Supplement to "*Market Design Considerations for Scarcity Pricing: A Stochastic Equilibrium Framework*"

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The present document is a supplement to the paper "Market Design Considerations for Scarcity Pricing: A Stochastic Equilibrium Framework" by Papavasiliou, Smeers, and de Maere d'Aertrycke. The supplement is organized as follows. In section 1 we present the models that are employed in the paper. In section 2 we discuss coherent risk measures. Section 3 describes the details of the case study that has been used for analyzing the Belgian electricity market. We analyze the implications of scarcity pricing on consumers in section 4. Section 5 presents additional simulation results beyond the main results that are presented in the paper.

1. Complementarity Formulation of Market Models

In this section we present the market equilibrium models for the two ends of the spectrum of market designs that we analyze in the paper, namely the US and EU design. We commence by presenting the notation that is used in the models, and proceed by presenting the models both from the point of view of an equilibrium (profit maximizing agents augmented by market clearing conditions) as well as in complementarity form.

1.1. Notation

Sets
G: set of generators
L: set of loads
RL^F: set of bids for operating fast reserve demand curve
RL^S: set of bids for operating slow reserve demand curve
Ω: set of scenarios in the second stage of the two-stage models
Ω_2: set of second-stage scenarios in the three-stage models

Ω_3: set of third-stage scenarios in the three-stage models
$C_3(\omega) \subset \Omega_3$: children of scenario $\omega \in \Omega_2$ in the three-stage models

Parameters

$P^{RT,+}_{g,\omega}$: maximum real-time production capacity of generator g in scenario ω
$P^{RT,-}_{g,\omega}$: minimum real-time production capacity of generator g in scenario ω
ϵ^+_g: marginal penalty to generators for excess real-time production relative to day-ahead production
ϵ^-_g: marginal penalty to generators for shortage in real-time production relative to day-ahead production
$P^{DA,+}_g$: day-ahead production capacity of generator g
C_g: marginal cost of generator g
$D^{RT,+}_{l,\omega}$: real-time demand of load l in scenario ω
$D^{DA,+}_l$: day-ahead demand of load l
V_l: valuation of load l
P_ω: probability of scenario ω in two-stage model
$P[(t+1,\omega')|(t,\omega)]$: transition probability from node (t,ω) to node $(t+1,\omega')$ in the three-stage models
K_g: activation cost of generator g
R^F_g: fast reserve capacity limit of generator or load g
R^S_g: slow reserve capacity limit of generator or load g
α_g: risk-aversion parameter of generator or load g
$V^{R,F}_l$: valuation of fast reserve capacity in segment l of ORDC
$V^{R,S}_l$: valuation of slow reserve capacity in segment l of ORDC
$D^{R,F}_l$: quantity of fast reserve demand for segment l of ORDC
$D^{R,S}_l$: quantity of slow reserve demand for segment l of ORDC

Primal variables

$p^{RT}_{g,\omega}$: real-time production of generator g in scenario ω
$s^{RT,+}_{g,\omega}$: excess of real-time production of generator g in scenario ω relative to day-ahead production
$s^{RT,-}_{g,\omega}$: excess of day-ahead production of generator g in scenario ω relative to real-time production
$d^{RT}_{l,\omega}$: real-time demand of consumer l in scenario ω
$r^{F,RT}_{g,\omega}$: fast reserve capacity supply of generator or load g in the real-time market under scenario ω
$r^{S,RT}_{g,\omega}$: slow reserve capacity supply of generator or load g in the real-time market under scenario ω

$d_{l,\omega}^{R,F,RT}$: fast reserve capacity demand of system operator for segment l of ORDC in the real-time market under scenario ω
$d_{l,\omega}^{R,S,RT}$: slow reserve capacity demand of system operator for segment l of ORDC in the real-time market under scenario ω
p_g^{DA}: production of generator g in the day-ahead market
$r_g^{F,DA}$: fast reserve capacity supply of generator or load g in the day-ahead market
$r_g^{S,DA}$: slow reserve capacity supply of generator or load g in the day-ahead market
d_l^{DA}: demand of load l in the day-ahead market
$CVaR_g$: conditional value at risk of agent g for the two-stage model
$CVaR2_{g,\omega}$: conditional value at risk of agent g in scenario ω of the second stage for the three-stage model
$CVaR1_g$: conditional value at risk of agent g in the first stage for the three-stage model
VaR_g: value at risk of agent g for the two-stage model
$VaR2_{g,\omega}$: value at risk of agent g in scenario ω of the second stage for the three-stage model
$VaR1_g$: value at risk of agent g in the first stage for the three-stage model
$u_{g,\omega}$: auxiliary variable recording risk-adjusted payoff of agent g under scenario ω for the two-stage model
$u2_{g,\omega}$: auxiliary variable recording risk-adjusted payoff of agent g under scenario ω of the second stage for the three-stage model
$u3_{g,\omega,\omega'}$: auxiliary variable recording risk-adjusted payoff of agent g under scenario ω' of the third stage with ancestor scenario ω for the three-stage model

Dual variables

λ^{RT}: real-time energy price under scenario ω
$\lambda_\omega^{R,F,RT}$: real-time fast reserve capacity price under scenario ω
$\lambda_\omega^{R,S,RT}$: real-time slow reserve capacity price under scenario ω
$\alpha^{G,RT,+}, \alpha^{G,RT,-}, \beta^{G,RT}, \gamma^{G,RT,+}, \gamma^{G,RT,-}$: dual multipliers of real-time constraints of generator g under scenario ω
$\alpha^{L,RT}, \beta^{L,F,RT}, \beta^{L,S,RT}, \gamma^{L,RT}, \delta^{L,F,RT}, \delta^{L,S,RT}$: dual multipliers of real-time constraints of load l under scenario ω
$\alpha_{l,\omega}^{R,F,RT}, \alpha_{l,\omega}^{R,S,RT}$: dual multipliers of real-time constraints of system operator for segment l of the ORDC under scenario ω
$q_{g,\omega}$: risk-adjusted probability of scenario ω for agent g for the two-stage model

$q2_{g,\omega}$: risk-adjusted probability of second-stage scenario ω for agent g for the three-stage model
$q3_{g,\omega,\omega'}$: risk-adjusted probability of third-stage scenario ω' with ancestor scenario ω for agent g for the three-stage model
λ^{DA}: day-ahead energy price
$\lambda^{R,F,DA}$: day-ahead fast reserve capacity price
$\lambda^{R,S,DA}$: day-ahead slow reserve capacity price
$\delta_g, \alpha_g^{G,DA}, \beta_g^{G,F,DA}, \beta_g^{G,S,DA}$: dual multipliers of day-ahead constraints of generator g
$\alpha_l^{R,F,DA}, \alpha_l^{R,S,DA}$: dual multipliers of day-ahead constraints of system operator for segment l of the ORDC

1.2. The US Model

This section presents the US model that is analyzed presented in the paper. The salient attributes of this design are the following: (i) there exists a real-time market for reserve capacity, (ii) energy and reserve capacity are cleared simultaneously in the day ahead, and (iii) virtual trading in energy is allowed.

1.2.1. Real Time

The generators are maximizing their profit in the real-time market. Thus, for every generator $g \in G$ and scenario $\omega \in \Omega$, we have the following real-time profit maximization:

$$
\begin{aligned}
\Pi_{g,\omega}^{RT}(y_g) = \quad & \max_{p_{g,\omega}^{RT}, r_{g,\omega}^{F,RT}, r_{g,\omega}^{S,RT}} (\lambda_\omega^{RT} - C_g) \cdot p_{g,\omega}^{RT} + \\
& \tilde{\lambda}_\omega^{R,F,RT} \cdot r_{g,\omega}^{F,RT} + \lambda_\omega^{R,S,RT} \cdot r_{g,\omega}^{S,RT} \\
(\alpha_{g,\omega}^{G,RT,+}): \quad & p_{g,\omega}^{RT} + r_{g,\omega}^{F,RT} + r_{g,\omega}^{S,RT} \leq P_{g,\omega}^{RT,+} \cdot y_g \\
(\alpha_{g,\omega}^{G,RT,-}): \quad & -p_{g,\omega}^{RT} \leq -P_{g,\omega}^{RT,-} \cdot y_g \\
(\beta_{g,\omega}^{G,F,RT}): \quad & r_{g,\omega}^{F,RT} \leq R_g^F \\
(\beta_{g,\omega}^{G,S,RT}): \quad & r_{g,\omega}^{S,RT} \leq R_g^S \\
& p_{g,\omega}^{RT} \geq 0, r_{g,\omega}^{F,RT} \geq 0, r_{g,\omega}^{S,RT} \geq 0
\end{aligned}
$$

Note that the real-time profit $\Pi_{g,\omega}^{RT}$ is a concave function of y_g, and its supergradient can be derived with elementary convex analysis arguments. Furthermore, note that we employ the notation $\tilde{\lambda}_\omega^{R,F,RT} = \lambda_\omega^{R,F,RT} + \lambda_\omega^{R,S,RT}$.

The KKT conditions of the generators are:

$$
\begin{aligned}
&0 \le p^{RT}_{g,\omega} \perp C_g - \lambda^{RT}_{\omega} + \alpha^{G,RT,+}_{g,\omega} - \alpha^{G,RT,-}_{g,\omega} \ge 0, g \in G, \omega \in \Omega \\
&0 \le r^{F,RT}_{g,\omega} \perp \alpha^{G,RT,+}_{g,\omega} + \beta^{G,F,RT}_{g,\omega} - \lambda^{R,F,RT}_{\omega} - \lambda^{R,S,RT}_{\omega} \ge 0, g \in G, \omega \in \Omega \\
&0 \le r^{S,RT}_{g,\omega} \perp \alpha^{G,RT,+}_{g,\omega} + \beta^{G,S,RT}_{g,\omega} - \lambda^{R,S,RT}_{\omega} \ge 0, g \in G, \omega \in \Omega \\
&0 \le \alpha^{G,RT,+}_{g,\omega} \perp P^{RT,+}_{g,\omega} \cdot y_g - p^{RT}_{g,\omega} - r^{F,RT}_{g,\omega} - r^{S,RT}_{g,\omega} \ge 0, g \in G, \omega \in \Omega \\
&0 \le \alpha^{G,RT,-}_{g,\omega} \perp p^{RT}_{g,\omega} - P^{RT,-}_{g,\omega} \cdot y_g \ge 0, g \in G, \omega \in \Omega \\
&0 \le \beta^{G,F,RT}_{g,\omega} \perp R^F_g - r^{F,RT}_{g,\omega} \ge 0, g \in G, \omega \in \Omega \\
&0 \le \beta^{G,S,RT}_{g,\omega} \perp R^S_g - r^{S,RT}_{g,\omega} \ge 0, g \in G, \omega \in \Omega
\end{aligned}
$$

Similarly, for every load $l \in L$ and scenario $\omega \in \Omega$, we have the following real-time profit maximization:

$$
\begin{aligned}
\Pi^{RT}_{l,\omega} = \quad & \max_{d^{RT}_{l,\omega}, r^{F,RT}_{l,\omega}, r^{S,RT}_{l,\omega} (V_l - \lambda^{RT}_{\omega}) \cdot d^{RT}_{l,\omega} +} \\
& \tilde{\lambda}^{R,F,RT}_{\omega} \cdot r^{F,RT}_{g,\omega} + \lambda^{R,S,RT}_{\omega} \cdot r^{S,RT}_{g,\omega} \\
(\alpha^{L,RT}_{l,\omega}): \quad & d^{RT}_{l,\omega} \le D^{RT,+}_{l,\omega} \\
(\beta^{L,F,RT}_{l,\omega}): \quad & r^{F,RT}_{l,\omega} \le R^F_l \\
(\beta^{L,S,RT}_{l,\omega}): \quad & r^{S,RT}_{l,\omega} \le R^S_l \\
(\gamma^{L,RT}_{l,\omega}): \quad & r^{S,RT}_{l,\omega} + r^{F,RT}_{l,\omega} - d^{RT}_{l,\omega} \le 0 \\
& d^{RT}_{l,\omega} \ge 0, r^{F,RT}_{l,\omega} \ge 0, r^{S,RT}_{l,\omega} \ge 0
\end{aligned}
$$

The KKT conditions of the loads are:

$$
\begin{aligned}
&0 \le d^{RT}_{l,\omega} \perp \lambda^{RT}_{\omega} - V_l + \alpha^{L,RT}_{l,\omega} - \gamma^{L,RT}_{l,\omega} \ge 0, l \in L, \omega \in \Omega \\
&0 \le r^{F,RT}_{l,\omega} \perp -\lambda^{R,F,RT}_{\omega} - \lambda^{R,S,RT}_{\omega} + \beta^{L,F,RT}_{l,\omega} + \gamma^{L,RT}_{l,\omega} \ge 0, l \in L, \omega \in \Omega \\
&0 \le r^{S,RT}_{l,\omega} \perp -\lambda^{R,S,RT}_{\omega} + \beta^{L,S,RT}_{l,\omega} + \gamma^{L,RT}_{l,\omega} \ge 0, l \in L, \omega \in \Omega \\
&0 \le \alpha^{L,RT}_{l,\omega} \perp D^{RT,+}_{l,\omega} - d^{RT}_{l,\omega} \ge 0, l \in L, \omega \in \Omega \\
&0 \le \beta^{L,F,RT}_{l,\omega} \perp R^F_l - r^{F,RT}_{l,\omega} \ge 0, l \in L, \omega \in \Omega \\
&0 \le \beta^{L,S,RT}_{l,\omega} \perp R^S_l - r^{S,RT}_{l,\omega} \ge 0, l \in L, \omega \in \Omega \\
&0 \le \gamma^{L,RT}_{l,\omega} \perp d^{RT}_{l,\omega} - r^{F,RT}_{l,\omega} - r^{S,RT}_{l,\omega} \ge 0, l \in L, \omega \in \Omega
\end{aligned}
$$

The system operator is procuring reserve in real time for every scenario $\omega \in \Omega$, according to a real-time operating reserve demand curve for fast and slow reserve:

$$
\begin{aligned}
\Pi_{\omega}^{SO,RT} = \quad & \max_{d_{l,\omega}^{R,F,RT}, d_{l,\omega}^{R,S,RT}} \sum_{l \in RL^F} (V_l^{R,F} - \lambda_{\omega}^{R,F,RT}) \cdot d_{l,\omega}^{R,F,RT} + \\
& \sum_{l \in RL^S} (V_l^{R,S} - \lambda_{\omega}^{R,S,RT}) \cdot d_{l,\omega}^{R,S,RT} \\
(\alpha_{l,\omega}^{R,F,RT}): \quad & d_{l,\omega}^{R,F,RT} \leq D_l^{R,F} \\
(\alpha_{l,\omega}^{R,S,RT}): \quad & d_{l,\omega}^{R,S,RT} \leq D_l^{R,S} \\
& d_{l,\omega}^{R,F,RT} \geq 0, d_{l,\omega}^{R,S,RT} \geq 0
\end{aligned}
$$

The KKT conditions of the system operator are:

$$
\begin{aligned}
& 0 \leq d_{l,\omega}^{R,F,RT} \perp -V_l^{R,F} + \lambda_{\omega}^{R,F,RT} + \alpha_{l,\omega}^{R,F,RT} \geq 0, l \in RL^F, \omega \in \Omega \\
& 0 \leq d_{l,\omega}^{R,S,RT} \perp -V_l^{R,S} + \lambda_{\omega}^{R,S,RT} + \alpha_{l,\omega}^{R,S,RT} \geq 0, l \in RL^S, \omega \in \Omega \\
& 0 \leq \alpha_{l,\omega}^{R,F,RT} \perp D_l^{R,F} - d_{l,\omega}^{R,F,RT} \geq 0, l \in RL^F, \omega \in \Omega \\
& 0 \leq \alpha_{l,\omega}^{R,S,RT} \perp D_l^{R,S} - d_{l,\omega}^{R,S,RT} \geq 0, l \in RL^S, \omega \in \Omega
\end{aligned}
$$

The market clearing conditions are:

$$
\begin{aligned}
& \sum_{g \in G} p_{g,\omega}^{RT} = \sum_{l \in L} d_{l,\omega}^{RT}, \omega \in \Omega \\
& \sum_{g \in G \cup L} r_{g,\omega}^{F,RT} = \sum_{l \in RL^F} d_{l,\omega}^{R,F,RT}, \omega \in \Omega \\
& \sum_{g \in G \cup L} (r_{g,\omega}^{S,RT} + r_{g,\omega}^{F,RT}) = \sum_{l \in RL^S} d_{l,\omega}^{R,S,RT}, \omega \in \Omega
\end{aligned}
$$

1.2.2. Day Ahead

In the day ahead, each generator $g \in G$ solves the following profit maximization, where we use conditional value at risk as the risk measure employed by producers.

$$
\begin{aligned}
& \max_{p_g^{DA}, r_g^{F,DA}, r_g^{S,DA}, y_g, CVaR_g, u_{g,\omega}} && -K_g \cdot y_g + \\
& && \lambda^{DA} \cdot p_g^{DA} + \tilde{\lambda}^{R,F,DA} \cdot r_g^{F,DA} + \lambda^{R,S,DA} \cdot r_g^{S,DA} + \\
& && VaR_g - \frac{1}{\alpha_g} \sum_{\omega \in \Omega} P_\omega \cdot u_{g,\omega} \\
& (\alpha_g^{G,F,DA}): && r_g^{F,DA} \leq R_g^F \\
& (\alpha_g^{G,S,DA}): && r_g^{S,DA} \leq R_g^S \\
& (\delta_g): && y_g \leq 1 \\
& (q_{g,\omega}): && u_{g,\omega} \geq VaR_g - (\Pi_{g,\omega}^{RT}(y_g) - \lambda_\omega^{RT} \cdot p_g^{DA} - \\
& && -\tilde{\lambda}_\omega^{R,F,RT} \cdot r_g^{F,DA} - \lambda_\omega^{R,S,RT} \cdot r_g^{S,DA}) \\
& && r_g^{F,DA} \geq 0, r_g^{S,DA} \geq 0, u_{g,\omega} \geq 0
\end{aligned}
$$

Note that we employ the notation $\tilde{\lambda}^{R,F,DA} = \lambda^{R,F,DA} + \lambda^{R,S,DA}$. In order to derive the KKT conditions for the day-ahead profit maximization problem of the generator, we need the expression for the subgradient of the real-time profit $\Pi_{g,\omega}^{RT}(y_g)$ with respect to y_g that we have derived above, as well as the chain rule for risk measures that we provide in equation (11) of the main paper.

The KKT conditions for the collection of generators are:

$$
\begin{aligned}
& 0 \leq u_{g,\omega} \perp \frac{P_\omega}{\alpha_g} - q_{g,\omega} \geq 0, g \in G, \omega \in \Omega \\
(p_g^{DA}): \quad & \sum_{\omega \in \Omega} q_{g,\omega} \cdot \lambda_\omega^{RT} - \lambda^{DA} = 0, g \in G \\
(r_g^{F,DA}): \quad & 0 \leq r_g^{F,DA} \perp \lambda^{R,F,DA} + \lambda^{R,S,DA} + \beta_g^{G,F,DA} \\
& - \sum_{\omega \in \Omega} q_{g,\omega} \cdot (\lambda_\omega^{R,F,RT} + \lambda_\omega^{R,S,RT}) \geq 0, g \in G \\
(r_g^{S,DA}): \quad & 0 \leq r_g^{S,DA} \perp \lambda^{R,S,DA} + \beta_g^{G,S,DA} \\
& - \sum_{\omega \in \Omega} q_{g,\omega} \cdot \lambda_\omega^{R,S,RT} \geq 0, g \in G \\
(VaR_g): \quad & \sum_{\omega \in \Omega} q_{g,\omega} = 1, g \in G \\
& 0 \leq y_g \perp \delta_g - \sum_{\omega \in \Omega} q_{g,\omega} \cdot P_{g,\omega}^{RT,+} \cdot \alpha_{g,\omega}^{G,RT} \\
& + K_g \geq 0, g \in G \\
& 0 \leq \delta_g \perp 1 - y_g \geq 0, g \in G \\
& 0 \leq \alpha_g^{G,F,DA} \perp R_g^F - r_g^{F,DA} \geq 0 \\
& 0 \leq \alpha_g^{G,S,DA} \perp R_g^S - r_g^{S,DA} \geq 0 \\
& 0 \leq q_{g,\omega} \perp u_{g,\omega} - VaR_g + \Pi_{g,\omega}^{RT} \\
& - \lambda_\omega^{RT} \cdot p_g^{DA} - (\lambda_\omega^{R,F,RT} + \lambda_\omega^{R,S,RT}) \cdot r_g^{F,DA} \\
& - \lambda_\omega^{R,S,RT} \cdot r_g^{S,DA} \geq 0, g \in G, \omega \in \Omega
\end{aligned}
$$

The day-ahead profit maximization of the loads is described as follows.

$$
\begin{aligned}
& \max_{d_l^{DA}, r_l^{F,DA}, r_l^{S,DA}, CVaR_l, u_{l,\omega}} && -\lambda^{DA} \cdot d_l^{DA} + \\
& && \tilde{\lambda}^{R,F,DA} \cdot r_l^{F,DA} + \lambda^{R,S,DA} \cdot r_l^{S,DA} + \\
& && VaR_l - \frac{1}{\alpha_l} \sum_{\omega \in \Omega} P_\omega \cdot u_{l,\omega} \\
& (\alpha_l^{L,F,DA}): && r_l^{F,DA} \leq R_l^F \\
& (\alpha_l^{L,S,DA}): && r_l^{S,DA} \leq R_l^S \\
& (q_{l,\omega}): && u_{l,\omega} \geq VaR_l - (\Pi_{l,\omega}^{RT} + \lambda_\omega^{RT} \cdot d_l^{DA} \\
& && -\tilde{\lambda}_\omega^{R,F,RT} \cdot r_l^{F,DA} - \lambda_\omega^{R,S,RT} \cdot r_l^{S,DA}) \\
& && r_l^{F,DA} \geq 0, r_l^{S,DA} \geq 0, u_{l,\omega} \geq 0
\end{aligned}
$$

The KKT conditions for the collection of loads are:

$$
\begin{aligned}
& && 0 \leq u_{l,\omega} \perp \frac{P_\omega}{\alpha_l} - q_{l,\omega} \geq 0, l \in L, \omega \in \Omega \\
& (d_l^{DA}): && \lambda^{DA} - \sum_{\omega \in \Omega} q_{l,\omega} \cdot \lambda_\omega^{RT} = 0, l \in L \\
& && 0 \leq r_l^{F,DA} \perp \lambda^{R,F,DA} + \lambda^{R,S,DA} + \alpha_l^{L,F,DA} \\
& && - \sum_{\omega \in \Omega} q_{l,\omega} \cdot (\lambda_\omega^{R,F,RT} + \lambda_\omega^{R,S,RT}) \geq 0, l \in L \\
& && 0 \leq r_l^{S,DA} \perp \lambda^{R,S,DA} + \beta_l^{L,S,DA} - \sum_{\omega \in \Omega} q_{l,\omega} \cdot \lambda_\omega^{R,S,RT} \geq 0, l \in L \\
& (VaR_l): && \sum_{\omega \in \Omega} q_{l,\omega} = 1, l \in L \\
& && 0 \leq \alpha_l^{L,F,DA} \perp R_l^F - r_l^{F,DA} \geq 0 \\
& && 0 \leq \alpha_l^{L,S,DA} \perp R_l^S - r_l^{S,DA} \geq 0 \\
& && 0 \leq q_{l,\omega} \perp u_{l,\omega} - VaR_l + \Pi_{l,\omega}^{RT} \\
& && +\lambda_\omega^{RT} \cdot d_l^{DA} - (\lambda_\omega^{R,F,RT} + \lambda_\omega^{R,S,RT}) \cdot r_l^{F,DA} \\
& && -\lambda_\omega^{R,S,RT} \cdot r_g^{S,DA} \geq 0, l \in L, \omega \in \Omega
\end{aligned}
$$

The system operator day-ahead profit maximization reads as follows:

$$
\begin{aligned}
& \max_{d_l^{R,F,DA}, d_l^{R,S,DA}} \sum_{l \in RL^F} (V_l^{R,F} - \lambda^{R,F,DA}) \cdot d_l^{R,F,DA} + \\
& \sum_{l \in RL^S} (V_l^{R,S} - \lambda^{R,S,DA}) \cdot d_l^{R,S,DA} + \\
& \sum_{\omega \in \Omega} P_\omega \cdot (\Pi_\omega^{SO,RT} + \\
& \sum_{l \in RL^F} \lambda_\omega^{R,F,RT} \cdot d_l^{R,F,DA} + \sum_{l \in RL^S} \lambda_\omega^{R,S,RT} \cdot d_l^{R,S,DA}) \\
(\alpha_l^{R,F,DA}): \quad & d_l^{R,F,DA} \leq D_l^{R,F} \\
(\alpha_l^{R,S,DA}): \quad & d_l^{R,S,DA} \leq D_l^{R,S} \\
& d_l^{R,F,DA} \geq 0, d_l^{R,S,DA} \geq 0
\end{aligned}
$$

The KKT conditions for the system operator are:

$$
\begin{aligned}
& 0 \leq d_l^{R,F,DA} \perp -V_l^{R,F} + \lambda^{R,F,DA} + \alpha_l^{R,F,DA} - \\
& \sum_{\omega \in \Omega} P_\omega \cdot \lambda_\omega^{R,F,RT} \geq 0, l \in RL^F \\
& 0 \leq d_l^{R,S,DA} \perp -V_l^{R,S} + \lambda^{R,S,DA} + \alpha_l^{R,S,DA} - \\
& \sum_{\omega \in \Omega} P_\omega \cdot \lambda_\omega^{R,S,RT} \geq 0, l \in RL^S \\
& 0 \leq \alpha_l^{R,F,DA} \perp D_l^{R,F} - d_l^{R,F,DA} \geq 0, l \in RL^F \\
& 0 \leq \alpha_l^{R,S,DA} \perp D_l^{R,S} - d_l^{R,S,DA} \geq 0, l \in RLS
\end{aligned}
$$

The market clearing conditions are:

$$
\begin{aligned}
& \sum_{g \in G} p_g^{DA} = \sum_{l \in L} d_l^{DA} \\
& \sum_{g \in G \cup L} r_g^{F,DA} = \sum_{l \in RLF} d_l^{R,F,DA} \\
& \sum_{g \in G \cup L} (r_g^{F,DA} + r_g^{S,DA}) = \sum_{l \in RLS} d_l^{R,S,DA}
\end{aligned}
$$

1.3. The EU Model

This section presents the EU model that is considered in the paper. The salient characteristics of this model are the following: (i) there exists no real-time market for reserve capacity, (ii) there is a sequential auctioning of reserve capacity and energy in the day ahead, and (iii) there is no virtual trading.

1.3.1. Stage 3: Real Time

The generator profit maximization reads as follows for a generator $g \in G$ in third-stage scenario $\omega' \in \Omega_3$, where $\omega \in \Omega_2$ corresponds to the parent of ω_3:

$$
\begin{aligned}
\Pi^{RT}_{g,\omega'}(y_g, p^{DA}_{g,\omega}) = \quad & \max_{p^{RT}_{g,\omega'}, s^{RT,+}_{g,\omega,\omega'}, s^{RT,-}_{g,\omega,\omega'}} (\lambda^{RT}_{\omega'} - C_g) \cdot p^{RT}_{g,\omega} - \\
& \epsilon^+_g \cdot s^{RT,+}_{g,\omega,\omega'} - \epsilon^-_g \cdot s^{RT,-}_{g,\omega,\omega'} \\
(\alpha^{G,RT,+}_{g,\omega,\omega'}): \quad & p^{RT}_{g,\omega'} \leq P^{RT,+}_{g,\omega'} \cdot y_{g,\omega} \\
(\alpha^{G,RT,-}_{g,\omega,\omega'}): \quad & -p^{RT}_{g,\omega'} \leq -P^{RT,-}_{g,\omega'} \cdot y_{g,\omega} \\
(\gamma^{G,RT,+}_{g,\omega,\omega'}): \quad & -s^{RT,+}_{g,\omega,\omega'} + p^{RT}_{g,\omega'} - p^{DA}_{g,\omega} \leq 0 \\
(\gamma^{G,RT,-}_{g,\omega,\omega'}): \quad & -s^{RT,-}_{g,\omega,\omega'} - p^{RT}_{g,\omega'} + p^{DA}_{g,\omega} \leq 0 \\
& p^{RT}_{g,\omega'} \geq 0, s^{RT,+}_{g,\omega,\omega'} \geq 0, s^{RT,-}_{g,\omega,\omega'} \geq 0
\end{aligned}
$$

The KKT conditions of the generators are:

$$
\begin{aligned}
& 0 \leq p^{RT}_{g,\omega'} \perp C_g - \lambda^{RT}_{\omega'} + \alpha^{G,RT,+}_{g,\omega,\omega'} - \alpha^{G,RT,-}_{g,\omega,\omega'} \\
& +\gamma^{G,RT,+}_{g,\omega,\omega'} - \gamma^{G,RT,-}_{g,\omega,\omega'} \geq 0, g \in G, \omega \in \Omega_2, \omega' \in C_3(\omega) \\
& 0 \leq s^{RT,+}_{g,\omega,\omega'} \perp \epsilon^+_g - \gamma^{G,RT,+}_{g,\omega,\omega'} \geq 0, g \in G, \omega \in \Omega_2, \omega' \in C_3(\omega) \\
& 0 \leq s^{RT,-}_{g,\omega,\omega'} \perp \epsilon^-_g - \gamma^{G,RT,-}_{g,\omega,\omega'} \geq 0, g \in G, \omega \in \Omega_2, \omega' \in C_3(\omega) \\
& 0 \leq \alpha^{G,RT,+}_{g,\omega,\omega'} \perp P^{RT,+}_{g,\omega'} \cdot y_{g,\omega} - p^{RT}_{g,\omega'} \geq 0, g \in G, \omega \in \Omega_2, \omega' \in C_3(\omega) \\
& 0 \leq \alpha^{G,RT,-}_{g,\omega,\omega'} \perp p^{RT}_{g,\omega'} - P^{RT,-}_{g,\omega'} \cdot y_{g,\omega} \geq 0, g \in G, \omega \in \Omega_2, \omega' \in C_3(\omega) \\
& 0 \leq \gamma^{G,RT,+}_{g,\omega,\omega'} \perp s^{RT,+}_{g,\omega,\omega'} - p^{RT}_{g,\omega'} + p^{DA}_{g,\omega} \geq 0, g \in G, \omega \in \Omega_2, \omega' \in C_3(\omega) \\
& 0 \leq \gamma^{G,RT,-}_{g,\omega,\omega'} \perp s^{RT,-}_{g,\omega,\omega'} - p^{DA}_{g,\omega} + p^{RT}_{g,\omega'} \geq 0, g \in G, \omega \in \Omega_2, \omega' \in C_3(\omega)
\end{aligned}
$$

For every load $l \in L$ and scenario $\omega' \in \Omega_3$, we have the following real-time profit maximization:

$$
\begin{aligned}
\Pi^{RT}_{l,\omega'} = \quad & \max_{d^{RT}_{l,\omega'}} (V_l - \lambda^{RT}_{\omega'}) \cdot d^{RT}_{l,\omega'} \\
(\alpha^{L,RT}_{l,\omega'}): \quad & d^{RT}_{l,\omega'} \leq D^{RT,+}_{l,\omega'} \\
& d^{RT}_{l,\omega'} \geq 0
\end{aligned}
$$

The KKT conditions of the loads are:

$$
\begin{aligned}
&0 \leq d^{RT}_{l,\omega'} \perp \lambda^{RT}_{\omega'} - V_l + \alpha^{L,RT}_{l,\omega'} \geq 0, l \in L, \omega' \in \Omega_3 \\
&0 \leq \alpha^{L,RT}_{l,\omega'} \perp D^{RT,+}_{l,\omega'} - d^{RT}_{l,\omega'} \geq 0, l \in L, \omega' \in \Omega_3
\end{aligned}
$$

The market clearing conditions are:

$$
\sum_{g \in G} p^{RT}_{g,\omega'} = \sum_{l \in L} d^{RT}_{l,\omega'}, \omega' \in \Omega_3
$$

1.3.2. Stage 2: Day-Ahead Energy

In the second stage, a generator decides on the amount of energy supply, on the basis of uncertain demand for power (which implies an uncertain real-time price for energy), and on the basis of past decisions that have been made on the amount of committed reserve capacity. We assume that the actual physical decision of committing a unit is made by generators in the day-ahead energy market. Concretely, every generator $g \in G$ solves the following profit maximization problem for every outcome $\omega \in \Omega_2$:

$$
\begin{aligned}
\Pi^{DA}_{g,\omega}(r^{F,DA}_g, r^{S,DA}_g) = \quad & \max_{y_{g,\omega}, p^{DA}_{g,\omega}, VaR2_{g,\omega}, u3_{g,\omega,\omega'}} \lambda^{DA}_\omega \cdot p^{DA}_{g,\omega} + \\
& VaR2_{g,\omega} - \frac{1}{\alpha_g} \sum_{\omega' \in C_3(\omega)} P[(3,\omega')|(2,\omega)] \cdot u3_{g,\omega,\omega'} \\
& - K_g \cdot y_{g,\omega} \\
(q3_{g,\omega,\omega'}): \quad & u3_{g,\omega,\omega'} \geq VaR2_{g,\omega} - (\Pi^{RT}_{g,\omega'}(y_{g,\omega}, p^{DA}_{g,\omega}) \\
& - \lambda^{RT}_{\omega'} \cdot p^{DA}_g), \omega' \in C_3(\omega) \\
(\delta_{g,\omega}): \quad & y_{g,\omega} \leq 1 \\
(\alpha^{G,DA,+}_{g,\omega}): \quad & p^{DA}_{g,\omega} + r^{F,DA}_g + r^{S,DA}_g \leq P^{DA,+}_g \cdot y_{g,\omega} \\
(\alpha^{G,DA,-}_{g,\omega}): \quad & -p^{DA}_{g,\omega} \leq -P^{DA,-}_g \cdot y_{g,\omega} \\
& y_{g,\omega} \geq 0, u3_{g,\omega,\omega'} \geq 0, p^{DA}_{g,\omega} \geq 0, \omega' \in C_3(\omega)
\end{aligned}
$$

The KKT conditions for the collection of generators are:

$$0 \leq u3_{g,\omega,\omega'} \perp \frac{P[(3,\omega')|(2,\omega)]}{\alpha_g} - q3_{g,\omega,\omega'} \geq 0, g \in G, \omega \in \Omega_2, \omega' \in C_3(\omega)$$
$$0 \leq p^{DA}_{g,\omega} \perp \sum_{\omega' \in C_3(\omega)} q3_{g,\omega,\omega'} \cdot \lambda^{RT}_{\omega'} + \alpha^{G,DA,+}_{g,\omega} + \alpha^{G,DA,-}_{g,\omega}$$
$$-\lambda^{DA}_{\omega} \geq 0, g \in G, \omega \in \Omega_2$$
$$0 \leq y_{g,\omega} \perp K_g + \alpha^{G,DA,-}_{g,\omega} \cdot P^{DA,-}_{g} - \alpha^{G,DA,+}_{g,\omega} \cdot P^{DA,+}_{g}$$
$$+ \sum_{\omega' \in C_3(\omega)} q3_{g,\omega,\omega'} \cdot (P^{RT,-}_{g,\omega} \cdot \alpha^{G,RT,-}_{g,\omega,\omega'} - P^{RT,+}_{g,\omega} \cdot \cdot \alpha^{G,RT,+}_{g,\omega,\omega'}) \geq 0, g \in G, \omega \in \Omega_2$$
$$\sum_{\omega' \in C_3(\omega)} q3_{g,\omega,\omega'} = 1, g \in G, \omega \in \Omega_2$$
$$0 \leq q3_{g,\omega,\omega'} \perp u3_{g,\omega,\omega'} - VaR2_{g,\omega} + \Pi^{RT}_{g,\omega'}$$
$$-\lambda^{RT}_{\omega'} \cdot p^{DA}_{g,\omega} \geq 0, g \in G, \omega \in \Omega_2, \omega' \in C_3(\omega)$$
$$0 \leq \delta_{g,\omega} \perp 1 - y_{g,\omega} \geq 0, g \in G, \omega \in \Omega_2$$
$$0 \leq \alpha^{G,DA,+}_{g,\omega} \perp P^{DA,+}_{g} \cdot y_{g,\omega} - p^{DA}_{g,\omega}$$
$$-r^{F,DA}_{g} - r^{S,DA}_{g} \geq 0, g \in G, \omega \in \Omega_2$$
$$0 \leq \alpha^{G,DA,-}_{g,\omega} \perp -P^{DA,-}_{g} \cdot y_{g,\omega} + p^{DA}_{g,\omega} \geq 0, g \in G, \omega \in \Omega_2$$

Compared to the two-stage model of section 1.2, the value at risk of the second stage, denoted by $VAR2$, are now indexed by ω, i.e. the node of the second stage which we find ourselves in. Similarly, $u3$, and $q3$ are now indexed by ω and ω', i.e. the specific path from the second to the third stage. The dual variable $q3_{g,\omega,\omega'}$ is the risk-adjusted conditional probability of transitioning to node ω' of stage 3 from node ω of stage 2. The constraints corresponding to the dual multipliers $\alpha^{G,DA,+}_{g,\omega}$ and $\alpha^{G,DA,-}_{g,\omega}$ enforce physical constraints on trading. In order for a resource to offer reserve and energy in the day-ahead market, it must be committed in the first place. If the resource is committed in the day ahead, then it must trade at least its technical minimum.

Note that the day-ahead reserve capacities, $r^{F,DA}_{g}$ and $r^{S,DA}_{g}$, are *parameters* for this problem. The profit $\Pi^{DA}_{g,\omega}(r^{F,DA}, r^{S,DA})$, is a concave function of $r^{F,DA}_{g}$ and $r^{S,DA}_{g}$. Its slope (supergradient) with respect to $r^{F,DA}$ is equal to $-\alpha^{G,DA}_{g,\omega}$. Similarly, its supergradient with respect to $r^{S,DA}$ is equal to $-\alpha^{G,DA}_{g,\omega}$.

We write the second-stage profit maximization problem of every load

$l \in L$ at every outcome $\omega \in \Omega_2$ as follows:

$$\begin{aligned}
\Pi^{DA}_{l,\omega}(r^{F,DA}_l, r^{S,DA}_l) = \quad & \max_{d^{DA}_{l,\omega}, VaR2_{l,\omega}, u3_{l,\omega,\omega'}} -\lambda^{DA}_{\omega} \cdot d^{DA}_{l,\omega} + \\
& VaR2_{l,\omega} - \frac{1}{\alpha_l} \sum_{\omega' \in C_3(\omega)} P[(3,\omega')|(2,\omega)] \cdot u3_{l,\omega,\omega'} \\
(q3_{l,\omega,\omega'}) : \quad & u3_{l,\omega,\omega'} \geq VaR2_{l,\omega} - (\Pi^{RT}_{l,\omega'} + \\
& \lambda^{RT}_{\omega'} \cdot d^{DA}_{l,\omega}), \omega' \in C_3(\omega) \\
(\alpha^{L,DA}_{l,\omega}) : \quad & d^{DA}_{l,\omega} \leq D^{DA,+}_l \\
(\gamma^{L,DA}_{l,\omega}) : \quad & r^{F,DA}_l + r^{S,DA}_l - d^{DA}_{l,\omega} \leq 0 \\
& u3_{l,\omega,\omega'} \geq 0, d^{DA}_{l,\omega} \geq 0, \omega' \in C_3(\omega)
\end{aligned}$$

The KKT conditions for the collection of loads are:

$$\begin{aligned}
& 0 \leq u3_{l,\omega,\omega'} \perp \frac{P[(3,\omega')|(2,\omega)]}{\alpha_l} - q3_{l,\omega,\omega'} \geq 0, l \in L, \omega \in \Omega_2, \omega' \in C_3(\omega) \\
& 0 \leq d^{DA}_{l,\omega} \perp - \sum_{\omega' \in C_3(\omega)} q3_{l,\omega,\omega'} \cdot \lambda^{RT}_{\omega'} + \lambda^{DA}_{\omega} \\
& +\alpha^{L,DA}_{l,\omega} - \gamma^{L,DA}_{l,\omega} \geq 0, l \in L, \omega \in \Omega_2 \\
(VaR2_{l,\omega}) : \quad & \sum_{\omega' \in \Omega_3} q3_{l,\omega,\omega'} = 1 \\
& 0 \leq q3_{l,\omega,\omega'} \perp u3_{l,\omega,\omega'} - VaR2_{l,\omega} + \Pi^{RT}_{l,\omega'} \\
& +\lambda^{RT}_{\omega'} \cdot d^{DA}_{l,\omega} \geq 0, l \in L, \omega \in \Omega_2, \omega' \in C_3(\omega) \\
& 0 \leq \alpha^{L,DA}_{g,\omega} \perp D^{DA,+}_l - d^{DA}_{l,\omega} \geq 0, l \in L, \omega \in \Omega_2 \\
& 0 \leq \gamma^{L,DA}_{l,\omega} \perp d^{DA}_{l,\omega} - r^{F,DA}_l - r^{S,DA}_l \geq 0, l \in L, \omega \in \Omega_2
\end{aligned}$$

Similarly to the generator case, this is a convex optimization problem. The objective function value, $\Pi^{DA}_{l,\omega}(r^{F,DA}, r^{S,DA})$, is concave with respect to $r^{F,DA}_l$ and $r^{S,DA}_l$. Its supergradient with respect to $r^{F,DA}_l$ is $-\gamma^{L,DA}_{l,\omega}$. Its supergradient with respect to $r^{S,DA}_l$ is $-\gamma^{L,DA}_{l,\omega}$.

The market clearing condition is:

$$\sum_{g \in G} p^{DA}_{g,\omega} = \sum_{l \in L} d^{DA}_{l,\omega}, \omega \in \Omega_2$$

1.3.3. Stage 1: Day-Ahead Reserve

We complete the description of the three-stage model by describing the first-stage equilibrium. To describe the first stage, we nest the result of the second-stage decisions into a first-stage optimization of agents' risk.

In particular, every generator $g \in G$ solves the following maximization problem:

$$
\begin{aligned}
& \max_{r_g^{F,DA}, r_g^{S,DA}, VaR1_g, u2_{g,\omega}} && \tilde{\lambda}^{R,F,DA} \cdot r_g^{F,DA} + \lambda^{R,S,DA} \cdot r_g^{S,DA} + \\
& && VaR1_g - \frac{1}{\alpha_g} \sum_{\omega \in \Omega_2} P[(2,\omega)|(1,1)] \cdot u2_{g,\omega} \\
& (q2_{g,\omega}): && u2_{g,\omega} \geq VaR1_g - \Pi_{g,\omega}^{DA}(r^{F,DA}, r^{S,DA}), \omega \in \Omega_2 \\
& (\beta_g^{G,F,DA}): && r_g^{F,DA} \leq R_g^F \\
& (\beta_g^{G,S,DA}): && r_g^{S,DA} \leq R_g^S \\
& && r_g^{F,DA} \geq 0, r_g^{S,DA} \geq 0, u2_{g,\omega} \geq 0, \omega \in \Omega_2
\end{aligned}
$$

Here, $VaR1_g$ corresponds to the value at risk at stage 1. The dual variable $q2_{g,\omega}$ is the risk-adjusted probability of outcome $\omega \in \Omega_2$ based on the risk preferences of generator g. Note that since $\Pi_{g,\omega}^{DA}$ is a concave function of $r_g^{F,DA}$ and $r_g^{S,DA}$, the constraint implicating $\Pi_{g,\omega}^{DA}$ in the problem above is a convex constraint.

The equivalent complementarity system for generators is:

$$
\begin{aligned}
& 0 \leq u2_{g,\omega} \perp \frac{P[(2,\omega)|(1,1)]}{\alpha_g} - q2_{g,\omega} \geq 0, g \in G, \omega \in \Omega_2 \\
& 0 \leq r_g^{F,DA} \perp \sum_{\omega \in \Omega_2} q2_{g,\omega} \cdot \alpha_{g,\omega}^{G,DA,+} \\
& +\beta_g^{G,F,DA} - \lambda^{R,F,DA} - \lambda^{R,S,DA} \geq 0, g \in G \\
& 0 \leq r_g^{S,DA} \perp \sum_{\omega \in \Omega_2} q2_{g,\omega} \cdot \alpha_{g,\omega}^{G,DA,+} \\
& +\beta_g^{G,S,DA} - \lambda^{R,S,DA} \geq 0, g \in G \\
& \sum_{\omega \in \Omega_2} q2_{g,\omega} = 1, g \in G \\
& 0 \leq q2_{g,\omega} \perp u2_{g,\omega} - VaR1_g + \Pi_{g,\omega}^{DA} \geq 0, g \in G, \omega \in \Omega_2 \\
& 0 \leq \beta_g^{G,F,DA} \perp R_g^F - r_g^{F,DA} \geq 0, g \in G \\
& 0 \leq \beta_g^{G,S,DA} \perp R_g^S - r_g^{S,DA} \geq 0, g \in G
\end{aligned}
$$

The first-stage problem for loads reads identically to that of generators. The equivalent complementarity system for loads is:

$$
\begin{aligned}
&0 \leq u2_{l,\omega} \perp \frac{P[(2,\omega)|(1,1)]}{\alpha_l} - q2_{l,\omega} \geq 0, l \in L, \omega \in \Omega_2 \\
&0 \leq r_l^{F,DA} \perp \sum_{\omega \in \Omega_2} q2_{l,\omega} \cdot (\sum_{\omega' \in C_3(\omega)} q3_{l,\omega,\omega'} \cdot \delta_{l,\omega'}^{L,F,RT} + \gamma_{l,\omega}^{L,DA}) \\
&+\beta_l^{L,F,DA} - \lambda^{R,F,DA} - \lambda^{R,S,DA} \geq 0, l \in L \\
&0 \leq r_l^{S,DA} \perp \sum_{\omega \in \Omega_2} q2_{l,\omega} \cdot (\sum_{\omega' \in C_3(\omega)} q3_{l,\omega,\omega'} \cdot \delta_{l,\omega'}^{L,S,RT} + \gamma_{l,\omega}^{L,DA}) \\
&+\beta_l^{L,S,DA} - \lambda^{R,S,DA} \geq 0, l \in L \\
&\sum_{\omega \in \Omega_2} q2_{g,\omega} = 1, l \in L \\
&0 \leq q2_{l,\omega} \perp u2_{l,\omega} - VaR1_l + \Pi_{l,\omega}^{DA} \geq 0, l \in L, \omega \in \Omega_2 \\
&0 \leq \beta_l^{L,F,DA} \perp R_l^F - r_l^{F,DA} \geq 0, l \in L \\
&0 \leq \beta_l^{L,S,DA} \perp R_l^S - r_l^{S,DA} \geq 0, l \in L
\end{aligned}
$$

The day-ahead optimization problem for the system operator follows the same reasoning as section 1.2. The complementarity conditions for the system operator problem read as follows:

$$
\begin{aligned}
&0 \leq d_l^{R,F,DA} \perp -V_l^{R,F} + \lambda^{R,F,DA} + \alpha_l^{R,F,DA} - \\
&\sum_{\omega \in \Omega_2} P[(2,\omega)|(1,1)] \cdot \sum_{\omega' \in \Omega_3} P[(3,\omega')|(2,\omega)] \cdot \lambda_{\omega'}^{R,F,RT} \geq 0, l \in RL^F \\
&0 \leq d_l^{R,S,DA} \perp -V_l^{R,S} + \lambda^{R,S,DA} + \alpha_l^{R,S,DA} - \\
&\sum_{\omega \in \Omega_2} P[(2,\omega)|(1,1)] \cdot \sum_{\omega' \in \Omega_3} P[(3,\omega')|(2,\omega)] \cdot \lambda_{\omega'}^{R,S,RT} \geq 0, l \in RL^S \\
&0 \leq \alpha_l^{R,F,DA} \perp D_l^{R,F} - d_l^{R,F,DA} \geq 0, l \in RL^F \\
&0 \leq \alpha_l^{R,S,DA} \perp D_l^{R,S} - d_l^{R,S,DA} \geq 0, l \in RL^S
\end{aligned}
$$

The market clearing conditions are:

$$
\begin{aligned}
\sum_{g \in G \cup L} r_g^{F,DA} &= \sum_{l \in RL^F} d_l^{R,F,DA} \\
\sum_{g \in G \cup L} (r_g^{F,DA} + r_g^{S,DA}) &= \sum_{l \in RL^S} d_l^{R,S,DA}
\end{aligned}
$$

Table 1: The real-time price scenarios and associated results for the illustrative CVaR example.

Scenario	Prob.	RT price λ_ω^{RT}	$\alpha_\omega^{RT,+}$ ($y=0.5$)	q_ω ($y=0.5$)	Profit Π_ω^{RT} ($y=0.5$)	Profit Π_ω^{RT} ($y=0.6$)
S1	0.25	50	0	0.417	0	0
S2	0.25	100	25	0.417	250	300
S3	0.25	150	75	0.167	750	900
S4	0.25	200	125	0	1250	1500

1.4. Remarks on the equilibrium models

Note that the resulting equilibrium models that are presented in section 1.2 and section 1.3 are *not* generalized Nash equilibrium problems, since competitors' decisions only affect the objective function of the players, not their constraints.

Note that the equilibrium models presented in this section correspond to an incomplete market, where we are limited to a forward market for reserve and energy. Therefore, the resulting equilibria will, in general, deviate from the equilibrium of a complete market, which can be expressed equivalently as the risk-averse optimization of system welfare (Ralph and Smeers, 2015).

2. Coherent Risk Measures

This section provides a more extensive discussion on coherent risk measures by illustrating the relations presented in section 3.2 of the main paper on a concrete example.

Consider a generator participating in a real-time market, where real-time prices are distributed uniformly between 50 and 200 €/MWh. The generator has a marginal cost of $C_g = 75$ €/MWh, and a capacity of $P_\omega^{RT,+} = 20$ MW. We can easily compute its real-time profit as the solution to the following linear program:

$$\Pi_\omega^{RT}(y) = \max_{p_\omega^{RT} \geq 0} (\lambda_\omega^{RT} - C_g) \cdot p_\omega^{RT}$$

$$(\alpha_\omega^{RT,+}): \qquad p_\omega^{RT} \leq P_\omega^{RT,+} \cdot y$$

The resulting profits for $y = 0.5$ (i.e. when half of the full capacity is committed) are presented in table 1. The dual multipliers corresponding to the different scenarios are also listed in the table.

Let us consider the $CVaR_{0.6}$ risk measure. The intuitive definition of this risk measure is the following: "find the expected profits, conditional on 60% of the worst possible outcomes occurring". The way to think about this statement is by plotting the distribution of profits, and then creating a new distribution which is biased towards accumulating 60% of the original mass, while collecting the worst possible outcomes. To illustrate this concept, suppose that we were to discretize the above table into 20 outcomes:

$$
\begin{aligned}
&(0, 0, 0, 0, 0, \\
&250, 250, 250, 250, 250, \\
&750, 750, 750, 750, 750 \\
&1250, 1250, 1250, 1250, 1250, 1250)
\end{aligned}
$$

Suppose, now, that we were forced to select 12 of these outcomes (60% of the mass), but in a way that biases our selection to the worst possible realizations. We would then select the following outcomes:

$$(0, 0, 0, 0, 0, 250, 250, 250, 250, 250, 750, 750)$$

The $CVaR_{0.6}$ risk measure computes the expected payoff of this distribution (assuming each of these pessimistic outcomes is equally likely). Concretely:

$$
\begin{aligned}
&CVaR_{0.6} = \\
&\frac{1}{12} \cdot (0 + 0 + 0 + 0 + 0 + 250 + 250 + 250 + 250 + 250 + 750 + 750) = \\
&229.167
\end{aligned}
$$

We now re-derive the same result by expressing $CVaR$ in its equivalent linear programming form (equation (9) of the main draft), based on risk sets. The risk set $\mathcal{M}$ for the $CVaR_\alpha$ risk measure is generally described as

$$\mathcal{M} = \{q_\omega \geq 0 : \sum_\omega q_\omega = 1, q_\omega \leq \frac{P_\omega}{\alpha}\}$$

The linear programming formulation of coherent risk measures aims at a worst-case expectation of the uncertain payoffs within the risk set. Applying this formulation to our simple example, we express $CVaR_{0.6}$ as follows.

$$CVaR_{0.6} = \min_{q_\omega \geq 0} \sum_\omega q_\omega \cdot \Pi_\omega^{RT}$$

$$\sum_\omega q_\omega = 1$$

$$q_\omega \leq \frac{P_\omega}{0.6}$$

This recovers the conditional value at risk, $CVaR_{0.6} = 229.167$, that was calculated above.

As a concrete illustration of equation (10) of the main draft of the paper, note that if we change the profit of the first scenario from 0 to 1 €, then the above linear program evaluates to 229.583 (when rounded to the third digit). The marginal change, 0.417, is equal to the risk-adjusted probability of the first scenario.

Let us now move to equation (11) of the main draft of the paper, and attempt to compute the subgradient of the CVaR with respect to the decision y, i.e. the decision about how much capacity to commit in the first stage. Numerically, we can estimate this subgradient by slightly perturbing $y = 0.5$ to $y = 0.6$, re-computing the resulting real-time profits, and then computing the implied change in CVaR. The real-time profits for $y = 0.6$ are presented in table 1. The resulting CVaR amounts to 275. Thus, the marginal change with respect to y amounts to $\frac{275-229.167}{0.1} = 458.33$.

The subgradient of the real-time profit with respect to y corresponds to the dual multiplier vector $(\alpha_\omega^{RT,+} \cdot P_\omega^{RT,+}, \omega \in \Omega)$. Equation (11) of the main draft states that the marginal change derived above for the CVaR with respect to the commitment variable y can be expressed alternatively as the expectation of $\alpha_\omega^{RT,+} \cdot P_\omega^{RT,+}$ with respect to the risk-neutral measure q_ω, namely:

$$\sum_{\omega \in \Omega} q_\omega \cdot \alpha_\omega^{RT,+} \cdot P_\omega^{RT,+} =$$
$$20 \cdot (0.41\overline{6} \cdot 0 + 0.41\overline{6} \cdot 25 + 0.16\overline{6} \cdot 75 + 0 \cdot 125) = 458.3\overline{3}$$

3. Case Study Setup

Loads. We assume that the day-ahead demand is equal to the historically observed net demand, after removing imports. We assume an inelastic load and we ignore transmission constraints. We will assume a value of lost load that is equal to 8300 €/MWh, based on an estimate of the Belgian Federal Planning Bureau Devogelaer (2014).

Generators. We consider the same mix of technologies as in previous research (Papavasiliou and Smeers, 2017; Papavasiliou et al., 2018): pumped storage, blast furnace, renewable[1], gas-oil, LVN, coal (3 units) and combined cycle gas turbines (11 units, of which 3 are placed in strategic reserve). The marginal cost consists of the fuel cost and the CO2 emissions cost. We use the CO2 prices, emissions rates, and fuel price data from previous research (Papavasiliou et al., 2018). The production of nuclear, wind, waste, and water are assumed to be price-inelastic, based on previous analyses (Papavasiliou and Smeers, 2017; Papavasiliou et al., 2018), and their production is subtracted directly from the system demand.

The fixed cost consists of startup cost and startup fuel, which we assume is incurred once per day (in the sense that for every hour that a unit is on, it must incur a cost which is 1/24 of the startup cost, so that if a unit is on for an entire day, it incurs a cost equal to its full startup cost). Additionally, we account for the minimum load fuel consumption of a generator.

Planned outages are accounted for in the data. The production capacity is scaled according to a capacity scaling factor which captures these forced outages. We ignore unplanned generator outages (assuming that they are captured implicitly in the imbalance scenarios), and simply set the real-time power generation capacity equal to the day-ahead capacity.

We set the ramp rate of all units except for CCGT units and pumped hydro units equal to zero. We assume that the fast ramp capacity is equal to half of the slow ramp capacity, which is equal to the 15-minute ramp rate of units. This is due to the fact that we associate fast capacity to secondary reserve, which is assumed to have a response time of 7.5 minutes. We associate slow capacity to tertiary reserve, which is assumed to have a response time of 15 minutes.

Pumped hydro. We account for pumped hydro resources by adding the six generators and four pumps that are located at the Coo pumped hydro facility in Belgium. In production mode, there exist three pairs of 144/215

[1]The fact that we are considering the renewable resources as price-responsive implies that the supply from these resources is not accepted as must-take in the system, but is rather inserted in the merit order curve of the system and only dispatched if it is economical to do so.

MW generators, and three pairs of 145/200 MW pumps. We then use the total water storage capacity of Coo (8450000 m^3), and the head height (245 m for each of the two pumped hydro reservoirs) to compute the total energy storage capacity.

We employ a separate model for pumped hydro resources, which we do not develop here in order to avoid overburdening the notation, and since it is not required for describing the equilibrium formulation. For the sake of simplicity, we assume that pumped hydro resources do not offer reserve, otherwise it would be necessary to employ a multi-stage stochastic program in order to properly account for the random activation of reserve during the day. This random activation could result in binding operational constraints, and would also use up water which has value that depends on market prices. In order to focus the paper on the formation of reserve prices under scarcity pricing, we do not attempt to model this level of complexity since it would distract from the main purpose of the paper. The assumption that pumped hydro resources are not contributing to reserve is of minor significance, since pumped hydro resources are still allowed to increase their production in real time up to the level of the maximum production capacity under tight system conditions. We note that our assumption that hydro resources do not commit capacity in reserve auctions is not far from empirical data. We have access to historical data from 2017, and note that pumped hydro resources contribute during some days to secondary reserve capacity, but only to a limited extent.

System operator. In order to design the ORDC, we need to account for the fact that fast and slow reserve can substitute for each other. As explained by Hogan (2013), the valuation of the system operator for fast and slow reserve capacity can be derived as follows:

$$\begin{aligned}
MBRF(r^F; r^{S,0}) &= (VOLL - \hat{MC}(\sum_g p_g)) \cdot \\
&\quad (0.5 \cdot LOLP_{7.5}(r^F) + 0.5 \cdot LOLP_{15}(r^{S,0} + r^F)) \\
MBRS(r^S; r^{F,0}) &= (VOLL - \hat{MC}(\sum_g p_g)) \cdot 0.5 \cdot LOLP_{15}(r^S + r^{F,0})
\end{aligned}$$

where r^F is the amount of fast reserve capacity, r^S is the amount of slow reserve capacity, $r^{F,0}$ and $r^{S,0}$ are reference values for these capacities[2], $VOLL$ is the value of lost load, $\hat{MC}(\sum_g p_g)$ is a proxy of the marginal cost of the

[2]These are reference values of system capacity around which we linearize the marginal

Table 2: Mean and standard deviation of 15-minute imbalance data used for the estimation of $LOLP_{15}$.

Seasons	Hours	Mean	St dev	Season	Hours	Mean	St dev
Winter	1,2,23,24	29.5	165.4	Summer	1, 2, 23, 24	20.1	133.1
	3-6	23.6	147.8		3-6	42.5	111.5
	7-10	16.6	181.3		7-10	25.8	132.1
	11-14	-20.9	224.1		11-14	34.8	154.4
	15-18	8.1	162.4		15-18	47.1	140.3
	19-22	9.8	147.2		19-22	13.5	108.8
Spring	1,2,23,24	28.4	147.9	Fall	1,2,23,24	29.2	138.7
	3-6	42.3	131.3		3-6	28.9	105.9
	7-10	27.8	151.3		7-10	-11.2	142.8
	11-14	68.4	174.9		11-14	18.5	164.9
	15-18	69.0	161.5		15-18	0.2	142.8
	19-22	9.0	134.3		19-22	-10.8	147.2

marginal unit[3], and $LOLP_t$ is the loss of load probability given the uncertainty that the system is facing in the following t minutes.

We consider an operating reserve demand curve which is identical in the day ahead and real time. In computing $LOLP_{7.5}$, we assume perfectly correlated increments of uncertainty, following the statistical analysis of Papavasiliou et al. (2018). We use the parameters of the imbalance distribution shown in table 2 in order to calibrate loss of load probabilities for every season and every four-hour block of every season.

In the case of the *ICH* tertiary reserve product, which is the Belgian tertiary reserve made available through demand response, it is straightforward to introduce it to the models. Namely, we augment the demand function for slow reserve, and we place a limit on the amount of reserve that demand response can offer which corresponds to the amount of *ICH* capacity. Since demand response reserve capacity is typically available at zero opportunity cost (according to the model developed in this paper, if a load is consuming, it can offer its consumption as demand response capacity), any extra reserve demand that is requested by the system will always be served first by demand

benefit to the system of additional capacity. A reasonable choice for the case of Belgium is to use the same requirements as in the previous studies, namely 350 MW of the *R3 production* tertiary reserve product and 140 MW of secondary reserve.

[3]We use 25 €/MWh for this study, although this can be refined to more closely approximate the real-time system lambda.

response. Thus, the effect of adding extra reserve demand and at the same time increasing the amount of reserve that can be satisfied by demand are two effects that cancel each other out.

Uncertainty. The overall 15-minute uncertainty in the system is characterized by the parameters of table 2. These parameters are based on the imbalance data of 2017. In order to derive real-time demand scenarios, we use this data in conjunction with the data of figure 8 of De-Vos et al. (2019). In that figure, we observe the distribution of dynamic sizing requirements. We will assume that these requirements correspond to a factor that inflates real-time system imbalance. We have eight possible scenarios of "inflation", where the inflation factor corresponds to first-stage uncertainty. In order to define scenarios for the second stage, we multiply this inflation factor by a discretized normal distribution of imbalances, which is calibrated using the data of table 2. The inflation factors and their corresponding probabilities can be observed directly in figure 8 of De-Vos et al. (2019).

The transition probabilities from the second to the third stage are chosen so that we capture outliers (2 scenarios with probability 0.1% each) and we discretize the remaining mass of the distribution in evenly spaced 'buckets' of mass.

Strategic reserve. We assume that strategic reserve capacity can contribute in real time at a very high cost, which is still below VOLL but above the marginal cost of the most expensive unit. We specifically assign a marginal cost of activation equal to 500 €/MWh for strategic reserve. This corresponds to a total capacity of 375 MW (Esche) + 485 MW (Seraing) + 385 MW (Vilvoorde), equal to 1245 MW in total. Our justification is that the commitment of strategic reserve capacity is the last resort before load shedding[4].

Other. We assume that reserve capacity is cleared daily, despite the fact that in the period of the case study the reserve auctions were weekly. The rationale for this choice is that, in practice, generators can trade their reserve obligations even after the week-ahead auction.

[4] http://www.elia.be/∼/media/files/Elia/Products-and-services/Strategic-Reserve/SFR-2017-18_fr_final.pdf

Table 3: Load profits under the different designs.

	Load profit decrease (€/MW-month)	Δ Profit / Δ Reserve (€/MW-month)	Break-even reserve capacity (MW)
EU	-	-	-
EU-inel.	5676	52819	926.4
RTReserve2	5227	57154	680.6
RTReserve	5111	66010	576.2
VT	5109	66126	575.0
US	5120	65877	578.4

4. Load Profits

Scarcity pricing implies a short-run increase in energy prices, but also an increase in the value of reserve capacity. Resources that are unable to respond to the real-time needs of the system and are a burden to the system in periods of scarcity will only experience the former effect. Resources that can respond to the real-time needs of the system will reap the benefit of this flexibility, and offset the resulting increase in energy prices. This is especially relevant for demand response, which is one of the most suitable options for offering fast-responding capacity to the system.

We summarize our results regarding the implications of scarcity pricing for the profitability of loads in table 3. The table reads as follows: (i) The second column presents the decrease in the profits of loads, relative to the EU design, under the assumption that loads do not offer any reserve to the market. Note that this is the total increase in the consumer bill from the introduction of scarcity pricing, divided by the total average demand during the study, which amounts to 7442 MW. (ii) The third column is the monthly increment in profit that loads enjoy by offering an additional MW of ramp capacity into the system. This increment is the result of their ability to offer additional capacity for secondary and/or tertiary reserve. (iii) The fourth column is the amount of reserve capacity that the loads would need to offer to the reserve market in order to offset their losses from the increase in energy prices which results from scarcity pricing. Any amount of capacity above this level would result in the market design in question creating a net benefit for loads, relative to the design with the lowest electricity price (EU).

As expected, the introduction of scarcity pricing reduces the profits of loads under the assumption that loads cannot offer reserves and monetize their flexibility in the reserve market. This can be seen by observing, in the second column, that the EU design, which is the design with the lowest energy prices and one of the two proxies of the current Belgian market design, results in the highest profits for loads. On the other extreme, the EU-inelastic design entails the greatest energy prices and the least profit for loads. The introduction of scarcity pricing in the RTReserve and RTReserve2 models lifts the prices of both energy and reserve. If loads cannot offer reserve to the market, this clearly entails a net loss. On the other hand, the introduction of the real-time reserve market under the RTReserve and RTReserve2 designs also revives the reserve price, and the third column of the table indicates the amount of reserve that loads would need to offer to the market in order to be able to offset the losses that they incur from the increase of energy price that results from scarcity pricing.

The ratio of the second and third column of table 3 indicate that, for every MW of load in the system, the additional expense that results from the introduction of scarcity pricing can be recuperated by making approximately 7.8% (for RTReserve, RTReserve2, VT, and US) to 10.7% (for EU-inelastic) of that capacity available in the reserve market. Any additional load capacity that can be made available in the reserve market stands to gain from the introduction of scarcity pricing.

5. Additional Simulation Results

In this section we provide additional simulation results, regarding the amount of reserved capacities, the capability of dealing with uncertainties, risk measures, and expected social welfare.

We quantify these metrics as follows: (i) Regarding the amount of reserved capacities, we report the average *day-ahead* reserved fast and slow reserve demand traded in the day-ahead market. (ii) Regarding the capability of dealing with uncertainties, we report the average *real-time* reserved fast and slow reserve demand traded in the real-time market, which corresponds to the remainder reserve capacity *after* activating generators in order to deal with imbalances. This measure is similar to the Available Reserve Capacity (ARC) measured by the Belgian system operator for quantifying instantaneous reserve capacity in the system. (iii) Regarding risk measures, we report the standard deviation of daily profits. (iv) Regarding expected social welfare, we report the total welfare and its split between generators, loads, system operator, hydro resources, and strategic reserve.

Table 4: Reserve capacity after (columns 2, 3) and before (columns 4, 5) activation for each market design.

	RT fast reserve (MW)	RT total (slow + fast) reserve (MW)	DA fast reserve (MW)	DA total (slow + fast) reserve (MW)
US	301.4	427.0	360.0	626.6
VT	301.6	427.2	357.6	627.7
RTReserve	301.8	427.4	358.2	621.5
RTReserve2	304.7	432.4	339.9	516.4
EU	73.8	224.9	328.0	770.2
EU-inel.	288.8	335.8	288.8	624.6

Table 4 demonstrates the depletion that takes place from the activation of reserves in real time. The table corroborates the general observation that the designs which include a real-time market for reserve capacity perform similarly to each other.

The average daily profit of each market participant is presented in table 5. We have included one line at the end of the table, which is the difference between the market welfare and the system operator welfare. This essentially corresponds to the difference between consumer benefit and producer cost, although for the RTReserve2, EU, and EU-inelastic policy these profit metrics incorporate the term which penalizes deviations from day-ahead positions. Among the three policies that do not penalize deviations between day-ahead and real-time positions, the US deign produces the highest market welfare (net of system operator profit), followed closely by the VT design. As a general remark, the inefficiencies arising from the separation of energy and reserve clearing in the day-ahead commitment stage relate to possibly over-committing reserve due to increased uncertainty at the reserve commitment stage Dominguez et al. (2019). The penalization of deviations between day-ahead and real-time positions also introduces inefficiency, since it distorts the marginal cost of resources at the real-time market clearing stage. The market welfare is dominated by consumer profits.

The average daily standard deviation of each market participant is presented in table 6. Loads exhibit a high variance in profits due to the fact that their daily demand is fluctuating. In terms of variance in CCGT profits, all designs that introduce a real-time market for reserve capacity attain comparable performance. By contrast, the EU design is generating more stable (but consistently lower) profit for CCGT resources, whereas the EU inelastic design is generating more volatile (but consistently higher) profit for CCGT resources.

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Table 5: Average daily profit (in €/day) for each market participant under every design.

	US	VT	RTReserve	RTReserve2	EU	EU-inel.
Blast furnace	50997	50997	50997	50867	14246	54588
Coal1	74638	74638	74638	74854	42603	78100
Coal2	74966	74966	74966	75158	42914	78517
Coal3	56029	56029	56029	56072	29442	59334
CCGT1	77299	77299	77299	77616	27146	81219
CCGT2	228328	228267	228349	229343	166043	234347
CCGT3	79269	79269	79269	79524	25986	83502
CCGT4	91044	91044	91044	91311	29035	95979
CCGT5	176984	176931	177027	177805	129307	181618
CCGT6	69739	69739	69739	69915	22260	73655
CCGT7	216177	216169	216202	217183	159391	221649
CCGT8	185190	185179	185201	186050	134338	189924
Non-wind renewable	14894	14890	14890	14491	7481	19269
Gas-oil	542	542	542	535	116	585
Nuclear	3108099	3106013	3097453	3108012	2563722	3162347
Turbojet	161	161	161	138	4	123
Water	75969	75932	75750	75962	63559	77170
Waste	223931	223822	223284	223914	188111	227392
Wind	695443	695109	693452	695424	584514	705807
Pumped hydro	63208	63276	63273	63463	30413	65942
Strategic reserve	0	0	0	0	0	0
Load	1475750301	1475753050	1475752321	1475726783	1477020435	1475612401
System operator	12731283	12731006	12730901	12729474	6403930	6294196
Total market	1494044489	1494044327	1494032785	1494023893	1487684999	1487597664
Total market except S.O.	1481313206	1481313321	1481301884	1481294420	1481281068	1481303468

Table 6: Average daily standard deviation (in €/day) for each market participant under every design.

	US	VT	RTReserve	RTReserve2	EU	EU-inel.
Blast furnace	141365	141365	141365	141067	47112	143246
Coal1	138299	138299	138299	138245	56326	139352
Coal2	138253	138253	138253	138205	56297	139277
Coal3	113590	113590	113590	113568	45224	114437
CCGT1	180398	180398	180398	180312	67220	182623
CCGT2	262005	262065	261990	261628	126164	263335
CCGT3	206166	206166	206166	206095	74318	207953
CCGT4	238827	238827	238827	238735	85501	240821
CCGT5	200059	200109	200030	199781	96166	201120
CCGT6	175485	175485	175485	174662	63764	175855
CCGT7	240930	240944	240911	240573	116206	242261
CCGT8	217195	217208	217188	216881	106290	218483
Non-wind renewable	34426	34428	34428	34233	12395	35202
Gas-oil	1616	1616	1616	1600	424	1659
Nuclear	2130890	2133231	2132410	2127327	1046470	2163394
Turbojet	761	761	761	672	24	604
Water	53506	53547	53534	53430	26273	53801
Waste	155403	155524	155490	155191	76751	156018
Wind	489457	489827	489706	488850	242919	489113
Pumped hydro	136331	136332	136323	136962	53151	140349
Strategic reserve	0	0	0	0	0	0
Load	145859633	145856248	145856414	370594900	147547913	145765420
System operator	108875	108758	108786	109486	8272	122444
Total market	149282843	149283116	149282829	149280532	149320038	149271494