An Analysis of the Multi-Period Optimal Power Flow Problem with Electric Vehicles under Emission Considerations

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December 8, 2020

Overview

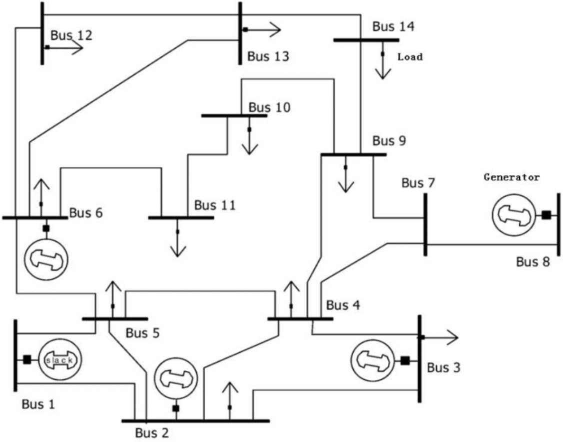
- Classical Optimal Power Flow
- Second-Order Cone Programming
- Multi-Period Optimal Power Flow with Electric Vehicles
    - Problem De nition
    - SOCP Formulation
    - Solution Approach
    - Computational Results

CLASSICAL OPTIMAL POWER FLOW PROBLEM

Optimal Power Flow Problem

- Aim: To supply electricity to all loads
- Objective Function: Minimization of generation cost
- Equality Constraints:
  - Power balance equations
  - Power ow equations
- Inequality Constraints:
  - Network operating limits

Figure 1: IEEE 14-bus network



PARAMETERS

For generator $i\ 2\ G$:

Active limit: $\underline{p}_i; \overline{p}_i$

Reactive limit: $\underline{q}_i; \overline{q}_i$

For bus $i\ 2\ B$:

Load: $p_i^d; q_i^d$

Voltage limit: $\underline{V}_i; \overline{V}_i$

Shunt: $g_{ii}; b_{ii}$

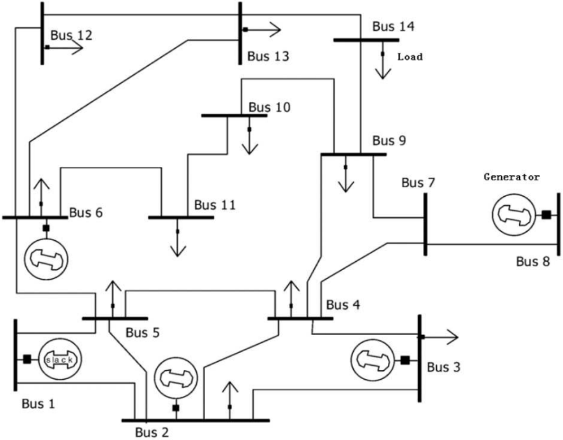
For line $(i; j)\ 2\ L$:

Flow limit: $\overline{S}_{ij}$

Angle limit: $_{ij}$

Admittance: $G_{ij}; B_{ij}$

Figure 1: IEEE 14-bus network



Introduction

PARAMETERS

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Active limit: $\underline{p}_i; \overline{p}_i$

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For bus i 2 B:

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For line $(i; j)$ 2 L:

Flow limit: $\overline{S}_{ij}$

Angle limit: $\underline{\ }_{ij}$

Admittance: $G_{ij}; B_{ij}$

DECISION VARIABLES

For generator i 2 G:

Active output: p_i^g

Reactive output: q_i^g

For bus i 2 B:

Voltage: V_i

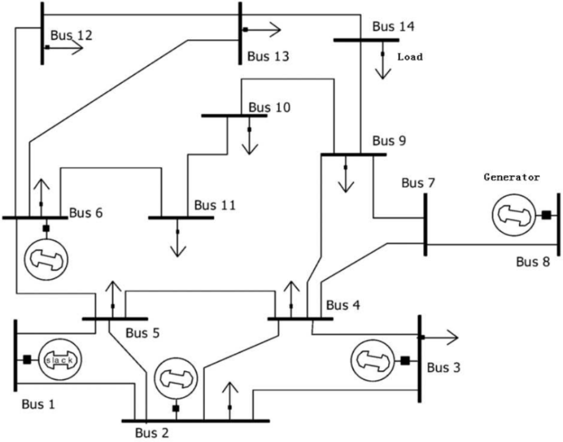
Angle: $_i$

For line $(i; j)$ 2 L:

Active flow: p_{ij}

Reactive flow: q_{ij}

Figure 1: IEEE 14-bus network



OPF Polar Formulation

$$\min \sum_{i \in \mathcal{G}} h(p_i^g)$$

$$
\begin{aligned}
\text{s.t.}\; p_i^g - p_i^d &= g_{ii}|V_i|^2 + \sum_{j \in \delta(i)} p_{ij} && i \in B \\
q_i^g - q_i^d &= -b_{ii}|V_i|^2 + \sum_{j \in \delta(i)} q_{ij} && i \in B
\end{aligned}
\quad \Bigg\} \text{ Power balance}
$$

$$
\begin{aligned}
p_{ij} &= G_{ij}|V_i|^2 + |V_i||V_j|[G_{ij}\cos(\theta_i - \theta_j) - B_{ij}\sin(\theta_i - \theta_j)] && (i,j) \in L \\
q_{ij} &= -B_{ij}|V_i|^2 - |V_i||V_j|[B_{ij}\cos(\theta_i - \theta_j) + G_{ij}\sin(\theta_i - \theta_j)] && (i,j) \in L
\end{aligned}
\quad \Big\} \text{ Power flow}
$$

$$\underline{V}_i \le |V_i| \le \overline{V}_i \qquad i \in B \quad \} \text{ Voltage limit}$$

$$
\begin{aligned}
|\theta_i - \theta_j| &\le \overline{\theta}_{ij} && (i,j) \in L \\
p_{ij}^2 + q_{ij}^2 &\le \overline{S}_{ij}^2 && (i,j) \in L \\
\underline{p}_i \le p_i^g &\le \overline{p}_i && i \in G \\
\underline{q}_i \le q_i^g &\le \overline{q}_i && i \in G.
\end{aligned}
\quad \Bigg\} \text{ Operational limits}
$$

The alternative formulation is first introduced in [1].

New Decision Variables:

- For each bus $i \in B$,

$$c_{ii} := |V_i|^2.$$

- For each line $(i,j) \in L$,

$$c_{ij} := |V_i||V_j|\cos(\theta_i - \theta_j)$$

$$s_{ij} := -|V_i||V_j|\sin(\theta_i - \theta_j)$$

$$\min \sum_{i \in G} h(p_i^g)$$

$$\text{s.t.}\; p_i^g - p_i^d = g_{ii} c_{ii} + \sum_{j \in \delta(i)} p_{ij} \qquad i \in B$$

$$q_i^g - q_i^d = -b_{ii} c_{ii} + \sum_{j \in \delta(i)} q_{ij} \qquad i \in B$$

Power balance

$$p_{ij} = G_{ij} c_{ii} + G_{ij} c_{ij} - B_{ij} s_{ij} \qquad (i,j) \in L$$

$$q_{ij} = -B_{ij} c_{ii} - B_{ij} c_{ij} - G_{ij} s_{ij} \qquad (i,j) \in L$$

Power flow

$$\underline{V}_i^2 \le c_{ii} \le \overline{V}_i^2 \qquad i \in B$$

Voltage limit

$$c_{ij}^2 + s_{ij}^2 = c_{ii} c_{jj} \qquad (i,j) \in L$$

$$\theta_j - \theta_i = \operatorname{atan2}(s_{ij}, c_{ij}) \qquad (i,j) \in L$$

Consistency

Operational Limits

OPF Challenges

- Network size
- Problem is nonlinear and nonconvex
- Locally optimal solutions exist in many OPF test cases [2].
- Local solvers do not guarantee global optimality.

Convex relaxations based on Semidefinite Programming and/or Second-Order Cone Programming may produce globally optimal solutions or provide valuable dual bounds.

SECOND-ORDER CONE PROGRAMMING

Second-Order Cone Programming

- $||\ ||_2$ is Euclidean norm: $||x||_2 = \sqrt{\sum_{i=1}^{n} x_i^2}$

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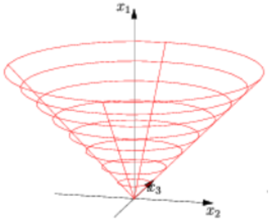
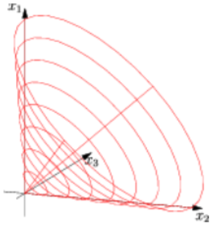
- Examples: $\sqrt{x_2^2 + x_3^2} \leq x_1$ Rotated cone: $x_3^2 \leq 2x_1x_2$





Second-Order Cone Programming

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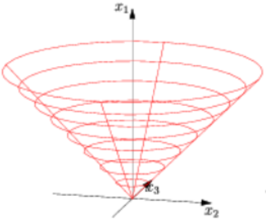
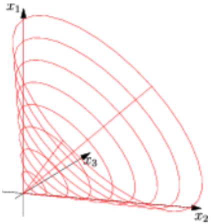




- Second Order Cone Programming (SOCP):

$$\min \quad f^T x$$
$$\text{s.t. } \|A_i x - b_i\|_2 \leq c_i^T x + d_i \quad i = 1, \ldots, m$$

$$\min \sum_{i \in G} h(p_i^g)$$

$$\text{s.t.}\; p_i^g - p_i^d = g_{ii} c_{ii} + \sum_{j \in \delta(i)} p_{ij} \qquad i \in B$$

$$q_i^g - q_i^d = -b_{ii} c_{ii} + \sum_{j \in \delta(i)} q_{ij} \qquad i \in B \qquad \text{Power balance}$$

$$p_{ij} = G_{ij} c_{ii} + G_{ij} c_{ij} - B_{ij} s_{ij} \qquad (i,j) \in L$$

$$q_{ij} = -B_{ij} c_{ii} - B_{ij} c_{ij} - G_{ij} s_{ij} \qquad (i,j) \in L \qquad \text{Power flow}$$

$$\underline{V}_i^2 \le c_{ii} \le \overline{V}_i^2 \qquad i \in B \qquad \text{Voltage limit}$$

$$c_{ij}^2 + s_{ij}^2 \le c_{ii} c_{jj} \qquad (i,j) \in L$$

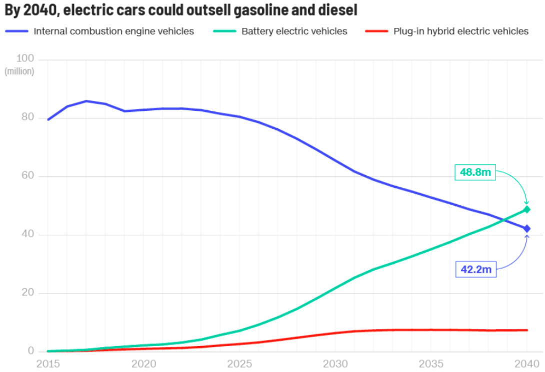
~~$\theta_j - \theta_i = \operatorname{atan2}(s_{ij}, c_{ij})$~~ $\qquad (i,j) \in L$ Relaxation!

Operational Limits

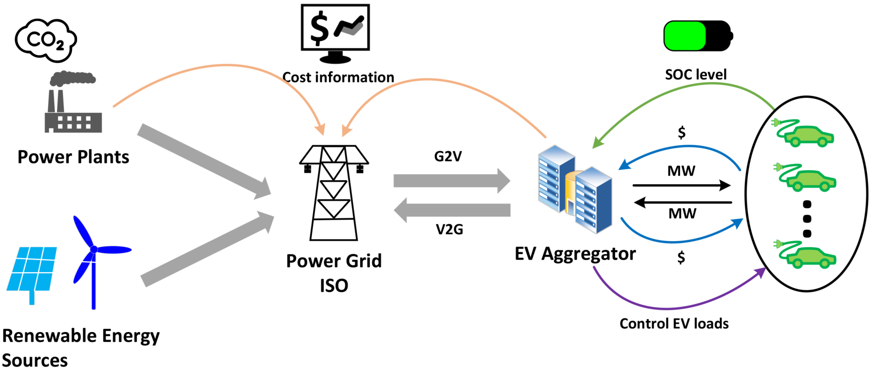
MULTI-PERIOD OPTIMAL POWER FLOW PROBLEM WITH ELECTRIC VEHICLES

Multi-Period Optimal Power Flow (MOPF) Problem

- Number of EVs: 3.3 million in 2018 ! 4.8 million in 2019
- Governments incentives + falling EV prices
- In 2040s, the number of EVs will be more than conventional vehicles.



MOPF with EVs under Emission Considerations



Contributions

- A new MOPF formulation
- Real datasets: electricity demand and EV driving profiles
- Emission considerations
- Global optimal solutions with very small optimality gaps even in large scale networks
- Computational results show that:
  - Marginal emissions can be signi cantly reduced, keeping cost constant.
  - V2G can save cost despite assuming hourly electricity prices are constant.

SOCP Relaxation of MOPF Formulation

Additional variables and parameters for each bus i 2 B and time t 2 T:

- Charging and discharging power: $a_{it}; b_{it}$
- EV battery stock variable: s_{it}
- Maximum allowable charging and discharging power: $\overline{a}_{it}; \overline{b}_{it}$
- Min energy requirement and max capacity of EV battery: $\underline{s}_{it}; \overline{s}_{it}$
- Charging efficiency: $_{i}$
- Initial battery state of charge (SOC): I_i
- Marginal emission parameter: e_t
- "Reference" power generation without any EVs (used to compute marginal emmisions): $\tilde{p}^{g}_{it}$
- Total marginal emission limit: $\overline{E}$

Additional EV related constraints:

$$\min \sum_{t 2 T} \sum_{i 2 G} h(p^g_{it})$$

s.t. $s_{it} + \; _i a_{it} \quad b_{it} \quad c_{it} = s_{i(t+1)}$	$i \; 2 \; B; t \; 2 \; f0g \; [\; T$ Stock
$\underline{s}_{it} \quad s_{it} \quad \overline{s}_{it}$	$i \; 2 \; B; t \; 2 \; T$ Stock limit
$s_{i0} = l_i \overline{s}_{i0}$	$i \; 2 \; B$) Initial battery SOC
$s_{i(T+1)} \quad l_i \overline{s}_{i(T+1)}$	$i \; 2 \; B$
$0 \quad a_{it} \quad \overline{a}_{it}$	$i \; 2 \; B; t \; 2 \; T$) Charging limit
$0 \quad b_{it} \quad \overline{b}_{it}$	$i \; 2 \; B; t \; 2 \; T$
$\sum_{i 2 B} \sum_{t 2 T} e_{it}(p^g_{it} \quad \tilde{p}^g_{it}) \quad \overline{E}$	Emission limit

$$p^g_{it} \quad p^d_{it} \;\; a_{it} + \;\; _i b_{it} = g_{ii} c_{iit} + \underset{j\,2\;\;(i)}{\mathrm{X}} p_{ijt} \qquad i\;2\;B;\,t\;2\;T$$

$$q^g_{it} \quad q^d_{it} = \;\; b_{ii} c_{iit} + \underset{j\,2\;\;(i)}{\mathrm{X}} q_{ijt} \qquad i\;2\;B;\,t\;2\;T$$

Power balance

$$p_{ijt} = G_{ij} c_{iit} + G_{ij} c_{ijt} \quad B_{ij} s_{ijt} \qquad (i;j)\;2\;L;\,t\;2\;T$$

$$q_{ijt} = \quad B_{ij} c_{iit} \quad B_{ij} c_{ijt} \quad G_{ij} s_{ijt} \qquad (i;j)\;2\;L;\,t\;2\;T$$

Power flow

$$\underline{V}_i^2 \quad c_{iit} \quad \overline{V}_i^2 \qquad i\;2\;B;\,t\;2\;T$$

Voltage limit

$$c^2_{ijt} + s^2_{ijt} \quad c_{iit} c_{jjt} \qquad (i;j)\;2\;L;\,t\;2\;T$$

Consistency

$$p^2_{ijt} + q^2_{ijt} \quad \overline{S}^2_{ij}; \quad j \;\; _{it} \quad _{jt} j \quad \bar{\ }_{ij} \qquad (i;j)\;2\;L;\,t\;2\;T$$

$$\underline{p}_i \quad p^g_{it} \quad \overline{p}_i; \quad \underline{q}_i \quad q^g_{it} \quad \overline{q}_i \qquad i\;2\;B;\,t\;2\;T$$

Operational limits

Input

We combine the following datasets to suggest practical solutions to cope with possible grid challenges:

- OPF instance
- Hourly electricity demand
- Marginal emission factors
- EV driving profiles

INPUT - OPF Test Case

Realistic OPF instance from the PG-OPF library.

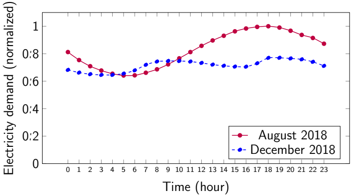
- Illinois 200-bus system
- South Carolina 500-bus system
- Texas 2000-bus system

Note from [3]: "These test cases are entirely synthetic, built from public information and a statistical analysis of real power systems. It bears no relation to the actual grid in this location, except that generation and load profiles are similar."

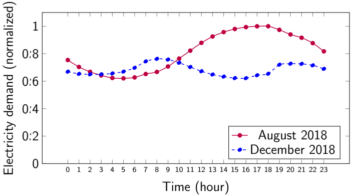
Input - Hourly Electricity Demand

Hourly electricity demand obtained from Energy Information Administration (EIA) is normalized by the maximum demand.

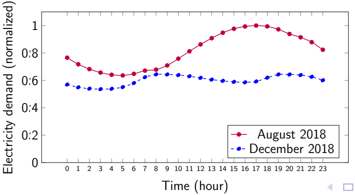
(a) Illinois 200-bus



(b) South Carolina 500-bus



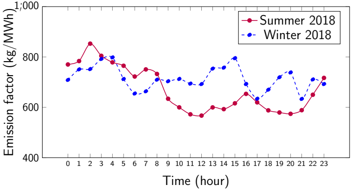
(c) Texas 2000-bus



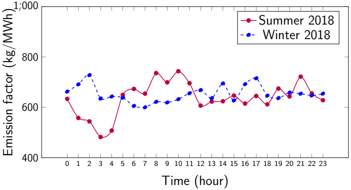
Input - Marginal Emission Factors

Marginal emission factors from the Climate and Energy Decision Making Center

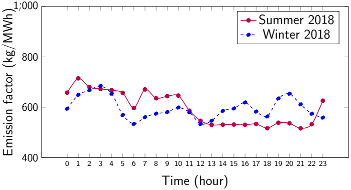
(a) Illinois 200-bus



(b) South Carolina 500-bus



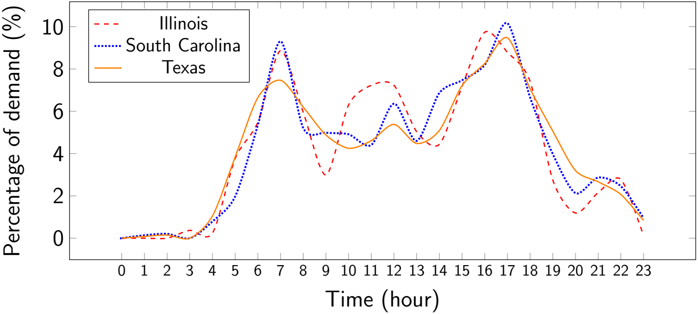
(c) Texas 2000-bus



Input - Driving Profiles

We use the National Household Travel Survey (NHTS) to obtain driving profiles.

Figure 4: Percentage of EV demand by time of day.



Solution Approach

Solution Approach

To obtain a lower bound: solve the SOCP relaxation (with Gurobi)

Fix the charging and discharging variables in the NLP model

To obtain an upper bound: solve the OPF problem (with IPOPT)

Compute optimality gap

Repeat this procedure for di erent \Total marginal emission limit"s

Computational Results

Table 1: Input data and parameter settings.

Settings	
Region	Illinois South Carolina Texas
Season	Winter Summer
Initial battery SOC	0% max%
Direction of ow	only G2V G2V and V2G

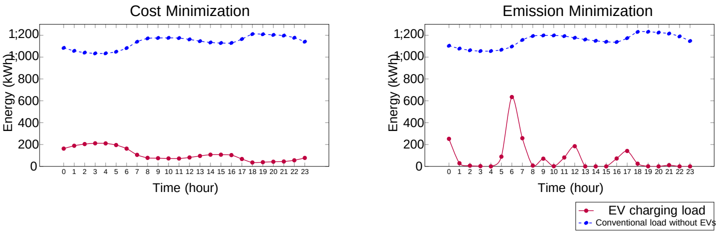
We compare the results of our optimization model with a simple BENCHMARK: Start charging the EV battery at midnight until it is charged enough or until its rst trip.

Computational Accuracy

Optimality gap of Illinois 200-bus instance is less than 05% for all cases.

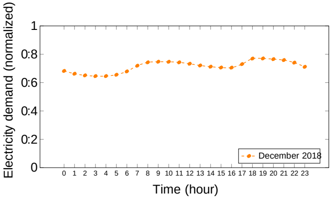
Hourly Load Variations

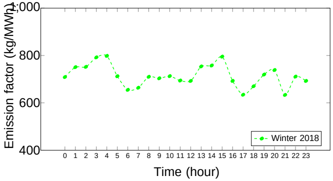
Figure 5: Illinois (200-bus system) in winter SOC=0%, only G2V.



(a) Hourly Electricity Demand

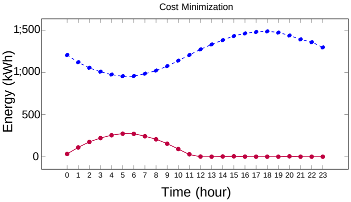
(b) Hourly Emission Factor



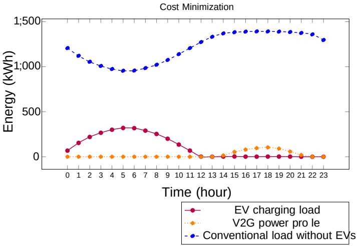


E ect of V2G for summer and winter (200-bus Illinois)

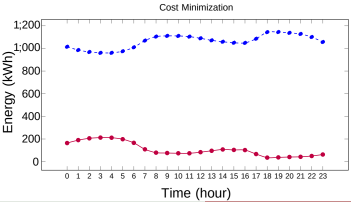
(a) Only G2V summer



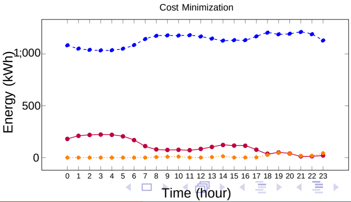
(b) G2V and V2G summer



(c) Only G2V winter

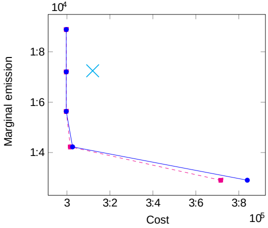


(d) G2V and V2G winter

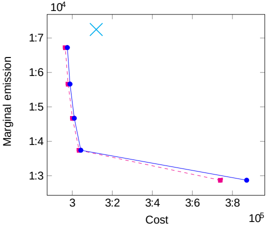


Total Cost vs. Marginal Emission

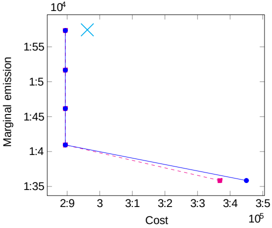
(a) only G2V summer



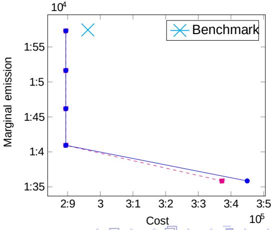
(b) G2V and V2G summer



(c) only G2V winter

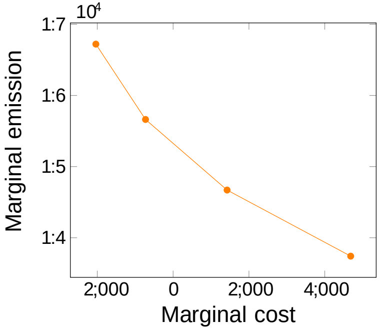


(d) G2V and V2G winter



Marginal Cost vs. Marginal Emission

Figure 10: V2G and G2V summer



Conclusions

- We proposed a new MOPF formulation with EV and emission considerations
- We combined di erent datasets (grid data, hourly demand, marginal emissions and driving pro les) to model the problem as realistically as possible
- Our SOCP-based algorithm provides (almost) globally optimal solutions with very small optimality gaps even in large scale networks
- Computational results showed that:
  - Marginal emissions can be signi cantly reduced, keeping cost constant.
  - V2G can save cost despite assuming hourly electricity prices are constant.

Optimality Gaps for MOPF

South Carolina 500-bus between 3.9 % and 6.4 %
Texas 2000-bus less than 0.7%

Bibliography

[1] A. G. Expósito and E. R. Ramos, "Reliable load flow technique for radial distribution networks," *IEEE Transactions on Power Systems*, vol. 14, no. 3, pp. 1063–1069, 1999.

[2] W. A. Bukhsh, A. Grothey, K. I. McKinnon, and P. A. Trodden, "Local solutions of the optimal power flow problem," *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 4780–4788, 2013.

[3] A. B. Birchfield, T. Xu, K. M. Gegner, K. S. Shetye, and T. J. Overbye, "Grid structural characteristics as validation criteria for synthetic networks," *IEEE Transactions on Power Systems*, vol. 32, no. 4, pp. 3258–3265, 2017.