Analytical Derivation of Optimal BSP / BRP Balancing Market Strategies

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1. Introduction

We are interested in proving analytical results on the optimal balancing strategy problem. We focus on analyzing four different designs:

- Design ($D1$): an imbalance price which is equal to the balancing price
- Design ($D2$): introducing the ELIA alpha term to the imbalance price
- Design ($D3$): introducing an ORDC adder to the imbalance price
- Design ($D4$): introducing an ORDC adder to the balancing and imbalance price, and also applying it to reserve imbalance (i.e. introducing a real-time market for reserve capacity)

2. The Model

2.1 Notation

The notation is as follows:

- λ^B: balancing price
- λ^I: imbalance price
- ai: active imbalance with leftover capacity not allocated to balancing auction (can be upward or downward, denoted respectively as ai^U and ai^D)
- qi: agent total imbalance (sum of passive and active imbalance)
- C: marginal cost

- qa: activated energy in balancing auction (denoted qa^+ and qa^- for upward and downward activated capacity, respectively)
- q: bid capacity in balancing auction (denoted q^+ and q^- for upward and downward respectively)

2.2 Sequence of Events

We have a two-stage interaction:

- An agent submits a BSP price-quantity bid in the platform
- An agent observes its imbalance, and decides how much of it to cover
- The TSO observes the system imbalance, activates BSPs, and produces a uniform clearing price.
- The TSO also computes an alpha component, which is added to the balancing price and is charged to BRPs.

The sequence of uncertainty realizations and decisions is described as follows, using MDP terminology:

- Stage 1
  - State: a single element
  - Action: P-Q offers in the balancing platform
  - No reward is collected at this stage
- Stage 2
  - State: (i) the bid price, (ii) the leftover BSP capacity after some capacity has been offered to the MARI auction, and (iii) the level of imbalance of an agent.
  - Action: How much of that imbalance to cover (this action must be limited to the leftover capacity that the BSP has not allocated to the reserve auction)
  - Reward: (i) BSP payment for upward/downward activation, (ii) BRP payment for imbalance settlement, and (iii) fuel costs related to self-balancing and BSP activation.

The realization of uncertainty which transpires when moving from stage 1 to stage 2 is the realization of the agent's portfolio imbalance.

3. Preview of Results

We summarize our main findings here, before presenting the full analysis. We essentially prove that $(D4)$ is the only design that (i) maintains the incentive of agents to bid their entire flexible capacity to the balancing auction, while also (ii) giving an incentive to agents to back-propagate the average scarcity price to day-ahead reserve auctions.

3.1 Design $(D1)$ Preview

It is always optimal for agents to bid their entire balancing capacity at the true marginal cost to the balancing auction. For agents with upward balancing capacity ($P^+ > 0$), the opportunity cost of bidding their capacity to the day-ahead reserve auction is zero. This is a pure strategy Nash equilibrium.

3.2 Design $(D2)$ Preview

In a system with independent and symmetric imbalances, it is optimal for agents to bid their entire balancing capacity at the true marginal cost to the balancing auction. For agents with upward balancing capacity ($P^+ > 0$), the opportunity cost of bidding their capacity to the day-ahead reserve auction is zero. This is a pure strategy Nash equilibrium.

3.3 Design $(D3)$ Preview

It is optimal for *a subset* agents to bid their entire balancing capacity at the true marginal cost to the balancing auction, whereas for a subset of the agents it is optimal to self-balance, and keep their flexible capacity out of the balancing auction. For agents with upward balancing capacity ($P^+ > 0$), the opportunity cost of bidding their capacity to the day-ahead reserve auction is less than or equal to the scarcity value $\mathbb{E}[\lambda^R]$.

This design is depressing the scarcity price in two ways: (i) agents who find it optimal to self-balance face an opportunity cost which is less than the scarcity price $\mathbb{E}[\lambda^R]$, and (ii) agents who find it optimal to bid their entire capacity to the balancing auction face an opportunity cost of zero for bidding reserve in the day ahead.

3.4 Design $(D4)$ Preview

It is always optimal for agents to bid their entire balancing capacity at the true marginal cost to the balancing auction. Agents have an incentive to bid the average scarcity price in the day-ahead reserve auction. This is a pure strategy Nash equilibrium.

4. Analysis

4.1 Generalities

Fringe assumption: Let us consider a fringe supplier, i.e. one with infinitesimal capacity. In this case, the bid price and quantity are in no position to influence the balancing price, and therefore the imbalance price.

We now demonstrate that there is no loss of generality in considering the case of an agent which has only downward capacity (i.e. $P^+ = 0$ and $P^- < 0$) or the case of an agent which has only upward capacity (i.e. $P^- = 0$ and $P^+ > 0$).

Consider the general case of an agent for which $P^+ > 0$ and $P^- < 0$. Suppose that the agent has offered $q^+ > 0$ of capacity in the balancing auction for upward energy, and $q^- < 0$ of capacity in the balancing auction for downward energy.

The balancing activation payoff is as follows:

$$z_B(\omega) = (\lambda^B - C) \cdot (qa^+(q^+) + qa^-(q^-))$$

The bid quantities obey the following constraints:

$$0 \leq q^+ \leq P^+$$
$$P^- \leq q^- \leq 0$$

Thus, the balancing payoff z_B is separable in q^+ and q^-.

Denote as ai^D the positive active imbalance (downward regulation), and ai^U as the negative active imbalance (upward regulation).

Given a second-stage active imbalance $ai = ai^D - ai^U$ (and an implied imbalance qi), the agent receives an imbalance payoff which is computed as follows:

$$-\lambda^B(\omega) \cdot qi - C \cdot (ai^U - ai^D)$$

with

$$qi = Imb + ai^D - ai^U$$

where Imb is the imbalance of the agent. The convention is that positive imbalance means that a portfolio is consuming more than it is producing.

Note that, the way we have defined the MDP, the imbalance price $\lambda^I(\omega) = \lambda^B(\omega)$ does not depend on qi. In practice, qi will affect the evolution of the imbalance price, based on which a new imbalance qi will be decided by the agent, and so on. Our assumption of focusing on a fringe supplier justifies the assumption of considering $\lambda^B(\omega)$ as not being influenced by the decisions of the agent.

By substituting out the imbalance and considering expectations, the active imbalance optimization is is written as

$$\begin{aligned}
z_I \quad = \quad & \max_{ai^D, ai^U} (\mathbb{E}[\lambda^B] - C) \cdot ai^U + (C - \mathbb{E}[\lambda^B]) \cdot ai^D - \mathbb{E}[\lambda^B \cdot Imb] \\
& ai^U + q^+ \leq P^+ \\
& ai^U \leq P^+ \\
& ai^D - q^- \leq -P^- \\
& ai^D \leq -P^- \\
& ai^D, ai^U \geq 0
\end{aligned}$$

Note that the upward active imbalance ai^U only interacts with the upward capacity bid q^+, and the downward active imbalance ai^D only interacts with the downward bid capacity q^-. Thus, the problem is separable in (ai^U, q^+) and in (ai^D, q^-), insofar as z_I is concerned. And since the payoff z_B is separable in q^+ and q^-, the desired conclusion follows.

In what follows, the imbalance payoff will be computed as follows for agents with $P^+ > 0$ (and therefore $q \geq 0$):

$$\begin{aligned}
& \max_{ai} (\mathbb{E}[\lambda^B] - C) \cdot ai - \mathbb{E}[\lambda^B \cdot Imb] \\
& ai + q \leq P^+ \\
& ai \geq 0
\end{aligned}$$

The imbalance payoff will be computed as follows for agents with $P^- < 0$ (and therefore $q \leq 0$):

$$\begin{aligned}
& \max_{ai} (C - \mathbb{E}[\lambda^B]) \cdot ai - \mathbb{E}[\lambda^B \cdot Imb] \\
& ai - q \leq -P^- \\
& ai \geq 0
\end{aligned}$$

4.2 Design $(D1)$

We start by fixing the bid (p, q) in the balancing market. Under the fringe assumption, we can ignore the influence of the agent decisions ai and the agent imbalance on the expected imbalance price. In the following calculations, we denote $D \triangleq -\mathbb{E}[\lambda^B \cdot Imb]$. This is not affected by the actions of the agent, and is therefore a constant offset to z_I.

We have two possible suppliers: (i) the ones for which $\mathbb{E}[\lambda^B] \geq C$, and (ii) the ones for which $\mathbb{E}[\lambda^B] < C$.

4.2.1 $(D1)$: $\mathbb{E}[\lambda^B] - C \geq 0$, $P^+ > 0$, $P^- = 0$

The imbalance payoff will be computed as follows for agents with $P^+ > 0$ (and therefore $q \geq 0$):

$$\begin{aligned}
&\max_{ai} (\mathbb{E}[\lambda^B] - C) \cdot ai - \mathbb{E}[\lambda^B \cdot Imb] \\
&ai + q \leq P^+ \\
&ai \geq 0
\end{aligned}$$

We have $ai^\star = P^+ - q$. The expected payoff z_I is then expressed as follows:

$$z_I \quad = \quad (\mathbb{E}[\lambda^B] - C) \cdot (P^+ - q) + D$$

The balancing payoff z_B can be expressed as follows:

- If $p > \lambda^B$, then $z_B(\omega) = 0$
- If $p = \lambda^B$, then $z_B(\omega) = (\lambda^B - C) \cdot qa$ for some qa which is selected by the auctioneer. We get rid of this case by assuming that the auctioneer always activates zero MW of the supplier when the bid is at the money. Since this is a fringe supplier, the auctioneer can always source the imbalance energy from alternative suppliers. Thus, we have $qa = 0$ and $z_B = 0$ in this case.
- If $p < \lambda^B$, then $z_B(\omega) = (\lambda^B - C) \cdot q$.

The realization ω corresponds to the realization of system imbalance. Note that $z_B(\omega)$ is random. In fact, the distribution of λ^B depends on the decisions of the agent, p and q. In the sequel, we denote the probability measure of the balancing price λ^B as μ.

The expected payoff can therefore be expressed as follows:

$$\begin{aligned} z_B &= \mathbb{E}[z_B(\omega)] \\ &= \int_{x>p} (x - C) \cdot q \cdot d\mu(x) \end{aligned}$$

The overall payoff of the agent can therefore be expressed as follows:

$$\begin{aligned} R(p,q) &= z_I + z_B \\ &= C_1 - C_2 \cdot q + C_3(p) \cdot q \end{aligned}$$

where the terms can be described as follows:

$$\begin{aligned} C_1 &= (\mathbb{E}[\lambda^B] - C) \cdot P^+ + D \\ C_2 &= \mathbb{E}[\lambda^B] - C \\ C_3(p) &= \int_{x>p} (x - C) \cdot d\mu(x) \end{aligned}$$

In order to determine the optimal bidding strategy, let us first fix the bid quantity q of the agent. We can express the first-order conditions with respect to p as:

$$\begin{aligned} \frac{\partial R(p,q)}{\partial p} &= C_3'(p) \cdot q \\ &= -\mu(p) \cdot (p - C) \cdot q \end{aligned}$$

We note that the payoff function $R(p,q)$ for fixed q is increasing in $(-\infty, C]$, zero at C, and decreasing in $[C, +\infty)$. Thus, for any q, an optimal strategy is to bid the true cost. And, given this strategy, the payoff becomes

$$R(C,q) = C_1 - C_2 \cdot q + C_3(C) \cdot q$$

We have

$$\begin{aligned} \frac{\partial R(C,q)}{\partial q} &= -C_2 + C_3(C) \\ &= -(\mathbb{E}[\lambda^B] - C) + C_3(C) \\ &= -(\int_{x \leq C} (x - C) \cdot d\mu(x) + \int_{x>C} (x - C) \cdot d\mu(x)) \\ &\quad + \int_{x>C} (x - C) \cdot d\mu(x) \\ &> 0 \end{aligned}$$

Therefore, it is optimal to bid $q^\star = P^+$ in the balancing auction, and $ai^\star = 0$. This reflects the fact that, when being in active imbalance, the agent takes the risk of producing power when being out of the money. Instead, the balancing market will only activate the agent when its marginal cost is lower than the balancing price. The fact that the balancing and imbalance price are equal sends the correct incentive to the agent for bidding its entire capacity to the balancing auction.

Note that every MW cleared in a forward reserve auction comes with an obligation to bid that MW in the balancing auction, so this is profit lost in the balancing and imbalance phase. Since the optimal strategy of the agent is to anyways bid its entire capacity in the balancing auction, there is no opportunity cost for the agent, the reserve price at which the agent would bid is zero.

4.2.2 $(D1)$: $\mathbb{E}[\lambda^B] - C \geq 0$, $P^+ = 0$, $P^- < 0$

The imbalance payoff will be computed as follows for agents with $P^- < 0$ (and therefore $q \leq 0$):

$$\max_{ai} (C - \mathbb{E}[\lambda^B]) \cdot ai - \mathbb{E}[\lambda^B \cdot Imb]$$
$$ai - q \leq -P^-$$
$$ai \geq 0$$

Since $\mathbb{E}[\lambda^B] - C \geq 0$, we have $ai^\star = 0$. The expected payoff z_I is then expressed as follows:

$$z_I = D$$

The expected balancing payoff can be expressed as follows:

$$\begin{aligned} z_B &= \mathbb{E}[z_B(\omega)] \\ &= \int_{x<p} (x - C) \cdot q \cdot d\mu(x) \end{aligned}$$

The overall payoff of the agent can therefore be expressed as follows:

$$\begin{aligned} R(p, q) &= z_I + z_B \\ &= C_1 + C_3(p) \cdot q \end{aligned}$$

where the terms can be described as follows:

$$C_1 = D$$

$$C_3(p) = \int_{x<p} (x - C) \cdot d\mu(x)$$

In order to determine the optimal bidding strategy, let us first fix the bid quantity q of the agent. We can express the first-order conditions with respect to p as:

$$\begin{aligned} \frac{\partial R(p,q)}{\partial p} &= C_3'(p) \cdot q \\ &= \mu(p) \cdot (p - C) \cdot q \end{aligned}$$

We note that the payoff function $R(p, q)$ for fixed q is increasing in $(-\infty, C]$, zero at C, and decreasing in $[C, +\infty)$. Thus, for any q, an optimal strategy is to bid the true cost. And, given this strategy, the payoff becomes

$$R(C, q) = C_1 + C_3(C) \cdot q$$

We further have

$$\frac{\partial R(C,q)}{\partial q} = C_3(C) < 0$$

We conclude that it is optimal to bid P^- in the balancing auction.

4.2.3 $(D1)$: $\mathbb{E}[\lambda^B] - C < 0$, $P^+ > 0$, $P^- = 0$

The imbalance payoff is computed as follows for agents with $P^+ > 0$ (and therefore $q \geq 0$):

$$\begin{aligned} &\max_{ai}(\mathbb{E}[\lambda^B] - C) \cdot ai - \mathbb{E}[\lambda^B \cdot Imb] \\ &ai + q \leq P^+ \\ &ai \geq 0 \end{aligned}$$

For $\mathbb{E}[\lambda^B] - C < 0$, we have $ai^\star = 0$. The expected payoff z_I is then expressed as follows:

$$z_I = D$$

The expected payoff is expressed as follows:

$$\begin{aligned} z_B &= \mathbb{E}[z_B(\omega)] \\ &= \int_{x>p} (x - C) \cdot q \cdot d\mu(x) \end{aligned}$$

The overall payoff of the agent can therefore be expressed as follows:

$$\begin{aligned} R(p,q) &= z_I + z_B \\ &= C_1 + C_3(p) \cdot q \end{aligned}$$

where the terms can be described as follows:

$$\begin{aligned} C_1 &= D \\ C_3(p) &= \int_{x>p} (x - C) \cdot d\mu(x) \end{aligned}$$

In order to determine the optimal bidding strategy, let us first fix the bid quantity q of the agent. We can express the first-order conditions with respect to p as:

$$\begin{aligned} \frac{\partial R(p,q)}{\partial p} &= C_3'(p) \cdot q \\ &= -\mu(p) \cdot (p - C) \cdot q \end{aligned}$$

We note that the payoff function $R(p,q)$ for fixed q is increasing in $(-\infty, C]$, zero at C, and decreasing in $[C, +\infty)$. Thus, for any q, an optimal strategy is to bid the true cost. And, given this strategy, the payoff becomes

$$R(C,q) = C_1 + C_3(C) \cdot q$$

We then have

$$\begin{aligned} \frac{\partial R(C,q)}{\partial q} &= C_3(C) \\ &= \int_{x>C} (x - C) \cdot d\mu(x) \\ &> 0 \end{aligned}$$

We thus conclude that it is optimal to bid P^+ in the balancing auction.

Note that every MW cleared in a forward reserve auction comes with an obligation to bid that MW in the balancing auction, so this is profit lost in the balancing and imbalance phase. Since the optimal strategy of the agent is to anyways bid its entire capacity in the balancing auction, there is no opportunity cost for the agent, the reserve price at which the agent would bid is zero.

4.2.4 $(D1)$: $\mathbb{E}[\lambda^B] - C < 0$, $P^+ = 0$, $P^- < 0$

The imbalance payoff will be computed as follows for agents with $P^- < 0$ (and therefore $q \leq 0$):

$$\max_{ai} (C - \mathbb{E}[\lambda^B]) \cdot ai - \mathbb{E}[\lambda^B \cdot Imb]$$
$$ai - q \leq -P^-$$
$$ai \geq 0$$

Since $\mathbb{E}[\lambda^B] - C < 0$, we have $ai^\star = -P^- + q$. The expected payoff z_I is then expressed as follows:

$$z_I = (\mathbb{E}[\lambda^B] - C) \cdot (P^- - q) + D$$

The expected balancing payoff can be expressed as follows:

$$\begin{aligned} z_B &= \mathbb{E}[z_B(\omega)] \\ &= \int_{x<p} (x - C) \cdot q \cdot d\mu(x) \end{aligned}$$

The overall payoff of the agent can therefore be expressed as follows:

$$\begin{aligned} R(p,q) &= z_I + z_B \\ &= C_1 - C_2 \cdot q + C_3(p) \cdot q \end{aligned}$$

where the terms can be described as follows:

$$\begin{aligned} C_1 &= (\mathbb{E}[\lambda^B] - C) \cdot P^- + D \\ C_2 &= \mathbb{E}[\lambda^B] - C \\ C_3(p) &= \int_{x<p} (x - C) \cdot d\mu(x) \end{aligned}$$

In order to determine the optimal bidding strategy, let us first fix the bid quantity q of the agent. We can express the first-order conditions with respect to p as:

$$\begin{aligned} \frac{\partial R(p,q)}{\partial p} &= C_3'(p) \cdot q \\ &= \mu(p) \cdot (p - C) \cdot q \end{aligned}$$

We note that the payoff function $R(p,q)$ for fixed q is increasing in $(-\infty, C]$, zero at C, and decreasing in $[C, +\infty)$. Thus, for any q, an optimal strategy is to bid the true cost. And, given this strategy, the payoff becomes

$$R(C,q) = C_1 - C_2 \cdot q + C_3(C) \cdot q$$

We have

$$\begin{aligned}
\frac{\partial R(C,q)}{\partial q} &= -C_2 + C_3(C) \\
&= -(\mathbb{E}[\lambda^B] - C) + C_3(C) \\
&= -(\int_{x \leq C} (x - C) \cdot d\mu(x) + \int_{x > C} (x - C) \cdot d\mu(x)) \\
&\quad + \int_{x \leq C} (x - C) \cdot d\mu(x) \\
&< 0
\end{aligned}$$

We conclude that it is optimal to bid P^- in the balancing auction.

4.2.5 Conclusion ($D1$)

We can state that it is always optimal for agents to bid their entire balancing capacity at the true marginal cost to the balancing auction. For agents with upward balancing capacity ($P^+ > 0$), the opportunity cost of bidding their capacity to the day-ahead reserve auction is zero. We have characterized a pure strategy Nash equilibrium.

4.3 Design ($D2$)

Denote $Imb^t \triangleq Imb^s + Imb - ai^U + ai^D$ as the total system imbalance. The alpha penalty will be embedded in the imbalance price:

$$\begin{aligned}
\lambda^I(\omega) &= \lambda^B(\omega) + \alpha(Imb^t) \cdot \mathbb{I}[Imb^t > UI] - \alpha(Imb^t) \cdot \mathbb{I}[Imb^t < LI] \\
&\simeq \lambda^B(\omega) + \alpha(Imb^s) \cdot \mathbb{I}[Imb^s > UI] - \alpha(Imb^s) \cdot \mathbb{I}[Imb^s < LI] \\
&= \lambda^B(\omega) + \alpha^+(\omega) - \alpha^-(\omega) \\
\alpha^+(\omega) &= \alpha(Imb^s) \cdot \mathbb{I}[Imb^s > UI] \\
\alpha^-(\omega) &= \alpha(Imb^s) \cdot \mathbb{I}[Imb^s < LI]
\end{aligned}$$

Here, Imb^s is the imbalance of the rest of the system (that does not include the agent), and $\alpha(x)$ is the alpha surcharge which applies when positive imbalances exceed the level UI, or when negative imbalances go below the level LI.

ELIA has decided to apply $UI = -LI = 150$ MW, and the following formula[1] for alpha:

$$\alpha(Imb^s|Imb^s_{t-1}) = \frac{200}{1+\exp(\frac{450-\frac{|Imb^s|+|Imb^s_{t-1}|}{2}}{65})}.$$

where we consider the imbalance of the previous balancing interval, Imb^s_{t-1}, as a fixed parameter. This is an increasing function of imbalances which lands smoothly to 200 €/MWh.

Note that the ELIA formula is symmetric, in the sense that $\alpha(-y|x) = \alpha(y|x)$ and $\alpha(y|-x) = \alpha(y|x)$. We also have $LI = -UI$.

The conditional expectation for the alpha penalties uses the conditional distribution

$$\nu(y|x) = \frac{\nu(x,y)}{\int_x \nu(x,y)dx}.$$

We then have:

$$\begin{aligned}
\alpha_x^- &= \int_{y<LI} \alpha(y|x)\cdot\nu(y|x)\cdot dy \\
\alpha_x^+ &= \int_{y>UI} \alpha(y|x)\cdot\nu(y|x)\cdot dy
\end{aligned}$$

Let us consider the following assumptions:

- Independent consecutive imbalances
- Symmetric imbalance distribution

We then have:

$$\begin{aligned}
\alpha_x^- &= \int_{y=-\infty}^{LI} \alpha(y|x)\cdot\nu(y)\cdot dy \\
&= -\int_{y'=\infty}^{-LI} \alpha(-y'|x)\cdot\nu(-y')\cdot dy' \\
&= \int_{y'=-LI}^{\infty} \alpha(y'|x)\cdot\nu(y')\cdot dy' \\
&= \int_{y'=UI}^{\infty} \alpha(y'|x)\cdot\nu(y')\cdot dy' \\
&= \alpha_x^+
\end{aligned}$$

[1] ELIA, “Tariffs for maintaining and restoring the residual balance of individual access responsible parties. Period 2020-2023.”.

4.3.1 $(D2)$: $\mathbb{E}[\lambda^B] - C \geq 0,\ P^+ > 0,\ P^- = 0$

The imbalance payoff will be computed as follows for agents with $P^+ > 0$ (and therefore $q \geq 0$):

$$\begin{aligned}
&(\lambda^B(\omega) - C) \cdot ai + \alpha^+(\omega) \cdot ai - \alpha^-(\omega) \cdot ai \\
&-(\lambda^B(\omega) + \alpha^+(\omega) - \alpha^-(\omega)) \cdot Imb = \\
&(\lambda^B(\omega) + \alpha^+(\omega) - \alpha^-(\omega) - C) \cdot ai - (\lambda^B(\omega) + \alpha^+(\omega) - \alpha^-(\omega) - C) \cdot Imb
\end{aligned}$$

The active imbalance optimization can be expressed as follows, when the agent observes x as the imbalance of the preceding interval, and assuming independent imbalances:

$$\begin{aligned}
&\max_{ai}(\mathbb{E}[\lambda^B] + \alpha_x^+ - \alpha_x^- - C) \cdot ai - \mathbb{E}[(\lambda^B + \alpha_x^+ - \alpha_x^-) \cdot Imb] \\
&ai + q \leq P^+ \\
&ai \geq 0
\end{aligned}$$

Under the assumption of independent symmetric imbalances, we have $\alpha_x^+ - \alpha_x^- = 0$, and the analysis reverts to that of $(D1)$.

4.3.2 $(D2)$: $\mathbb{E}[\lambda^B] - C < 0,\ P^+ > 0,\ P^- = 0$

The active imbalance optimization can be expressed as follows, when the agent observes x as the imbalance of the preceding interval, and assuming independent imbalances:

$$\begin{aligned}
&\max_{ai}(\mathbb{E}[\lambda^B] + \alpha_x^+ - \alpha_x^- - C) \cdot ai - \mathbb{E}[(\lambda^B + \alpha_x^+ - \alpha_x^-) \cdot Imb] \\
&ai + q \leq P^+ \\
&ai \geq 0
\end{aligned}$$

Under the assumption of independent symmetric imbalances, we have $\alpha_x^+ - \alpha_x^- = 0$, and the analysis reverts to that of $(D1)$.

4.3.3 $(D2)$: $C - \mathbb{E}[\lambda^B] \geq 0,\ P^+ = 0,\ P^- < 0$

The imbalance payoff will be computed as follows for agents with $P^- < 0$ (and therefore $q \leq 0$):

$$\begin{aligned}
&(C - \lambda^B(\omega)) \cdot ai - (\alpha^+(\omega) - \alpha^-(\omega)) \cdot ai - (\lambda^B(\omega) + \alpha^+(\omega) - \alpha^-(\omega)) \cdot Imb = \\
&(C - \lambda^B(\omega) - \alpha^+(\omega) + \alpha^-(\omega)) \cdot ai - (\lambda^B(\omega) + \alpha^+(\omega) - \alpha^-(\omega)) \cdot Imb
\end{aligned}$$

The active imbalance optimization can be expressed as follows, when the agent observes x as the imbalance of the preceding interval, and assuming independent imbalances:

$$\max_{ai}(C - \mathbb{E}[\lambda^B] - \alpha_x^+ + \alpha_x^-) \cdot ai - \mathbb{E}[(\lambda^B + \alpha_x^+ - \alpha_x^-) \cdot Imb]$$
$$ai - q \leq -P^-$$
$$ai \geq 0$$

Under the assumption of independent symmetric imbalances, we have $\alpha_x^+ - \alpha_x^- = 0$, and the analysis reverts to that of $(D1)$.

4.3.4 $(D2)$: $C - \mathbb{E}[\lambda^B] < 0$, $P^+ = 0$, $P^- < 0$

The active imbalance optimization can be expressed as follows, when the agent observes x as the imbalance of the preceding interval, and assuming independent imbalances:

$$\max_{ai}(C - \mathbb{E}[\lambda^B] - \alpha_x^+ + \alpha_x^-) \cdot ai - \mathbb{E}[(\lambda^B + \alpha_x^+ - \alpha_x^-) \cdot Imb]$$
$$ai - q \leq -P^-$$
$$ai \geq 0$$

Under the assumption of independent symmetric imbalances, we have $\alpha_x^+ - \alpha_x^- = 0$, and the analysis reverts to that of $(D1)$.

4.3.5 Conclusion $(D2)$

Under the assumption of independent symmetric imbalances, we can state that it is always optimal for agents to bid their entire balancing capacity at the true marginal cost to the balancing auction. For agents with upward balancing capacity ($P^+ > 0$), the opportunity cost of bidding their capacity to the day-ahead reserve auction is zero. We have characterized a pure strategy Nash equilibrium.

4.4 Design $(D3)$

We now consider a design in which the imbalance price is the balancing price plus an ORDC adder. We use the same approach as in the previous section:

$$\lambda^I = \lambda^B + \lambda^R.$$

Here, λ^R corresponds to the ORDC adder, which depends on the level of stress in the system, i.e. the amount of leftover capacity in the system.

4.4.1 $(D3)$: $C < \mathbb{E}[\lambda^B + \lambda^R] - \int_{x>C}(x - C) \cdot d\mu(x)$, $P^+ > 0$, $P^- = 0$

The imbalance payoff will be computed as follows for agents with $P^+ > 0$ (and therefore $q \geq 0$):

$$\begin{aligned}
&\max_{ai}(\mathbb{E}[(\lambda^B + \lambda^R)] - C) \cdot ai - \mathbb{E}[(\lambda^B + \lambda^R) \cdot Imb] \\
&ai + q \leq P^+ \\
&ai \geq 0
\end{aligned}$$

We have $ai^\star = P^+ - q$. The expected payoff z_I is then expressed as follows:

$$z_I = (\mathbb{E}[\lambda^B + \lambda^R] - C) \cdot (P^+ - q) + E$$

where $E \triangleq -\mathbb{E}[(\lambda^B + \lambda^R) \cdot Imb]$.

The expected balancing payoff can be expressed as follows:

$$\begin{aligned}
z_B &= \mathbb{E}[z_B(\omega)] \\
&= \int_{x>p} (x - C) \cdot q \cdot d\mu(x)
\end{aligned}$$

The overall payoff of the agent can therefore be expressed as follows:

$$\begin{aligned}
R(p, q) &= z_I + z_B \\
&= C_1 - C_2 \cdot q + C_3(p) \cdot q
\end{aligned}$$

where the terms can be described as follows:

$$\begin{aligned}
C_1 &= (\mathbb{E}[\lambda^B + \lambda^R] - C) \cdot P^+ + E \\
C_2 &= \mathbb{E}[\lambda^B + \lambda^R] - C \\
C_3(p) &= \int_{x>p} (x - C) \cdot d\mu(x)
\end{aligned}$$

We can express the first-order conditions with respect to p as:

$$\begin{aligned}
\frac{\partial R(p, q)}{\partial p} &= C_3'(p) \cdot q \\
&= -\mu(p) \cdot (p - C) \cdot q
\end{aligned}$$

We note that the payoff function $R(p, q)$ for fixed q is increasing in $(-\infty, C]$, zero at C, and decreasing in $[C, +\infty)$. Thus, for any q, an optimal strategy is to bid the true cost. And, given this strategy, the payoff becomes

$$R(C, q) = C_1 - C_2 \cdot q + C_3(C) \cdot q$$

We have

$$\begin{aligned}\frac{\partial R(C,q)}{\partial q} &= -C_2 + C_3(C) \\ &= -(\mathbb{E}[\lambda^B + \lambda^R] - C) + C_3(C) \\ &< 0\end{aligned}$$

Therefore, it is optimal to bid $q^\star = 0$ in the balancing auction. This implies that for agents with low marginal costs[2], the incentive is to self-balance. The opportunity cost of bidding in a reserve auction is equal to $C_2 - C_3(C)$, which can be rewritten as:

$$\begin{aligned}&\mathbb{E}[\lambda^B + \lambda^R] - C - \int_{x>C} (x - C) \cdot d\mu(x) \\ &\mathbb{E}[\lambda^R] + \int (x - C) \cdot d\mu(x) - \int_{x>C} (x - C) \cdot d\mu(x) \\ &\leq \mathbb{E}[\lambda^R]\end{aligned}$$

4.4.2 (*D3*): $\mathbb{E}[\lambda^B + \lambda^R] - \int_{x>C}(x-C) \cdot d\mu(x) \leq C \leq \mathbb{E}[\lambda^B + \lambda^R]$, $P^+ > 0$, $P^- = 0$

The imbalance payoff will be computed as follows for agents with $P^+ > 0$ (and therefore $q \geq 0$):

$$\begin{aligned}&\max_{ai}(\mathbb{E}[(\lambda^B + \lambda^R)] - C) \cdot ai - \mathbb{E}[(\lambda^B + \lambda^R) \cdot Imb] \\ &ai + q \leq P^+ \\ &ai \geq 0\end{aligned}$$

We have $ai^\star = P^+ - q$. The expected payoff z_I is then expressed as follows:

$$z_I = (\mathbb{E}[\lambda^B + \lambda^R] - C) \cdot (P^+ - q) + E$$

where $E \triangleq -\mathbb{E}[(\lambda^B + \lambda^R) \cdot Imb]$.

The expected balancing payoff can be expressed as follows:

$$\begin{aligned}z_B &= \mathbb{E}[z_B(\omega)] \\ &= \int_{x>p} (x - C) \cdot q \cdot d\mu(x)\end{aligned}$$

[2]Note that the condition for this case can be re-expressed as $\int_{x \leq C} C \cdot d\mu(x) < \mathbb{E}[\lambda^B + \lambda^R]$, which is an increasing function of C.

The overall payoff of the agent can therefore be expressed as follows:

$$\begin{aligned} R(p,q) &= z_I + z_B \\ &= C_1 - C_2 \cdot q + C_3(p) \cdot q \end{aligned}$$

where the terms can be described as follows:

$$\begin{aligned} C_1 &= (\mathbb{E}[\lambda^B + \lambda^R] - C) \cdot P^+ + E \\ C_2 &= \mathbb{E}[\lambda^B + \lambda^R] - C \\ C_3(p) &= \int_{x>p} (x - C) \cdot d\mu(x) \end{aligned}$$

We can express the first-order conditions with respect to p as:

$$\begin{aligned} \frac{\partial R(p,q)}{\partial p} &= C_3'(p) \cdot q \\ &= -\mu(p) \cdot (p - C) \cdot q \end{aligned}$$

We note that the payoff function $R(p,q)$ for fixed q is increasing in $(-\infty, C]$, zero at C, and decreasing in $[C, +\infty)$. Thus, for any q, an optimal strategy is to bid the true cost. And, given this strategy, the payoff becomes

$$R(C,q) = C_1 - C_2 \cdot q + C_3(C) \cdot q$$

We have

$$\begin{aligned} \frac{\partial R(C,q)}{\partial q} &= -C_2 + C_3(C) \\ &= -(\mathbb{E}[\lambda^B + \lambda^R] - C) + C_3(C) \\ &> 0 \end{aligned}$$

Therefore, it is optimal to bid $q^\star = P^+$ in the balancing auction. Since the optimal strategy of the agent is to anyways bid its entire capacity in the balancing auction, there is no opportunity cost for the agent, the reserve price at which the agent would bid is zero.

4.4.3 $(D3)$: $C > \mathbb{E}[\lambda^B + \lambda^R]$, $P^+ > 0$, $P^- = 0$

The imbalance payoff is computed as follows for agents with $P^+ > 0$ (and therefore $q \geq 0$):

$$\max_{ai} (\mathbb{E}[(\lambda^B + \lambda^R)] - C) \cdot ai - \mathbb{E}[(\lambda^B + \lambda^R) \cdot Imb]$$

$$
\begin{aligned}
ai + q &\leq P^+ \\
ai &\geq 0
\end{aligned}
$$

We have $ai^\star = 0$. The expected payoff z_I is then expressed as follows:

$$z_I = E$$

The expected payoff is expressed as follows:

$$
\begin{aligned}
z_B &= \mathbb{E}[z_B(\omega)] \\
&= \int_{x>p} (x - C) \cdot q \cdot d\mu(x)
\end{aligned}
$$

The overall payoff of the agent can therefore be expressed as follows:

$$
\begin{aligned}
R(p, q) &= z_I + z_B \\
&= C_1 + C_3(p) \cdot q
\end{aligned}
$$

where the terms can be described as follows:

$$
\begin{aligned}
C_1 &= E \\
C_3(p) &= \int_{x>p} (x - C) \cdot d\mu(x)
\end{aligned}
$$

We express the first-order conditions with respect to p as:

$$
\begin{aligned}
\frac{\partial R(p, q)}{\partial p} &= C_3'(p) \cdot q \\
&= -\mu(p) \cdot (p - C) \cdot q
\end{aligned}
$$

We note that the payoff function $R(p, q)$ for fixed q is increasing in $(-\infty, C]$, zero at C, and decreasing in $[C, +\infty)$. Thus, for any q, an optimal strategy is to bid the true cost. And, given this strategy, the payoff becomes

$$R(C, q) = C_1 + C_3(C) \cdot q$$

We then have

$$
\begin{aligned}
\frac{\partial R(C, q)}{\partial q} &= C_3(C) \\
&= \int_{x>C} (x - C) \cdot d\mu(x) \\
&> 0
\end{aligned}
$$

We thus conclude that it is optimal to bid P^+ in the balancing auction. There is no opportunity cost for the agent, the reserve price at which the agent would bid is zero.

4.4.4 $(D3)$: $\mathbb{E}[\lambda^B + \lambda^R] - C \geq 0$, $P^+ = 0$, $P^- < 0$

The imbalance payoff will be computed as follows for agents with $P^- < 0$ (and therefore $q \leq 0$):

$$\max_{ai}(C - \mathbb{E}[\lambda^B + \lambda^R]) \cdot ai - \mathbb{E}[(\lambda^B + \lambda^R) \cdot Imb]$$
$$ai - q \leq -P^-$$
$$ai \geq 0$$

Since $\mathbb{E}[\lambda^B + \lambda^R] - C \geq 0$, we have $ai^\star = 0$. The expected payoff z_I is then expressed as follows:

$$z_I = E$$

The expected balancing payoff can be expressed as follows:

$$\begin{aligned} z_B &= \mathbb{E}[z_B(\omega)] \\ &= \int_{x<p} (x - C) \cdot q \cdot d\mu(x) \end{aligned}$$

The overall payoff of the agent can therefore be expressed as follows:

$$\begin{aligned} R(p,q) &= z_I + z_B \\ &= C_1 + C_3(p) \cdot q \end{aligned}$$

where the terms can be described as follows:

$$\begin{aligned} C_1 &= E \\ C_3(p) &= \int_{x<p} (x - C) \cdot d\mu(x) \end{aligned}$$

We express the first-order conditions with respect to p as:

$$\begin{aligned} \frac{\partial R(p,q)}{\partial p} &= C_3'(p) \cdot q \\ &= \mu(p) \cdot (p - C) \cdot q \end{aligned}$$

We note that the payoff function $R(p,q)$ for fixed q is increasing in $(-\infty, C]$, zero at C, and decreasing in $[C, +\infty)$. Thus, for any q, an optimal strategy is to bid the true cost. And, given this strategy, the payoff becomes

$$R(C,q) = C_1 + C_3(C) \cdot q$$

We further have

$$\frac{\partial R(C,q)}{\partial q} = C_3(C) < 0$$

We conclude that it is optimal to bid P^- in the balancing auction.

4.4.5 $(D3)$: $\mathbb{E}[\lambda^B + \lambda^R] - C < 0$, $P^+ = 0$, $P^- < 0$

The imbalance payoff will be computed as follows for agents with $P^- < 0$ (and therefore $q \leq 0$):

$$\begin{aligned}
&\max_{ai}(C - \mathbb{E}[\lambda^B + \lambda^R]) \cdot ai - \mathbb{E}[(\lambda^B + \lambda^R) \cdot Imb] \\
&ai - q \leq -P^- \\
&ai \geq 0
\end{aligned}$$

Since $\mathbb{E}[\lambda^B + \lambda^R] - C < 0$, we have $ai^\star = -P^- + q$. The expected payoff z_I is then expressed as follows:

$$z_I = (\mathbb{E}[\lambda^B + \lambda^R] - C) \cdot (P^- - q) + E$$

The expected balancing payoff can be expressed as follows:

$$z_B = \int_{x<p} (x - C) \cdot q \cdot d\mu(x)$$

The overall payoff of the agent can therefore be expressed as follows:

$$\begin{aligned}
R(p,q) &= z_I + z_B \\
&= C_1 - C_2 \cdot q + C_3(p) \cdot q
\end{aligned}$$

where the terms can be described as follows:

$$\begin{aligned}
C_1 &= (\mathbb{E}[\lambda^B + \lambda^R] - C) \cdot P^- + E \\
C_2 &= \mathbb{E}[\lambda^B + \lambda^R] - C \\
C_3(p) &= \int_{x<p} (x - C) \cdot d\mu(x)
\end{aligned}$$

In order to determine the optimal bidding strategy, let us first fix the bid quantity q of the agent. We can express the first-order conditions with respect to p as:

$$\begin{aligned}
\frac{\partial R(p,q)}{\partial p} &= C_3'(p) \cdot q \\
&= \mu(p) \cdot (p - C) \cdot q
\end{aligned}$$

We note that the payoff function $R(p,q)$ for fixed q is increasing in $(-\infty, C]$, zero at C, and decreasing in $[C, +\infty)$. Thus, for any q, an optimal strategy is to bid the true cost. And, given this strategy, the payoff becomes

$$R(C,q) = C_1 - C_2 \cdot q + C_3(C) \cdot q$$

We have

$$\begin{aligned}
\frac{\partial R(C,q)}{\partial q} &= -C_2 + C_3(C) \\
&= -(\mathbb{E}[\lambda^B + \lambda^R] - C) + C_3(C) \\
&= -(\int_{x \le C} (x - C) \cdot d\mu(x) + \int_{x > C} (x - C) \cdot d\mu(x)) \\
&\quad + \int_{x \le C} (x - C) \cdot d\mu(x) - \mathbb{E}[\lambda^R] \\
&< 0
\end{aligned}$$

We conclude that it is optimal to bid P^- in the balancing auction.

4.4.6 Conclusion ($D3$)

We can state that it is sometimes, but not always, optimal for agents to bid their entire balancing capacity at the true marginal cost to the balancing auction. For agents with upward balancing capacity ($P^+ > 0$), the opportunity cost of bidding their capacity to the day-ahead reserve auction is less than or equal to the scarcity value $\mathbb{E}[\lambda^R]$. We have not characterized a pure strategy Nash equilibrium, since some agents find it optimal to self-balance.

This design is depressing the scarcity price in two ways: (i) agents who find it optimal to self-balance face an opportunity cost which is less than the scarcity price $\mathbb{E}[\lambda^R]$, and (ii) agents who find it optimal to bid their entire capacity to the balancing auction face an opportunity cost of zero for bidding reserve in the day ahead.

4.5 Design ($D4$)

For the US design, we have the following additional term in the objective function, which reflects the remuneration for leftover real-time reserve capacity:

$$\lambda^R \cdot (P^+ - qa^+(q^+) - ai^U + ai^D).$$

Moreover, both the balancing price and imbalance price are augmented by the scarcity adder. Thus, we replace λ^B with $\lambda^B + \lambda^R$ and λ^I with $\lambda^B + \lambda^R$ in the following analysis.

Finally, we have the agent buying back its real-time reserve capacity, which is reflected with the following term:

$$-\lambda^R \cdot qa^R$$

The balancing activation payoff is as follows:

$$z_B(\omega) = (\lambda^B + \lambda^R - C) \cdot (qa^+(q^+) + qa^-(q^-)) + \lambda^R \cdot (P^+ - qa^+(q^+))$$

The bid quantities obey the following constraints:

$$0 \leq q^+ \leq P^+$$
$$P^- \leq q^- \leq 0$$

Thus, the balancing payoff z_B is separable in q^+ and q^-.

Given a second-stage active imbalance $ai = ai^D - ai^U$ (and an implied imbalance qi), the agent receives an imbalance payoff which is computed as follows:

$$-(\lambda^B + \lambda^R) \cdot qi - C \cdot (ai^U - ai^D) - \lambda^R \cdot (ai^U - ai^D)$$

with

$$qi = Imb + ai^D - ai^U$$

Note that, the way we have defined the MDP, the reserve price $\lambda^R(\omega)$ does not depend on qi. In practice, qi will affect the evolution of the reserve price, based on which a new imbalance qi will be decided by the agent, and so on. Our assumption of focusing on a fringe supplier justifies the assumption of considering $\lambda^R(\omega)$ as not being influenced by the decisions of the agent.

By substituting out the imbalance and considering expectations, the active imbalance optimization is is written as

$$\begin{aligned} z_I \quad = \quad & \max_{ai^D, ai^U} (\mathbb{E}[\lambda^B] - C) \cdot ai^U + (C - \mathbb{E}[\lambda^B]) \cdot ai^D - \mathbb{E}[(\lambda^B + \lambda^R) \cdot Imb] \\ & ai^U + q^+ \leq P^+ \\ & ai^U \leq P^+ \\ & ai^D - q^- \leq -P^- \\ & ai^D \leq -P^- \\ & ai^D, ai^U \geq 0 \end{aligned}$$

Note that the upward active imbalance ai^U only interacts with the upward capacity bid q^+, and the downward active imbalance ai^D only interacts with the downward bid capacity q^-. Thus, the problem is separable in (ai^U, q^+) and in (ai^D, q^-), insofar as z_I is concerned. And since the payoff z_B is separable in q^+ and q^-, it follows that we can consider the problem separately for the case of upward and downward balancing capacity.

4.5.1 $(D4)$: $\mathbb{E}[\lambda^B] - C \geq 0$, $P^+ > 0$, $P^- = 0$

The imbalance payoff will be computed as follows for agents with $P^+ > 0$ (and therefore $q \geq 0$):

$$\max_{ai} (\mathbb{E}[\lambda^B] - C) \cdot ai - \mathbb{E}[(\lambda^B + \lambda^R) \cdot Imb]$$
$$ai + q \leq P^+$$
$$ai \geq 0$$

We have $ai^\star = P^+ - q$. The expected payoff z_I is then expressed as follows:

$$z_I = (\mathbb{E}[\lambda^B] - C) \cdot (P^+ - q) + E$$

The balancing payoff z_B can be expressed as follows:

- If $p > \lambda^B$ then $qa = 0$, and $z_B(\omega) = \lambda^R \cdot P^+$
- If $p = \lambda^B$, then $z_B(\omega) = (\lambda^B - C) \cdot qa + \lambda^R \cdot (P^+ - qa)$ for some qa which is selected by the auctioneer. We get rid of this case by assuming that the auctioneer always activates zero MW of the supplier when the bid is at the money. Since this is a fringe supplier, the auctioneer can always source the imbalance energy from alternative suppliers. Thus, we have $qa = 0$ and $z_B = \lambda^R \cdot P^+$ in this case.
- If $p < \lambda^B$, then $qa = q$ and $z_B(\omega) = (\lambda^B + \lambda^R - C) \cdot q + \lambda^R \cdot (P^+ - q) = (\lambda^B - C) \cdot q + \lambda^R \cdot P^+$.

The expected payoff can therefore be expressed as follows:

$$\begin{aligned} z_B &= \mathbb{E}[z_B(\omega)] \\ &= \int_{x>p} (x - C) \cdot q \cdot d\mu(x) + \mathbb{E}[\lambda^R] \cdot P^+ \end{aligned}$$

The overall payoff of the agent can therefore be expressed as follows:

$$\begin{aligned} R(p, q) &= z_I + z_B \\ &= C_1 - C_2 \cdot q + C_3(p) \cdot q \end{aligned}$$

where the terms can be described as follows:

$$\begin{aligned} C_1 &= (\mathbb{E}[\lambda^B] - C) \cdot P^+ + \mathbb{E}[\lambda^R] \cdot P^+ + E \\ C_2 &= \mathbb{E}[\lambda^B] - C \end{aligned}$$

$$C_3(p) = \int_{x>p} (x - C) \cdot d\mu(x)$$

We express the first-order conditions with respect to p as:

$$\begin{aligned} \frac{\partial R(p,q)}{\partial p} &= C_3'(p) \cdot q \\ &= -\mu(p) \cdot (p - C) \cdot q \end{aligned}$$

We note that the payoff function $R(p,q)$ for fixed q is increasing in $(-\infty, C]$, zero at C, and decreasing in $[C, +\infty)$. Thus, for any q, an optimal strategy is to bid the true cost. And, given this strategy, the payoff becomes

$$R(C,q) = C_1 - C_2 \cdot q + C_3(C) \cdot q$$

We have

$$\begin{aligned} \frac{\partial R(C,q)}{\partial q} &= -C_2 + C_3(C) \\ &= -(\mathbb{E}[\lambda^B] - C) + C_3(C) \\ &= -(\int_{x \le C} (x - C) \cdot d\mu(x) + \int_{x>C} (x - C) \cdot d\mu(x)) \\ &\quad + \int_{x>C} (x - C) \cdot d\mu(x) \\ &> 0 \end{aligned}$$

Therefore, it is optimal to bid $q^\star = P^+$ in the balancing auction.

4.5.2 $(D4)$: $\mathbb{E}[\lambda^B] - C \ge 0$, $P^+ = 0$, $P^- < 0$

The imbalance payoff will be computed as follows for agents with $P^- < 0$ (and therefore $q \le 0$):

$$\begin{aligned} &\max_{ai} (C - \mathbb{E}[\lambda^B]) \cdot ai - \mathbb{E}[(\lambda^B + \lambda^R) \cdot Imb] \\ &ai - q \le -P^- \\ &ai \ge 0 \end{aligned}$$

Since $\mathbb{E}[\lambda^B] - C \ge 0$, we have $ai^\star = 0$. The expected payoff z_I is then expressed as follows:

$$z_I = E$$

The expected balancing payoff can be expressed as follows:

$$z_B \quad = \quad \int_{x<p} (x - C) \cdot q \cdot d\mu(x) + \mathbb{E}[\lambda^R] \cdot P^+$$

The overall payoff of the agent can therefore be expressed as follows:

$$\begin{aligned} R(p,q) &= z_I + z_B \\ &= C_1 + C_3(p) \cdot q \end{aligned}$$

where the terms can be described as follows:

$$\begin{aligned} C_1 &= E + \mathbb{E}[\lambda^R] \cdot P^+ \\ C_3(p) &= \int_{x<p} (x - C) \cdot d\mu(x) \end{aligned}$$

In order to determine the optimal bidding strategy, let us first fix the bid quantity q of the agent. We can express the first-order conditions with respect to p as:

$$\begin{aligned} \frac{\partial R(p,q)}{\partial p} &= C_3'(p) \cdot q \\ &= \mu(p) \cdot (p - C) \cdot q \end{aligned}$$

We note that the payoff function $R(p,q)$ for fixed q is increasing in $(-\infty, C]$, zero at C, and decreasing in $[C, +\infty)$. Thus, for any q, an optimal strategy is to bid the true cost. And, given this strategy, the payoff becomes

$$R(C,q) = C_1 + C_3(C) \cdot q$$

We further have

$$\frac{\partial R(C,q)}{\partial q} \quad = \quad C_3(C) < 0$$

We conclude that it is optimal to bid P^- in the balancing auction.

4.5.3 $(D4)$: $\mathbb{E}[\lambda^B] - C < 0$, $P^+ > 0$, $P^- = 0$

The imbalance payoff is computed as follows for agents with $P^+ > 0$ (and therefore $q \geq 0$):

$$\max_{ai} (\mathbb{E}[\lambda^B] - C) \cdot ai - \mathbb{E}[(\lambda^B + \lambda^R) \cdot Imb]$$

$$ai + q \leq P^+$$
$$ai \geq 0$$

For $\mathbb{E}[\lambda^B] - C < 0$, we have $ai^\star = 0$. The expected payoff z_I is then expressed as follows:

$$z_I = E$$

The expected payoff is expressed as follows:

$$\begin{aligned} z_B &= \mathbb{E}[z_B(\omega)] \\ &= \int_{x>p} (x - C) \cdot q \cdot d\mu(x) + \mathbb{E}[\lambda^R] \cdot P^+ \end{aligned}$$

The overall payoff of the agent can therefore be expressed as follows:

$$\begin{aligned} R(p, q) &= z_I + z_B \\ &= C_1 + C_3(p) \cdot q \end{aligned}$$

where the terms can be described as follows:

$$\begin{aligned} C_1 &= E + \mathbb{E}[\lambda^R] \cdot P^+ \\ C_3(p) &= \int_{x>p} (x - C) \cdot d\mu(x) \end{aligned}$$

In order to determine the optimal bidding strategy, let us first fix the bid quantity q of the agent. We can express the first-order conditions with respect to p as:

$$\begin{aligned} \frac{\partial R(p, q)}{\partial p} &= C_3'(p) \cdot q \\ &= -\mu(p) \cdot (p - C) \cdot q \end{aligned}$$

We note that the payoff function $R(p, q)$ for fixed q is increasing in $(-\infty, C]$, zero at C, and decreasing in $[C, +\infty)$. Thus, for any q, an optimal strategy is to bid the true cost. And, given this strategy, the payoff becomes

$$R(C, q) = C_1 + C_3(C) \cdot q$$

We then have

$$\begin{aligned} \frac{\partial R(C, q)}{\partial q} &= C_3(C) \\ &= \int_{x>C} (x - C) \cdot d\mu(x) \\ &> 0 \end{aligned}$$

We thus conclude that it is optimal to bid P^+ in the balancing auction.

4.5.4 $(D4)$: $\mathbb{E}[\lambda^B] - C < 0$, $P^+ = 0$, $P^- < 0$

The imbalance payoff will be computed as follows for agents with $P^- < 0$ (and therefore $q \leq 0$):

$$\begin{aligned}
&\max_{ai}(C - \mathbb{E}[\lambda^B]) \cdot ai - \mathbb{E}[(\lambda^B + \lambda^R) \cdot Imb] \\
&ai - q \leq -P^- \\
&ai \geq 0
\end{aligned}$$

Since $\mathbb{E}[\lambda^B] - C < 0$, we have $ai^\star = -P^- + q$. The expected payoff z_I is then expressed as follows:

$$z_I \;=\; (\mathbb{E}[\lambda^B] - C) \cdot (P^- - q) + E$$

The expected balancing payoff can be expressed as follows:

$$z_B \;=\; \int_{x<p} (x - C) \cdot q \cdot d\mu(x) + \mathbb{E}[\lambda^R] \cdot P^+$$

The overall payoff of the agent can therefore be expressed as follows:

$$\begin{aligned}
R(p,q) &= z_I + z_B \\
&= C_1 - C_2 \cdot q + C_3(p) \cdot q
\end{aligned}$$

where the terms can be described as follows:

$$\begin{aligned}
C_1 &= (\mathbb{E}[\lambda^B] - C) \cdot P^- + E + \mathbb{E}[\lambda^R] \cdot P^+ \\
C_2 &= \mathbb{E}[\lambda^B] - C \\
C_3(p) &= \int_{x<p} (x - C) \cdot d\mu(x)
\end{aligned}$$

We express the first-order conditions with respect to p as:

$$\begin{aligned}
\frac{\partial R(p,q)}{\partial p} &= C_3'(p) \cdot q \\
&= \mu(p) \cdot (p - C) \cdot q
\end{aligned}$$

We note that the payoff function $R(p,q)$ for fixed q is increasing in $(-\infty, C]$, zero at C, and decreasing in $[C, +\infty)$. Thus, for any q, an optimal strategy is to bid the true cost. And, given this strategy, the payoff becomes

$$R(C,q) = C_1 - C_2 \cdot q + C_3(C) \cdot q$$

We have

$$\begin{aligned}
\frac{\partial R(C,q)}{\partial q} &= -C_2 + C_3(C) \\
&= -(\mathbb{E}[\lambda^B] - C) + C_3(C) \\
&= -(\int_{x \leq C} (x-C) \cdot d\mu(x) + \int_{x>C} (x-C) \cdot d\mu(x)) \\
&\quad + \int_{x \leq C} (x-C) \cdot d\mu(x) \\
&< 0
\end{aligned}$$

We conclude that it is optimal to bid P^- in the balancing auction.

4.5.5 Conclusion ($D4$)

We can state that it is always optimal for agents to bid their entire balancing capacity at the true marginal cost to the balancing auction. We have characterized a pure strategy Nash equilibrium.

Essentially, the analysis follows exactly as in the case of section 4.2. The only thing that changes, ultimately, is the term E instead of the term D and the additional term $\mathbb{E}[\lambda^R] \cdot P^+$.

Crucially, the additional element of the US design is the fact that the day-ahead contracted reserve capacity is bought back at a real-time price. Therefore, the opportunity cost is zero (as in section 4.2) plus $\mathbb{E}[\lambda^R]$.

5. Numerical Illustration

We illustrate the concept in a simple example. Denote by V^U the ceiling of the balancing auction, which occurs when the system runs out of capacity (and correspondingly denote V^D as the floor of the balancing auction). Denote by K^U the total upward capacity of the system, and by K^D the total downward capacity of the system.

We consider a system with the following characteristics:

- The system imbalance is normally distributed with a mean $\mu = 0$ and a standard deviation $\sigma = 91.5$
- The supply function of the system is affine, and expressed as $\lambda^B = a + b \cdot Imb$, where Imb is the total system balancing activation. We choose $a = 50$, and $b = 0.1109$.

- The system has a total capacity of upward capacity $K^U = 301$ MW and a total capacity of downward capacity $K^D = -350$ MW.
- The market has a price ceiling of $V^U = 120$ €/MWh and a price floor of $V^D = -120$ €/MWh.
- We consider an agent with $C = 51$ €/MWh, and a capacity of $P^+ = 1$ MW.
- Suppose that the non-agent system imbalance and the imbalance of the agent are independent.
- We assume that the mean imbalance of the agent is $\mu_a = 0$, and the standard deviation is $\sigma_a = 0.4082$.

The expected balancing price can be computed as:

$$\mathbb{E}[\lambda^B] \quad = \quad \mathbb{P}[q < K^D] \cdot V^D + \int_{K^D \leq x \leq K^U} (a + b \cdot x) \cdot \phi_{\mu,\sigma}(x) \cdot dx + \mathbb{P}[q > K^U] \cdot V^U$$

In our case study, we have $\mathbb{E}[\lambda^B] = 50.01$ €/MWh.
Regarding design ($D2$), we assume the following:

- $\alpha^U = 120$ €/MWh
- $\alpha^L = 120$ €/MWh
- $UI = 0.75 \cdot P^{+,total} = 225.75$ MW
- $LI = 0.75 \cdot P^{-,total} = -262.5$ MW

Regarding designs ($D3$) and ($D4$), we have

$$\lambda^R = (VOLL - \lambda^B) \cdot LOLP(P^{+,tot} - Imb) \cdot \mathbb{I}[P^{+,total} - Imb \geq 0] + (VOLL - C^{max}) \cdot \mathbb{I}[P^{+,total} - Imb < 0]$$

where $C^{max} = a + b \cdot P^{+,total}$ is the marginal cost of the most expensive upward capacity, namely $C^{max} = 83.37$ €/MWh.
We assume:

- $VOLL = 1000$ €/MWh

Table 1: The results produced by the analytical model.

	$(D1)$	$(D2)$	$(D3)$	$(D4)$
Profit	4.05	4.05	9.49	13.55
$p^\star$ [€/MWh]	51	51	51	51
$q^\star$ [MW]	1	1	0	1
$dR(C, q^\star)/dq$ [€/MWh]	0	0	5.44	9.50

We further compute the $LOLP$ as follows:

$$LOLP(R) = \mathbb{P}[Imb > P^{+,total} - q]$$
$$= 1 - \Phi_{\mu,\sigma}(P^{+,total} - q)$$

The average scarcity adder is computed as follows:

$$\mathbb{E}[\lambda^R] = \int_{x \leq P^{+,total}} (VOLL - a - b \cdot x) \cdot (1 - \Phi_{\mu,\sigma}(P^{+,total} - x)) \cdot \phi_{\mu,\sigma}(x) \cdot dx$$
$$+\mathbb{P}[Imb > P^{+,total}] \cdot (VOLL - C^{max})$$

For the specific values used in this analysis, we have $\mathbb{E}[\lambda^R] = 9.50$ €/MWh.

5.1 Design $(D1)$

We have $C > \mathbb{E}[\lambda^B]$, $P^+ > 0$, and $P^- = 0$. Looking up the corresponding case of section 4.2, we have the profit of the agent expressed as

$$R(C, P^+) = C_1 + C_3(C) \cdot P^+$$
$$C_1 = D$$
$$C_3(C) = \int_{x > C} (x - C) \cdot d\mu(x)$$
$$= + \int_{\frac{C-a}{b} \leq x \leq K^U} (a + b \cdot x - C) \cdot \phi_{\mu,\sigma}(x) \cdot dx + \mathbb{P}[q > K^U] \cdot (V^U - C)$$

We compute D as follows, where Imb_r denotes the rest of the system imbalance (not related to the agent), which is independent of the agent imbalance Imb_a:

$$\begin{aligned} -D &= \mathbb{E}[\lambda^B \cdot Imb_a] \\ &= \mathbb{E}[(a + b \cdot (Imb_r + Imb_a)) \cdot Imb] \\ &= \mathbb{E}[a \cdot Imb_a + b \cdot Imb_a \cdot Imb_r + b \cdot (Imb_a)^2] \end{aligned}$$

$$
\begin{aligned}
&= b \cdot \mathbb{E}[(Imb_a)^2] \\
&= b \cdot \sigma_a^2
\end{aligned}
$$

For the specific numbers used in our example, we have $D = -0.02$ €, therefore a negligible financial loss, which is due to the fact that the imbalance of the agent undermines the profit of the agent.

We compute the overall profit, and report the result in table 1.

5.2 Design $(D2)$

Since we assume symmetric independent imbalances, the results are identical to those of $(D1)$. The profits are presented in table 1.

5.3 Design $(D3)$

Since $C < \mathbb{E}[\lambda^B + \lambda^R] - \int_{x>C}(x-C) \cdot d\mu(x)$, $P^+ > 0$, $P^- = 0$, it is optimal for the agent to self-balance and withhold capacity from the balancing market (i.e. $q = 0$), and we can express the optimal payoff of the agent as follows:

$$
\begin{aligned}
&R(C, 0) = C_1 \\
&C_1 = (\mathbb{E}[\lambda^B + \lambda^R] - C) \cdot P^+ + E
\end{aligned}
$$

In order to simplify the calculations, we assume that the scarcity price λ^R and the agent imbalance are independent (this is justified from the fringe assumption), which implies that $E \simeq D$ when $\mathbb{E}[Imb] = 0$, which is the case for the example that we are studying here.

The opportunity cost for bidding reserve is computed as

$$
\begin{aligned}
&C_2 - C_3(C) \\
&C_2 = \mathbb{E}[\lambda^R + \lambda^B] - C \\
&C_3(C) = \int_{x>C} (x - C) \cdot d\mu(x)
\end{aligned}
$$

The results are reported in table 1.

5.4 Design $(D4)$

We have $C > \mathbb{E}[\lambda^B]$, $P^+ > 0$, and $P^- = 0$. Looking up the corresponding case of section 4.5, we have the profit of the agent expressed as

$$
R(C, P^+) = C_1 + C_3(C) \cdot P^+
$$

$$C_1 = E + \mathbb{E}[\lambda^R] \cdot P^+$$
$$C_3(C) = \int_{x>C} (x - C) \cdot d\mu(x)$$

We compute E under the same assumption as in design ($D3$). We report the results in table 1.

6. Conclusion

The design ($D4$) is the only design that (i) maintains the incentive of agents to bid their entire flexible capacity to the balancing auction, while also (ii) giving an incentive to agents to back-propagate the average scarcity price to day-ahead reserve auctions.

Therefore, a proper implementation of scarcity pricing requires a real-time market for reserve capacity. Equating the balancing price with the imbalance price has been shown to be part of the appropriate market design. We have found no market design that uses a different balancing price and imbalance price, and that is still capable of back-propagating reserve prices.

References