Hierarchical Coordination of Transmission and Distribution System Operations in European Balancing Markets

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***Abstract*—We propose a hierarchical scheme for the coordination of transmission and distribution system operations, which is inspired by nested decomposition and tailored for integration in upcoming European balancing markets. We compare our proposal to a number of recent European research projects and pilots. We demonstrate the effectiveness of our proposal in resolving a number of transmission-distribution coordination dilemmas, including conflict resolution between network operators, self-healing, sharing charges in mutually beneficial actions, and the recursive integration of distributed resources in pan-European balancing markets. We implement a proof-of-concept market clearing platform that can match transmission and distribution system market orders, cope with non-convexities in market offers and power flow constraints, scale to systems of realistic size, and respect the decentralization of communications and information sharing which is required in European electricity markets. We demonstrate the effectiveness of our platform on large-scale instances of the Italian and Danish power systems based on data sourced from the EU Horizon 2020 SmartNet project. We demonstrate the superiority of our approach to alternative coordination schemes in terms of economic efficiency, alignment of incentives and system security.**

***Index Terms*—Transmission-distribution coordination, flexibility markets, local energy markets, balancing, congestion management, optimal power flow.**

I. Introduction

There has been considerable activity recently both in the academic community as well as among practitioners in the mobilization of *flexibility* from resources that are connected to medium and low-voltage distribution networks. Flexibility is to be understood as the ability of resources that are capable of rapidly responding to real-time conditions for the needs of balancing the system and relieving network constraints. In the context of the present work, we specifically refer to flexible resources as resources that can offer manual frequency restoration reserve and congestion management services[1] to the system.

The increasing integration of both distributed renewable resources (e.g. rooftop solar) as well as flexible resources in medium and low-voltage grids (e.g. electric vehicles or heat pumps) has recently motivated the active engagement of distribution system operators (DSOs) in short-term operations. Our focus in the present work is on real-time operations, i.e. the balancing (in EU terminology) or real-time market. There are a number of barriers that we are interested in addressing in this work, which include institutional boundary conditions, information privacy, complex network constraints and the large number of resources that are involved. We discuss each of these challenges in the context of European balancing markets in the sequel.

A. European Initiatives

Table I-A summarizes a number of completed or ongoing European projects on local energy markets and TSO-DSO coordination, and classifies them according to salient characteristics. There are a number of concerns related to existing proposals [1]. (i) **Unambiguous real-time prices.** As indicated in the last column, certain projects lack a clear interface between the proposed products that are traded on these platforms and European balancing markets, in particular balancing prices and imbalance settlement[2] [1]–[5]. Proposals which disconnect imbalance settlement from balancing prices can induce inefficient bidding (see section III-F). (ii) **Dispatch efficiency and baselining.** Platforms which are not integrated to broader electricity market functions fail to guarantee efficient resource dispatch, and suffer from baselining requirements, which can be manipulated. (iii) **Inc-dec gaming.** Platforms which are limited to forward reservation of capacity with an inadequate representation of network constraints are prone to inc-dec gaming [6].

The application of locational marginal pricing to real-time markets is a coherent framework for overcoming the aforementioned challenges. (i) A unique locational real-time energy price precludes arbitrage between imbalance settlement and balancing prices. (ii) An integrated treatment of balancing and congestion management in real-time operations treats these inter-dependent functions simultaneously by design, and avoids the suboptimality which is implied by a sequential optimization of interdependent tasks. The first principles of two-settlement systems imply that real-time prices settle deviations from forward positions, thereby overcoming baselining

[1] The artificial distinction between balancing and congestion management is an endemic feature of European market design. The feature that mFRR and congestion management share in common which is relevant to our current work is the fact that these resources are committed in advance to offer said services, and are activated close to real time in order to resolve system imbalance and network overloads. Their expected response time to TSO dispatch instructions is in the range of a few (e.g. fifteen) minutes.

[2] As indicated in [1], "*A key point of difference within flexibility market design is that between platforms that act as self-contained local markets for congestion management and platforms that act as intermediaries to existing national (and international) wholesale and system services markets.*"

TABLE I
FEATURES OF VARIOUS EUROPEAN FLEXIBILITY PLATFORMS.

	Countries	Cong. mgmt.	Balancing	DA or earlier	Intraday	Cont. trading	Closed-gate
Enera [2]	DE	X	X		X	X	
GOPACS [2]	NL	X			X	X	
NODES [1], [2]	NO, UK	X					
Piclo Flex [2]	UK	X		X			X
Centrica [3]	UK	X	X	X	X		X
Soteria [4]	BE	X	X			X	X
CoordiNet [5]	ES, GR, SE	X	X	X	X		X
DA/RE [1]	DE	X		X			
Crowd balancing platform [1]	NL, IT, DE, AT, CH	X	X				
INTERRFACE [1]	LV, EE, FI	X	X				
Our proposal (FEVER)	DE, ES	X	X			X	X

challenges. (iii) Pricing consistently with network constraints is the starting point for overcoming inc-dec gaming. Ignoring network constraints in real time compromises system security and induces a patchwork of congestion management mechanisms which, according to empirical evidence [6], induce gaming opportunities.

In section I-B we discuss the extant literature on locational marginal pricing in distribution networks, while in section I-C we discuss the literature on decomposition approaches. We argue that neither provides a clear connection to existing European balancing market design. Our proposal is specifically designed to bridge this gap, as described in section II.

B. Distribution Locational Marginal Pricing

Distribution locational marginal pricing [7]–[12] is studied widely in the literature as a mean of overcoming the aforementioned difficulties in TSO-DSO coordination. In this respect, the common approach adopted in the literature is the following: (i) wholesale market prices are determined by clearing transmission markets, and (ii) DSOs clear local markets in order to determine distribution locational marginal prices (DLMPs) at the level of individual distribution nodes, given wholesale prices.

This top-down coordination can in principle be applied in the context of the problem that the aforementioned European platforms attempt to tackle. However, the exact translation of these first principles to European balancing market operations is unclear. In particular, it is unclear what DLMPs imply for balancing prices, imbalance prices, and the roles and responsibilities of TSOs, DSOs, balancing service providers (BSPs) and balancing responsible parties (BRPs).

C. Decomposition Methods

Decentralized optimization methods for solving the optimal power flow problem [13] are a natural framework to consider for our application, since they present advantages in terms of computations and data privacy. The typical setup of a decentralized approach relies on an Augmented Lagrangian relaxation of the lines that interconnect regions, which can be solved through a variety of algorithms. Common approaches that can be found in the literature include the Alternative Direction Method of Multipliers (ADMM) [13], [14], the Auxiliary Problem Principle [13], [15] or Analytical Target Cascading [15]. These approaches cover a wide range of network models (AC or DC models as well as SDP and SOCP relaxations), with certain papers employing different network models for different parts of the network [16]. Recent work [17] also focuses on developing decentralized algorithms that include convergence guarantees (to stationary points) for non-convex network models. Molzahn et al. [15] provide an extensive review of the literature in this field.

The overall decomposition framework considered in the aforementioned literature [13]–[17] could be applicable to our context if institutional barriers were not present. However, none of these references discusses how these various decomposition mechanisms are expected to interface with existing or future markets. More specifically, those decomposition approaches universally rely on an *iterative* exchange of data between different regions of the network. To the best of our understanding, and given that foreseen balancing platforms such as MARI are expected to operate as closed-gate auctions, this is not compatible with foreseen market evolutions in Europe. Instead, our proposed ”hierarchical” coordination scheme relies on a *single round* of exchange of information between the DSO and the TSO.

D. Contributions and Organization of the Paper

The are two principal contributions in our work: (i) to outline a well-defined market design for flexibility platforms that aims at aligning with integrated European balancing market evolutions, and (ii) to propose a computational infrastructure for implementing the proposed market design.

The remainder of the paper is structured as follows: In section II we describe our proposed coordination scheme which relies on the concept of a *Residual Supply Function* (RSF). Section III applies our proposed coordination scheme on a number of illustrative TSO-DSO coordination scenarios. We present the market clearing model in sections IV and V. Section VI outlines the algorithm that we propose for solving the market clearing problem at large scale. Section VII presents results from three large-scale instances that have been developed in the EU H2020 project SmartNet. We conclude in Section VIII.

II. Proposed Coordination Scheme

A. Hierarchical Coordination

Our proposed hierarchical approach aims at bridging the gap between the physical details of the grid constraints (that include the DSO grid) and the real-time market, operated by the TSO, which typically ignores most of the (DSO) grid constraints, notably for institutional reasons. In brief, the approach consists of representing *implicitly* the DSO grid constraints in the form of "grid-secure" bids (later called a *residual supply function*) that are submitted to the TSO balancing market. The approach preserves the independence of the DSO while accounting for its grid constraints in the market operated by the TSO.

The transmission network that we consider is meshed, and we use a standard DC approximation which is considered adequate for high-voltage grids in market clearing models. Distribution networks are assumed to be radial in our model, and we consider AC power flow in order to account for losses, voltage limits, and reactive flows, which are more relevant in medium and low-voltage grids. Reserve capacity [7] is considered as being out of scope for the present work but is the subject of ongoing research.

Fig. 1 introduces the timeline of our hierarchical approach. The roles and responsibilities of different market actors in the hierarchical design can be summarized as follows.

Transmission System Operator (TSO). The TSO operates the real-time power market. The TSO procures real power from BSPs that are connected in the transmission network, as well as real power from the DSO at the transmission-distribution interface (e.g. the medium-to-high-voltage feeders). The TSO collects payments from BRPs for real power, as well as payments from the DSO for the exchange of real power at the T&D interface.

Balancing Service Provider (BSP). In terms of the mathematical models that are presented in the sequel, we will identify BSPs as price-responsive resources.

Balancing Responsible Party (BRP). Distribution network BRPs face a location-specific price for real power and reactive power, against which their imbalances are settled. Transmission network BRPs face a price for real power. In terms of the mathematical models that are presented in the sequel, we identify BRPs as price-inelastic resources.

DSO Aggregation-Disaggregation System (ADS). The ADS implements the decentralization of T&D system operations by aggregating distribution system BSP offers, trading energy at the T&D interface, and subsequently disaggregating the interface flow to BSP activations within the distribution network. The flexibility platform also determines the location-specific real and reactive prices of the distribution network.

Market timeline (Fig. 1) and Residual Supply Function (RSF). The residual supply function [18], [19] evaluates the cost of exporting real power from the distribution network at the level of the interface. One residual supply function is defined for every T&D interface. An important advantage of residual supply functions is their direct interpretation as bids in the balancing market operated by the TSO, which masks the complexity of the underlying distribution network. The ADS computes a residual supply function by collecting the market bids of all BSPs that are connected to the distribution network of a given DSO, as well as the relevant network parameters from the DSO. The residual supply function can then be bid into the wholesale market, and can compete on equal footing with the BSP offers of transmission-level BSPs. The clearing of the market implies a net position for the DSO. This net position needs to be disaggregated to individual BSPs at the distribution network by the disaggregation function of the ADS.

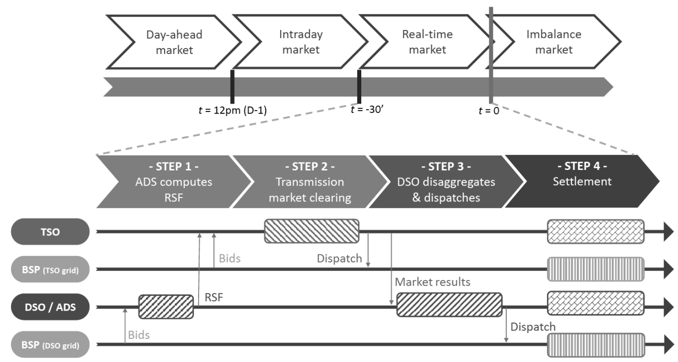

Fig. 1. Sequence of events in our proposed market platform.

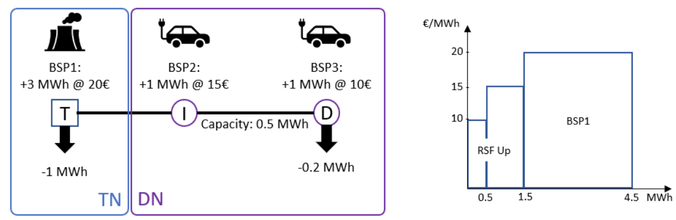

Fig. 2. Illustrative T&D example of section II-B. Note that we are only presenting the upward balancing offers in the figure.

B. Illustrative Example of T&D Coordination Based on RSFs

We describe the interaction of the RSF with our envisioned hierarchical market design by employing the illustrative example of Fig. 2. In this example, BSP 1 bids 3 MWh at a price of 20 e/MWh while BSPs 2 and 3 bid 1 MWh each at a price of 15 e/MWh and 10 e/MWh respectively. These bids can be partially or totally accepted. Note that we ignore losses and reactive power in this illustrative example. We also consider a real power flow limit of 0.5 MWh on the line connecting BSP 2 and BSP 3. There is an imbalance of -1 MWh on the single transmission bus and an imbalance of -0.2 MWh at bus 3.

The centralized optimal solution can be described as:

BSP 1: activate 0 MWh at a clearing price of 15 e/MWh.
BSP 2: activate 0.5 MWh at a clearing price of 15 e/MWh.
BSP 3: activate 0.7 MWh at a clearing price of 10 e/MWh.

A decentralized solution of the problem using an RSF proceeds as follows (following the steps of Fig. 1). The aggregation of the DN would need to capture the BSP offers and physics of the DNs, while sharing a minimal amount of information with the TSO. The cheapest distribution BSP, BSP 3, can only deliver 0.5 MWh at the T&D interface because of

TABLE II
SETTLEMENT TABLE OF THE EXAMPLE OF FIG. 2.

	Transmission			Distribution			
Settlements	BSP 1	BRP 1	TSO	ADS	BSP 2	BSP 3	BRP 3
TM-BSP	0 e	-	15 e	+15 e	-	-	-
Quantity (MW)			1	+1			
Price (e/MWh)			15	15			
ADS Dis.	-	-	-	14:5 e	+7:5 e	+7 e	-
Quantity (MW)				1.2	+0.5	+0.7	
Price (e/MWh)				[10;15]	15	10	
TM-BRP	-	15 e	+17 e	-	-	-	2 e
Quantity (MW)		1	+1.2				0.2
Price (e/MWh)		15	[10;15]				10
ADS Rebal.	-	-	2 e	+2 e	-	-	-
Quantity (MW)			0.2	+0.2			
Price (e/MWh)			10	10			
Total	0 e	15 e	0 e	+2:5 e	+7:5 e	+7 e	2 e

the line limit connecting nodes 2 and 3. BSP 2 can produce 1 MWh and make it available at the interface. Additional imbalance can be covered by the most expensive asset, BSP 1. This information is aggregated in the RSF represented in the right-hand side of Fig. 2. By only communicating the RSF, the TSO has no access to grid properties (BSP capacities and line limits). This computation corresponds to 'Step 1' in Fig. 1.

The next step is for the TSO to clear the balancing market ('Step 2' in Fig. 1). In the balancing market, the system imbalance is matched against +1 MWh of the RSF. The TSO then buys 1 MWh from the ADS at the balancing price of 15 e/MWh (which is determined by the ADS offer which is at the money).

In 'Step 3' of the procedure, the ADS then disaggregates the +1 MWh bought by the TSO. The disaggregation consists of a primal and a dual step. In the primal step, the ADS computes the least-cost way in which it can cover the required imbalances. The distribution dispatch activates 0.5 MWh from BSP 2 and 0.7 MWh from BSP 3 to cover the +1 MWh requested by the TSO at the interface and the 0.2 MWh load at bus 3. In the dual / pricing step, the ADS computes LMPs for distribution BSPs. The idea is to compute these prices in a decentralized fashion while preserving consistency with the prices determined in the balancing market. The procedure is explained in detail in section V-B. Intuitively, it relies on treating the export as a variable with a marginal cost determined by the balancing market. This is in contrast to the primal step of the ADS, which treats the export as a fixed parameter.

In Table II, we report the financial position of the different T&D actors according to our proposed market design, which takes place in 'Step 4' of Fig. 1. Rows 'Transmission market BSP' (TM-BSP) and 'ADS disaggregation' (ADS Dis.) correspond to the cash flows of Steps 2 and 3 of Fig. 1. Concerning the BRP settlement, the functioning is slightly different: we assume that the TSO collects the BRP payments (TM-BRP) and then redistributes distribution BRP payments (ADS Rebalancing). The last row reports the payment of each participant. As expected, BSPs and BRPs are remunerated according to their consumption / generation and their LMP. The TSO and the ADS share the congestion rent. The procedure thus reproduces the outcome of a perfectly coordinated transmission-distribution co-optimization in a decentralized fashion.

An important future source of flexibility will be from the residential sector. Gerard [20] outlines how priority service pricing [21] and multilevel demand subscription [22] can be combined with the hierarchical procedure proposed in this paper in order to engage retail consumers in flexibility markets.

III. TSO-DSO COORDINATION SCENARIOS

In this section we illustrate the versatility of our approach by describing how it addresses a number of TSO-DSO coordination scenarios.

A. *Efficiency versus Security*

The example of section II-B illustrates how our proposal ensures the prioritization of system security over economic efficiency. This is exactly what one would expect from a well-functioning operation of the system: economic welfare maximization is desirable, but system constraints need to be respected first.

Concretely, notice how the BSPs that are located in the distribution grid in this example are more economical in terms of offering upward balancing energy. Thus, from the point of view of the TSO it would be preferable to deploy them for resolving negative system imbalance. However, doing so would result in an overload of the distribution line connecting BSP2 to BSP3. This is internalized in the offer that is submitted to the balancing market (the purple part of the supply curve in Fig. 2).

B. *Conflict Resolution*

This scenario is depicted[3] in Fig. 3. BSP2 in this scenario is a resource that can offer both upward as well as downward balancing energy (e.g. a half-charged battery), at the same offer price. This scenario presents a conflict between the TSO and the DSO. BSP2 is less costly than BSP1, thus from the point of view of the TSO, which is responsible for balancing the system, this resource should be activated upward since the total system imbalance is -1 MWh, and thus the system is short. However, from the point of view of the DSO, BSP2 should be activated downward in order to absorb the imbalance in node D, which would otherwise overload the line D-T, which has no remaining capacity in the direction from D to T.

The balancing offers that arrive to the balancing market using the RSF computation procedure are presented in the right part of Fig. 3. Upward balancing bids are presented in the upper right, while downward balancing bids are presented in the lower right. Note that the upward capacity of BSP2 is not present in the upper right, because any export of upward balancing energy from the distribution network is infeasible for the distribution network. The downward offer curve consists of one segment for 0.5 MWh valued at the price cap of

[3]There is a non-trivial step in transforming the data that is actually monitored by network operators in real time (system imbalance and actual network flows) to the data input that is required for our models (nodal imbalances). This transformation is discussed in further detail in [23]. In this paper, we assume that nodal imbalances are measurable in real time. The remaining network capacities depicted in the illustrative examples correspond to the baseline flows before the occurrence of any real-time imbalance.

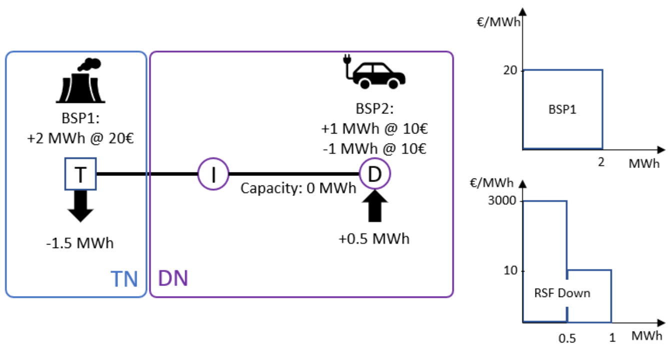

Fig. 3. A scenario where the TSO and the DSO have conflicting interests (left). The upward offers (upper right) and downward offers (lower right) in the balancing market result from the hierarchical computation of an RSF.

the balancing market (for instance, 3,000 e/MWh in this example), and another 0.5 MWh valued at 10 e/MWh. The first segment of the demand curve reflects the fact that the distribution network operator is price inelastic for the first 0.5 MWh of consumption, since the distribution network would otherwise experience an overload.

Notice how the RSF resolves this TSO-DSO conflict by design: the price-taking segment of the RSF ensures that the distribution network will not be overloaded. Using the hierarchical approach, BSP2 turns out being activated downward by 0.5 MWh in order to resolve congestion, and BSP1 is activated upward by 1.5 MWh in order to resolve system imbalance.

C. Remuneration of Mutually Beneficial Actions

An interesting question which emerges in the context of TSO-DSO coordination is whether the TSO or DSO should be liable for remunerating BSPs that offer mutually beneficial services to both the TSO and the DSO. Such a scenario is illustrated in Fig. 4. In both cases considered in this figure, the system is long (and it is the responsibility of the TSO to resolve this issue through the balancing market), but the distribution line is overloaded (and that is a problem that falls under the responsibility of the DSO). The upward activation of BSP2 is beneficial for both the TSO (since it resolves the negative imbalance) as well as the DSO (since it relieves congestion in the T-D direction). Who, then, should remunerate BSP2: the TSO or the DSO?

The hierarchical design highlights that the answer to the question of who pays in the case of a mutually beneficial activation depends on whether the imbalance exceeds the network congestion or not. In the upper right part of Fig. 4, we present the residual supply function for upward balancing energy. It is equal to the price floor for the first 0.5 MWh, because the distribution line is overloaded by 0.5 MWh in the T-D direction, and the DSO is a price taker in exporting as much power as needed to decongest its network. This residual supply function is identical for both the upper and lower left panel of Fig. 4.

The upper left panel represents a case where the system imbalance exceeds the network congestion. The system imbalance crosses the merit order of upward balancing energy at

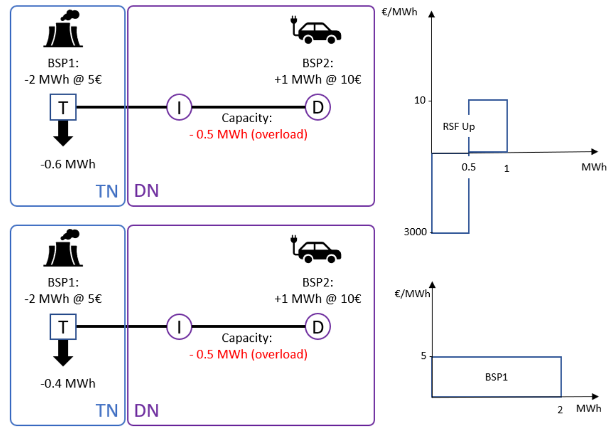

Fig. 4. A scenario where the TSO and the DSO have mutual interests. In the upper left, the network overload does not exceed the system imbalance. In the lower left, the network overload exceeds the system imbalance. The upward offers (upper right) and downward offers (lower right) in the balancing market result from the hierarchical computation of an RSF.

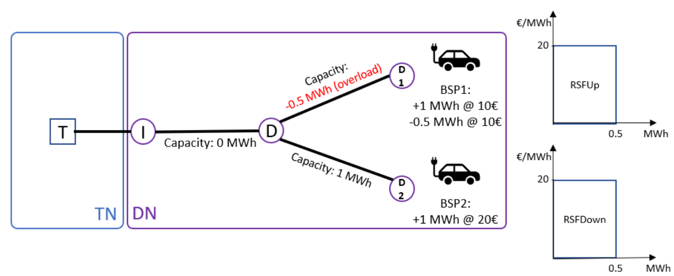

Fig. 5. A "self-healing" scenario where the distribution network resolves its own overload (left). The upward offers (upper right) and downward offers (lower right) in the balancing market result from the hierarchical computation of an RSF.

0.6 MWh, and the price is 10 e/MWh across the network. In this case, the DSO would not be liable for any net payment, since the ADS would pass its payment from the balancing market to BSP2. The lower left panel corresponds to a case where system imbalance is less than the network overload[4]. In this case, the balancing price is set by BSP1 at 5 e/MWh, whereas the disaggregation step of the ADS dispatches BSP2 at 0.5 MWh and sets the distribution price at 10 e/MWh. The DSO then incurs a net payment in order to resolve congestion in its grid.

D. Self-Healing

Figure 5 presents a scenario in which the system is not experiencing any imbalance, but where the distribution network is overloaded at the outset. The distribution network has at its disposal both upward and downward flexibility, and one would expect that a platform should be capable of resolving this overload using locally available resources.

This is precisely what the hierarchical approach achieves. The balancing market receives upward and downward bids that are driven by BSP2. Since the system is balanced and there are no profitable counter-activations of balancing offers,

[4]Despite the fact that this would be rare in practice, it serves to highlight the point of the example.

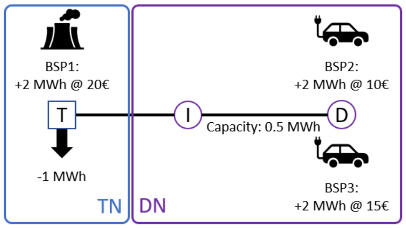

Fig. 6. A scenario which could lead to gaming when distribution system resources are settled at the balancing price. If the balancing price is used for settling distribution system resources, BSP3 engages in a "price war" with BSP2, to the point that the balancing market cannot tell them apart. The hierarchical approach induces BSP3 to not underbid BSP2.

a market-clearing solution is to not match any of the upward and downward bids. The disaggregation step then resolves the overload and produces a local price of 10 €/MWh for D1 and 20 €/MWh for D2. The DSO finds itself paying 10 €, in order to resolve the overload.

E. Recursive Integration in Pan-European Balancing Markets

It is worth noting that a recursive application of the hierarchical coordination concept can be applied for interconnecting national European balancing markets to pan-European balancing platforms such as MARI [19], [24]. This paves the way for distribution system resources to engage actively in pan-European balancing market coordination. A concrete numerical illustration is presented in [23].

The original motivation for considering the application of the hierarchical approach presented in this paper to the context of coupling national markets with MARI is the fact that MARI may be prone to causing overloads due to the fact that it is expected to rely on a transportation network which does not account for internal constraints of participating European Member States [24]. This has been workable in the European market in the context of day-ahead market clearing for years, however the propagation of this misrepresentation of network physics close to real time jeopardizes network security, since little time remains available for re-dispatch with MARI clearing a few minutes before real-time operation. The application of the hierarchical approach to this context restores secure system operation [24], [25].

F. Gaming Incoherent Platform Prices

We conclude this section by highlighting an undesirable effect of ignoring the locational nature of equilibrium prices in TSO-DSO coordination platforms. Whereas inc-dec gaming exploits the arbitrage opportunities that emerge from a sequence of zonal market clearing followed by re-dispatch, this example unfolds in a one-shot auction.

The example is presented in Fig. 6. Let us consider a scenario in which the price disaggregation step of Fig. 1 is skipped, which would imply that distribution system flexibility would be remunerated at the balancing price. This is an option that is under consideration by certain European stakeholders in order to avoid distribution LMPs. Note that the balancing price in this example is set by BSP1 at 20 €/MWh. If this price is used for settling resources in distribution node D, then BSP3 is induced to engage in underbidding BSP2 and vice versa, to the point that the two resources would bid at the price floor. This would imply that the two BSPs become indistinguishable, despite the fact that BSP3 is 50% more costly than BSP2. It also implies that both BSPs have a very strong incentive to produce at their maximum and cause an overload on the line. The hierarchical approach overcomes these perverse incentives by setting a price at D which is determined at the disaggregation step by the marginal cost of the marginal unit. Thus, if BSP3 underbids BSP2, it is matched in the balancing market and incurs a loss since its local price is below its marginal cost. In section VII we report lost opportunity cost as a measure of such perverse incentives.

IV. The Primal Market Clearing Problem

We now focus on tackling realistic TSO-DSO case studies that have been compiled in the context of European Horizon-2020 research projects [26]. These realistic instances, which are based on the national power grids of Italy and Denmark, are large scale and involve non-convex market offers and non-convex distribution network constraints.

The topology of the networks considered in this paper is presented in Fig. 5. We denote by $\mathcal{B} = \mathcal{TB} \cup \mathcal{DB}$ the set of buses which is the union of the set of transmission buses and distribution buses (and similarly, the set of generators $\mathcal{G} = \mathcal{TG} \cup \mathcal{DG}$) and $\mathcal{L} = \mathcal{TL} \cup \{l' = (i', j')\} \cup \mathcal{DL}$ is the set of directed lines. The set of lines is the union of the transmission lines, interconnection lines and distribution lines.

A. Optimal Power Flow Constraints

We first present the market clearing constraints in a single time period.

1) Transmission: We assume that the DC approximation of the power flow equations holds in the TN. We use the $B - \theta$ formulation. The variables of interest are real power injections p, real power flows $f^p_{\mathcal{TL}}$, and voltage angles θ. The transmission constraints are described in compact notation as follows:

$$F_i^{DC}(x^T) = 0, \qquad \forall i \in \mathcal{TB} \tag{1a}$$
$$(f^p_{\mathcal{TL}}, \theta) \in \mathcal{OC}^{DC} \tag{1b}$$
$$p_g \in \mathcal{GC}_g^{DC}, \qquad \forall g \in \mathcal{TG} \tag{1c}$$

where $x^T = (p_{\mathcal{TG}}, f^p_{\mathcal{TL}}, \theta)$. Constraint (1a) describes the transmission real power balance, constraint (1b) describes the operational transmission constraints (typically line limits) and (1c) relates to transmission generation constraints (such as generation capacity for example). The constraints are described in detail in [27].

2) Distribution: We rely on the AC power flow equations for representing the distribution network. We employ the quadratic relaxation [28], [29], which is exact for radial networks and adapts easily for the second-order cone (SOC) relaxation [28]–[32]. The classical formulation of AC-OPF relies on variables (p, q, f, v, θ): real power injection, reactive power injection, (real and reactive) flows, voltage magnitude and voltage angle. By introducing c and s variables (as $c_{ij} = v_i v_j \cos(\theta_i - \theta_j)$ and $s_{ij} = -v_i v_j \sin(\theta_i - \theta_j)$), we can derive the quadratic formulation [31]. Using analogous notation to Eqs. (1), distribution constraints are described as:

$$F_i(x^D) = 0, \qquad \forall i \in \mathcal{DB} \tag{2a}$$
$$(c, s, f^p_{\mathcal{DL}}, f^q) \in \mathcal{OC} \tag{2b}$$
$$(p_g, q_g) \in \mathcal{GC}_g, \qquad \forall g \in \mathcal{DG} \tag{2c}$$

where $x^D = (p_{\mathcal{DG}}, q, c, s, f^p_{\mathcal{DL}}, f^q)$. Constraint (2a) describes the distribution real and reactive power balance, constraint (2b) describes the operational distribution constraints (typically line and voltage limits) and (2c) relates to distribution generation constraints. When using the SOC relaxation later in the paper, we relax the operational constraints $\mathcal{OC}$ and use the notation $\mathcal{OC}^{SOC}$. Details can be found in [27].

3) Interconnection: At the interface, we assume (without loss of generality[5]) that there is a lossless line defined by a single variable $f^p_{l^\theta}$. For ease of modeling later in the paper, we duplicate this variable in both networks by introducing $f^{p,T}_{l^\theta}$ and $f^{p,D}_{l^\theta}$. The interconnection constraints are then:

$$f^p_{l^\theta} = f^{p,T}_{l^\theta},\ f^p_{l^\theta} = f^{p,D}_{l^\theta},\ \underline{f}^p_{l^\theta} \le f^p_{l^\theta} \le \overline{f}^p_{l^\theta}$$

B. Non-Convex Offers

The bids that we consider in this paper are based on the market product specifications of the SmartNet project (section 3 of [33], see also [34]). They thus obey the logic of portfolio bidding, similar to the bids used in the single day-ahead European market coupling model [35]. Note that the European real-time market such as MARI also foresees "indivisible" bids.

In order to incorporate complex bids, we introduce binary variables y_g for each generator g. The set $\mathcal{GC}_g$ is now extended and can incorporate the following binary constraints:

$$(p_g, q_g, y_g) \in \mathcal{GC}_g,\ y_g \in \{0,1\}^{d_g}, \quad \forall g \in \mathcal{DG}$$

The set $\mathcal{GC}_g$ can incorporate various features. These include linked bids, exclusive acceptance, minimum acceptance duration, and so on. These bids therefore introduce binary variables and inter-temporal constraints. The same holds for $\mathcal{GC}^{DC}_g$. For more details concerning the bid structure and a precise mathematical formulation, the reader is referred to [34].

C. The Integrated T&D Real-Time Market Clearing Model

Before writing the complete problem, note that our model is multi-period on a time horizon $\mathcal{T} = \{1, \ldots, t_f\}$. In a multi-period setting, the power balance constraints and operational constraints must be satisfied at every time-step and generation constraints involve time-coupling (such as ramp constraints). The details of the bid and coupling constraints are provided in [34]. We make use of the subscript t to refer to a certain time period. The formulation of the integrated T&D market clearing problem is as follows:

$$\min_{(x,y,f^p_{l^\theta})} \quad \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} C_{g,t} p_{g,t} \tag{3a}$$
$$s.t. \quad F^{DC}_{i,t}(x^T_t) = 0,\ (\lambda_{i,t}) \quad \forall i \in \mathcal{TB},\ t \in \mathcal{T} \tag{3b}$$
$$(\theta_t, f^p_{\mathcal{TL},t}) \in \mathcal{OC}^{DC}, \quad \forall t \in \mathcal{T} \tag{3c}$$
$$(p_g, y_g) \in \mathcal{GC}^{DC}_g, \quad \forall g \in \mathcal{TG} \tag{3d}$$
$$f^p_{l^\theta,t} = f^{p,T}_{l^\theta,t}, \quad \forall t \in \mathcal{T} \tag{3e}$$
$$f^p_{l^\theta,t} = f^{p,D}_{l^\theta,t}, \quad \forall t \in \mathcal{T} \tag{3f}$$
$$\underline{f}^p_{l^\theta,t} \le f^p_{l^\theta,t} \le \overline{f}^p_{l^\theta,t}, \quad \forall t \in \mathcal{T} \tag{3g}$$
$$F_{i,t}(x^D_t) = 0,\ (\lambda_{i,t}) \quad \forall i \in \mathcal{DB},\ t \in \mathcal{T} \tag{3h}$$
$$(c_t, s_t, f^p_{\mathcal{DL},t}, f^q_t) \in \mathcal{OC}, \quad \forall t \in \mathcal{T} \tag{3i}$$
$$(p_g, q_g, y_g) \in \mathcal{GC}_g, \quad \forall g \in \mathcal{DG} \tag{3j}$$
$$y_g \in \{0,1\}^{d_g}, \quad \forall g \in \mathcal{G} \tag{3k}$$

This problem is a mixed integer non-linear problem and its linear relaxation is non-convex. Integrality results from the introduction of binary variables y. In addition to binary variables, non-convexities originate from constraint (3i) which models power flow equations in the DN. Inter-temporal constraints appear in two constraints: (3d) and (3j). Note that, even if we do not explicitly define them in problem (3), slack variables are introduced and are penalized at a high cost.

V. Pricing

Model (3) constitutes the primal dispatch problem. Our goal is to develop a platform that, in addition to dispatch decisions, also computes market clearing prices that can support a competitive equilibrium. Since the primal problem is non-convex (in both the network model as well as the market orders), there is no guarantee that such uniform prices exist. The approach that we develop relies on O'Neill pricing [36] and is detailed in section VI. The general idea of O'Neill pricing is to (i) solve a market problem with binary and continuous variables, (ii) fix the binary variables to their optimal values, and (iii) solve the resulting continuous problem and deduce prices as dual variables of the constraints of interest. Here, the variables of interest are T&D power balance constraints (3b)–(3h) and prices are denoted by λ (λ^p (resp. λ^q) associated with active (resp. reactive) LMPs).

An LMP $\lambda_{i,t}$ is defined for a specific location $i \in \mathcal{B}$ and a time-period $t \in \mathcal{T}$. As a metric of the quality of our derived prices, we introduce lost opportunity costs (LOCs) [37], [38].

A. Lost Opportunity Cost

Computing LOC is a way of ensuring that the primal solution (dispatch variables) and the dual solution (prices) are consistent and fair in the sense of aligning agents' selfish profit-maximizing actions with the market dispatch. We differentiate two types of LOC: one for the generators, and one for the network. Generators' LOC is referred to here as *generator side-payment* and the network LOC is called *potential congestion revenue shortfall* [39]. Since this concept is common in

[5] If the interconnection were to consider losses, the *physical* line could be included in the DSO network while adding an additional *artificial* lossless line to the model, that would correspond to the described interconnection.

electricity markets, we do not repeat the optimization models for computing LOCs. The reader is referred to [27], [39] (section 5.4.1) for the precise definition.

B. Decentralized Computation of Dual Optimal Multipliers

A central challenge of pricing in a decentralized fashion is to ensure spatial price equilibrium between the transmission and distribution network. We explain our approach for computing decentralized prices (or dual optimal multipliers) using the example of section II-B and Fig. 2. The centralized problem can be expressed as follows:

$$\begin{aligned} \min \quad & 20p_1 + 15p_2 + 10p_3 \\ s.t. \quad & p_1 - 1 = f_{12}^p && (\lambda_1) \\ & p_2 = -f_{12}^p + f_{23}^p && (\lambda_2) \\ & p_3 - 0.2 = -f_{23}^p && (\lambda_3) \\ & 0 \leq p_1 \leq 3,\ 0 \leq p_2 \leq 1,\ 0 \leq p_3 \leq 1 && (\eta^-, \eta^+) \\ & -0.5 \leq f_{23}^p \leq 0.5 && (\gamma_{23}^-, \gamma_{23}^+) \end{aligned}$$

The primal optimal decision is $p_1 = 0, p_2 = 0.5, p_3 = 0.7, f_{12}^p = -1, f_{23}^p = -0.5$. The dual optimal decision is $\lambda_1 = 15, \lambda_2 = 15, \lambda_3 = 10, \eta_1^- = 5, \gamma_{23}^- = 5$ (other dual variables are equal to zero). The multipliers λ correspond to the market clearing prices.

In order to compute the LMPs in a decentralized fashion, let us first consider a sub-problem where the interface real power flow decision, f_{12}^p, is fixed to $f_{12}^p = -1$ (obtained from 'Step 2' of Fig. 1):

$$\begin{aligned} \min \quad & 20p_1 && && \text{(4a)} \\ s.t. \quad & p_1 - 1 = f_{12}^p, && (\lambda_1) && \text{(4b)} \\ & 0 \leq p_1 \leq 3, && (\eta_1^-, \eta_1^+) && \text{(4c)} \\ & f_{12}^p = -1 && (\mu_{12}) && \text{(4d)} \end{aligned}$$

The primal solution of this problem is $\hat{p}_1 = 0, \hat{f}_{12}^p = -1$, and is consistent with the centralized solution. From the KKT conditions of this sub-problem, we can infer the following:

$$\eta_1^+ = 0, \quad 20 + \lambda_1 + \eta_1^- = 0, \quad \lambda_1 + \mu_{12} = 0, \ \eta_1^- \geq 0$$

This system of equations has 4 variables and 3 equality constraints which results in a non-unique dual solution. Although the centralized prices *can* form a solution to this KKT system, there are also other solutions which deviate from the price equilibrium of the fully coordinated problem. For example, solving problem (4) with Mosek leads to $\lambda_1^0 = 20, \mu_{12}^0 = -20, \eta_1^{-0} = 0, \eta_1^{+0} = 0$. In particular, the price cleared at bus 1 ($\lambda_1^0 = 20$) is different from the centralized price ($\lambda_1 = 15$).

The above discussion demonstrates that fixing the interface flow in the pricing sub-problem can result in inconsistent prices. Instead, what we propose is to solve a pricing sub-problem which treats interface flow as a variable, the marginal cost of which is equal to that determined by step 2 of Fig. 1, denoted as $\hat{\mu}_{12} = 15$€/MWh:

$$\begin{aligned} \min \quad & 20p_1 - 15f_{12}^p && && \text{(5a)} \\ s.t. \quad & p_1 - 1 = f_{12}^p, && (\lambda_1) && \text{(5b)} \\ & 0 \leq p_1 \leq 3, && (\eta_1^-, \eta_1^+) && \text{(5c)} \end{aligned}$$

This problem yields the same primal solution as (4) (even if this is not the case in general). Crucially, from the KKT conditions of this model we can conclude the following:

$$\eta_1^+ = 0, \quad 20 + \lambda_1 + \eta_1^- = 0, \quad \lambda_1 + 15 = 0, \ \eta_1^- \geq 0$$

The same LMPs as the ones obtained centrally are obtained in a decentralized fashion. The same idea can be applied to the distribution problem, in order to obtain the distribution dispatch decisions and LMPs. These ideas form the basis of our reasoning for Algorithm 1. The procedure is described and analyzed in detail in [27].

VI. Proposed Market Clearing Algorithm

We now describe the application of the procedure of Fig. 1 in our large-scale realistic models. We detail the different steps in the following paragraphs.

a) Computing the RSF (Step 1): As explained in section II-B, the RSF corresponds to the gradient of the least cost at which a given amount of real power can be exported from a DN to the T&D interface. In order to compute the RSF, we solve distribution problems for different levels of real power flow exports. Note that, in principle, the RSF is $|T|$-dimensional. Indeed, the total cost of export depends on the flow level *for every* time period of the considered horizon. By contrast, real-time energy markets admit one-dimensional bids (a marginal cost curve for each market time unit). In order to overcome this issue, we rely on the fact that the interface flow may not vary drastically from one time period to another. We therefore consider the projection of the total cost function at the vector of equal exports for all time intervals, when computing the RSF.

Concretely, we compute the RSF on N equally spaced points $E_{l'}^n$ (time-independent) between the lower and upper interconnection flow limits ($E_{l'}^1 = \underline{f}_{l'}^p$ and $E_{l'}^N = \overline{f}_{l'}^p$). In order to derive the associated value of the RSF, $\hat{\mu}_{l',t}$, which is time dependent, we solve the following SOCP problem:

$$\begin{aligned} \min \quad & \sum_{g \in DG} \sum_{t \in T} C_{g,t} p_{g,t} && \text{(6a)} \\ s.t. \quad & \text{(3f)} - \text{(3h)},\ \text{(3j)} && \text{(6b)} \\ & (c_t, s_t, f_{DL,t}^p, f_t^q) \in OC^{SOC}, \ \forall t \in T && \text{(6c)} \\ & f_{l',t}^p = E_{l'}^n, \quad (\mu_{l',t}) \quad \forall t \in T && \text{(6d)} \\ & y_g \in [0,1]^{d_g}, \quad \forall g \in DG && \text{(6e)} \end{aligned}$$

This problem provides a primal-dual solution. Using a sensitivity argument, the RSF slope $\hat{\mu}_{i'j',t}^n$ is obtained as the dual optimal value associated with constraint (6d) for real power flow level $E_{i'j'}^n$. Note that problem (6) relies on the SOCP relaxation of the OPF constraints as well as a continuous relaxation of the binary variables y.

b) Transmission market clearing (Step 2): Once the RSF is computed, it is explicitly bid into the transmission market clearing model. This results in the following model for step 2:

$$\min \quad \sum_{g \in TG} \sum_{t \in T} C_{g,t} p_{g,t} + \sum_{t \in T} \sum_{n=1}^{N} \hat{\mu}^n_{l',t} f^{p,n}_{l',t} \tag{7a}$$

$$s.t. \quad (3b) - (3e) \tag{7b}$$

$$f^p_{l',t} = \sum_{n=1}^{N} f^{p,n}_{l',t}, \quad \forall t \in T \tag{7c}$$

$$E^{n-1}_{l'} \leq f^{p,n}_{l',t} \leq E^n_{l'}, \ \forall n = 2, \ldots, N, \ t \in T \tag{7d}$$

$$y_g \in \{0,1\}^{d_g}, \quad \forall g \in TG \tag{7e}$$

Note, therefore, that distribution system resources are not bid explicitly into the wholesale market. They are rather aggregated into the RSF, which is computed by the ADS (or DSO), since its derivation requires information about the DN. Note also that, since the interconnection is part of the DN, constraint (3g) is not explicitly expressed in (7) but should be captured by the RSF.

From (7), we deduce binary variables $\hat{y}_g, g \in TG$ for transmission system resources. After fixing the binary variables, we solve the following LP:

$$\min \quad \sum_{g \in TG} \sum_{t \in T} C_{g,t} p_{g,t} + \sum_{t \in T} \sum_{n=1}^{N} \hat{\mu}^n_{l',t} f^{p,n}_{l',t} \tag{8a}$$

$$s.t. \quad (3b) - (3c), \ (3e), \ (7d) \tag{8b}$$

$$(p_g, \hat{y}_g) \in GC^{DC}_g, \quad \forall g \in TG \tag{8c}$$

$$f^p_{l',t} = \sum_{n=1}^{N} f^{p,n}_{l',t}, \quad (\mu_{l',t}) \quad \forall t \in T \tag{8d}$$

From this problem, we deduce dispatch decisions for the TN $\hat{x}^T$, interface flows $\hat{f}^p_{l'}$, and interface prices $\hat{\mu}$.

Given the solutions of the previous problems, we arrive to a complete binary solution $\hat{y}^T$ as well as a primal solution $\hat{x}^T$ for the TN. The pricing step aims at deriving LMPs λ that are coherent with the primal market clearing solution. We refer to coherent prices as prices that keep the LOC as low as possible. For this purpose, we use the idea that is developed in section V-B. Adapting this idea to our context leads to the following TN subproblem, which is an LP:

$$\min \quad \sum_{g \in TG} \sum_{t \in T} C_{g,t} p_{g,t} + \sum_{t \in T} \hat{\mu}^n_{i'j',t} f^p_{i'j',t} \tag{9a}$$

$$s.t. \quad (3b) - (3c), \ (7c) - (7d), \ (8c) \tag{9b}$$

The LMPs $\hat{\lambda}^T$ are derived as the dual multipliers of constraints (3b).

c) Disaggregating the distribution decisions (Step 3): Given a target export quantity ($\hat{f}^p_{l'}$ decided by the TSO), the DSO disaggregates this export level to individual distribution resources by solving the following MISOCP:

$$\min \quad \sum_{g \in DG} \sum_{t \in T} C_{g,t} p_{g,t} \tag{10a}$$

$$s.t. \quad (3h), \ (3j), \ (6c) \tag{10b}$$

$$\hat{f}^p_{l',t} = f^{p,D}_{l',t}, \qquad \forall t \in T \tag{10c}$$

$$y_g \in \{0,1\}^{d_g}, \qquad \forall g \in DG \tag{10d}$$

This problem yields distribution binary variables $\hat{y}_g, g \in DG$ and distribution dispatch decisions $\hat{x}^D$. If the distribution dispatch decisions are not physically feasible, the following continuous non-linear problem is solved:

$$\min \quad \sum_{g \in DG} \sum_{t \in T} C_{g,t} p_{g,t} \tag{11a}$$

$$s.t. \quad (3h) - (3i), \ (10c) \tag{11b}$$

$$(p_g, q_g, \hat{y}_g) \in GC_g, \quad \forall g \in DG \tag{11c}$$

Prices are deduced with the same technique as in the TN. We employ the interface price $\hat{\mu}$ in order to value the exports (or imports) by solving the following SOCP:

$$\min \quad \sum_{g \in DG} \sum_{t \in T} C_{g,t} p_{g,t} - \sum_{t \in T} \hat{\mu}^n_{i'j',t} f^p_{i'j',t} \tag{12a}$$

$$s.t. \quad (3g) - (3h), \ (6c), \ (11c) \tag{12b}$$

The LMPs $\hat{\lambda}^D$ are derived as the dual multipliers of constraints (3h).

Overall algorithm: The steps of the overall process are summarized in Algorithm 1. Each step of Fig. 1 corresponds to one or two steps in Algorithm 1.

Algorithm 1 Algorithm of the RSF approach.

1: **Step 1:** Compute the RSF by solving (6) for N flow levels.
2: **Step 2.1:** Solve the transmission system primal problem (7) and derive $\hat{y}_g, \ g \in TG$.
3: **Step 2.2:** Fix the transmission binary decisions and solve (8) to derive $\hat{x}^T$, $f^p_{l'}$ and $\hat{\mu}$.
4: **Step 2.3:** Solve (9) to obtain transmission LMPs $\hat{\lambda}^T$.
5: **Step 3.1:** Solve the distribution system primal problem (10) to derive $\hat{y}_g, \ g \in DG$ and $\hat{x}^D$.
6: **Step 3.2:** If $\hat{x}^D$ is not feasible, solve (11) and update $\hat{x}^D$.
7: **Step 3.3:** Solve (12) to obtain distribution LMPs $\hat{\lambda}^D$.

Parallelization: A number of steps of the approach can be executed in parallel. Step 1 can be parallelized in two ways: each distribution network can compute the RSF independently, and each of the N flow levels can be computed separately. Using the decomposability of the T&D network, Steps 3.1, 3.2 and 3.3 can also be executed in parallel, for each DN separately.

Convergence: Note that our algorithm converges to a *feasible solution* but that there is no guarantee about finding a solution within a certain optimality gap since the problem is mixed integer non-convex and since we limit ourselves to a *single* iteration between the DSO and the TSO (unlike certain iterative decomposition methods [17]).

VII. Numerical Illustration

A. Test Cases

The test cases that we consider were developed in the European project SmartNet (http://smartnet-project.eu/). For

TABLE III
OVERVIEW OF THE ITALIAN AND DANISH TEST CASES

Test Case	`MediumIt`	`Italy1`	`Italy2`	`Denmark`
jTj	4	3	3	4
jTBj	27	4,236	4,236	209
jDBj	175	1,822	1,822	2,981
# DNs	4	50	50	73
# Bids	1,667	12,318	26,578	25,923
# Binary	4,222	29,660	51,510	90,688
# Variables	25,342	244,052	298,226	391,040
# Constraints	26,401	248,864	310,272	429,931

TABLE IV
COMPARISON OF THE RSF WITH THE CENTRALIZED APPROACHES

Test case	Approach	Objective	Gap (e)	MV	LOC (e)	Time (s)
`MediumIt`	Relaxation	7:872e^{+3}	-	1e^{-6}	0.59	5.39
	Hybrid	7:872e^{+3}	0.01	3e^{-7}	0.53	10.17
	RSF	7:855e^{+3}	8.09	1e^{-7}	20.27	85.90
`Italy1`	Relaxation	1:644e^{+3}	-	2e^{-4}	14.66	85.54
	Hybrid	1:648e^{+3}	3.79	5e^{-7}	14.30	89.23
	RSF	1:781e^{+3}	136.52	9e^{-7}	86.37	662.5
`Italy2`	Relaxation	7:998e^{+3}	-	1e^{-4}	29.68	160.7
	Hybrid	8:001e^{+3}	2.57	4e^{-7}	32.25	166.9
	RSF	8:147e^{+3}	149.22	1e^{-6}	166.01	823.2
`Denmark`	Relaxation	1:023e^{+4}	-	2e^{-6}	30.30	362.6
	Hybrid	9:951e^{+3}	274.92	8e^{-7}	33.41	375.4
	RSF	9:518e^{+3}	707.97	9e^{-7}	592.57	1,340

each of the test cases that we present, we receive as input the topology of the T&D network, the bids associated with the generators at each bus, and a time horizon of 3 or 4 time periods. Given that each market time unit of the European balancing market corresponds to a 15-minute period, the horizon of the problem corresponds to 45 minutes or 1 hour.

We consider networks from the Italian and Danish power systems. For the Italian system, we consider three test cases: a medium-sized instance, called `MediumIt`, and two other instances based on the same network topology, referred to as `Italy1` and `Italy2`. For the Danish instances, we consider only one test case, which we refer to as `Denmark`. Instance `MediumIt` is a medium-sized example, which serves towards validating our approaches, while `Italy1`, `Italy2` and `Denmark` are more realistic instances. An overview of the Italian and Danish test systems is provided in Table III, where jTBj/jDBj are the number of transmission/distribution buses.

B. Examined Approaches

We compare our proposed RSF approach with two extreme cases. In the first case, the DSO-connected flexibility bids are not considered in the TSO market (No-DSO-Bids approach), while in the second case, all the DSO-connected bids are passed over to the TSO market *but* without accounting for the DSO grid constraints (No-DSO-Network approach). These two alternatives, although "extreme", are compatible with the EU market design and are considered by practitioners [24].

We also consider two *centralized* approaches and compare them with the *decentralized* RSF approach. Although not compatible with the institutional separation of DSOs and TSOs, centralized approaches provide an interesting "idealized" benchmark for a computational analysis. The first alternative, referred to as Relaxation, relies on the SOCP relaxation in order to provide dispatch decisions and LMPs. Indeed, in this approach, we solve the MISOCP version of (3) ((3i) is replaced with (6c)) and deduce the primal decisions x and y. Binary variables y are fixed to their optimal value and the remaining SOCP is solved in order to compute the LMPs λ. Note that the Relaxation approach provides a lower bound for problem (3) but does not guarantee AC feasibility in the DN. For this reason, we also consider an alternative centralized approach, namely Hybrid. This method recovers feasibility in the DNs if the dispatch is not AC feasible by solving (11).

C. Comparison of the Approaches

Computational setting: The clearing algorithms are implemented in Julia using JuMP on a MacBook Pro 2016, with a 2.9 GHz Dual-Core Intel Core i5 processor. LPs, SOCPs, MILPs, MISOCPs, are solved using Mosek[6]; NLPs are solved using IPOPT.

Tolerance: We set the feasibility tolerance of the solvers to $\epsilon = 1e\ ^{6}$. When presenting the results, only the Relaxation approach can be violating constraints since it is based on the SOC relaxation. The maximum constraint violation (MV) is then reported in Table IV.

EU Benchmarks: Table V compares the results of the proposed RSF approach with the two extreme cases. The *No-DSO-Bids* approach safeguards the DSO from any network constraint violations since it prevents the TSO from using the DSO-connected resources. This is reflected in the "Slack" column, which measures (in MWh) the degree of network constraint violation. Nevertheless, this approach leads to a higher system cost since the TSO cannot access some of the more economical bids from the DSO grid. On the other hand, the *No-DSO-Network* approach provides full access to the DSO-connected bids to the TSO. Since some of the bids passed to the TSO are not grid-secured, the approach leads to constraint violations (as observed in the Slack column), translating into the DSO having to perform an ex-post redispatch, which harms total cost. While the RSF approach achieves a better trade-off by (i) providing the TSO with the access to DSO welfare enhancing bids (cf. *Cost*) but (ii) in a grid-secure way so as to avoid DSO grid constraints violations (cf. *Slack*). It overall provides more accurate economic signals, highlighted by the lower LOC value in all cases.

Centralized Benchmarks and Computational Analysis: The results are presented in Table IV. From left to right, the columns display the name of the test case, the approach used, the objective value of the primal problem, the absolute gap to the lower bound (obtained using the Relaxation approach), the maximum constraint violation, the LOC, and the solve time in seconds. Note that the RSF is computed with 300 points for all the test systems.

The results on the medium-sized example, `MediumIt`, demonstrate that all the approaches provide similar results in

[6]The motivation for using Mosek is the fact that it was empirically observed to reach solutions to (MI)SOCPs much faster for our target feasibility tolerance (1e^{-6}), while providing competitive performance to that of Gurobi for LPs and MILPs for the instances that were examined in our case studies.

TABLE V
COMPARISON OF THE RSF APPROACH WITH THE EU BENCHMARKS

Test case	Approach	Cost	Slack (MWh)	LOC (e)
MediumIt	No-DSO-Bids	7:730e+3	0:00	11:55e+3
	RSF	7:855e+3	0:00	20:27
	No-DSO-Network	8:360e+3	9:93	176:09
Italy1	No-DSO-Bids	2:646e+3	0:00	969:84
	RSF	1:787e+3	0:00	90:79
	No-DSO-Network	7:908e+3	56:66	3:19e+3
Italy2	No-DSO-Bids	13:606e+3	0:00	5:614e+3
	RSF	8:142e+3	0:00	159:28
	No-DSO-Network	10:939e+3	2:26	3:038e+3
Denmark	No-DSO-Bids	9:347e+3	0:00	937:49
	RSF	10:073e+3	0:11	265:96
	No-DSO-Network	9:443e+3	16:55	1:209e+3

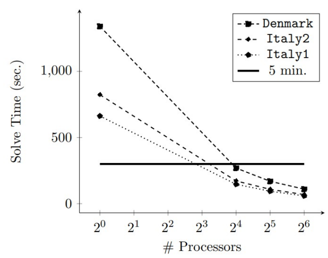

Fig. 7. Impact of parallelization on the solve time of the RSF approach.

TABLE VI
SETTLEMENT TABLE OF THE DENMARK TEST CASE.

	Transmission			Distribution		
Settlements	BSPT	BRPT	TSO	ADS	BSPD	BRPD
TM-BSP	+11;371e	-	8;311e	3;061e	-	-
ADS Dis.	-	-	-	568e	+568e	-
TM-BRP	-	803;062e	+808;713e	-	-	5;651e
ADS Rebal.	-	-	5;651e	+5;651e	-	-
Total	11;371e	803;062e	+794;751e	+2;022e	+568e	5;651e

TABLE VII
POWER PRODUCED AND CONSUMED FOR THE DENMARK TEST CASE.

Transmission		Distribution	
Power injected	Power withdrawn	Power injected	Power withdrawn
19,756 MW	-19,561 MW	550 MW	-723 MW

terms of objective value. For the largest instances (Italy1, Italy2 and Denmark), the objective values of the Relaxation and Hybrid approaches confirm the quality of the SOCP relaxation of the AC-OPF on radial networks. Note, however, that for the Danish test system, even if the Relaxation solution is almost feasible ($MV = 2^{\ 6}$), both Hybrid and RSF provide solutions for which the objective is significantly different. We further note that the solve time for all the approaches is significant for the Danish test case (more than 5 minutes). Executing the RSF sequentially also leads to time-consuming computations. Fig. 7 presents how parallelization can decrease substantially the execution time of the RSF approach[7]. In particular, using 16 processors ensures execution times of less than 5 minutes for the three largest test cases. On the contrary, it is not possible to parallelize centralized schemes. This underlines the potential weaknesses of considering centralized schemes in addition to not preserving privacy.

When analyzing the solution of the RSF approach compared to the Hybrid approach, we observe that in the Italian test cases (Italy1 and Italy2) the RSF approach underestimates what can be withdrawn from the DN. In addition to underestimating what the DN could cover in terms of power balance, one of the assumptions on which we rely is an approximation: indeed, assuming that real power flows at the interface are merely changing from one time period to another leads to inaccuracies. Consequently, some decisions are notably different from those obtained from solving the monolithic model. The improvement of the approximation of the RSF in the multi-period setting will be considered in future research.

Despite the approximation error introduced by the RSF, the difference in performance relative to methods that ensure a feasible dispatch (Hybrid and RSF) is quite small. The same comment applies for LOC: although its value increases when using the RSF approach relative to the centralized approaches, the increase is empirically observed to be negligible.

[7]The estimates presented in the figure are a worst-case approximation, because for steps of the algorithm which require a number of subproblem evaluations that exceed the number of available processors we assume a worst-case balancing, in the sense that the most computationally intensive subproblems are assigned to the same processor.

D. Settlements in the Danish Case Study

We illustrate the cash flows implied by our proposed design by presenting a settlement table that describes the financial position of each market participant. We discuss the insights of the settlement table in the Danish test system (Table VI).

In Table VI, we aggregate the transmission (resp. distribution) BSPs (BSPT resp. BSPD) and BRPs (BRPT resp. BRPD). The total payments are reported in the same way as for the example of section II-B. In this test case, power is mostly flowing from the TN to the different DNs: the ADS is buying power from the TN for a total payment of 3,061 e. The ADS compensates the distribution BSPs when disaggregating. The TSO is collecting payments from BRPs in both transmission and distribution, before reallocating the BRPD payments to the ADS. In total, the TSO and the ADS are collecting congestion revenue, with the TSO revenue being significantly more important. Even if the number of buses is larger in the DN in this test case, we observe that most of the power is consumed and produced in the TN (Table VII), which explains the scale of the revenues.

To understand why the TSO is collecting an important congestion revenue, we draw a box plot representation of the T&D LMPs computed for the first time period ($t = 1$) on the Denmark test case in Fig. 8. When comparing the LMPs of both networks, we notice that even if the lower, middle and upper quartiles are of the same scale, the spread of the transmission LMPs is more important, which indicates significant congestion at the transmission level.

VIII. CONCLUSIONS AND PERSPECTIVES

In this paper, we propose a market design for T&D coordination and we implement the related market clearing platform capable of respecting several technical requirements. These requirements include the consistency with existing EU

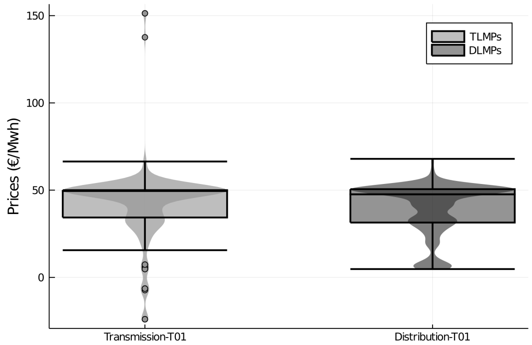

Fig. 8. A box plot representation of the LMPs in the Danish test case.

markets, preserving TSO and the DSO privacy by exchanging only border information, AC feasibility in the DN, clearing of coherent T&D LMPs, decomposability and parallelization.

We test our market design experimentally on large-scale systems. We especially benchmark it against two alternatives currently considered by European practitioners and we show the superiority of our design in terms of both cost minimization and grid constraint compliance. We also benchmark it against an idealized centralized (incompatible with EU markets) benchmark which highlights the satisfactory performance of our approach.

Finally, our work identified a number of potential improvements and open questions which will be considered in future research. (i) The selection of the points of interest on the RSF could possibly be improved through market experience and historical data. (ii) Moreover, in our analysis, the LMPs were cleared using IP pricing while a comparison of different pricing techniques is of interest. For instance, extending [39] by considering large-scale test systems could be an avenue of future research. (iii) The development of the RSF approach in the context of hierarchical balancing in markets with zonal pricing [19] is an additional direction of research, where stakeholder engagement is currently underway. (iv) Finally, the present work assumes truthful bidding. In that respect, modeling the strategic behavior of the agents in order to assess the vulnerability of the market design to gaming opportunities would be another avenue of research.

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References

[1] Frontier Economics (for ENTSO-E), "Review of Flexibility Platforms," 2021, available online [Link].

[2] T. Schittekatte and L. Meeus, "Flexibility Markets: Q&A with Project Pioneers," *Utilities policy*, vol. 63, p. 101017, 2020.

[3] C. Edmunds, S. Galloway, and S. Gill, "Distributed Electricity Markets and Distribution Locational Marginal Prices: A Review," in *2017 52nd International Universities Power Engineering Conference (UPEC)*. IEEE, 2017, pp. 1–6.

[4] Soteria, 2021. [Online]. Available: https://www.ioenergy.eu/soteria/

[5] C. Madina, I. Gómez-Arriola, M. Santos-Mugica, J. Jimeno, K. Kessels, D. Trakas, J. P. Chaves, and Y. Ruwaida, "Flexibility Markets to Procure System Services. CoordiNet Project," in *2020 17th International Conference on the European Energy Market (EEM)*. IEEE, 2020, pp. 1–6.

[6] L. Hirth and I. Schlecht, "Market-Based Redispatch in Zonal Electricity Markets," 2018.

[7] M. Caramanis, E. Ntakou, W. W. Hogan, A. Chakrabortty, and J. Schoene, "Co-Optimization of Power and Reserves in Dynamic T&D Power Markets with Nondispatchable Renewable Generation and Distributed Energy Resources," *Proceedings of the IEEE*, vol. 104, no. 4, pp. 807–836, 2016.

[8] P. M. Sotkiewicz and J. M. Vignolo, "Nodal Pricing for Distribution Networks: Efficient Pricing for Efficiency Enhancing DG," *IEEE Transactions on Power Systems*, vol. 21, no. 2, pp. 1013–1014, 2006.

[9] Y. Ding, S. Pineda, P. Nyeng, J. Ostergaard, E. Larsen, and Q. Wu, "Real-Time Market Concept Architecture for EcoGrid EU - A Prototype for European Smart Grids," *IEEE Transactions on Smart Grid*, vol. 4, no. 4, pp. 2006–2016, 2013.

[10] R. Li, Q. Wu, and S. S. Oren, "Distribution Locational Marginal Pricing for Optimal Electric Vehicle Charging Management," *IEEE Transactions on Power Systems*, vol. 29, no. 1, pp. 203–211, 2014.

[11] R. A. Verzijlbergh, L. J. D. Vries, and Z. Lukszo, "Renewable Energy Sources and Responsive Demand. Do We Need Congestion Management in the Distribution Grid?" *IEEE Transactions on Power Systems*, vol. 29, no. 5, pp. 2119–2128, September 2014.

[12] S. Huang, Q. Wu, S. S. Oren, R. Li, and Z. Lu, "Distribution Locational Marginal Pricing through Quadratic Programming for Congestion Management in Distribution Networks," *IEEE Transactions on Power Systems*, vol. 30, no. 4, pp. 2170–2178, 2015.

[13] B. H. Kim and R. Baldick, "A Comparison of Distributed Optimal Power Flow Algorithms," *IEEE Transactions on Power Systems*, vol. 15, no. 2, pp. 599–604, 2000.

[14] T. Erseghe, "A Distributed Approach to the OPF Problem," *EURASIP Journal on Advances in Signal Processing*, vol. 2015, no. 1, pp. 1–13, 2015.

[15] D. K. Molzahn, F. Dörfler, H. Sandberg, S. H. Low, S. Chakrabarti, R. Baldick, and J. Lavaei, "A Survey of Distributed Optimization and Control Algorithms for Electric Power Systems," *IEEE Transactions on Smart Grid*, vol. 8, no. 6, pp. 2941–2962, 2017.

[16] Z. Li, Q. Guo, H. Sun, and J. Wang, "Coordinated Transmission and Distribution AC Optimal Power Flow," *IEEE Transactions on Smart Grid*, vol. 9, no. 2, pp. 1228–1240, 2016.

[17] K. Sun and X. A. Sun, "A Two-level ADMM Algorithm for AC-OPF with Convergence Guarantees," *IEEE Transactions on Power Systems*, 2021.

[18] A. Papavasiliou and I. Mezghani, "Coordination Schemes for the Integration of Transmission and Distribution System Operations," in *2018 Power Systems Computation Conference (PSCC)*. IEEE, 2018, pp. 1–7.

[19] A. Papavasiliou, M. Bjørndal, G. Doorman, and N. Stevens, "Hierarchical Balancing in Zonal Markets," in *2020 17th International Conference on the European Energy Market (EEM)*. IEEE, 2020, pp. 1–6.

[20] C. Gérard, "Mobilizing Flexible Demand in Electric Power Systems Through Service Quality Differentiation," Ph.D. dissertation, UCL-Université Catholique de Louvain, 2021.

[21] H.-p. Chao and R. Wilson, "Priority Service: Pricing, Investment, and Market Organization," *The American Economic Review*, pp. 899–916, 1987.

[22] H.-p. Chao, S. S. Oren, S. A. Smith, and R. B. Wilson, "Multilevel Demand Subscription Pricing for Electric Power," *Energy economics*, vol. 8, no. 4, pp. 199–217, 1986.

[23] N-SIDE, "Market Approaches for TSO-DSO Coordination in Norway," 2021, available online [Link.

[24] ——, "Study – System Balancing Solutions with Detailed Grid Data – Phase 2 – Core Analysis," 2020, available online [Link.

[25] A. Papalexopoulos, R. Frowd, and A. Birbas, "On the Development of Organized Nodal Local Energy Markets and a Framework for the TSO-DSO Coordination," *Electric Power Systems Research*, vol. 189, p. 106810, 2020.

[26] G. Migliavacca, M. Rossi, D. Six, M. Džamarija, S. Horsmanheimo, C. Madina, I. Kockar, and J. M. Morales, "SmartNet: H2020 Project Analysing TSO–DSO Interaction to Enable Ancillary Services Provision from Distribution Networks," *CIRED-Open Access Proceedings Journal*, vol. 2017, no. 1, pp. 1998–2002, 2017.

[27] I. Mezghani, "Coordination of Transmission and Distribution System Operations in Electricity Markets," Ph.D. dissertation, UCLouvain, 2021, [Link].

[28] A. G. Expósito and E. R. Ramos, "Reliable Load Flow Technique for Radial Distribution Networks," *IEEE Transactions on Power Systems*, vol. 14, no. 3, pp. 1063–1069, 1999.

[29] R. A. Jabr, "Radial Distribution Load Flow using Conic Programming," *IEEE Transactions on Power Systems*, vol. 21, no. 3, pp. 1458–1459, 2006.

[30] M. Farivar and S. H. Low, "Branch Flow Model: Relaxations and Convexification - Part I," *IEEE Transactions on Power Systems*, vol. 28, no. 3, pp. 2554–2564, 2013.

[31] B. Kocuk, S. S. Dey, and X. A. Sun, "Strong SOCP Relaxations for the Optimal Power Flow Problem," *Operations Research*, vol. 64, no. 6, pp. 1177–1196, 2016.

[32] L. Bobo, A. Venzke, and S. Chatzivasileiadis, "Second-Order Cone Relaxations of the Optimal Power Flow for Active Distribution Grids," *arXiv preprint arXiv:2001.00898*, 2020.

[33] G. Leclercq, M. Pavesi, T. Gueuning, A. Ashouri, P. Sels, F. Geth, R. D'hulst, and H. Le Cadre, "Network and Market Models," *SmartNet Deliverable D*, vol. 2, 2019.

[34] I. Mezghani and A. Papavasiliou, "A Mixed Integer Second Order Cone Program for Transmission-Distribution System Co-Optimization," in *2019 IEEE Milan PowerTech*. IEEE, 2019, pp. 1–6.

[35] NEMO Committee, "EUPHEMIA Public Description – Single Price Coupling Algorithm," 2020, [Link].

[36] R. P. O'Neill, P. M. Sotkiewicz, B. F. Hobbs, M. H. Rothkopf, and W. R. Stewart Jr, "Efficient Market-Clearing Prices in Markets with Nonconvexities," *European Journal of Operational Research*, vol. 164, no. 1, pp. 269–285, 2005.

[37] D. A. Schiro, T. Zheng, F. Zhao, and E. Litvinov, "Convex Hull Pricing in Electricity Markets: Formulation, Analysis, and Implementation Challenges," *IEEE Transactions on Power Systems*, vol. 31, no. 5, pp. 4068–4075, 2015.

[38] B. Hua, D. A. Schiro, T. Zheng, R. Baldick, and E. Litvinov, "Pricing in Multi-Interval Real-Time Markets," *IEEE Transactions on Power Systems*, vol. 34, no. 4, pp. 2696–2705, 2019.

[39] M. Garcia, H. Nagarajan, and R. Baldick, "Generalized Convex Hull Pricing for the AC Optimal Power Flow Problem," *IEEE Transactions on Control of Network Systems*, vol. 7, no. 3, pp. 1500–1510, 2020.

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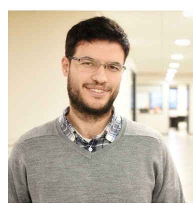

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