, the allocation (c^*, q^*, x^*, f^*) satisfies condition (ii) in Definition 3. The issue is that there may be no price π that fulfils condition (i), provided this allocation. Assuming that the market agents maximize their profit (Definitions 1 and 2), the violation of condition (i) is measured by the *lost opportunity cost* (LOC).

Definition 4 (*Lost Opportunity Cost*)**.** The lost opportunity cost is the difference between the maximum profit and the as-cleared profit under price π. It is defined hereafter for each supplier g (Eq. (4)), for the network (Eq. (5)) and in total (Eq. (6)).

$$LOC_g^{gen}(\pi) = \max_{\substack{(c,q,x)_g\\ \in\mathcal{X}_g}} \mathcal{P}_g(c,q,x,\pi) - \mathcal{P}_g(c^*,q^*,x^*,\pi) \tag{4}$$

$$LOC^{net}(\pi) = \max_{f\in\mathcal{F}} \mathcal{P}_N(f,\pi) - \mathcal{P}_N(f^*,\pi) \tag{5}$$

$$LOC(\pi) = \sum_{g\in\mathcal{G}} LOC_g^{gen}(\pi) + LOC^{net}(\pi) \tag{6}$$

The lost opportunity cost measures the financial incentives that each profit-maximizing agent has for deviating from the allocation decided by the auctioneer. Having a price that is *incentive compatible* is important, both in order to ensure truthful bidding *before* the market clears, as well as to ensure that the participants would follow the dispatch instructions *after* the market has cleared. Concretely, incentive compatibility is related to the notion of *self-scheduling*: a positive LOC means that the price does not support the dispatch, thereby implying an opportunity for the concerned agents to self-schedule, thus deviating from the dispatch (c^*, q^*, x^*) that is cleared in the auction. As far as the network LOC is concerned, Garcia et al. (2020) interpret it as a potential congestion revenue shortfall, meaning a possible inadequacy between the FTR payments and the congestion revenue that the system operator collects. More generally, it can be interpreted as an incentive for the grid operator, given the market prices, to organize the flows on the network in a manner that deviates from its efficient usage. For example, let us consider two nodes connected by a line. The two nodes receive different prices, but the line is not congested. This could arguably be contemplated as an undesirable configuration. Formally, there is a network LOC: the cleared flows do not maximize the value of the network.

Certain researchers and practitioners have advocated that the price should not only aim at being *incentive-compatible*, as measured by the LOC, but that it should also ensure a *non-confiscatory* outcome: the price should at least enable the cleared bids to recover their costs (Madani and Papavasiliou, 2022; Bichler et al., 2022; EPRI, 2019). The latest is measured by *revenue shortfall*.

Definition 5 (*Revenue Shortfall*)**.** The revenue shortfall (RS) corresponds to the payments that are required in order to ensure a non-negative profit. It is defined for each supplier (Eq. (7)), for the network (Eq. (8)) and in total (Eq. (9)).

$$RS_g^{gen}(\pi) = -\min\left(0,\ \mathcal{P}_g(c^*, q^*, x^*, \pi)\right) \tag{7}$$

$$RS^{net}(\pi) = -\min\left(0,\ \mathcal{P}_N(f^*, \pi)\right) \tag{8}$$

$$RS(\pi) = \sum_{g\in\mathcal{G}} RS_g^{gen}(\pi) + RS^{net}(\pi) \tag{9}$$

Needless to say that the LOC and RS are non-negative numbers. Let us notice that lost opportunity cost and revenue shortfall are sometimes referred to, respectively, as "uplift payments" and "make-whole payments" in the literature. However, this terminology is misleading. Because of the absence of a competitive equilibrium, the auctioneer may indeed resort to some sort of out-of-market discriminatory payments that complement the uniform energy price. For example, several US ISOs pay make-whole payments, while ISO-NE pays lost opportunity costs for committed units (EPRI, 2019). Nonetheless, denoting the LOC as "uplift *payment*" suggests that the LOC only matters for the markets that are actually paying them. Instead, the LOC is a crucial indicator, independently from the *actual* payments that are paid by a particular auctioneer.

3. Pricing scheme proposals

There is no straightforward solution to the absence of competitive prices. We consider hereafter four pricing mechanisms that are proposed in the literature. They all correspond to a certain convex reformulation (either a *relaxation* or a *restriction*) of the non-convex problem (1), cf. the discussion in Madani and Papavasiliou (2022). A first option is to rely on *marginal pricing*, also called Integer Programming (IP) pricing (O'Neill et al., 2005). This pricing scheme is theoretically meaningful to study since it is widely used in economics. It is also practically relevant, given its historical usage in US power auctions, and considering that it is a serious candidate currently on the table for the EU market.

Definition 6 (*Marginal Pricing*)**.** The marginal (IP) prices are the dual variables π^{IP} associated with the market clearing constraint in problem (1) in which the binary variables x have been fixed to their optimal value x^*.

It effectively corresponds to taking the price as the subgradient of the total cost curve with binary variables fixed.

A second approach – central for our paper – is Convex Hull Pricing (CHP), which has been proposed in Hogan and Ring (2003) and Gribik et al. (2007). We adopt here the primal formulation of CHP (Hua and Baldick, 2017).

Definition 7 (*Convex Hull Pricing*)**.** The convex hull prices are the dual variables π^{CH} that are associated to the market clearing constraints in problem (1), in which the sets $\mathcal{X}_g$ are replaced by $\mathrm{conv}(\mathcal{X}_g)$.

It is worth noting – besides the peculiar name – the natural interpretation of this pricing approach. The very problem of non-convexities is the inexistence of a competitive equilibrium. The logic of this approach is to compute the prices of the *closest convex economy*, in which a competitive equilibrium exists. Remarkably, although most of the economic theory neglects non-convexities, Starr (1969) and Arrow and Hahn (1971), who studied non-convexities in the theory of general equilibrium, adopted convex hull pricing – albeit they do not use this term. The main property of CHP which has justified its interest in power auctions is that it minimizes the LOC (Gribik et al., 2007): they are the prices that are "as incentive-compatible as possible", i.e. that are as close as possible to a competitive equilibrium.

Proposition 1 (*CHP*)**.** *CH prices minimize the total lost opportunity costs, as defined in* (6)*.*

All the proofs are in Appendix A. From Lagrangian duality theory, one can observe that the LOC corresponds to the *duality gap* between the primal solution z^* and the Lagrangian dual function in which the market-clearing constraint (1b) is relaxed. Proposition 1 then states that CHP is the price (the Lagrangian multiplier) that minimizes the duality gap.

Convex hull prices are notably difficult to compute (Schiro et al., 2015). Therefore, an *approximation* of CHP, called ELMP, has been proposed and is already implemented by several ISOs, as explained in Section 1.

Definition 8 (*Extended Locational Marginal Pricing*)**.** The extended locational marginal prices are the dual variables π^{ELMP} that are associated to the market clearing constraints in problem (1), in which the sets $\mathcal{X}_g$ are replaced by $\mathcal{X}_g^{[0,1]}$, i.e. the binary constraints on x are relaxed to $[0,1]$.

In case $\mathcal{X}_g^{[0,1]} = \mathrm{conv}(\mathcal{X}_g)$, ELMP would correspond to the *exact* CHP approach. This is the main justification for ELMP: it is viewed as a *tractable approximation* of CHP. Nonetheless, even though the above equality, $\mathcal{X}_g^{[0,1]} = \mathrm{conv}(\mathcal{X}_g)$, can be guaranteed in certain simple cases, there are some constraints, such as the ramp constraints, for which the equality is not straightforward to obtain, and reaching a *tight* formulation in these cases may require the introduction of a substantial number of valid inequalities (Hua and Baldick, 2017).

In a similar spirit as CHP, which minimizes the LOC, a number of researchers have advocated for a price that minimizes the revenue shortfall. In multiple works, O'Neill has proposed the Average Incremental Cost (AIC) pricing (Chen et al., 2020; O'Neill et al., 2023), which aims at finding a "*zero* make-whole payment price" for the suppliers. However, this is not an achievable target for both the suppliers and the loads if the latter are elastic. Indeed, it cannot be guaranteed that we can find a uniform price that ensures *zero* revenue shortfall for all the market participants in a two-sided auction.

Example 1 (*Impossibility of Zero RS with Elastic Load*)**.** Let us consider an hourly market with a non-convex supplier producing at maximum 200 MW for 50€/MWh, and at minimum 100 MW. Let us also consider two convex and *elastic* loads: one is willing to consume 90 MW for 10,000€/MWh, the other is willing to consume 20 MW for 20€/MWh. Because of the minimum output constraint of the supplier (the non-convexity of the present example), the optimum solution is to produce 100 MW and to clear respectively 90 and 10 MW of the loads. Any price π would result in either a RS for the loads or for the supplier. Indeed, the non-negative as-cleared profit condition implies $\pi \geq 50$ for the supplier and $\pi \leq 20$ for the load, so the set of prices ensuring *zero* RS is empty.[5] We notice that, in this example, CHP, ELMP, MMWP or AIC pricing all result in a market clearing price of 50€/MWh, which implies a RS of 300€ for the second load.

[5] The European SDAC clearing rule achieves *zero* revenue shortfall in a two-sided auction. The difference with Example 1 is that the SDAC rule does not fix the optimal dispatch: it allows a change in the dispatch, and tolerates a possible loss of social welfare, in order to find a price that ensures zero RS.

Instead of AIC pricing, we shall consider, as the fourth pricing scheme of this paper, a method that aims at *minimal* make-whole payments (MMWP), proposed by Madani and Papavasiliou (2022), that works with both elastic and inelastic loads.[6] Two variants of MMWP will later be discussed in Section 5.

Definition 9 (*Minimal Make-Whole Payments Pricing*). The minimal make-whole payments prices are the dual variables π^{MMWP} associated to the market clearing constraints in the following problem:

$$\min_{k_g^{gen},k^f} \sum_{g\in\mathcal{G}} k_g^{gen} c_g^* \tag{10a}$$

$$(\pi_{i,t}^{MMWP}) \quad \sum_{g\in\mathcal{G}_i} k_g^{gen} q_{g,t}^* - D_t^i = \tag{10b}$$

$$k^f\Big(\sum_{\substack{l\in\\ from(i)}} f_{l,t}^* - \sum_{\substack{l\in\\ to(i)}} f_{l,t}^*\Big) \quad \forall i\in\mathcal{N}, t\in\mathcal{T} \tag{10c}$$

$$0 \le k_g^{gen}, k^f \le 1 \tag{10d}$$

Proposition 2 (*MMWP*). *MMWP prices minimize the total revenue shortfall, as defined in* (9).

Given Problem (1) assumes inelastic load, MMWP will in fact lead to *zero* revenue shortfall.

We conclude the section with three general remarks. Firstly, among the four pricing approaches, IP, ELMP and MMWP are computationally straightforward to obtain, while CHP is notably more challenging to compute. In this paper, we calculate it using the Level Algorithm which has demonstrated its ability to compute *exact* CHPs for realistic market sizes (Stevens, 2016; Stevens and Papavasiliou, 2022). Secondly, CHP, IP and MMWP are formulation-*independent*, while ELMP is formulation-*dependent*. Two equivalent formulations of the sets $\mathcal{X}_g$ could result in different ELMPs.[7] Thirdly, we notice that both CHP and ELMP keep primal and dual computations distinct, while IP and MMWP do not. As highlighted by Schiro et al. (2015), this implies that an off-line unit could set the price under CHP or ELMP. It is, nonetheless, unclear to what extent this is an undesirable feature. For example, the principle of a second-price auction, which is contemplated in economics as a sound manner to clear an auction, is that the first *losing* bid sets the price.

4. Agents' incentives: Distributional analysis

The main property of CHP (Proposition 1) informs us on the *total* LOC, which is guaranteed to be lower under CHP than under any alternative price. But it says nothing about how the total LOC is distributed among the market participants. This section studies the main properties that can be established mathematically and observed in the numerical simulations. In general, nothing can be said a priori about how each agent will be affected *individually*, depending on the pricing scheme: although the *total* LOCs are lower under CHP, a supplier *may* have a higher LOC under CHP than under the other prices. Nonetheless, some properties can be established about the split of LOC among the three following categories of market participants: the network, the convex suppliers ($g \in \mathcal{G}^C$) and the non-convex suppliers ($g \in \mathcal{G}^{NC}$, with $\mathcal{G} = \mathcal{G}^C \cup \mathcal{G}^{NC}$). Let us notice that both the European auction and the US markets include a convex network and convex suppliers.

[6] The original presentation of the method by Madani and Papavasiliou (2022) includes elastic loads. We extend the approach to include a network and inelastic loads.

[7] Zhao et al. (2021) have challenged the "formulation-independence" of CHP. However, their usage of the term "formulation" departs from ours. By "formulation", we mean here the textbook definition (Wolsey, 1998): let $\mathcal{X}_g \subseteq \mathbb{R}^n \times \mathbb{Z}^m$, then P_1 and P_2 are two *formulations* of $\mathcal{X}_g$ (e.g. two ways to write ramp constraints) if $\mathcal{X}_g = P_1 \cap (\mathbb{R}^n \times \mathbb{Z}^m) = P_2 \cap (\mathbb{R}^n \times \mathbb{Z}^m)$.

Proposition 3 (*LOC of Convex Agents in IP*). *Under IP pricing, all the convex market participants (the convex suppliers $g \in \mathcal{G}^C$ and the network) have a zero LOC.*

Proposition 4 (*RS of Convex Agents in IP*). *Assuming $\mathbf{0} \in \mathcal{X}_g\ \forall g \in \mathcal{G}^C$ and $\mathbf{0} \in \mathcal{F}$, then both the convex suppliers and the network have a zero revenue shortfall under IP pricing.*

These properties follow from the fact that IP prices reflect the marginal cost of on-line units. Since a *convex* supplier is always on-line and does not bear fixed costs, its LOCs are null under marginal prices. Furthermore, since the primal and the IP pricing problems are coupled so that the flows are equal in both problems, the (convex) network does not bear a LOC. These properties are not shared with the other pricing rules.

Proposition 5 (*Non-Zero LOC of Convex Agents*). *Under CHP, ELMP or MMWP, the convex market participants (both the convex suppliers and the network) may have a positive LOC.*

For the sake of completeness, the following result can also be deduced from Propositions 1 and 3.

Proposition 6 (*LOC of Non-Convex Agents*). *Under CHP, the total lost opportunity cost of the non-convex suppliers ($\sum_{g\in\mathcal{G}^{NC}} LOC_g^{gen}(\pi)$) is lower than under IP prices.*

Intuitively, CHP permits to increase the LOC of the network and the convex generators in order to reduce the total LOC.[8] We shall discuss these Propositions in parallel with the results of the numerical simulations. As announced in the introduction, we use two different datasets, each having their merits for the properties we seek to illustrate. The first, later denoted as "FERC dataset", is based on public data (Knueven et al., 2020; Krall et al., 2012). The underlying unit commitment model includes minimum up and down time constraints, ramp constraints (including start-up and shut-down ramps), time-dependant start-up costs, no-load costs, and piecewise linear production costs. The model gathers almost 1000 power units, but has no network. This is a market of realistic size, except for the absence of the network. We conduct our analysis over 11 net-load scenarios of 24 periods each, with hourly time step. The second dataset, later denoted as "CWE dataset", is based on non-public data assembled by our team (Aravena and Papavasiliou, 2016; Stevens and Papavasiliou, 2022). It includes a network of 30 bidding zones and 74 power units. The suppliers are modelled using a simpler unit commitment model than the FERC dataset (essentially simplifying the cost structure). We simulate 12 different load profiles (half of which correspond to 24 periods and the half of which correspond to 96 periods). Tables 1 and 2 report the average results of the FERC and CWE simulations respectively. The detailed results per load scenario are available in Appendix C. We will focus on IP, CHP and ELMP, and delay the analysis of MMWP until the next section.

As far as the suppliers are concerned, the FERC data include both a share of convex (14%) and non-convex (86%) suppliers. The CWE data only include non-convex suppliers. We observe that the convex suppliers in the FERC case as well as the network in the CWE case have zero LOC under IP pricing (Proposition 3). They also have a null RS (Proposition 4). We also observe that CHP outperforms the other prices on the total LOC (Proposition 1) as well as on RS, although the latter is not guaranteed by the theory. Tables 1 and 2 also report the proportion of suppliers impacted by LOC as well as the average LOC carried by these suppliers. On the FERC data, we observe that CHP reduces both figures. On the CWE case, the share of suppliers impacted by LOC is

[8] Similarly, in case all the suppliers are convex and the network is non-convex, then IP pricing guarantees zero LOC for the suppliers, while CHP transfers some of the LOC from the network to the convex generators, in order to ensure a minimum total LOC (Garcia et al., 2020).

Table 1
Incentives of market agents on the FERC dataset depending on the price (average over 11 scenarios). All figures are in US$. Since all the suppliers have the possibility of inaction, $RS_g^{\notin LOC}(\pi) = 0$. The lost opportunity costs (LOC), the revenue shortfall (RS), and the foregone opportunities (FO) are reported for the convex (Conv.) and non-convex (Non-Conv.) suppliers as well as in total (Tot.).

		IP	CHP	ELMP	MMWP	MMWP*	MMWP**
Dispatch cost					29,780,000		
Av. Price [$/MWh]		28.8	28.7	28.8	56.3	26.8	28.9
Num. Suppl. with LOC		3.4%	1.8%	7.5%	79.2%	24.7%	9.5%
Av. LOC per Suppl.		628	19	37	148,232	4577	94
LOC	Tot.	37,576	323	2801	130,147,114	1,176,050	14,217
	Conv.	0	67	94	1,978,501	5268	79
	Non-Conv.	37,576	257	2707	128,168,613	1,170,782	14,137
RS (in LOC)	Tot.	669	19	206	0	0	0
	Conv.	0	0	3	0	0	0
	Non-Conv.	669	19	203	0	0	0
FO	Tot.	36,907	304	2,596	130,147,114	1,176,050	14,217
	Conv.	0	66	91	1,978,501	5268	79
	Non-Conv.	36,907	238	2505	128,168,613	1,170,782	14,137

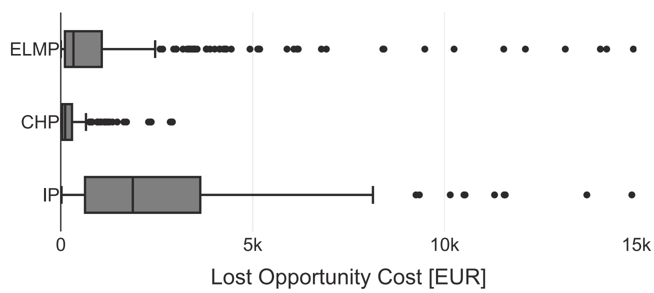

Fig. 1. Distribution of the LOC across suppliers for IP, CHP and ELMP (aggregate of all the CWE cases).

similar between CHP and IP pricing, but CHP significantly reduces the average LOC carried by each supplier (see also Fig. 1). Interestingly, in both the FERC and CWE datasets, ELMP tends to spread the LOC over a higher share of suppliers.

As far as the network is concerned, we stress two observations. Firstly, Zhao et al. (2021) questions the validity of CHP on the basis that minimizing the network LOC is off-target. As Propositions 3 and 5 indicates, it could be argued that CHP minimizes the network LOC to a smaller extent than IP pricing. Secondly, if the concept of network LOC has already been analysed in the literature (Garcia et al., 2020), the concept of network RS has been less discussed. Under some prices, not only could the network bear a LOC (a potential FTR shortfall), but it could also have a shortfall of revenue, i.e. a *negative* congestion rent. The following example illustrates this possibility, although it does not materialize in our simulations. Indeed, Table 2 shows that the system operator has positive LOC under CHP, ELMP and MMWP. But the network RS is null under all prices.

Example 2 (*Network RS*). Let us consider a simple network with two nodes (A and B) connected by a line with a capacity of 100 MW. There is an hourly demand of 200 MW at 100€/MWh in both nodes as well as a flexible supplier of 400 MW at 50€/MWh in node A and an inflexible supplier of 1000 MW (all-or-nothing) at 10€/MWh in node B. The welfare-maximizing allocation is to produce 300 MWh in node A: 200 MWh is consumed in A while 100 MWh is consumed in B and the line is congested. Under IP pricing, the prices (π^{IP}) at A and B are 50 and 100€/MWh, respectively and the congestion rent is 5000€. Under CHP or ELMP, the prices ($\pi^{CHP} = \pi^{ELMP}$) at A and B are 50 and 10€/MWh and the congestion rent is −4000€.

5. LOC vs. make-whole payments controversy

As mentioned in Section 2, some advocate that incentive-compatibility (measured by LOC) is not the adequate target for a price, that should instead aim at being non-confiscatory (measured by RS). Schiro et al. (2015) particularly stress that, in some cases, the revenue shortfall may be lower with IP pricing than with CHP, casting some doubt about the validity of the latter. Although CHP reduces the RS *on average* in our numerical simulations (Tables 1 and 2), there are indeed instances in both datasets where CHP turns out to modestly increase the RS, cf. Appendix C. In order to discuss rigorously the controversy "LOC vs. RS", it is first worth clarifying the relationship between LOC and RS.

Proposition 7 (*Relationship between RS and LOC*). *If all the market agents have the possibility of inaction ($\mathbf{0} \in \mathcal{X}_g\ \forall g \in \mathcal{G}$, $\mathbf{0} \in \mathcal{F}$), then $RS_g^{gen}(\pi) \leq LOC_g^{gen}(\pi)\ \forall g$ and $RS^{net}(\pi) \leq LOC^{net}(\pi)$.*

Which is to say that, given the possibility of inaction,[9] the lost opportunity costs can be viewed as the sum of the revenue shortfall and the foregone opportunities (FO):

$$LOC_g^{gen}(\pi) = RS_g^{gen}(\pi) + FO_g^{gen}(\pi) \quad \forall g \in \mathcal{G}$$
$$LOC^{net}(\pi) = RS^{net}(\pi) + FO^{net}(\pi)$$

The RS is a certain type of LOC in which the cleared profit is negative and the opportunity is to self-schedule at 0, while the FO denotes the remaining "lost opportunities". If the as-cleared profit is zero (as for a unit that is not operating), or positive, the RS is null and the LOC equals the FO, which corresponds to the *additional* profit that the supplier could gain by deviating from the cleared volumes. If the as-cleared profit is negative, the foregone opportunities are the maximal profit *above zero* that the supplier could earn.

Although the possibility of inaction is a standard assumption in economics, there are cases when it does not hold. This happens when there are barriers of exit, for instance, in the presence of must-run constraints (this is the case in Example 7 presented by Schiro et al. (2015)), or in case a supplier that is initially on-line faces a binding "minimum up time" or a ramp constraint that prevents it to be switched off. In these circumstances, Proposition 7 does not hold: a unit could produce at a loss ($RS_g^{gen} > 0$) without having any opportunity to act differently ($LOC_g^{gen} = 0$). More specifically, the revenue shortfall could be further dissected into two quantities: $RS_g^{\in LOC}$ (the part of RS which can be expressed as an LOC) and $RS_g^{\notin LOC}$ (the part which cannot be expressed as an LOC, roughly speaking the revenue shortfall due to a barrier of exit). For example, a supplier having an as-cleared profit of −200€ and a maximum profit of 100€, has an LOC of 300€. The latter corresponds to a RS of 200€ as well as a FO of 100€. Alternatively, a supplier which does not have possibility of inaction and who has an as-cleared profit of −200€ and a maximum profit of −100€, has an LOC of 100€ with $RS = RS^{\in LOC} + RS^{\notin LOC} = 100 + 100 = 200$€.

[9] This is the case for all suppliers in the European DA market.

Table 2
Incentives of market agents on the CWE dataset depending on the price (average over 12 scenarios). All figures are in €. The LOC, RS and FO are reported for the suppliers (Suppl.), the network (Net.) and in total (Tot.).

		IP	CHP	ELMP	MMWP	MMWP*	MMWP**
Dispatch cost					5,489,000		
Av. Price [€/MWh]		42.8	43.4	47.3	27.7	23.8	52.6
Num. Suppl. with LOC		33.2%	35.9%	45.3%	83.6%	63.4%	64.3%
Av. LOC per Suppl.		3528	278	1285	141,834	29,326	27,066
LOC	Tot.	83,543	8093	42,948	98,681,795	41,808,171	20,789,079
	Suppl.	83,543	6810	39,006	8,746,513	1,350,259	1,250,017
	Net.	0	1282	3942	89,935,282	40,457,912	19,539,062
RS (in LOC)	Tot.	10,550	1987	8508	0	0	0
	Suppl.	10,550	1987	8508	0	0	0
	Net.	0	0	0	0	0	0
FO	Tot.	72,993	6106	34,440	98,681,795	41,808,171	20,789,079
	Suppl.	72,993	4823	30,499	8,746,513	1,350,259	1,250,017
	Net.	0	1282	3942	89,935,282	40,457,912	19,539,062
RS (not in LOC)	Tot.	897,653	877,040	730,234	0	0	0
	Suppl.	897,653	877,040	730,234	0	0	0
	Net.	0	0	0	0	0	0

Definition 10 (*RS & FO*)**.** The revenue shortfall (Definition 5) and the foregone opportunities can be further characterized as follows[10]:

$$RS_g^{\notin LOC}(\pi) = \max(0,\ RS_g^{gen}(\pi) - LOC_g^{gen}(\pi))$$
$$RS_g^{gen}(\pi) = RS_g^{\in LOC}(\pi) + RS_g^{\notin LOC}(\pi)$$
$$FO_g^{gen}(\pi) = LOC_g^{gen}(\pi) - RS_g^{\in LOC}(\pi)$$

Under possibility of inaction, $RS_g^{\notin LOC}(\pi) = 0$

CHP minimizes the total lost opportunity costs. Under the possibility of inaction, this means that CHP minimizes the revenue shortfall as long as it does not exacerbate the foregone opportunities. In case the possibility of inaction does not hold, some of the revenue shortfalls ($RS_g^{\notin LOC}(\pi)$) would not enter into what is minimized by CHP. Following those remarks, the LOC-RS controversy, as raised by Schiro et al. (2015), could be formulated as follows:

- Under the possibility of inaction, is it desirable to minimize the RS *at all cost*?
- In case the possibility of inaction does not hold, is it desirable to minimize $RS_g^{\notin LOC}(\pi)$?

We shall present several arguments against both. To address both questions, we rely on the comparison of CHP with MMWP, which is precisely the price that minimizes the RS.

Firstly, is it desirable to minimize the revenue shortfall? A major concern when dealing with MMWP is price indeterminacy: the MMWP prices are typically not unique. This also happens for CHP or IP pricing, as well as for a convex case in which multiple prices could support a competitive equilibrium. Nonetheless, the indeterminacy is expected to be more severe under MMWP than for the other pricing rules. Indeed, minimizing the revenue shortfall is a mild requirement: in a load-inelastic case, *any* price that is high enough would guarantee zero revenue shortfall – e.g. fixing the price at the market price cap would certainly make each cleared bid whole. Mathematically, in problem (10), π^{MMWP} belongs to a set that ranges from the smallest price ensuring profitability for all the committed units to infinity.

This indeterminacy is observed in our numerical results. In Table 1, the MMWP prices meet their objective of zero revenue shortfall. But this is achieved with prices that are excessively high – two times the CHP on average – which, in turn, leads to extravagant LOC – four times the total system cost. This makes the "vanilla" version of MMWP (Definition 9) impracticable. Load elasticity would certainly mitigate the indeterminacy, but it would likely not solve it entirely. If one chooses to proceed with MMWP prices, this then raises the question of how to choose the right price among the *many* MMWP prices. We shall consider two possibilities. The first one, that we shall denote as MMWP*, is to select the *smallest price* that minimizes the revenue shortfall.

Definition 11 (*MMWP**)**.** The MMWP* prices are the optimal variables π of the following problem:

$$\min_{\pi} \|\pi\|_2 \tag{11a}$$
$$\mathcal{P}_g(c^*, q^*, x^*, \pi) \geq 0 \quad \forall g \in \mathcal{G} \tag{11b}$$
$$\mathcal{P}_N(f^*, \pi) \geq 0 \tag{11c}$$

Constraints (11b)–(11c) require that the price π results in zero revenue shortfall, while the objective (11a) resolves the eventual indeterminacy over π by selecting the smallest price that satisfies the required constraints. This method is *akin to* average cost pricing, at least when the load is inelastic, since the smallest price that ensures zero RS is essentially the highest average cost among the committed units. A similar proposal is described by Liberopoulos and Andrianesis (2016).

Bichler et al. (2022) propose another formulation, which we refer to later in the paper as MMWP**, in which, among the possible MMWP prices, the one that minimizes the LOC is selected. Their model relies on a bi-level optimization problem which is intractable. Consequently, they introduce an approximation of this bi-level model, which consists of finding a price that is as close as possible to ELMP while minimizing the RS.[11]

Definition 12 (*MMWP***)**.** The MMWP** prices are the optimal variables π of problem (11) in which the objective function (Eq. (11a)) is replaced by $\|\pi - \pi^{ELMP}\|_2$.

Concretely, MMWP* and MMWP** are linked with MMWP as follows. If $\Pi^{MMWP} = \{\pi \text{ solving } (10)\}$, then $\pi^{MMWP*}, \pi^{MMWP**} \in \Pi^{MMWP}$. Finally, we notice that the two previous models are straightforward to extend to a configuration that includes elastic loads, by relying on slack variables in constraints (11b)–(11c), cf. Bichler et al. (2022).

As far as the numerical results are concerned, we observe in Table 1 that, as expected, both MMWP* and MMWP** prices reach zero

[10] Although we define them for the suppliers, these concepts could also be transposed to the network.

[11] The actual model of Bichler et al. (2022) slightly differs from ours: they compute the price that minimizes the RS *for every hour*, as opposed to our model, that minimizes the RS *over the entire market horizon*.

revenue shortfall. They also both significantly improve the LOC as compared to the vanilla MMWP. Nonetheless, MMWP* is still widely outperformed by the alternative pricing methods. It illustrates that resolving the price indeterminacy that is inherent in MMWP is by no means obvious. This leaves MMWP** as the only serious competitor for IP, CHP and ELMP. We shall nonetheless see later in this section some shortcomings of MMWP** in the CWE case. The question remains: is it desirable to minimize the revenue shortfall *at all cost*? On the FERC simulations, the average total RS under CHP is 19$. Under MMWP**, it drops to zero, but the total LOC increases from 323$ with CHP to 14,217$ with MMWP**. Are the 19$ savings in RS worth the loss of ~14,000$ in LOC? More generally, in the hypothetical case that lowering the RS of 1€ would induce a LOC of 1 M€, should we take the stance that minimizes RS? In contrast with MMWP which minimizes the RS *at all cost*, convex hull pricing offers an appealing trade-off: it minimizes the revenue shortfall as long as it does not exacerbate more the foregone opportunities. This is not to say that RS are irrelevant, but since they are unavoidable in two-sided auctions (cf. Example 1), considering the above discussion, it may appear more appropriate to handle them through side-payments instead of through the uniform price (as demonstrated by Madani and Papavasiliou (2022), there *always* exist "zero-sum transfers" that can finance the make-whole payments while guaranteeing *revenue-adequacy* for the auctioneer).

Secondly, is it desirable to implement a price that aims at minimizing the RS *including* $RS_g^{\notin LOC}(\pi)$ (the three MMWP approaches described so far minimize the total RS, including $RS_g^{\notin LOC}(\pi)$)? Let us first look at the question from the viewpoint of a *convex* market. Actually, having $RS_g^{\notin LOC}(\pi) > 0$ is not specific to non-convexities. Indeed, while $LOC = 0$ is guaranteed in a convex market, it is straightforward to design an instance of a *convex* market (e.g. with a must-run constraint) with a competitive equilibrium, in which some agents have $RS_g^{\notin LOC}(\pi) > 0$. Remarkably, CHP, IP and ELMP would boil down to the classic competitive prices in a convex market, while MMWP would not. Then, the numerical results also highlight another shortcoming of MMWP prices. In the FERC case, all suppliers have the possibility of inaction, and therefore $RS_g^{\notin LOC}(\pi) = 0$. In the CWE case, 36% of the suppliers do not have possibility of inaction because of binding constraints. Consequently, we observe positive $RS_g^{\notin LOC}(\pi)$ in Table 2 for all the pricing methods except the three MMWP approaches. We observe that mitigating the $RS_g^{\notin LOC}(\pi)$ through the uniform price of energy comes with a substantial effect on the lost opportunity costs. Intuitively, in order to ensure zero revenue shortfall for suppliers which are in any case not willing to deviate from the market schedule, MMWP raises the prices, which in turn exacerbate the foregone opportunities of the other suppliers. MMWP** which, although disputable, is still competitive in the FERC cases, is simply impracticable in the CWE cases. Again, we are not arguing that the $RS_g^{\notin LOC}(\pi)$ are irrelevant, but according to the evidences of this section, they are not specific to the topic of pricing *non-convexities* and it is not clear that they should be settled through the uniform price of energy, as MMWP does.

6. The limits of approximating CHP

The previous section has focused on MMWP. In the present section, we turn to ELMP. As outlined in Section 3, the main economic justification for ELMP is that it is viewed as a scalable approximation of CHP which comes with the remarkable Proposition 1 (Chao, 2019). This analogy with CHP suggests that ELMP would achieve a lower lost opportunity cost than IP pricing, as it "approximately minimizes LOC". Tables 1 and 2 confirm this intuition. On average, ELMP roughly cuts by ten (resp. two) the lost opportunity costs in the FERC dataset (resp. CWE dataset) as compared with IP pricing. This is also observed in other works (PJM, 2017; Hua and Baldick, 2017; Yu et al., 2020). Nonetheless, if *empirical* evidences show that ELMP reduces the LOC as compared to IP pricing, it is worth noting that, in general, there is no *theoretical* guarantee that this will be the case.

Table 3
Supplier data in Example 3. The columns stand for the initial commitment, the no-load cost (€/h), the marginal cost (€/MWh), the production limits (MW) and the ramp limits (MW).

Suppliers	x^0	NLC	MC	Q^{max}	Ramp
G1	1	0	80	500	500
G2	0	1950	78	600	300
G3	0	5920	74	600	100
G4	0	0	130	500	105

Table 4
Hourly demand (MW), commitments/schedules (MW) and prices (€/MWh) in Example 3.

D	G1	G2	G4	IP	ELMP	CHP
350	1/350	1/0	0/0	80	80	80
500	1/200	1/300	1/0	80	80	80
950	1/255	1/600	1/95	80	82.5	82.5
1300	1/500	1/600	1/200	180	95.1	145.27

Proposition 8 (*ELMP vs. IP LOC*). *Given a feasible primal solution of problem (1), ELMP does not guarantee a lower total LOC than IP pricing.*

Example 3 (*LOC ELMP vs. IP*). Designing a stylized example with $LOC(\pi^{IP}) < LOC(\pi^{ELMP})$ is not trivial, since it firstly requires that ELMP differs from CHP. Let us consider a market with four suppliers (Table 3) and four hourly periods with an inelastic load (Table 4). The suppliers do not have a minimal production limit, but they have a no-load cost and a ramp constraint (the detailed model is in Appendix B). The optimal schedule is reported in Table 4. The cheapest way to meet the load in $t = 2$ is using G1. Nonetheless, due to binding ramp constraints, G2 has to be started in $t = 1$, and to produce in $t = 2$ in order to meet the ramp from period 2 to 3. Similarly, the cheapest way to satisfy the load in $t = 3$ is using G1 and G2. Because of the ramp from period 3 to 4, G4 produces in $t = 3$. The total production cost is 267,550€. The binding ramp constraints make ELMP different from CHP. The crux of the example is that G4 has zero no-load cost, as opposed to G3. The optimal schedule commits G4, which has a higher MC, implying a high IP price. In the ELMP pricing problem, since integers are relaxed, the no-load cost of G3 does not have to be borne entirely in periods 2, 3 and 4, rendering it economically more attractive than G4. This drives the ELMP price downward, resulting in a significant revenue shortfall for G4. The prices are reported in Table 4 (the intuition about these prices is discussed in Appendix B). They lead to a total LOC of 10,670, 12,105 and 3675€ for, respectively, IP, ELMP and CHP.

This is not merely a phenomenon that occurs in a pathological example. In our simulations, there are instances in both datasets where ELMP induces a higher LOC than IP pricing (one instance in both datasets, cf. Appendix C). As discussed in Section 3, ELMP is formulation-dependent. If the formulation of ELMP is tight, then $\pi^{CH} = \pi^{ELMP}$, which implies from Proposition 1 that $LOC(\pi^{ELMP}) \leq LOC(\pi^{IP})$. The above discussion highlights that the previous inequality is not guaranteed *in general* for any ELMP, regardless of the tightness of the formulation. This highlights the advantage of *exact* CHP over ELMP, not only for the average reduction of LOC, but also for the theoretical guarantees surrounding CHP. Let us stress that, according to the evidence from Stevens and Papavasiliou (2022), computing *exact* CHP is expected to be feasible for the European market, although this should be confirmed by simulation on the actual order book.

7. Minimizing the costs or the LOC

The last two sections focus on a comparison of IP pricing with CHP, and stress two properties. Again, IP pricing is the candidate currently envisioned by SDAC for the European day-ahead market (MCSC, 2022).

Table 5
Sensitivity of lost opportunity cost to the primal optimality gap, depending on the price. The simulations are performed on CWE dataset (Spring WD 24). All figures are in €.

Opt. gap	Tot. cost	IP LOC	ELMP LOC	CHP LOC
0.1%	5,213,357	115,043	43,346	12,611
0.09%	5,212,947	101,212	42,937	12,201
0.08%	5,212,121	194,521	42,111	11,375
0.07%	5,212,121	194,521	42,111	11,375
0.06%	5,211,690	129,455	41,680	10,944
0.05%	5,211,057	119,929	41,047	10,312
0.04%	5,210,885	119,579	40,875	10,140
0.03%	5,210,743	119,360	40,733	9997
0.02%	5,210,685	119,351	40,675	9940
0.01%	5,210,685	119,351	40,675	9940

Firstly, convex hull pricing minimizes the LOC, not only for the optimal allocation (c^*, q^*, x^*, f^*) of problem (1), but for *any* feasible allocation. In this section, we briefly revisit the interplay between primal and dual (prices) results, also studied in previous works (Sioshansi et al., 2008; Eldridge et al., 2019; Byers and Hug, 2022).

Proposition 9 (*LOC-Primal Relationship 1*). *Under CHP or ELMP, the total LOC decreases monotonically with the optimality gap of the primal solution. More specifically, let* $(c, q, x, f)_1$ *and* $(c, q, x, f)_2$ *denote two feasible solutions of problem* (1)*, with objectives* z_1 *and* z_2 *and lost opportunity cost* LOC_1 *and* LOC_2*, respectively. Then:*

$$LOC_1(\pi) - LOC_2(\pi) = z_1 - z_2$$

This result immediately follows the interpretation of the LOC as the duality gap, explained in Section 3. Under convex hull pricing, the objective of minimizing the primal optimality gap is consistent with both the minimization of the total costs and the minimization of the lost opportunity costs. Let us notice that Proposition 9 holds even if the computation of CHP is not exact. Proposition 9 also implies that there is no other allocation that could make the agents better off than the welfare-maximizing allocation. This is notably different under IP or MMWP.

Proposition 10 (*LOC-Primal Relationship 2*). *Under IP or MMWP, the total LOC does not decrease monotonically with the optimality gap of the primal solution.*

Intuitively, as far as IP pricing is concerned, a suboptimal solution commits costlier suppliers which, if entailing higher variable production cost, pulls the IP price upward, which in turn might reduce the LOC. We want to emphasize the dilemma that this might create when it comes to picking the "best" solution among a set of feasible solutions. The dilemma is illustrated on the numerical results of Table 5.[12] Here, one instance of the CWE dataset is solved for various optimality gaps. As expected from Proposition 9, the LOC associated with CHP and ELMP diminishes monotonically with the primal optimality gap: the improvement in LOC corresponds exactly to the improvement in total cost. Under IP prices, a suboptimal solution (optimality gap of 0.09%) achieves the best LOC. This creates inconsistent incentives for the primal and the pricing problems: going from an optimality gap of 0.09% to 0.01% reduces the total cost by 2262€ while it increases the lost opportunity cost by 18,139€. Which solution should be preferred? More radically: going from the gap 0.09% to 0.08% reduces the total cost by 826€ while it increases the lost opportunity cost by 93,309€. CHP makes such dilemmas irrelevant.

[12] Since the comparison of this section focuses on CHP and IP pricing, we omit the three MMWP schemes from Table 5. Nonetheless, the reader may find the related results for MMWP in Appendix A (Table A.1).

8. The curse or blessing of market size

Convex hull pricing does not only minimize the lost opportunity cost, it is also guaranteed to remain *bounded*, so that it does not grow with the market size. This remarkable property, which builds on works from the theory of general equilibrium (Starr, 1969; Arrow and Hahn, 1971), can be expressed for the market model (1), in order to derive a theoretical bound on the LOC (Chao, 2019).

Proposition 11 (*LOC Bound 1*). *Under CHP or ELMP, the total LOC is bounded. The bound depends on the shape of* $\mathcal{X}_g$*, but is independent of* $|\mathcal{G}|$: $\lim_{|\mathcal{G}|\to\infty} LOC(\pi) < \Gamma$.

The surprising feature of Proposition 11 is that the LOC does not depend on the market size: if the market grows (increasing the number of suppliers as well as load), given that the LOC remains bounded, its relative importance shrinks ($LOC(\pi^{CH})/z^* \to 0$). The strength of Proposition 11 is better captured when contrasted to alternatives prices (see also the discussion in Stevens et al. (2024)).

Proposition 12 (*LOC Bound 2*). *Under IP or MMWP pricing, the total LOC is not necessarily bounded: it could be that* $\lim_{|\mathcal{G}|\to\infty} LOC(\pi) \to \infty$.

Propositions 11 and 12 highlight the theoretically sound behaviour of CHP, as opposed to IP pricing. Stylized examples as well as numerical illustrations of these Propositions have nonetheless been scarce in the literature. Example 4 aims at providing intuition about the Propositions, while the subsequent numerical simulations and the related discussion explore their practical implications.

Example 4 (*LOC Bounded or Unbounded*). Consider a session of the European day-ahead market with one hourly period and the following supply orders: one divisible stepwise curve of 100 MW at 50€/MWh and a set of N fully indivisible block orders of 100 MW at 100€/MWh. Let us assume a divisible demand of 250 MW at 1000€/MWh. The welfare maximizing allocation is to clear 2 blocks and 50 MW of the stepwise curve. Under IP pricing, the price is 50€/MWh and the two cleared blocks have a revenue shortfall of 10,000€. Let us now assume that the demand grows to 550 MW. The IP price remains the same while the revenue shortfall is now 25,000€. This quantity will keep growing with the demand. Under CHP, the price is 100€/MWh. Only the stepwise supply curve has an LOC (in this case, a foregone opportunity) of $50 \times 50 = 2,500$€, whether the demand is 250 or 550 MW. This shall remain bounded if the demand keeps growing.

In order to further illustrate the theoretical Propositions, we conduct the following experiment on the FERC dataset over one load profile (`2015-08-01_lw`). First, we randomly select 50 power units out of the 1000. Then, we adapt the load profile accordingly, in order to make the problem feasible. Under these settings, we compute the welfare maximizing allocation as well as the marginal prices and the convex hull prices together with their associated lost opportunity costs. Finally, we gradually increase the market size by duplicating x times the 50 units and multiplying the load accordingly. The results are reported in Table 6. We proceed with certain observations. Proposition 11 establishes that, when the market size increases, the LOC under convex hull pricing remains bounded and the bound is not affected by the number of plants. Thus, the ratio of the LOC relative to some measure of the market size (e.g. the relative duality gap) is expected to shrink with the market size. This is what we observe in Table 6 where the ratio of the LOC relative to the total system cost ranges from 0.62% to 0.01% while the number of power plants grows from 50 to 1000. On the other hand, Proposition 12 establishes that the LOC under IP pricing is *not* subject to such a bound and *could* therefore increase with the market size so that the relative importance of LOC remains largely unaffected. Concretely, what we observe in Table 6 (last column), is that the ratio of LOC relative to the total system cost remains around 15%, regardless of the market size.

Table 6
Results of CHP and IP pricing on FERC datasets (load profile 2015-08-01_lw) depending on the market size. The initial 50-unit market is multiplied by a factor ranging from 2 to 20.

Market size			Convex hull pricing		Marginal pricing	
Number of plants	Av. hourly load (MW)	Tot. cost ($)	LOC ($)	LOC (% Tot. cost)	LOC ($)	LOC (% Tot. cost)
50	4900	1,820,308	11,222	0.62%	276,383	15.18%
100	9800	3,631,286	13,114	0.36%	538,713	14.84%
150	14,700	5,444,099	16,841	0.31%	805,370	14.79%
200	19,600	7,245,546	9202	0.13%	1,060,574	14.64%
250	24,500	9,052,185	6756	0.07%	1,320,763	14.59%
300	29,400	10,857,007	2492	0.02%	1,579,297	14.55%
350	34,300	12,666,418	2817	0.02%	1,842,613	14.55%
400	39,200	14,475,824	3136	0.02%	2,105,629	14.55%
450	44,100	16,290,191	8417	0.05%	2,373,870	14.57%
500	49,000	18,099,571	8711	0.05%	2,636,708	14.57%
1000	98,000	36,183,999	2280	0.01%	5,258,840	14.53%

We make two more remarks on the Propositions and the numerical results: the first regards the mathematical bound in Proposition 11, the second concerns the practical implications of the propositions. As far as the bound is concerned, the mathematical expression of Γ is provided in Appendix A. This expression can be used to calculate the bound on the FERC dataset: $\Gamma = 21.9$M\$. From Table 6, we observe that this bound is far from tight, since the actual LOC amounts to a few thousand dollars per day. Although the trend expected from Propositions 11 and 12 materializes in the numerical results, the practical usefulness of the bound itself appears to be limited.

As far as the practical implications are concerned, it is of course unrealistic to expect the market to grow by a factor of ten in most US markets or in Europe. We nevertheless stress that the variations of volume traded in the market do not necessarily represent a physical change of generation. In a country such as India, in which the day-ahead market has been created in 2008, and which has recently adopted a similar pricing rule as in Europe (N-SIDE, 2021), such an increase is not far from reality. Indeed, since its creation, the market daily average traded volume has increased by a factor of ten (IEX, 2020). Similarly, in Japan, the traded volume in the day-ahead market was multiplied by more than ten since the implementation of liberalization policies in 2016 (JPEX, 2023). In Europe, if the growth of the day-ahead market is more modest (+1.5% of daily traded volume between 2018 and 2021, with a notable increase of +7% in the number of non-convex block orders over the same period, cf. NEMO Committee (2022)), the traded volume in a market session can vary significantly. As an example, the daily average traded volume in 2020 ranges from 3.83 to 5.82 TWh (NEMO Committee, 2022).

9. Conclusion

We have reviewed and analysed six pricing methods from the literature. They are all potential candidates for reforming the current European pricing rule. Marginal pricing could be an upgrade as compared to the current SDAC pricing rule, given the likely improvement in both welfare and scalability. Nonetheless, the fact that many US markets have exhibited the tendency to move away from marginal pricing during the last ten years is something that stakeholders may wish to pay attention to in Europe, given the favourable alternatives that are on the table.

In the paper, we have attempted to highlight some of the advantages of convex hull pricing over several dimensions. With respect to IP pricing, the fact that CHP incorporates the lumpy costs in the price signal improves significantly the incentives faced by the market agents (Section 4). CHP is also accompanied by appealing theoretical guarantees, both in terms of consistency between cost and LOC minimization (Section 7) as well as in terms of the bound it ensures on the LOC (Section 8). While ELMP would be a significant first step in the direction of CHP – a step that several US ISO have made – we have tried to highlight some limits of this approximation. In particular, ELMP does not safeguard all the theoretical guarantees of CHP (Section 6), nor does it achieve the same performance in terms of LOC minimization. Finally, while minimizing the revenue shortfall – or "make-whole payments" – may sound like a reasonable target, we have shown that it may also result in unbearable (and unbounded, cf. Section 8) lost opportunity costs (Section 5).

CRediT authorship contribution statement

Nicolas Stevens: Conceptualization, Formal analysis, Methodology, Software, Writing – original draft. **Anthony Papavasiliou:** Conceptualization, Funding acquisition, Supervision, Writing – review & editing. **Yves Smeers:** Conceptualization, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Proofs of the propositions

Proof (*Proposition 1*). Building on Lagrangian duality theory (Wolsey, 1998), CHP as defined in Definition 7 is equivalent to solving the following Lagrangian relaxation (Hua and Baldick, 2017).

$$L(\pi) = \min_{\substack{(c,q,x)_g \in \mathcal{X}_g \\ \forall g \in \mathcal{G},\ f \in \mathcal{F}}} \sum_{g\in\mathcal{G}} c_g - \sum_{\substack{i\in\mathcal{N} \\ t\in\mathcal{T}}} \pi_{i,t} \Big(\sum_{g\in\mathcal{G}_i} q_{g,t} - D_t^i - \sum_{\substack{l\in \\ from(i)}} f_{l,t} + \sum_{\substack{l\in \\ to(i)}} f_{l,t} \Big) \tag{A.1a}$$

$$\pi^{CH} = \arg\max_{\pi} L(\pi) \tag{A.1b}$$

Hence π^{CH} minimizes the following duality gap:

$$\sum_{g\in\mathcal{G}} c_g^* - \max_{\pi} L(\pi) = \sum_{g\in\mathcal{G}} c_g^* - \max_{\pi} \Bigg[\sum_{i\in\mathcal{N},t\in\mathcal{T}} \pi_{i,t} D_t^i - \sum_{g\in\mathcal{G}} \max_{(c,q,x)_g\in\mathcal{X}_g} \Big\{ \sum_{t\in\mathcal{T}} q_{g,t}\pi_{i(g),t} - c_g \Big\} - \max_{f\in\mathcal{F}} \Big\{ \sum_{i\in\mathcal{N},t\in\mathcal{T}} -\pi_{i,t} \Big(\sum_{\substack{l\in \\ from(i)}} f_{l,t} - \sum_{l\in to(i)} f_{l,t} \Big) \Big\} \Bigg]$$

Replacing D_t^i by $\sum_{g\in\mathcal{G}_i} q^*_{g,t} - \sum_{l\in from(i)} f^*_{l,t} + \sum_{l\in to(i)} f^*_{l,t}$ (using (1b)) and rearranging terms, the previous expression is equivalent to $\min_\pi \left\{\sum_{g\in\mathcal{G}} LOC^{gen}(\pi) + LOC^{net}(\pi)\right\}$.

Proof (*Proposition* 2)**.** Using a similar result from Lagrangian duality theory as in the CHP approach, computing the prices π^{MMWP} from problem (10) is equivalent to solving the Lagrangian relaxation of problem (1) in which the sets of constraints are changed from $\mathcal{X}_g$ to $\widehat{\mathcal{X}_g} = \{(0,0,0),\ (c^*,q^*,x^*)_g\}$ and from $\mathcal{F}$ to $\widehat{\mathcal{F}} = \{0,\ f^*\}$. Indeed, the previously defined sets can be modelled with binary variables k. Since solving the Lagrangian relaxation amounts to finding the convex hull of the non-relaxed constraints, and since $\mathrm{conv}(\{0,1\}) = [0,1]$, this leads to problem (10). The Lagrangian relaxation is expressed as follows:

$$\min_\pi \left\{ \max_{f\in\widehat{\mathcal{F}}} \left\{ \sum_{i\in\mathcal{N}}\sum_{t\in\mathcal{T}} -\pi_{i,t}\left(\sum_{l\in from(i)} f_{l,t} - \sum_{l\in to(i)} f_{l,t}\right)\right\} + \sum_{g\in\mathcal{G}} \max_{(c,q,x)_g\in\widehat{\mathcal{X}_g}} \left\{\sum_{t\in\mathcal{T}} \pi_{i(g),t} q_{g,t} - c_g\right\} - \sum_{i\in\mathcal{N}}\sum_{t\in\mathcal{T}} \pi_{i,t} D_t^i \right\}$$

Let us replace $D_t^i = \sum_{g\in\mathcal{G}_i} q^*_{g,t} - (\sum_{l\in from(i)} f^*_{l,t} - \sum_{l\in to(i)} f^*_{l,t})$ into the previous expression and let us add the constant $\sum_{g\in\mathcal{G}} c^*_g$. The Lagrangian relaxation then corresponds to:

$$\min_\pi \left\{ \sum_{g\in\mathcal{G}} \left(\max_{(c,q,x)_g\in\widehat{\mathcal{X}_g}} \mathcal{P}_g(c,q,x,\pi) - \mathcal{P}_g(c^*,q^*,x^*,\pi)\right) + \max_{f\in\widehat{\mathcal{F}}} \mathcal{P}_N(f,\pi) - \mathcal{P}_N(f^*,\pi) \right\}$$

which, from the definition of the modified sets, corresponds to the total revenue shortfall (Definition 5).

Proof (*Proposition* 3)**.** Let us consider the Lagrangian relaxation $L^{IP}(\pi)$ of the problem of Definition 6 in which the market clearing constraint is relaxed. Since the problem is convex, the duality gap is zero and $\pi^{IP} = \arg\max_\pi L^{IP}(\pi)$. Furthermore, the optimum dispatch of both the primal problem (1) (z^*) and the IP problem of Definition 6 (z^*_{IP}) is the same: $\sum_{g\in\mathcal{G}} c^*_g = z^* = z^*_{IP}$. We then write:

$$\begin{aligned} 0 &= \sum_{g\in\mathcal{G}} c^*_g - \max_\pi L^{IP}(\pi) = \sum_{g\in\mathcal{G}} c^*_g - L^{IP}(\pi^{IP}) \\ &= \underbrace{\sum_{g\in\mathcal{G}^C} \max_{(c,q,x^*)_g\in\mathcal{X}_g} \mathcal{P}_g(c,q,x,\pi^{IP}) - \mathcal{P}_g(c^*,q^*,x^*,\pi^{IP})}_{=LOC_g^{gen}\geq 0} \\ &+ \underbrace{\sum_{g\in\mathcal{G}^{NC}} \max_{(c,q,x^*)_g\in\mathcal{X}_g} \mathcal{P}_g(c,q,x,\pi^{IP}) - \mathcal{P}_g(c^*,q^*,x^*,\pi^{IP})}_{\geq 0,\ \text{but}\ \neq LOC_g^{gen}} \\ &+ \underbrace{\max_{f\in\mathcal{F}} \mathcal{P}_N(f,\pi^{IP}) - \mathcal{P}_N(f^*,\pi^{IP})}_{=LOC^{net}\geq 0} \end{aligned}$$

From which we conclude that $LOC^{net} = 0$ and $LOC_g^{gen} = 0\ \forall g\in\mathcal{G}^C$.

Proof (*Proposition* 4)**.** The result follows from Propositions 3 and 7.

Proof (*Proposition* 7)**.** Let us consider the case where $RS_g > 0$ (the unit g faces a revenue shortfall – the case for which $RS_g = 0$ is trivial since $LOC_g \geq 0$):

$$\begin{aligned} LOC_g^{gen} &= \overbrace{\max_{(c,q,x)_g\in\mathcal{X}_g} \left\{\sum_{t\in\mathcal{T}} q_{g,t}\pi_{i(g),t} - c_g\right\}}^{\geq 0 \text{ by assumption of possibility of inaction}} - (\sum_{t\in\mathcal{T}} q^*_{g,t}\pi_{i(g),t} - c^*_g) \\ &\geq -(\sum_{t\in\mathcal{T}} q^*_{g,t}\pi_{i(g),t} - c^*_g) = RS_g^{gen} \end{aligned}$$

The same reasoning applies to the network.

Table A.1
Sensitivity of the LOC under MMWP pricing with respect to the primal optimality gap. The simulations are performed on CWE dataset (Spring WD 24). All figures are in €.

Opt. Gap	MMWP LOC	MMWP* LOC	MMWP** LOC
0.1%	127,174,509	46,374,970	25,487,688
0.09%	128,078,572	46,380,214	25,477,625
0.08%	128,503,837	46,534,306	25,374,010
0.07%	128,503,837	46,534,306	25,374,010
0.06%	129,671,679	46,505,121	25,366,511
0.05%	129,665,324	46,286,384	25,414,900
0.04%	129,855,305	46,366,246	25,411,805
0.03%	127,937,157	46,371,886	25,411,662
0.02%	127,677,227	46,360,290	25,411,605
0.01%	127,677,227	46,360,290	25,411,605

Proof (*Proposition* 9)**.** This follows the interpretation of the LOC as the duality gap (cf. Proposition 1):

$$LOC_1(\pi) - LOC_2(\pi) = \sum_{g\in\mathcal{G}} c_g^1 - L(\pi) - \sum_{g\in\mathcal{G}} c_g^2 + L(\pi) = z_1 - z_2$$

where $L(\pi)$ is the Lagrangian function defined in (A.1a). The equality follows from the fact that CHP and ELMP prices are not affected by a change of primal solution, so the $L(\pi)$ cancel out.

Proof (*Proposition* 10)**.** The proof for IP pricing derives from the mere observation of Table 5. The proof for the three MMWP pricing schemes is straightforward from the observation of Table A.1, which reports the results of MMWP for the same experience as in Table 5. In Table A.1, we observe that the LOC under MMWP pricing evolves non-monotonically with respect to the primal optimality gap.

Proof (*Proposition* 11)**.** Ignoring the network, the bound takes the following form:

$$\sum_{g\in\mathcal{G}} LOC_g(\pi^{CH}) \leq \rho|\mathcal{T}|$$

with $\rho = \max_{g\in\mathcal{G}} \rho_g$ and ρ_g defined as follows:

$$\rho_g = \max_{(\hat{c},\hat{q},\hat{x})_g\in\mathrm{conv}(\mathcal{X}_g)} \left\{\tilde{c}_g(\hat{q},\hat{x}) - (\hat{c}_g)\right\}$$

$$\tilde{c}_g(\hat{q},\hat{x}) = \min_{\substack{(c,q,x)_g\in\mathcal{X}_g \\ q_{g,t}\geq \hat{q}_{g,t}}} c_g$$

The proof, deriving from an application of the Shapley–Folkman theorem, can be found in Chao (2019) or in Stevens et al. (2024).

Proof (*Proposition* 12)**.** The proof for IP pricing follows from Example 4. A similar stylized example can prove the Proposition for MMWP. Let us consider an hourly market with one fully indivisible block order A of 50 MW at 100€/MWh and $N = 3$ block orders B_i of 100 MW at 75€/MWh with a minimum acceptance of 90 MW. The demand is 240 MW at 1000€/MWh. The welfare maximizing allocation (with or without free disposal) is to clear A as well as two blocks B (one produces 100 MW, the other 90 MW). In order to ensure zero revenue shortfall, $\pi^{MMWP} = 100$€/MWh. At this price, the blocks B which are not cleared have a foregone opportunity. Clearly, if $N \rightarrow \infty$, $LOC(\pi^{MMWP}) \rightarrow \infty$.

Appendix B. Model of Example 3

The model is the following:

$$\begin{aligned} \min_{q,x,v,w} \quad & \sum_{g\in\mathcal{G},t\in\mathcal{T}} MC_g q_{g,t} + NLC_g x_{g,t} \\ & \sum_{g\in\mathcal{G}} q_{g,t} = D_t && \forall t\in\mathcal{T} \\ & 0 \leq q_{g,t} \leq Q_g^{max}(x_{g,t} - v_{g,t}) && \forall g\in\mathcal{G}, t\in\mathcal{T} \end{aligned}$$

$$q_{g,t+1} \leq q_{g,t} + Ramp_g \quad \forall g \in \mathcal{G}, t < |\mathcal{T}|$$
$$q_{g,t+1} \geq q_{g,t} - Ramp_g \quad \forall g \in \mathcal{G}, t < |\mathcal{T}|$$
$$v_{g,t} - w_{g,t} = x_{g,t} - x_{g,t-1} \quad \forall g \in \mathcal{G}, t > 1$$
$$v_{g,1} - w_{g,1} = x_{g,1} - x_g^0 \quad \forall g \in \mathcal{G}$$
$$x_{g,t}, v_{g,t}, w_{g,t} \in \{0, 1\} \quad \forall g \in \mathcal{G}, t \in \mathcal{T}$$

where x, v and w stand respectively for the commitment, the start-up and shut-down decision variables. We notice that IP prices (Table 4) can be interpreted as follows. G1 is marginal in $t \in \{1, 2, 3\}$, so $\pi^{IP} =$ 80€/MWh. Increasing the demand of ϵ in $t = 4$ requires to increase the production of G4 in $t = 4$ as well as to substitute production of G1 by G4 in $t = 3$, because of the ramp. So $\pi_4^{IP} = 130+(130-80) = 180$€/MWh.

ELMP prices (Table 4) are less straightforward to interpret, as it is necessary to resort to the KKT conditions of the ELMP problem. To provide some intuition, we look at the price of the third period. The average cost of G2 is $MC + NLC/Q^{max} = 81.25$€/MWh. Increasing the demand of ϵ in $t = 3$ requires to increase the production and commitment of G2 in $t = 3$ as well as to substitute production from G1 by G2 in $t = 2$, so $\pi_3^{ELMP} = 81.25 + (81.25 - 80) = 82.5$€/MWh.

Appendix C. Detailed numerical results

Tables C.1 and C.2 provide the detailed results (per load scenario) of Tables 1 and 2.

Table C.1
Incentives of market agents on the FERC dataset depending on the pricing scheme (detailed figures per scenario).

		IP	CHP	ELMP	MMWP	MMWP*	MMWP**
2015-02-01-hw	Av. Price	23.1	23.7	23.1	247	20.3	23.2
	Num. Suppl.	7.4%	3.3%	7.2%	99.9%	17.2%	9.1%
	Av. LOC/Suppl.	476	22	53	623,873	8853	52
	LOC	32,858	673	3522	582,073,937	1,425,275	4421
	RS (in LOC)	0	41	88	0	0	0
	FO	32,858	633	3434	582,073,937	1,425,275	4421
2015-04-01-hw	Av. Price	19.3	18.9	19.2	27.1	17.3	19.3
	Num. Suppl.	3.4%	1.2%	8.4%	51.7%	16.7%	8.7%
	Av. LOC/Suppl.	265	18	74	41,666	1546	75
	LOC	8734	229	6084	21,082,899	252,047	6360
	RS (in LOC)	2426	0	831	0	0	0
	FO	6307	229	5253	21,082,899	252,047	6360
2015-05-01-hw	Av. Price	24.8	24.7	24.8	23.1	23.1	24.8
	Num. Suppl.	1.3%	1.3%	4.1%	60.4%	17.9%	4.1%
	Av. LOC/Suppl.	68	4	12	40,060	1488	12
	LOC	888	60	471	23,675,307	260,410	471
	RS (in LOC)	499	0	0	0	0	0
	FO	389	60	471	23,675,307	260,410	471
2015-06-01-hw	Av. Price	27.2	27.4	27.1	23.1	23.1	27.2
	Num. Suppl.	2.4%	1.8%	5.3%	64.5%	19.9%	5.2%
	Av. LOC/Suppl.	344	15	20	43,962	6790	18
	LOC	7906	271	1026	27,739,814	1,323,996	923
	RS (in LOC)	0	5	32	0	0	0
	FO	7906	265	995	27,739,814	1,323,996	923
2015-07-01-lw	Av. Price	32.8	32.9	32.8	49.6	32.8	32.9
	Num. Suppl.	1.2%	1.1%	4.4%	83.7%	31.6%	5.1%
	Av. LOC/Suppl.	231	21	29	38,983	4543	35
	LOC	2772	241	1273	31,926,833	1,403,848	1733
	RS (in LOC)	21	0	5	0	0	0
	FO	2751	241	1268	31,926,833	1,403,848	1733
2015-07-01-hw	Av. Price	28.6	27.8	28.7	26.4	27.1	28.7
	Num. Suppl.	2.4%	3.3%	8.8%	60.4%	23.8%	8.8%
	Av. LOC/Suppl.	608	13	42	36,043	2611	42
	LOC	13,978	427	3583	21,301,239	608,479	3583
	RS (in LOC)	3922	0	0	0	0	0
	FO	10,056	427	3583	21,301,239	608,479	3583
2015-08-01-hw	Av. Price	28	27.2	28.1	23.1	26.4	28.1
	Num. Suppl.	3.2%	1.4%	11.0%	81.3%	31.8%	11.1%
	Av. LOC/Suppl.	749	24	31	75,557	2156	30
	LOC	23,217	336	3341	60,067,559	670,588	3327
	RS (in LOC)	229	12	38	0	0	0
	FO	22,988	324	3303	60,067,559	670,588	3327
2015-09-01-lw	Av. Price	43.3	43	43.4	34.3	41	43.6
	Num. Suppl.	4.1%	2.7%	9.6%	83.1%	44.7%	14.9%
	Av. LOC/Suppl.	168	18	26	49,988	7778	99
	LOC	6719	468	2461	40,640,061	3,399,179	14,509
	RS (in LOC)	233	0	82	0	0	0
	FO	6486	468	2379	40,640,061	3,399,179	14,509

(continued on next page)

Table C.1 (*continued*).

		IP	CHP	ELMP	MMWP	MMWP*	MMWP**
2015-09-01-hw	Av. Price	35.2	36.9	35.3	78.6	33.2	35.8
	Num. Suppl.	8.8%	1.3%	12.7%	99.2%	33.7%	21.9%
	Av. LOC/Suppl.	3579	29	35	241,607	5830	511
	LOC	307,764	383	4318	234,358,740	1,923,877	109,366
	RS (in LOC)	0	71	435	0	0	0
	FO	307,764	313	3883	234,358,740	1,923,877	109,366
2015-10-01-lw	Av. Price	30	30.3	30	61	27.4	30.2
	Num. Suppl.	2.4%	1.4%	4.6%	90.8%	22.1%	6.4%
	Av. LOC/Suppl.	366	26	46	105,984	7301	57
	LOC	8053	341	1973	89,874,328	1,503,907	3403
	RS (in LOC)	0	0	91	0	0	0
	FO	8053	341	1882	89,874,328	1,503,907	3403
2015-12-01-hw	Av. Price	23.8	23.8	23.8	26.1	23.2	23.9
	Num. Suppl.	1.0%	1.0%	6.9%	96.1%	12.2%	9.0%
	Av. LOC/Suppl.	50	14	43	332,826	1447	99
	LOC	447	128	2763	298,877,534	164,941	8286
	RS (in LOC)	26	85	660	0	0	0
	FO	421	43	2104	298,877,534	164,941	8286

Table C.2
Incentives of market agents on the CWE dataset depending on the pricing scheme (detailed figures per scenario).

		IP	CHP	ELMP	MMWP	MMWP*	MMWP**
SpringWE-24	Av. Price	35.6	36.4	43.8	25.3	24.2	50.3
	Num. Suppl.	28.4%	25.7%	37.8%	78.4%	60.8%	60.8%
	Av. LOC/Suppl.	4047	278	2631	137,534	36,071	38,786
	LOC	84,978	7189	75,852	95,284,255	45,775,853	27,790,203
	RS (in LOC)	1207	1145	15,035	0	0	0
	FO	83,771	6043	60,816	95,284,255	45,775,853	27,790,203
	RS (not in LOC)	1,241,106	1,262,137	937,572	0	0	0
AutumnWE-24	Av. Price	38	38.6	45.2	30.8	24.4	51.1
	Num. Suppl.	29.7%	23.0%	36.5%	78.4%	55.4%	56.8%
	Av. LOC/Suppl.	4245	195	2164	228,052	36,306	36,682
	LOC	93,398	4814	61,033	134,946,417	45,300,887	25,886,284
	RS (in LOC)	9812	1364	10,898	0	0	0
	FO	83,586	3450	50,135	134,946,417	45,300,887	25,886,284
	RS (not in LOC)	1,165,408	1,177,396	880,050	0	0	0
SummerWE-24	Av. Price	34.5	34.5	42.4	25.1	23.9	49.6
	Num. Suppl.	29.7%	23.0%	37.8%	78.4%	63.5%	63.5%
	Av. LOC/Suppl.	5549	621	2861	115,019	36,695	40,142
	LOC	122,078	12,606	82,506	88,196,688	46,897,168	29,419,777
	RS (in LOC)	6111	4985	23,074	0	0	0
	FO	115,967	7621	59,431	88,196,688	46,897,168	29,419,777
	RS (not in LOC)	1231,897	1312,780	997,636	0	0	0
SummerWE-96	Av. Price	44.3	44.4	46.8	24.9	21.1	51.4
	Num. Suppl.	41.9%	36.5%	52.7%	81.1%	70.3%	71.6%
	Av. LOC/Suppl.	2286	212	627	99,952	22,600	17,941
	LOC	70,879	6406	25,444	76,307,676	35,056,506	15,473,856
	RS (in LOC)	3814	2858	7065	0	0	0
	FO	67,065	3547	18,379	76,307,676	35,056,506	15,473,856
	RS (not in LOC)	634,481	654,700	577,425	0	0	0
SummerWD-24	Av. Price	35.3	34.2	42.9	23.2	24.2	49.9
	Num. Suppl.	25.7%	25.7%	39.2%	79.7%	60.8%	63.5%
	Av. LOC/Suppl.	3612	357	2450	106,374	37,784	39,207
	LOC	68,620	7707	73,911	86,363,710	46,951,981	29,096,219
	RS (in LOC)	7190	3832	21,504	0	0	0
	FO	61,430	3875	52,408	86,363,710	46,951,981	29,096,219
	RS (not in LOC)	1,319,226	1,288,743	984,371	0	0	0
AutumnWD-24	Av. Price	47.9	43.4	49.6	33.6	26.7	55
	Num. Suppl.	32.4%	40.5%	41.9%	81.1%	58.1%	56.8%
	Av. LOC/Suppl.	4839	464	817	198,139	32,992	33,969
	LOC	116,130	18,723	42,626	134,179,378	45,844,078	23,812,415
	RS (in LOC)	67,061	649	1025	0	0	0
	FO	49,070	18,074	41,601	134,179,378	45,844,078	23,812,415
	RS (not in LOC)	1,048,066	888,203	832,867	0	0	0

(*continued on next page*)

Table C.2 (*continued*).

		IP	CHP	ELMP	MMWP	MMWP*	MMWP**
AutumnWD-96	Av. Price	53	52.6	54	30.1	25.8	58.1
	Num. Suppl.	35.1%	55.4%	58.1%	94.6%	70.3%	71.6%
	Av. LOC/Suppl.	1999	152	388	126,419	20,139	14,407
	LOC	51,962	6,625	21,179	90,412,716	39,833,338	13,737,122
	RS (in LOC)	7886	1194	1146	0	0	0
	FO	44,076	5430	20,033	90,412,716	39,833,338	13,737,122
	RS (not in LOC)	538,906	532,300	507,761	0	0	0
SpringWE-96	Av. Price	44.8	45.3	47.3	25.7	21.1	51.7
	Num. Suppl.	37.8%	44.6%	50.0%	86.5%	67.6%	68.9%
	Av. LOC/Suppl.	2841	131	626	198,060	22,316	17,651
	LOC	79,560	5196	24,688	94,436,434	35,076,443	15,125,326
	RS (in LOC)	7511	633	4893	0	0	0
	FO	72,049	4563	19,795	94,436,434	35,076,443	15,125,326
	RS (not in LOC)	627,309	640,835	561,709	0	0	0
SummerWD-96	Av. Price	46.7	46.1	49.1	25.7	23.2	53.5
	Num. Suppl.	36.5%	43.2%	55.4%	83.8%	71.6%	73.0%
	Av. LOC/Suppl.	2238	165	556	90,557	22,047	15,174
	LOC	60,430	5989	24,358	80,534,617	38,674,968	14,967,433
	RS (in LOC)	3356	1339	4493	0	0	0
	FO	57,074	4650	19,865	80,534,617	38,674,968	14,967,433
	RS (not in LOC)	645,930	629,168	556,458	0	0	0
SpringWD-24	Av. Price	44.5	42.9	47.4	30.4	25	53.3
	Num. Suppl.	28.4%	32.4%	37.8%	82.4%	52.7%	54.1%
	Av. LOC/Suppl.	5683	400	1185	182,800	37,165	36,427
	LOC	119,351	9940	40,675	127,677,227	46,360,290	25,411,605
	RS (in LOC)	6463	3829	8973	0	0	0
	FO	112,889	6111	31,702	127,677,227	46,360,290	25,411,605
	RS (not in LOC)	1,110,721	969,030	859,078	0	0	0
AutumnWE-96	Av. Price	46	45.2	48.3	25.4	21.6	52.6
	Num. Suppl.	39.2%	39.2%	45.9%	83.8%	63.5%	64.9%
	Av. LOC/Suppl.	2581	167	643	88,274	25,366	18,805
	LOC	74,843	5591	23,722	82,089,816	36,154,282	14,545,232
	RS (in LOC)	5296	1116	2911	0	0	0
	FO	69,546	4475	20,811	82,089,816	36,154,282	14,545,232
	RS (not in LOC)	625,250	614,743	543,490	0	0	0
SpringWD-96	Av. Price	50	49.6	51	32.2	24.2	55.1
	Num. Suppl.	33.8%	41.9%	50.0%	94.6%	66.2%	66.2%
	Av. LOC/Suppl.	2412	189	467	130,828	22,438	15,598
	LOC	60,292	6328	19,382	93,752,601	39,772,253	14,203,481
	RS (in LOC)	895	898	1079	0	0	0
	FO	59,397	5430	18,303	93,752,601	39,772,253	14,203,481
	RS (not in LOC)	583,540	554,445	524,396	0	0	0

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