

#### Why energy communities?

#### **Context**

- Communities may bring about significant gains:
  - Energy transition: facilitates the decentralization of energy systems
    - Local management of load, on-site consumption: less DSO upgrades, more flexibility
    - Restoration of some returns to scale
    - Unlocking private capital
  - Social innovation:
    - Better alignement of product with consumer preferences
    - Increased consumer participation
    - Increased sense of community

#### "Energy communities": a strong political will

**European commission winter package:** "Consumers are active and central players on the energy markets of the future"

Promote: Collective self-consumption, Energy communities, Peer-to-peer trading...

#### Two separate laws:

- Renewable Energy Communities (Renewable Energy Directive (EU) 2018/2001)
- Citizen energy communities (Internal Electricity Market Directive (EU) 2019/944)

a legal entity that (...) is based on voluntary and open participation and is effectively controlled by members or shareholders, (...) has for its primary purpose to provide environmental, economic or social community benefits to its members

We take a case that fits all of these definitions: Several households in a given building decide to use a single meter, and potentially jointly invest in PV + batteries.

#### **Research questions**

Communities are formed following cooperation, while the interaction with the rest of the system is non-cooperative.

- 1. Can energy communities be stable/viable, and self managed? (recall of previous paper "On the viability of Energy Communities")
  - A subset of the community may find it profitable to exit the community and create one of their own
  - Stability is key to success for long-term investment decisions
- 2. Is there a snowball effect in the formation of energy communities?
  - CEER 2019 : Issues of grid cost recovery
  - How should the DSO adapt to the formation of Energy Communities?
     Connexion fee? Capacity based? Volume based?

#### **Existing literature**

#### Cooperative game theory:

- Seminal papers: Shapley (1953, 1971), Young (2014), Moulin and Shenker (2001), Moulin (2002)
- In energy:
  - Allocation of network costs: Contreras et al (2009), Kattuman et al (2004)
  - CO2 emissions: Kellner (2013), Pierru (2007)
  - LNG: Massol and Tchung-Ming (2010)

#### Decentralized energy systems

- Basak et al. (2012), Lopes (2016), Lidula and Rajapaske (2011), Lo prete et al. (2012), Costa et al. (2008)...
- Lo prete et al. (2016) and Lee et al. (2014) tackle both. But focus is on gain sharing between community and rest of the system.

# Efficient tariffs in power systems

Borenstein and Davis (2012), Borestein (2013).

# **SUMMARY**

Part 1.

Are energy communities viable?

Part 2.

The snowball effect

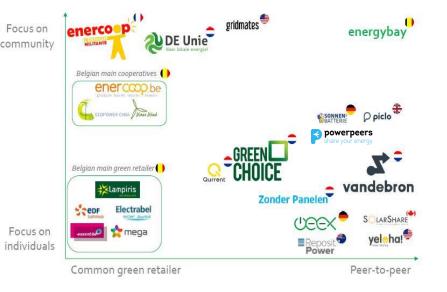
Part 3.

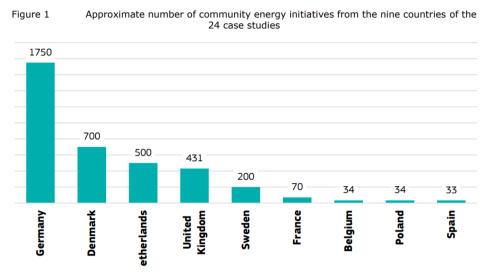
Some results

#### A large potential

The European Commission estimates that more than 3500 renewable energy communities are active today in Europe.

The trend indicates an increase. European Council, 2016: By 2030, 38% of installed capacity could be owned by energy communities.





Source: European Commission

### « Energy communities » in the paper

#### Our definition:

Several households in a given building decide to use a single meter, and potentially jointly install PV.



#### **Base model**

A set of households (i.e. consumers)  $I = \{1, 2 \dots n\}, n > 1$ , consider joining an energy community.

- They consume energy.
- May install PV panels at a cost on a shared roof.
- Save on grid tariffs: we assume here a general structure of grid tariffs.
- Electricity consumed locally if possible. Benefit is the retail price.
- Excess sold to system at market price.

#### **Base model**

The total value of the energy community is:

$$\alpha \left[ \sum_{i \in S} \operatorname{Max}_{l}(f_{i}(t)) - \operatorname{Max}_{l} \left( \sum_{i \in S} (f_{i}(t)) \right) \right] + \delta(s - 1)$$

$$+ \sum_{t=1}^{T} \beta(t) \left( \sum_{i \in S} f_{i}(t) - \left( \sum_{i \in S} (f_{i}(t) - k_{i}(\overline{\mu})g(t)) \right)^{+} \right)$$

$$+ \sum_{t=1}^{T} \gamma(t) \left( \sum_{i \in S} (k_{i}(\overline{\mu})g(t) - f_{i}(t)) \right)^{+}$$

$$- c \left( \sum_{i \in S} k_{i}(\overline{\mu}) \right)$$
(PV sold to system)
$$- c \left( \sum_{i \in S} k_{i}(\overline{\mu}) \right)$$
(PV costs)

We assume that PV and battery capacity are built to optimize this value. Other utility functions can be derived.

## Stability of the community, notion of core and the Shapley value

**Definition 2.** The core of the game Ker(I) is the set of all allocations  $x(v) = (x_1(v), x_2(v), ...x_n(v)) \in \mathbb{R}^n$  such that:

$$\forall S \subset I, \quad \sum_{i \in S} x_i(v) \ge v(S)$$
  
$$\sum_{i=1}^n x_i(v) = v(I)$$

- A community is said to be stable if it has a non-empty core.
- Assessing if the core of game is empty can be difficult:
  - Theory: stylized
  - Numerical application: more realistic

**Definition 3** The Shapley value  $x^s(v)$  is the unique allocation rule satisfying symmetry, linearity and Pareto-optimality:

$$\forall i \in \{1, 2, ..., n\}, x_i^s(v) = \sum_{i \in S \subset I} \left( v(S) - v(S / \{i\}) \right) \frac{(n-s)!(s-1)!}{n!}$$

#### **Numerical application**

- Simulation of several buildings/neighbourhoods composed of 6 households each.
- Abstract away from grid costs (focus on gain sharing)
- Sources:
  - —Load from www.loadprofilegenerator.de
  - —PV costs calibrated on latest observed panel prices
  - —PV gains set at German retail tariffs/ market prices
- We investigate if the following allocations are stable:
  - —Per-capita
  - —Pro-rata of volume
  - —Pro-rata of peak demand
  - —Shapley
  - —Minvar (allocation rule in the core that minimizes the inequality of gains)

#### Convex problem: heterogenous building

Table: benefit of investing in PV either individually or jointly (in €/annum)

	Couple Working	Family Working One child	Man Work from home	$\operatorname{Student}$	Storekeeper	Retired Couple	Total	In the core?
Annual demand (kWh)	2623	2613	1601	1563	4003	1747	9930	
Peak demand (kW)	10.1	6.7	2.1	5.4	1.4	7.0	36.3	
Individual value	40.9	23.2	15.2	26.6	196.7	32	334.6	n/a
per capita allocation	82.5	82.5	82.5	82.5	82.5	82.5	495.3	no
per volume allocation	91.8	91.5	56	54.7	140.1	61.1	495.3	no
per capacity allocation	108.4	112.4	55.9	83.3	41.2	94.1	495.3	no
Shapley	63.2	58	33.5	38.7	231.8	70.1	495.3	Yes
MinVar	70	70.7	39.8	41.1	197.4	76.2	495.3	Yes

Core is non-empty?	Yes
Total value	495.3
Strength of stability	0.5

Installed PV (no coalition): 3.7 kW Installed PV (grand coalition): 4.7 kW

- Heterogenous load profiles => heterogenous rewards
- Usual, simple allocation rules fail to provide stability
- Casts doubts on desirability of strong retail rate control
- All these results hold for different kinds of buildings differeing in their size, composition and level of demand.

#### Key take-aways: equity vs simplicity

- An energy community can create some value: aggregation benefit and autoconsumption.
- The way this value is shared will be key to its long-term stability: we show that simple allocation rules fail to stabilize the community.
- More subtle and stable allocations exist, like the Shapley vlaue: they are fair but might be complicated to implement.
- We could drastically simplify the Shapley value to ease its implementation while keeping it stable.
- Despite this, we believe that some effort has to be done by public authorities to foster the development of Energy Communities:
  - 1- Educate community managers on the importance of equity and stability.
  - 2- Communicate on the existence of stable allocation rules.
  - 3- Elaborate (even via the private sector) dedicated software that automatically calculates the sharing rule. Such solution already exists for financial products.

# **SUMMARY**

Part 1. Are energy communities viable?

Part 2. The snowball effect

Part 3. Some results

#### The snowball effect at play

# The "snowball effect":

- Given existing grid tariffs, some communities form but not all players join a community.
- Given community formation, grid tariffs are modified.
- Following this modification, new communities form or existing communities increase in size.

#### The setting

- We consider a neighborhood of B buildings
- In each building, energy communities can form but not necessarily and not necessarily only one.
- A community mutualizes an investment in PV and battery to maximize its utility.

#### The grid tariffs

We assume that the DSO has three levers to recover his costs

A fixed part tariff:  $\delta$ 

A capacity based tariff:  $\alpha$ 

A volume based tariff:  $\phi$ 

Entity with a yearly net consumption pattern f(t) pays a yearly fee:

$$\alpha \max f(t) + \phi \sum_{t} f(t) + \delta$$

# Given the structures of EC in the neighborhood, how much does the DSO earn?

The DSO earns grid charges:

DSO<sub>Payoff</sub> 
$$(P_1, P_2, ..., P_B) = \alpha \sum_{b=1}^{B} \sum_{k=1}^{p_b} \text{Max}_t \left( \sum_{i_b \in S_b^k} f_{i_b}(t) - aut(S_b^k)_t \right)$$
  
  $+\phi \sum_{b=1}^{B} \sum_{k=1}^{p_b} \sum_{t=1}^{T} \left( \sum_{i_b \in S_b^k} f_{i_b}(t) - aut(S_b^k)_t \right)$   
  $+\delta \sum_{b=1}^{B} p_b$ 

The DSO adjusts its tariffs  $\delta$ ,  $\alpha$ ,  $\phi$  in reaction to the structure of energy communities

#### **Community formation**

- Cooperative game: We consider that community S<sub>b</sub> will exist only if it manages to share its value in a way that satisfies all smaller coalitions.
  - When communities corresponding to a whole building fail to materialize, we look for the optimal (stable) partitioning.
- Non-cooperative game: Grid cost recovery constraint

The system is in an **equilibrium** if and only if all buildings are partitioned optimally, and the DSO recovers its costs.

# **SUMMARY**

Part 1. Are energy communities viable?

Part 2. Illustration of the snowball effect

Part 3. Some results

## A case study

We consider two buildings of a neighborhood in Northwest Germany

	(1	Building 1		Building 2 (Homogeneous household)		
	1)	Mixed household)		(HOI		/
Household	Tuno	Annual demand	Peak demand	Trmo	Annual demand	Peak demand
number	Type	(kWh)	(kW)	Type	(kWh)	(kW)
1	Couple, working	2623	10.1	Retired adult	1101	5.4
2	Family, working 1 child	2613	6.7	Retired adult	1016	5.1
3	Adult, work from home	1601	2.1	Retired couple	2680	8.2
4	Student	1563	5.4	Retired couple	2088	7.3
5	Storekeeper	4003	1.4	Retired couple	1747	7.0
6	Retired couple	1747	7.0	Retired couple	1747	7.0
	Whole building	14150	32.7	Whole building	10379	40.0

Table 1: Composition of buildings 1 and 2  $\,$ 

	Iteration	0 (no community)	1	2	3	4	5 (final)
Grid cost	α (€/kW)	52.5	196.7	265.41	268.9	269.1	269.1
Building	Optimal partition	{1}{2} {3}{4} {5}{6}	{1,4} {2,3} {5,6}	{1,5,6} {2,3,4}	$\{1,5,6\}$ $\{2,3,4\}$	{1,5,6} {2,3,4}	{1,5,6} {2,3,4}
1	Value (€)	629.3	805.2	2509.5	3352.8	3396.4	3397.8
	PV (kW)	7.1	7.6	7.9	8.1	8.2	8.2
	Battery (kWh)	11.9	11.6	12.0	12.5	12.6	12.6
Building	Optimal partition	{1}{2} {3}{4} {5}{6}	$\{1,4\}$ $\{2,6\}$ $\{3,5\}$	$\{2\}$ $\{3,6\}$ $\{1,4,5\}$	$\{2\}$ $\{3,5\}$ $\{1,4,6\}$	{2} {3,5} {1,4,6}	{2} {3,5} {1,4,6}
2	Value (€)	573.6	770.1	2792.1	3764.9	3815.2	3816.9
	PV (kW)	6.0	5.9	6.1	6.1	6.1	6.1
	Battery(kWh)	13.8	11.3	11.9	12.3	12.3	12.3
	Number of coalitions	12	6	5	5	5	5
Total	Value	1202.9	1575.3	5301.6	7117.7	7211.6	7214.7
	PV(kW)	13.1	13.5	14.0	14.2	14.3	14.3
	Battery(kWh)	25.7	22.9	23.9	24.8	24.9	24.9

Table 2: Community building over iterations with capacity-based grid cost recovery

Iteration	0 (no community)	1	2	3	4	5 (final)
Grid cost α (€/kW	52.5	196.7	265.41	268.9	269.1	269.1

As a reaction to the formation of more and more energy communities, the DSO has to increase the grid charge, which triggers the formation of new communities.

Building	partition	{5}{4} {5}{6}	$\{2,0\}$ $\{3,5\}$	$\{3,0\}$ $\{1,4,5\}$	{3,3} {1,4,6}	{3,3} {1,4,6}	$\{3,5\}$ $\{1,4,6\}$
2	Value (€)	573.6	770.1	2792.1	3764.9	3815.2	3816.9
	PV (kW)	6.0	5.9	6.1	6.1	6.1	6.1
	Battery(kWh)	13.8	11.3	11.9	12.3	12.3	12.3
	Number of coalitions	12	6	5	5	5	5
Total	Value	1202.9	1575.3	5301.6	7117.7	7211.6	7214.7
	PV(kW)	13.1	13.5	14.0	14.2	14.3	14.3
	Battery(kWh)	25.7	22.9	23.9	24.8	24.9	24.9

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	Iteration	0 (no community)	1	2	3	4	5 (final)
Grid cost	α (€/kW)	52.5	196.7	265.41	268.9	269.1	269.1
Building	Optimal partition	{1}{2} {3}{4} {5}{6}	{1,4} {2,3} {5,6}	{1,5,6} {2,3,4}	{1,5,6} {2,3,4}	{1,5,6} {2,3,4}	{1,5,6} {2,3,4}
1	Value (€) PV (kW) Battery (kWh)	629.3 7.1 11.9	805.2 7.6 11.6	2509.5 7.9 12.0	3352.8 8.1 12.5	3396.4 8.2 12.6	3397.8 8.2 12.6
Building 2	Optimal partition Value (€)	{1}{2} {3}{4} {5}{6} 573.6	{1,4} {2,6} {3,5} 770.1	{2} {3,6} {1,4,5} 2792.1	{2} {3,5} {1,4,6} 3764.9	{2} {3,5} {1,4,6} 3815.2	{2} {3,5} {1,4,6} 3816.9
2	PV (kW) Battery(kWh)	6.0 13.8	5.9 11.3	6.1 11.9	6.1	6.1	6.1

The structure of EC at the equilibrium is not trivial.

	Iteration	0 (no community)	1	2	3	4	5 (final)
Grid cost	<i>α</i> (€/kW)	52.5	196.7	265.41	268.9	269.1	269.1
		{1}{2}	{1.4}				

Two opposite drivers that explain the evolution of PV with community formation.

- 1- Effect 1: the creation of EC can bring households having some consumption around noon, when PV produces, which might increase the interest to build PV.
- 2- Effect 2: the aggregation of demand profiles increases the value extracted from a single panel.

Overall the total PV installed increases with the formation of EC in our buildings.

		coantions							
	Total	Value	1202.9	1575.3	5301.6	7117.7	7211.6	7214.7	
		PV(kW)	13.1	13.5	14.0	14.2	14.3	14.3	
,		Battery(kWh)	25.7	22.9	23.9	24.8	24.9	24.9	_

Table 2: Community building over iterations with capacity-based grid cost recovery

	Iteration	0 (no community)	1	2	3	4	5 (final)
Grid cost	<i>α</i> (€/kW)	52.5	196.7	265.41	268.9	269.1	269.1
Building	Optimal partition	{1}{2} {3}{4} {5}{6}	{1,4} {2,3} {5,6}	$\{1,5,6\}$ $\{2,3,4\}$	$\{1,5,6\}$ $\{2,3,4\}$	$\{1,5,6\}$ $\{2,3,4\}$	$\{1,5,6\}$ $\{2,3,4\}$
1	Value (€)	629.3	805.2	2509.5	3352.8	3396.4	3397.8

There are similarly two opposite drivers explaining the evolution of battery investment.

Overall the total the investment in batteries decreases with the formation of EC in our buildings.

Battery(kWh)	25.7	22.9	23.9	24.8	24.9	24.9	

Table 2: Community building over iterations with capacity-based grid cost recovery

We calculated the equilibria for the three possible tariffs  $\delta$ ,  $\alpha$  and  $\beta$ . Combinations of tariffs were not explored for technical reasons.

	Handshake $cost = 10 \in$						
	<i>α</i> (€/kW)	$\delta$ ( $\leq$ /connection)	φ (€/kWh)				
Initial value	52.5	143.7	0.07				
Final value	309.4	862.5	0.20				
Number of iterations	5	2	4				
Final partition build.1	$\{1,2,3,4,5,6\}$	$\{1,2,3,4,5,6\}$	$\{1,4,5\}\{2,3,6\}$				
Final partition build.2	${3,5}{1,2,4,6}$	$\{1,2,3,4,5,6\}$	$\{1,3,6\}\{2,4,5\}$				
# of communities	3	2	4				
Final PV (kW)	14.1	9.6	17.0				
Final battery (kWh)	22.3	6.6	28.0				
Final value (€)	8956	9206	3667				
Final welfare (€)	735.4	881.2	516.8				
Final welfare coor (€)	515.4	581.2	396.8				
Best PV (kW)	9.2	9.2	9.2				
Best battery (kWh)	5.4	5.4	5.4				
Best welfare $(\in)$	941.4	941.4	941.4				

We calculated the equilibria for the three possible tariffs  $\alpha$ ,  $\delta$  and  $\phi$ . Combinations of tariffs were not explored for technical reasons.

	Handshake $cost = 10 \in$		
	$\alpha \in (kW)$	$\delta$ ( $\in$ /connection)	φ (€/kWh)
Initial value	52.5	143.7	0.07
Final value	309.4	862.5	0.20

We report on the welfare in the different cases.

As a benchmark, we also report on the optimal welfare obtained by a social planner

Final value (€)	8956	9206	3667
Final welfare (€)	735.4	881.2	516.8
Final welfare coor (€)	515.4	581.2	396.8
Best PV (kW)	9.2	9.2	9.2
Best battery (kWh)	5.4	5.4	5.4
Best welfare (€)	941.4	941.4	941.4

We calculated the equilibria for the three possible tariffs  $\delta$ ,  $\alpha$  and  $\beta$ . Combinations of tariffs were not explored for technical reasons.

	Handshake cost = 10€		
	α (€/kW)	$\delta \ (\in / \text{connection})$	φ (€/kWh)
Initial value	52.5	143.7	0.07
Final value	309.4	862.5	0.20
Number of iterations	5	2	4
Final partition build 1	{123456}	£1 2 3 4 5 6}	{1.4.5}{2.3.6}

In all cases, the DSO has to increase her tariff at equilibrium to recover his cost.

- Our results indicate that setting a fixed part tariff brings the system the closest to the optimal.
- Inherent effect: this is the tariff structure that creates the biggest communities.
- Investments do not allow for gaming on  $\delta$ .

	rumber of recramons		<u> -</u>	1
	Final partition build.1	{1,2,3,4,5,6}	$\{1,2,3,4,5,6\}$	$\{1,4,5\}\{2,3,6\}$
١	Final partition build.2	${3,5}{1,2,4,6}$	$\{1,2,3,4,5,6\}$	$\{1,3,6\}\{2,4,5\}$
Ī	# of communities	3	2	4
	Final PV (kW)	14.1	9.6	17.0
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	Best welfare $(\leqslant)$	941.4	941.4	941.4
		•		

#### **Conclusions**

- We have embedded a cooperative game theoretical framework, into a non-cooperative one.
  - Simple sharing rules fail to induce stability
  - 2. Energy communities over-invest in RES technologies.
  - 3. Snowball: conflict between mutual and public interest
- Grid tariff design is key. Depending on the agenda of policy makers:
  - If the objective is to foster RES investments for individuals: use volume tariff
  - If the objective is to bring the system close to optimality: use connections fees.
    - In line with Borenstein (2013).

- Overall, Energy communities over-invest in PV and batteries.
  - The motivation to avoid grid charges has a negative effect on investments.
- Here again, setting a fixed part component to the grid charge limits this overinvestment.

	Handshake cost = 10€		
	<i>α</i> (€/kW)	$\delta$ ( $\in$ /connection)	φ (€/kWh)
Initial value	52.5	143.7	0.07
Final value	309.4	862.5	0.20
Number of iterations	5	2	4
Final partition build.1	{1,2,3,4,5,6}	$\{1,2,3,4,5,6\}$	$\{1,4,5\}\{2,3,6\}$
Final partition build.2	${3,5}{1,2,4,6}$	$\{1,2,3,4,5,6\}$	$\{1,3,6\}\{2,4,5\}$
# of communities	3	2	4
Final PV (kW)	14.1	9.6	17.0
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Final value (€)	8956	9206	3667
Final welfare (€)	735.4	881.2	516.8
Final welfare coor (€)	515.4	581.2	396.8
Best PV (kW)	9.2	9.2	9.2
Best battery (kWh)	5.4	5.4	5.4
Best welfare (€)	941.4	941.4	941.4

#### Some limitations of our work

- Possible positive externalities due to the creation of ECs are overlooked (avoided generation and grid capacity investments).
- Only financial incentives are considered (this can be easily relaxed).
- Combinations of grid tariffs by the DSO were not considered.
- Our illustration is quite simple (only two buildings that are quite small).
- Technicalities related to the production of PV (random) and the functioning of batteries are ignored.
- Etc.

#### **Convex case: results**

**Theorem 1.** When the investment cost is strictly concave and players are either symmetric or anti-symmetric, the coalition game is convex.

- This implies the core is non-empty, and Shapley is in the core
  - —Shapley: (symmetric, linear, pareto-optimal: reflects marginal contribution of players to coalitions)

$$\forall i \in \{1, 2..., n\}, \ x_i^s(v) = \sum_{i \in S \subset I} (v(S) - v(S/\{i\})) \frac{(n-s)!(s-1)!}{n!}$$

- Such communities are always stable (phew!)
  - —However, basic sharing rules (pro-rata) unlikely to be suitable

# The "value" of a community

$$v(S_{b}, \alpha, \phi, \delta) = \text{Max} \qquad \alpha \left( \sum_{i_{b} \in S_{b}} \text{Max}_{t} \left( f_{i_{b}}(t) \right) - \text{Max}_{t} \left( \sum_{i_{b} \in S_{b}} f_{i_{b}}(t) - aut(S_{b})_{t} \right) \right) \\ + \phi \sum_{t=1}^{T} aut(S_{b})_{t} + \delta(s_{b} - 1) \\ + \beta \sum_{t=1}^{T} aut(S_{b})_{t} \\ + \gamma \sum_{t=1}^{T} inj(S_{b})_{t} \\ - c_{PV} \left( \mu(S_{b}) \frac{\sum_{i_{b} \in S_{b}} \sum_{t=1}^{T} f_{i_{b}}(t)}{\sum_{t=1}^{T} g(t)} \right) - c_{Bat} \left( Bat(S_{b}) \right) \\ - c_{coo}(S_{b}) \\ \text{s.t.} \qquad 0 \leq \mu(S_{b}) \leq \bar{\mu} \\ 0 \leq aut(S_{b})_{t}, \ 0 \leq inj(S_{b})_{t} \\ 0 \leq Bat(S_{b}), \ 0 \leq st(S_{b})_{t}, \ 0 \leq with(S_{b})_{t}, \ 0 \leq l(S_{b})_{t} \\ \forall t = 1, ..., T \quad \mu(S_{b}) \frac{\sum_{i_{b} \in S_{b}, t} f_{i_{b}}(t)}{\sum_{t=1}^{T} g(t)} g(t) + with(S_{b})_{t} - st(S_{b})_{t} = aut(S_{b})_{t} + inj(S_{b})_{t} \\ \forall t = 2, ..., T \quad l(S_{b})_{t} \leq l(S_{b})_{t-1} + st(S_{b})_{t} - with(S_{b})_{t} \\ \forall t = 1, ..., T \quad st(S_{b})_{t} \leq ch \ Bat(S_{b}) \\ \forall t = 1, ..., T \quad with(S_{b})_{t} \leq ch \ Bat(S_{b}) \\ \forall t = 1, ..., T \quad with(S_{b})_{t} \leq dch \ Bat(S_{b}) \\ \forall t = 1, ..., T \quad with(S_{b})_{t} \leq dch \ Bat(S_{b})$$