A note on a revenue adequate pricing scheme that minimizes make-whole payments

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Abstract— **In this paper, we compare convex hull pricing to an alternative pricing scheme minimizing make-whole payments, in contrast to minimizing lost opportunity costs. We show how to compute prices that minimize make-whole payments using the so-called primal approach proposed for convex hull pricing. Building further on this analogy, a pricing framework is proposed. The framework is general enough to describe virtually all pricing rules proposed so far in the literature or used in industry, including current European market rules, and is meant to ease comparisons. Minimum make-whole payment pricing and convex hull pricing are compared in terms of deviation from a uniform pricing scheme, revenue adequacy for market operators, and computational hardness. Finally, we briefly recall why both pricing methods may encounter challenges in sending adequate price signals in case of congestion or scarcity of supply or demand.**

Index Terms—**Convex hull pricing, make-whole payments, revenue adequacy, day-ahead electricity markets.**

I. Introduction

It is well known that landmark microeconomic theory on the existence of market equilibria relies on key convexity hypotheses, see e.g. [1] or [2], Chapter 3. On the other hand, the liberalization of electricity markets since the 1980s has fostered research on partial near-equilibria in the presence of non-convexities [3,4], which was a requirement in such markets in order to model either technical constraints or cost structures.

Debates around pricing in day-ahead electricity markets regain interest both in the US and in Europe. In Europe, "non-uniform pricing", which is departing from the current European design, has recently been contemplated from an R&D standpoint as a possible future design evolution. This is pointed out in the 2020 CACM Annual Report released in July 2021 [5] and in the Market Coupling Consultative Group meeting of June 2022 [21]. Convex hull pricing and LMP (also named IP Pricing) are among the main pricing schemes that are considered as being in scope when it comes to real-world market design, and the authors have engaged actively in these policy debates.

The context considered in this paper is the following: a Market Operator (MO) is in charge of collecting supply (or offer) and demand bids from market participants. It must then compute a market outcome and decide which quantities will be exchanged, what the electricity market prices are, and what the settlements are, i.e., the money transfers between market parties. Besides impacting the magnitude of side payments required to ensure non-confiscatory or equilibrium outcomes, market prices have their own importance as underlying prices of reference for contingent claims of derivative products such as Financial Transmission Rights or futures.

In this paper, we compare convex hull pricing to an alternative that seeks to minimize make-whole payments, in contrast to minimizing lost opportunity costs. We show how to compute prices that minimize make-whole payments (MMW Pricing, denoted below MMWP) via the primal approach initially proposed for computing Convex Hull Prices (CHP) [6, 7, 8]. The idea relies on a standard MIP Lagrangian duality result [9, 10], and appropriately modifying the feasible sets of market participants at the pricing stage. This primal approach for MMWP admits a clear economic interpretation in terms of convexification corresponding to allowing non-increasing returns to scale (see Debreu's Theory of Value [2] p.40) in the pricing problem. The pricing scheme has associated economic interpretations: a unit earns profits solely thanks to market prices (i.e. without side-payments) if its production capacity is scarce for the system in this convexified context. The topic is further discussed in Section II. Building further on analogies between MMWP and CHP, a pricing and settlement framework is proposed. The framework is general enough to describe virtually all pricing rules proposed so far in the literature or used in industry and is meant to ease comparisons.

MMW Pricing and CHP are compared in terms of deviation from a uniform pricing scheme, revenue adequacy for market operators, and computational hardness. Revenue adequacy means that side payments can always be financed with market payments based on market prices that are charged to market participants, without incurring losses. This prevents any missing money for Market Operators, which is considered an institutionally important requirement in European day-ahead

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markets where power exchange operations are separated from system operation. With MMW Pricing, side payments correspond to non-confiscatory make-whole transfers between market players, minimized by construction. For this reason, MMW Pricing achieves smaller side payments than convex hull pricing. On the other hand, the outcome may be further away from a market equilibrium compared to convex hull pricing.

MMW Pricing is also revenue adequate, while an example illustrates why convex hull pricing is not revenue adequate, though the conditions that drive this example are not likely to occur in practice.

From a computational standpoint, MMW Pricing is shown to be tractable, even when detailed technical and cost characteristics of generation units are simultaneously considered (minimum up and down times, ramp constraints, startup and no-load costs, etc.).

Finally, we discuss why both MMWP and CHP may encounter challenges in sending adequate price signals in case of congestion or scarcity of supply.

The paper is structured as follows. We first introduce MMWP in Section II, highlighting differences with CHP, and showing how prices can be efficiently computed via the primal approach initially proposed for CHP. Corresponding economic interpretations evoked above are also presented in this section. Section III introduces a pricing and settlement framework aimed at allowing a general comparison of pricing rules in non-convex markets (both economic and computational). In this section we also formally discuss revenue adequacy for market operators. We argue that the main pricing rules proposed in the literature can be cast in this framework. Section IV specifically compares MWWP and CHP from a revenue adequacy and computational hardness perspective, and discusses the question of price signals. Finally, Section V concludes the paper.

II. Minimizing Make-whole payments

A. *Make-whole payments versus uplifts*

We consider a welfare maximization program (denoted SWP) described as:

$$\max_{(u,z,x)} \sum_c V_c\,(u_c, z_c, x_c)$$

$$:= \sum_{c,t} (\,P^{c,t} Q_{c,t} x_{c,t} - F_c u_{c,t} - N_{c,t} z_{c,t}) \quad (1)$$

Subject to

$$\sum_c Q_{c,t}\, x_{c,t} = 0 \quad \forall t \in T \quad [\pi_t] \quad (2)$$

$$(u_c, z_c, x_c) \in X_c \quad \forall c \in C \quad (3)$$

For generators, the quantities $Q_{c,t}$ are negative, $V_c < 0$ are the cost functions, and X_c encodes minimum up/down times, ramp constraints and possibly other generator technical characteristics such as time-dependent start-up costs [11]. For demand bids, $Q_{c,t} > 0$, $V_c > 0$ are the utility functions, and X_c can be chosen so as to model a standard stepwise demand curve, or an inelastic demand.

Before discussing Minimum Make-whole Payments Pricing and an appropriate choice for the modification of the feasible sets of market participants, we first underline the difference between make-whole payments and uplifts. Convex hull pricing finds prices π minimizing the sum of the so-called uplifts, defined for each unit or market order c as a function of the market prices π. Uplifts amount to the difference between the best profit that the market participant could have achieved by choosing its own commitment and dispatch decisions given the market prices, and the profit made given the decisions of the market operator. It is computed as:

$$D_c := uplift_{(u_c^*, z_c^*, x_c^*)}(\pi)$$

$$:= \left(\max_{(u_c, z_c, x_c) \in X_c} \left[V_c(u_c, z_c, x_c) - \sum_t \pi_t\, Q_{c,t} x_{c,t} \right] \right)$$

$$- \left(V_c(u_c^*, z_c^*, x_c^*) - \sum_t \pi_t\, V_{c,t} x_{c,t}^* \right) \quad (4)$$

It is shown in [12] that Convex Hull Prices can be obtained by solving the Lagrangian Dual of (1)-(3) where the balance constraints (2) are dualized. In turn, leveraging a well-known MIP Lagrangian Duality result [9, 10], the references [6, 7, 8] show that Convex Hull Prices can be calculated alternatively by solving the primal problem (1)-(3) but where in (3), the sets X_c are replaced by their convex hull, see (5)-(7). We assume here that the sets X_c are non-empty, compact, mixed integer linear sets and that the functions V_c are linear. Concretely, we compute convex hull prices by solving the following problem:

$$\max_{(u,z,x)} \sum_c V_c\,(u_c, z_c, x_c)$$

$$:= \sum_{c,t} (\,P^{c,t} Q_{c,t} x_{c,t} - F_c u_{c,t} - N_{c,t} z_{c,t}) \quad (5)$$

Subject to

$$\sum_c Q_{c,t}\, x_{c,t} = 0 \quad \forall t \in T \quad [\pi_t] \quad (6)$$

$$(u_c, z_c, x_c) \in conv(X_c) \quad \forall c \in C \quad (7)$$

Contrary to uplifts in the case of convex hull pricing, make-whole payments consist in side payments covering only negative profits (i.e. actual losses):

$$MakeWhole_{(u_c^*, z_c^*, x_c^*)}(\pi)$$

$$:= -\min\left[0\,;\left(V_c(u_c^*, z_c^*, x_c^*) - \sum_t \pi_t\, Q_{c,t} x_{c,t}^* \right) \right] \quad (8)$$

Compensating for these losses ensure a non-confiscatory market outcome. The amount of side payments depends on the market prices π. We show in Section IV that whatever these market prices are, compensating only negative profits can always be financed using payments of market players, i.e., the pricing and settlement scheme is revenue adequate for Market Operators. This hence applies to convex hull pricing in case only the negative profit parts of the uplifts are compensated.

B. Minimum Make-whole payments

The following result leads to an interpretation relating prices minimizing make-whole payments to average pricing and the virtual possibility for the Market Operator to uniformly scale down the accepted production of a unit c over all periods by the same factor $k_c \in [0,1]$. Minimizing make-whole payments is obtained by considering, for an order c, the total quantities $Q_{c,t}x^*_{c,t}$ accepted by the Market Operator solving (1)-(3) as the quantities initially offered at a limit price equal to the total costs of production.

$$\max_k \sum_c V_c\,(k_c u^*_c, k_c z^*_c, k_c x^*_c) :=$$

$$\max_k \sum_c k_c \left(\sum_t P^{c,t}\, Q_{c,t} x^*_{c,t} - \sum_t F_c\, u^*_{c,t} - \sum_t N_{c,t}\, z^*_{c,t} \right) \quad (9)$$

$$\sum_c (\,Q_{c,t} x^*_{c,t}) k_c = 0 \quad \forall t \in T \qquad [\pi_t] \qquad (10)$$

$$k_c \le 1 \qquad \forall c \in C \quad [s_c] \qquad (11)$$

$$k_c \ge 0 \qquad \forall c \in C \qquad (12)$$

The basic idea is to rely on (5)-(7) with modified feasible sets for each market participant: $\tilde{X}_c := \{(0,0,0)\} \cup \{(u^*_c, z^*_c, x^*_c)\}$. The solution (u^*_c, z^*_c, x^*_c) is still a welfare maximizing solution for (1)-(3) if the X_c are replaced by $\tilde{X}_c$. Moreover, assuming that $V_c(0,0,0) = 0$ (i.e., that profits/losses of the market player c are null if $(u_c, z_c, x_c) = (0,0,0)$), the uplifts defined in (4) correspond in that case to the make-whole payments defined in (8), because of the definition of the new set $\tilde{X}_c$. The fact that prices π obtained as optimal dual variables of (10) provide prices minimizing make-whole payments then follows from the primal approach discussed above and the fact that $conv(\tilde{X}_c) = \{(k_c u^*_c, k_c z^*_c, k_c x^*_c) \mid 0 \le k_c \le 1\}$.

The convexification of the initial welfare maximization problem consists in allowing the matched quantity of each bid to be arbitrarily scaled down, a property called non-increasing returns to scale in [2], p.40, in case of production.

Let us now consider the dual of (9)-(12):

$$\min_{\pi,s} \sum_c s_c \qquad (13)$$

$$s_c + \sum_t \pi_t\,(Q_{c,t} x^*_{c,t}) \qquad (14)$$

$$\ge (\sum_t P^{c,t}\, Q_{c,t} x^*_{c,t} - \sum_t F_c\, u^*_{c,t} - \sum_t N_{c,t}\, z^*_{c,t}) \quad \forall c \in C \ (15)$$

$$s_c \ge 0 \qquad \forall c \in C \ (16)$$

The dual and complementarity conditions lead to interesting economic interpretations about the pricing properties of this scheme.

Suppose $0 < k_c < 1$, then

$$\sum_t \pi_t\,(Q_{c,t} x^*_{c,t})$$

$$= (\sum_t P^{c,t}\, Q_{c,t} x^*_{c,t} - \sum_t F_c\, u^*_{c,t} - \sum_t N_{c,t}\, z^*_{c,t}).$$

This means that plants the production of which would be uniformly scaled down with production costs scaled down accordingly (if this were allowed) are those making zero profits or losses in the computed market outcome. Only plants that would fully keep their production level for that convexification (i.e., those with $k_c = 1$) are making profits at the market prices. Moreover, those plants receiving make-whole payments are among those that would not produce anything ($k_c = 0$).

III. GENERAL PRICING AND SETTLEMENT FRAMEWORK

The pricing and settlement framework introduced below generalizes both convex hull pricing and the bilevel marginal pricing models considered in [13, 14, 15]. It has been developed independently from the unified approach to pricing with non-convexities proposed in [16]. Both share common characteristics which emerge naturally when pricing in non-convex markets is considered, such as convexifications of the initial allocation problem. The latter reference also informally evokes the idea to modify feasible sets of units at the pricing stage, which can be traced back already to [17]. Compared to the framework presented here, the unified approach in [16] considers constraints defining uplifts that can later be considered in a scalarized multi-objective optimization problem. On the other hand, the framework in [16] is not sufficiently general to consider the current European pricing paradigm or the bilevel marginal pricing models in [13, 14, 15].

Virtually all pricing rules proposed over the last two decades in the literature rely in one way or another on a convexification of the welfare maximization problem (resp. cost minimization problem to meet a specified load). This is considered here by noting that constraint (20) below in the proposed pricing framework (17)-(21) is, if the X_c are compact linear feasible sets, equivalent to asking that prices be obtained as marginal prices of a convexified welfare maximization problem (see [6,7,8] describing the so-called "primal approach" or [9,10] for the original underlying theorems). In (20), the feasible set X_c of each market participant is replaced by a modified version $\widetilde{X_c}(u,z,x)$ that can depend on the values of the primal variables (u,z,x) in the upper optimization problem (see (17)-(21)).

$$\max_{(u,z,x,\pi,\eta)} F(u,z,x,\pi,\eta) \qquad (17)$$

$$\sum_c Q_{c,t}\, x_{c,t} = 0 \qquad \forall t \in T\ [\pi_t] \quad (18)$$

$$(u_c, z_c, x_c) \in X_c \qquad \forall c \in C \qquad (19)$$

$$\pi \in argmin_\pi \begin{bmatrix} \max_{(\widetilde{u_c}, \widetilde{x_c})} \sum_c V_c\,(\widetilde{u_c}, \widetilde{z_c}, \widetilde{x_c}) - \sum_t \pi_t \sum_c Q_{c,t}\, \widetilde{x_{c,t}} \\ s.t.\ (\widetilde{u_c}, \widetilde{z_c}, \widetilde{x_c}) \in \widetilde{X_c}(u,z,x), c \in C \end{bmatrix} (20)$$

$$G(u,z,x,\pi,\eta) \le 0 \qquad (21)$$

Here, F is the objective function of the Market Operator, which can correspond for example to welfare maximization or price minimization, while G is used to model price-based constraints linking upper-level primal variables u, z, x to dual variables π.

Convex hull pricing can be described by setting $G := 0$, $\widetilde{X_c}(u, z, x) := X_c$ and choosing F as the welfare maximizing objective. In that case, as $G = 0$, the lower-level pricing problem is independent from the upper-level optimization problem and can be solved in a second "price stage". MMWP corresponds to $\widetilde{X_c}(u, z, x) := \{(u, z, x),\ (0,0,0)\}$.

IP Pricing (or Locational Marginal Pricing) is obtained by setting $G := 0$, $\widetilde{X_c}(u, z, x) := \{(\widetilde{u_c}, \widetilde{z_c}, \widetilde{x_c}) \in X_c \mid \widetilde{u_c} = \mathrm{u_c}\}$, i.e., the convexification consists in modifying feasible sets by fixing binary variables $\widetilde{u_c}$ to their value $\mathrm{u_c}$ in the upper level (hence in the optimal primal solution), and choosing F as the welfare maximizing objective. Again, as $G = 0$, the pricing stage can be decoupled from the calculations that are required for obtaining the optimal allocations.

Current European market rules applicable to block orders (which cannot be "paradoxically accepted", i.e., lead to negative profits), are special cases of price-based constraints and are typical of current EU markets. These rules are considered for example in [15]. In this case, the optimal allocation as well as the market prices are constrained by $G \neq 0$, and price calculations cannot be fully decoupled from the calculations that are required for obtaining the optimal primal allocation (bid matchings).

Beyond pricing, the market operator calculates settlements resulting from the pricing. These settlements depend on the market prices but also on side payments D_c that can compensate market players, and which are funded by contributions M_c made by market participants themselves. A settlement rule defines the actual payments made (resp. received) by buyers (resp. sellers) to/from the Market Operator, and are decomposed as follows in payments depending on the market price(s) π_t and side payments $D_c(V_c, X_c, Q_c, \pi), M_c(V_c, X_c, Q_c, \pi)$:

- each seller c *is paid* $\sum_t \pi_t\,(-Q_{c,t} x_{c,t}) + D_c - M_c$.
- each buyer c *pays* $\sum_t \pi_t\,(Q_{c,t} x_{c,t}) - D_c + M_c$

with $D_c, M_c \geq 0$. Let us also recall the sign convention according to which $Q_{c,t} > 0$ for buy bids and $Q_{c,t} < 0$ for sell bids.

A settlement rule is non-confiscatory (i.e., ensures cost recovery for market participants) if:

$$\forall c \in C_{sellers}, \sum_t \pi_t\,(-Q_{c,t} x_{c,t}) + D_c - M_c \geq V_c(u_c, z_c, x_c)$$

$$\forall c \in C_{buyers}, \sum_t \pi_t\,(Q_{c,t} x_{c,t}) - D_c + M_c \leq V_c(u_c, z_c, x_c)$$

A settlement rule (π, D, M) is revenue adequate *if it is non-confiscatory - see above - and*:

$$\sum_c D_c \leq \sum_c M_c.$$

This means that one can guarantee a non-confiscatory settlement by relying only on transfers between market parties taking place besides the transfers directly depending on market prices, i.e., without relying on out-of-market payments. Revenue adequacy is hence guaranteed both for market parties and the market operator. We show below with an example that settlements associated with CHP cannot always be revenue adequate in the above sense, while a revenue adequate settlement rule can always be defined if only non-negative profits shall be compensated, whatever is the underlying pricing rule used to compute the market prices of reference.

IV. REVENUE ADEQUACY, COMPUTATIONAL HARDNESS AND PRICE SIGNALS

In this Section, CHP and MWWP are compared from the point of view of revenue adequacy and computational hardness.

A. *Revenue Adequacy*

Example 1 below recalls that CHP is not revenue adequate in the sense described above, and is adapted from [17] to consider a more realistic case where welfare resulting from the primal allocation is strictly positive. Note that the Modified Convex Hull approach proposed in [17] is also not revenue adequate. On the other hand, any price and settlement system that only compensates for negative profits will be revenue adequate, under the assumption that the welfare resulting from the primal allocation is non-negative.

The example is composed of four bids for a single location, in a single-period market. It shows that the sum of the uplifts as defined for classical CHP cannot be financed by payments made by market participants without incurring losses to some of them. Hence, no non-confiscatory settlement rule exists which allows to finance the corresponding side-payments with payments from market participants.

Table 1: Input data of example 1

Bids	*Quantity (MW)*	*Limit price (€/MWh)*	*Min. Acc. Ratio*
A - Buy bid	10	60	-
B - Sell bid	10	10	-
C - Buy bid	50	50	4/5
D - Sell bid	25	20	-

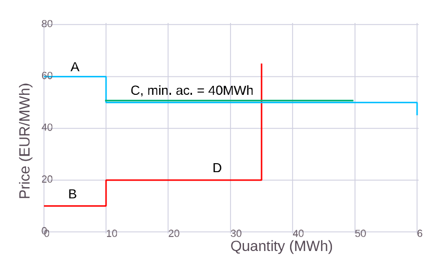

Figure 1. Input data of example 1

The welfare maximizing solution is to fully accept bids A and B. Note that it is impossible to accept bid C as at least $40\ MWh$ should then be accepted, while the total offer is $35\ MWh$. Using the primal approach [6, 7, 8], one can easily show that the Convex Hull price is obtained by considering the continuous relaxation of the welfare maximization problem, and is set by the marginal bid. Here, bid C sets the price at $50€/MWh$. The market outcome is given in Table 2.

Table 2: Results of example 1, showing that CHP is not revenue adequate

Bids	*Accepted Quantity (MW)*	*Uplifts*	*Surplus*
A - Buy bid	10	0	100
B - Sell bid	10	0	400
C - Buy bid	0	0	0
D - Sell bid	0	750	0

Here, bid D requires an uplift D_D of 750 €. However, to ensure a non-confiscatory outcome (no loss incurred to A and B, see definition above), the contribution to side payments M_A from participant A could be at most 100 €, and the contribution from market participant B, M_B, at most 400 €. Hence, it is not possible to define payments and side payments such that the definition of a revenue adequate settlement rule is satisfied.

On the other hand, one can easily show that any settlement rule that compensates only for negative profits will be revenue adequate for market operators. We formalize below this fact.

Consider the following side-payments, which essentially amount to reverting back to a pay-as-bid scheme (recall that D_c corresponds to a side-payment received by the market participant, while M_c corresponds to a contribution to their financing).

$$D_c := -\min\{0; V_c(u_c, z_c, x_c) - \sum_t \pi_t \, Q_{c,t} x_{c,t}\}$$

$$M_c := \max\{0; V_c(u_c, z_c, x_c) - \sum_t \pi_t \, Q_{c,t} x_{c,t}\}$$

One can directly check that, assuming that the total welfare $\sum_c V_c\,(u_c, z_c, x_c)$ is positive or null, $\sum_c D_c \le \sum_c M_c$. In practice, to be as close as possible to a pay-as-clear (i.e., uniform price) scheme, an objective is to minimize the D_c and M_c.

B. *Computational hardness*

We consider in this note 5 medium-size instances with 90 units and a two-node network. The units feature minimum up and down times, as well as ramp constraints. The models and algorithms have been implemented in Julia (version 0.6.2) using the packages JuMP.jl (version 0.18.0) and CPLEX.jl (version 0.2.8), on a computer with an i7-8550u CPU (4 cores @ max 4 GHz) and 16 GB of RAM running on Windows 10, using CPLEX 12.7.1 as the underlying MIP solver. Convex Hull Prices are computed using the compact extended formulation described in [18]. The problem to solve is a large-scale linear program, solved here directly, though efficient decompositions methods can be shown to scale well when applied to this formulation, see [19]. The level method has also been shown to perform well for computing convex hull prices in such a setup [20]. As minimizing make-whole payments requires to solve a small-scale linear program with one variable per unit, balance constraints and bounds on variables, solving the price problem is extremely fast. Runtimes are respectively reported in columns 'Run CHP' and 'Run MMWP'.

Columns 'Uplifts CHP' and 'MW Payments' respectively report the total uplifts in the convex hull pricing case, and the total of make-whole payments when these are specifically minimized. Note that, although make-whole payments (compensating only negative profits) are always smaller than uplifts (which compensate all lost opportunity costs including "missed profits"), they can also define prices leading to larger total lost opportunity costs, i.e., to a more significant deviation from a competitive equilibrium.

Table 3: Orders, prices and settlements in simulations of section IV-B

# inst	# units	# steps in bid curves	Welfare	Run UC	Run CHP (sec.)	Run MMWP (sec.)	Uplifts CHP (€)	MW Paym. (€)
1	90	14309	115630168	12.01	28.26	0.101	26.35	0.00
2	91	13986	107736913	8.04	32.41	0.005	46.31	0.00
3	91	14329	114397755	9.81	222.72	0.004	201.45	0.00
4	92	14594	110146166	9.66	189.24	0.005	1935.97	360
5	89	14370	107351341	7.85	26.80	0.005	1786.78	0.00

C. *Price signals*

In terms of price signals, both CHP and MMWP can lead to situations of locational price differences even in the absence of congestion, as already observed in [7]. From an economic standpoint, this implies that transmission resources receive a strictly positive value (given by locational price differences) even if they are not scarce, leaving arbitrage opportunities. In Europe, interpretability of zonal price differences is an institutionally important topic, especially as zones mostly correspond until today to countries, and price differentiation between countries in the absence of congestion according to the zonal network approximation may not be acceptable. Also, both CHP and MMWP can lead to situations where e.g., demand is curtailed but where the market price doesn't reach the price cap (equal to the limit price of the "price-taking demand" which is curtailed). In these situations, the market price does not necessarily signal the scarcity of supply.

V. CONCLUSIONS

This note has shown how to adapt the primal approach initially proposed for computing convex hull prices, to compute market prices minimizing make-whole payments and discussed related economic interpretations. It has also compared convex hull pricing and the pricing rule minimizing make-whole payments in terms of revenue adequacy and computational hardness. The authors are actively engaged in stakeholder debates on introducing pricing schemes that respect important institutional requirements for EU day-ahead market clearing. These requirements include computational tractability, as well as a variety of business rules that relate to congestion and non-convexities. The consideration of non-uniform pricing in EU market design can result in various computational benefits, and raises numerous novel questions related to pricing and settlement that are currently under investigation, see [5,21].

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