Self-Commitment of Combined Cycle Units under Electricity Price Uncertainty

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***Abstract*—Day-ahead energy market clearing relies on a deterministic equivalent model with a limited time horizon, which may lead to inefficient scheduling of generating units from the point of view of generators. For this reason, generators may wish to assume the risk of self-committing their units with the hope of securing greater profits. This phenomenon may reduce the room for economic signals in the day-ahead market. In this paper we investigate the influence of risk aversion and price volatility on the decision of generators to self-commit units. We present a stochastic programming model for self-committing combined cycle units under price uncertainty with a conditional value at risk criterion. We use Benders decomposition to solve the problem and present results on a case study to draw conclusions.**

***Index Terms*—Self-Commitment, Combined Cycle Units, Conditional Value at Risk, Benders Decomposition**

I. Introduction

The clearing of day-ahead markets relies on a deterministic equivalent model where uncertainty is represented through its expected value. In addition, the horizon of day-ahead market models is often too short to account for operating constraints that couple the operations of units from one day to the next[1]. These two factors may lead to the inefficient commitment of conventional units. This is especially true for combined cycle units due to the increased complexity of their technical constraints and cost characteristics.

The self-commitment of units reduces the room for economic signals since generators that would otherwise contribute to setting the market clearing price fix their commitment independently of price. The large-scale integration of renewable resources with the resulting increase of real-time price uncertainty[2], and the increased complexity of combined cycle units[3], exacerbates the weakness of a deterministic day-ahead market model. On the other hand, the risk aversion of generators provides a strong incentive to participate in the day-ahead market. In this paper we investigate how these two factors influence the decision of generators to self-commit combined cycle units.

The use of combined cycle units in short-term balancing is becoming increasingly important due to the better controllability of these units, their modularity, and their flexibility in terms of the fuel they consume [4]. The proliferation of these units has increased the need to represent their operations and costs accurately in day-ahead commitment models. This need was underscored in a recent presentation by Ott [5] for the Pennsylvania Jersey Maryland (PJM) system. A detailed survey of literature on combined cycle unit modeling is presented by Anders et al. [6].

The difficulty of modeling combined cycle units stems from the fact that they consist of multiple components, each of which has independent technical and cost characteristics, as well as dependencies with the other components of the unit. Combined cycle units typically consist of multiple combustion turbines. The waste heat from the combustion turbines can be used for fueling steam turbines. The operation of these units can be represented either by a bottom-up modeling of the components (combustion and steam turbines) of the units, or a reduced modeling of the modes (the combination of combustion and steam turbines that are operational). Liu et al. [7] present a detailed comparison of a component model and a mode model for combined cycle units. Both approaches result in a mixed integer linear program which is substantially more complex than simplified models of conventional units.

Early work on the modeling of combined cycle units is presented by Cohen and Ostrowski [8]. Subsequent deterministic models of combined cycle units include the work of Lu and Shahidehpour, [4], [9]. In [4] the authors present a model for combined cycle units with combustion turbines and steam turbines that they incorporate in a security constrained economic dispatch model. In [9] the authors extend their model to account for the operation of flexible generating units of three types, mixed fuel units, combined cycle units and dual fuel units. Li and Shadidehpour [10] compare the solution of unit commitment models with combined cycle units based on Lagrange relaxation and branch and bound methods. Simoglou et al. [11] present a detailed model for self-scheduling combined cycle units, however they also do not account for uncertainty.

The modeling of uncertainty in self-scheduling has been addressed by various authors. Cerisola et al. [12] present a stochastic unit commitment model for deciding on the day-ahead and balancing market trades and production quantities

[1]The time horizon of the California ISO Integrated Forward Market is 24 hours [1].

[2]In the California ISO market, there will be a high demand for flexible resources as renewable generation is integrated increasingly in the market. Self-commitment and more so self-scheduling undermine this purpose by rendering combined cycle units as inflexible resources from the point of view of the system operator, although these resources are inherently flexible. An initiative is underway from the California ISO and the California Public Utilities Commission to enforce flexible capacity (flexible resource adequacy) in addition to the current generic resource adequacy requirement on California utilities [2].

[3]An additional reason for self-commitment is the fact that certain cost components of combined cycle units are not accurately represented in the ISO model due to bid cap rules. For instance, in the California ISO the fixed startup cost is capped by twice the cost of startup fuel [3] which is not sufficient to recover the true actual startup cost of combined cycle units, that exceeds cost factors not related to fuel.

of an owner of a hydrothermal portfolio under electricity price uncertainty. Tseng and Zhu [13] also present a model for self-scheduling generators subject to uncertainty in electricity prices. Garces and Conejo [14] present a stochastic programming model for a generator that decides on self-scheduling units, forward contracting and offering bids in the pool. The authors also account for risk aversion in the model through the use of the conditional value at risk (CVaR), using the theory provided by Rockafellar and Uryasev [15]. Although this work focuses on self-commitment, none of the aforementioned papers account for combined cycle unit operations and uncertainty simultaneously. In addition, with the exception of Garces and Conejo [14], none of the papers account for risk aversion.

The purpose of this paper is to investigate the influence of risk aversion and price volatility on the decision of generators to self-commit combined cycle units. This requires the integration of uncertainty and risk aversion in the self-commitment model. The previously cited literature either addresses self-commitment under uncertain price conditions without accounting for the complexity of combined cycle unit operations, or addresses the complex operations of combined cycle units without accounting for uncertainty and risk aversion. In addition to introducing a model that accounts for these features simultaneously, in this paper we develop a methodology that exploits this model in order to analyze the influence of risk aversion and price volatility on the willingness of generators to self-commit units in the day-ahead time frame. Following the previously referenced literature, we will focus on an open-loop model that optimizes utility operations.

A simple example that motivates our analysis is presented in Sect. II. In Sect. III we describe our methodology and the models used in our analysis. A solution algorithm for the risk-averse self-commitment model is presented in Sect. IV. We use the case study of Sect. V in order to draw conclusions, which are presented in Sect. VI.

II. A Motivating Example

In this paper we assume a two-settlement system with a day-ahead power pool where market agents submit multipart bids and the system operator solves a unit commitment problem and provides side payments in order for generators to voluntarily follow the centralized unit commitment schedule [16]. Our model is specifically inspired by the California Market Redesign and Technology Upgrade (MRTU) [1]. In this market design, self-commitment refers to the situation where a resource is modeled as being online in the day-ahead and real-time market unit commitment applications, but is not eligible for recovery of startup and minimum load costs for the intervals when the resource self-commits [1], §2.5.2.1. The paper will focus on self-commitment, assuming that units bid truthfully in the power pool. A game theoretic analysis of bidding in the power pool is therefore outside the scope of the paper.

In order to clarify how self-commitment can result in a different outcome from simply bidding into the market, we examine a simple example. Consider the owner of a conventional unit with capacity P, constant marginal cost C and minimum load cost K, facing an uncertain real-time price λ. We assume that the generator is risk-neutral.

The unit will be kept off, unless it receives uplift payments, if the day-ahead price cannot support the commitment of the generator, namely

$$\lambda_{DA} \le C + \frac{K}{P}. \tag{1}$$

Suppose that the conventional unit considers self-committing in order to capture real-time profit opportunities. The conventional unit then solves the following problem:

$$\begin{aligned} &\max \mathbb{E}[(\lambda_{RT} - C) \cdot p] - K \cdot u \\ &0 \le p \le P \cdot u \\ &u \in \{0, 1\} \end{aligned}$$

where p is the production of the unit according to the realized price and the expectation is taken with respect to the belief of the generator about the distribution of real-time prices. If the generator self-commits and real-time price does not exceed marginal cost, $\lambda_{RT} \le C$, then the generator will limit its output to zero and incur the fixed cost K. If, on the other hand, the real-time price exceeds marginal cost, $\lambda_{RT} > C$, then the generator will produce at maximum output and earn a profit of $(\lambda_{RT} - C) \cdot P - K$. The optimal solution to the self-commitment problem, then, is to commit the unit if the conditional average of the real-time price, for $\lambda_{RT} \ge C$, can compensate the average cost of the unit when the unit produces, namely:

$$C \cdot \mathbb{P}[\lambda_{RT} \ge C] + \frac{K}{P} \le \mathbb{E}[\lambda_{RT} | \lambda_{RT} \ge C] \tag{2}$$

We note that if both conditions of Eqs. (1) and (2) hold simultaneously, then the generator would prefer to keep the unit on, whereas the day-ahead market would keep the unit off. The interpretation of these conditions is quite intuitive. The right-hand side of Eq. (1) is the average hourly cost, per MW of output, incurred by the generator. According to Eq. (1), the unit is kept off if the day-ahead price is not sufficient to cover the average cost of operating the unit. The interpretation is identical in Eq. (2). The left-hand side is the average hourly cost per MW of output, where fuel cost is only incurred when the real-time price is high enough to induce the generator to produce. The right-hand side is the average price that the generator receives conditional on the price being high enough to induce the generator to produce. The generator is self-committed provided that the average price that it receives when it produces exceeds its average cost.

The simultaneous occurrence of the conditions described in Eqs. (1), (2) is certainly possible. Note that, given a probability that price exceeds marginal cost $\mathbb{P}[\lambda_{RT} \ge C]$, an increased volatility of real-time prices can increase the incentive of the generator to self-commit by resulting in a higher conditional average, $\mathbb{E}[\lambda_{RT} | \lambda_{RT} \ge C]$.

In order to simplify the discussion in the example, we have considered a single-period unit commitment model. We have also assumed that the generator does not account for the influence of its actions on price. Although these simplifying assumptions do not reflect realistic practice, they illuminate the difference between self-commitment and bidding in the day-ahead market and they highlight conclusions that are also confirmed in a realistic example in the case study of Sect. V.

III. Model

The decision of a generator on whether or not to self-commit in the day-ahead time frame depends on its assessment of real-time risk, and is depicted in Fig. 1. Throughout the paper we focus on uncertainty in the real-time price of electricity, denoted λ_{st}[4]. Here $s \in S$ indexes a discrete set of scenarios that represents the possible realizations of uncertainty and $t \in T$ indexes the set of time periods over which the evaluation of the self-commitment decision is performed. We consider a two-stage model. In the first stage generators commit their units. Subsequently, generators are dispatched against the real-time prices given the fixed day-ahead commitment decisions. From Fig. 1 it is evident that the modeling of the day-ahead and real-time market in a two-settlement system is necessary in our analysis since it determines the default profit against which a generator can improve by self-commitment. Once this baseline is established, it is possible to quantify the incremental benefits of self-commitment.

The day-ahead market model is presented in Sect. III-A. The day-ahead market model also determines the day-ahead profit of generators, which is a secured profit that can only increase if the generator identifies profit opportunities in the real-time market. The real-time profits are computed by running the real-time dispatch model, which is presented in Sect. III-C. The profits are computed over a large number of samples, and the resulting distribution of profits is transformed through the CVaR risk criterion in order to compute the risked profits. The CVaR criterion is defined mathematically in Sect. B of the appendix.

CVaR has been used increasingly in modeling risk aversion due to its robustness with respect to input and the fact that it satisfies the axioms of coherent risk measures [15]. The computational advantage of using the CVaR risk criterion, demonstrated by Rockafellar and Uryasev [15], is the fact that it can be computed by solving a linear program. In addition, as we will show in Sect. IV, this linear program can be incorporated within a Benders decomposition scheme for solving the risk-constrained self-commitment problem. The risk-constrained self-commitment problem is presented in Sect. III-B.

A. Market Model

A feature that makes the modeling of combined cycle units complex is the fact that the components of the units can only be fired up in sequence. This can be depicted through a state transition diagram which represents the sets of permissible transitions, as in Fig. 2. The set of permissible states $x \in X$ and transitions $y \in Y$ in this diagram correspond to a generating unit that consists of three combustion turbines. Within each state there are various operating constraints that need to be respected. Costs are incurred for operating within each state and also transitioning between states. Transitions between states are modeled through the indicator variable v_{yt}, equal to 1 if there is a transition y in period t and zero otherwise. The indicator variable u_{xt} indicates whether the unit is in state x (in which case $u_{xt} = 1$) or not (in which case $u_{xt} = 0$) in period t.

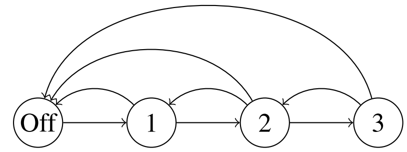

Fig. 2. The state transition diagram for a combined cycle unit with three combustion turbines.

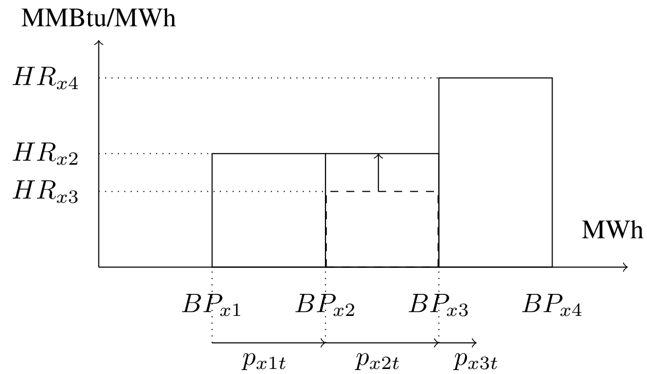

Fig. 3. The incremental heat rate curve within each operating state of a combined cycle unit.

Each state obeys a non-linear incremental heat rate curve, such as the one depicted in Fig. 3. The entire operating range of a certain state x is separated in a set of segments $\{1, \ldots, M\}$. The incremental heat rate is constant within each segment and the total production p_{xt} within the specific state x is equal to the minimum load of the specific state BP_{x1} plus the production within each segment $p_{x,m-1,t}$. Fig. 3 explains the notation that is used in the mixed integer linear program. We note that the heat rate curve needs to be increasing in order to avoid adding binary variables indicating the segment within which each state is operating. This is not always obeyed in practice, in which case we 'convexify' the cost curve before solving the model, as shown in Fig. 3.

The objective function of the market model is given by Eq. (3). Congestion is internalized in the model through locational marginal prices. Locations that suffer from congestion will exhibit large price volatility and risk, which will be accounted for by the self-commitment model and the economic dispatch. The models presented in this section describe the problem faced by a single generator in a single location, which is valid in the absence of utility-scale coupling constraints (e.g. self-provision of reserves). Coupling constraints and correlations among locational marginal prices can be incorporated in the

[4]The model can also account for demand uncertainty, as we will explain in Sect. III-B.

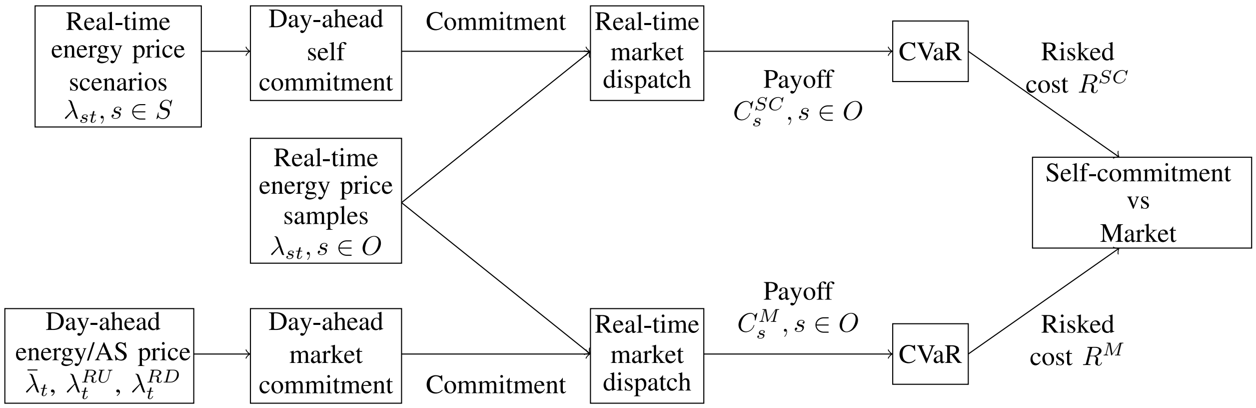

Fig. 1. The decision of self-committing or bidding a unit in the day-ahead market.

model.

The cost of operating a combined cycle unit consists of fuel costs, variable operating and maintenance (VOM) costs, fixed operating costs and transition costs. Fuel costs are computed using the non-linear heat rate curve of Fig. 3, where F is the price of fuel, HR_{xm} is the heat rate in segment m of state x and BP_{xm} is the upper breakpoint of segment m in the heat rate curve. The fixed operating charge that is incurred every hour that a unit runs is denoted as OC_x. The fixed cost of a transition y from a certain state $F(y)$ to another state $T(y)$ is denoted as TC_y. VOM_x is the VOM cost in state x. The generator also has the option of buying ($b_t > 0$) or selling ($b_t < 0$) energy at the day-ahead price $\bar{\lambda}_t$ and selling regulation up and down services, denoted r^+_{xt} and r^-_{xt} respectively, at prices $\lambda_t^{RU}, \lambda_t^{RD}$ respectively. Lower value spinning reserve can similarly be incorporated in the model, and is ignored here in order to simplify the exposition.

$$\begin{aligned}\min \sum_{t \in T} \bar{\lambda}_t b_t - \sum_{x \in X, t \in T} (\lambda_t^{RU} r^+_{xt} + \lambda_t^{RD} r^-_{xt}) \\ + \sum_{x \in X, m \in 1 \ldots M-1, t \in T} HR_{x,m+1} \cdot F \cdot p_{xmt} \\ + \sum_{x \in X, t \in T} VOM_x p_{xt} \\ + \sum_{x \in X, t \in T} (OC_x + F \cdot HR_{x1} \cdot BP_{x1}) u_{xt} \\ + \sum_{y \in Y, t \in T} TC_y v_{yt} \end{aligned} \tag{3}$$

The utility that owns the combined cycle unit may also function as a load serving entity, in which case it can serve its demand D_t either from the combined cycle unit or by procuring power in the market, as in Eq. (4).

$$\sum_{x \in X} p_{xt} + b_t = D_t, t \in T \tag{4}$$

The total power output in a certain state x is calculated as in Eq. (5).

$$p_{xt} = BP_{x1} u_{xt} + \sum_{m=1}^{M-1} p_{xmt}, x \in X, t \in T \tag{5}$$

Limits on the production of each segment are imposed in Eq. (6).

$$\begin{aligned} p_{xmt} \leq (BP_{x,m+1} - BP_{xm}) u_{xt}, x \in X, \\ 1 \leq m \leq M-1, t \in T \end{aligned} \tag{6}$$

Eqs. (7), (8) impose ramp rate limits in each state. R^+_x and R^-_x are the ramp up and ramp down rates of mode x respectively. The ramp rate constraints apply both for transitions from state to state as well as ramp rates within a state.

$$\begin{aligned} p_{xt} - p_{x,t-1} + r^+_{xt} \leq (2 - u_{x,t-1} - u_{xt}) BP_{x1} + \\ (1 + u_{x,t-1} - u_{xt}) R^+_x, x \in X, 2 \leq t \leq N \end{aligned} \tag{7}$$

$$\begin{aligned} p_{x,t-1} - p_{xt} + r^-_{xt} \leq (2 - u_{x,t-1} - u_{xt}) BP_{x1} \\ + (1 + u_{x,t-1} - u_{xt}) R^-_x, x \in X, 2 \leq t \leq N \end{aligned} \tag{8}$$

The allocation of capacity in energy or reserves is described in Eqs. (9), (10).

$$p_{xt} + r^+_{xt} \leq BP_{xM} u_{xt}, x \in X, t \in T \tag{9}$$

$$p_{xt} - r^-_{xt} \geq BP_{x1} u_{xt}, x \in X, t \in T \tag{10}$$

The provision of reserves is limited by the ramp rate of the unit within the interval at which the reserves need to be offered, as in Eqs. (11), (12).

$$r^+_{xt} \leq R^+_x, x \in X, t \in T \tag{11}$$

$$r^-_{xt} \leq R^-_x, x \in X, t \in T \tag{12}$$

Eq. (13) dictates that the combined cycle unit can perform no more than one state transition from period to period.

$$\sum_{y \in Y} v_{yt} \leq 1, t \in T \tag{13}$$

Eq. (14) describes the dynamics of the transition.

$$\begin{aligned} u_{xt} = u_{x,t-1} + \sum_{a \in A: T(a) = x} v_{at} - \sum_{a \in A: F(a) = x} v_{at}, \\ x \in X, 2 \leq t \leq N \end{aligned} \tag{14}$$

Eq. (15) requires that at each period the unit be in exactly one state.

$$\sum_{x \in X} u_{xt} = 1, t \in T \tag{15}$$

Minimum up and down times for each state are enforced in Eqs. (16), (17). The minimum up and down times of state x are denoted by UT_x, DT_x respectively.

$$\sum_{\tau=t-UT_x+1}^{t} \sum_{y \in Y: T(y)=x} v_{y\tau} \leq u_{xt}, \quad x \in X, UT_x \leq t \leq N \tag{16}$$

$$\sum_{\tau=t+1}^{t+DT_x} \sum_{y \in Y: T(y)=x} v_{y\tau} \leq 1 - u_{xt}, \quad x \in X, 1 \leq t \leq N - DT_x \tag{17}$$

Eq. (18) defines a binary variable u_t that indicates whether the combined cycle unit is off or not, and Eq. (19) defines a binary variable v_t that indicates whether the unit has started up or not.

$$u_t = \sum_{x \in X - \{\text{'Off'}\}} u_{xt}, t \in T \tag{18}$$

$$v_t = \sum_{y \in Y: F(y)=\text{'Off'}} v_{yt}, t \in T \tag{19}$$

Using these variables we define overall minimum up and down time constraints in Eqs. (20), (21), where UT and DT denote the minimum up and down times respectively[5].

$$\sum_{\tau=t-UT+1}^{t} v_\tau \leq u_t, UT \leq t \leq N \tag{20}$$

$$\sum_{\tau=t+1}^{t+DT} v_\tau \leq u_t, 1 \leq t \leq N - DT \tag{21}$$

Lower and upper bounds and integrality constraints are defined in Eqs. (22), (23).

$$v_{yt} \leq 1, y \in Y, t \in T \tag{22}$$

$$u_{xt} \in \{0, 1\}, \quad u_t, v_t, v_{yt}, p_{xt}, p_{xmt}, r^+_{xt}, r^-_{xt} \geq 0, x \in X, y \in Y, t \in T \tag{23}$$

We denote as $\bar{b}_t$ the amount of energy procured in the day-ahead market. Similarly, we denote as $\bar{r}^+_t, \bar{r}^-_t$ the amount of regulation up and down.

B. Self-Commitment Model

When a unit self-commits at a certain state, it has the freedom to produce between its technical minimum and maximum output for the given state[6]. Self-commitment exposes the generator to real-time electricity price risk. This risk is a function of the self-commitment decision, which is taken in the day-ahead time frame. In this paper we assume that the attitude of the decision-maker towards risk can be represented by the CVaR criterion. The role of risk aversion in the model is explained in Fig. 1. The formal definition of value at risk and conditional value at risk are provided in Sect. B of the appendix[7].

Consider a risk-averse generator that evaluates the real-time market payoff $Q(w, \lambda_s)$ over a set of price scenarios $s \in S$ according to the CVaR criterion, where $w = (u_{xt}, v_{yt}, u_t, v_t)$ is the set of first-stage commitment and transition decisions and $\lambda_s = (\lambda_{st}, t \in T)$ is the vector of real-time electricity prices for scenario s. The influence of virtual bidding can be incorporated in the methodology presented in the paper by requiring that real-time prices converge to day-ahead energy prices [1], [18]. The results presented in this paper rely on the 2012 California hub data, which exhibit this pattern[8]. Theorem 10 of Rockafellar and Uryasev [15] guarantees that the $CVaR_a$ of the random payoff $Q(w, \lambda_s)$ for a given first-stage decision w can be computed as the optimal objective function value of the following optimization:

$$\min_\zeta \zeta + \frac{1}{1-a} \sum_{s \in S} \pi_s (Q(w, \lambda_s) - \zeta)^+, \tag{24}$$

where $(x)^+ = \max(x, 0)$. In addition, the theorem guarantees that the VaR of the random payoff $Q(w, \lambda_s)$ is given by the optimal value of ζ.

In the case of the self-commitment model, the computation of $Q(w, \lambda_s)$ is a linear program that represents the reaction of generators in the real-time market, given a day-ahead self-commitment decision $w = (u_{xt}, v_{yt}, u_t, v_t)$. In particular, the risk averse self-commitment model can be described by the following optimization problem:

$$\begin{aligned}
\min & \sum_{t \in T} (\bar{\lambda}_t \bar{b}_t - \lambda^{RU}_t \bar{r}^+_t - \lambda^{RD}_t \bar{r}^-_t) \qquad (25)\\
& \sum_{x \in X, t \in T} (OC_x + F \cdot HR_{x1} \cdot BP_{x1}) u_{xt} \\
& + \sum_{y \in Y, t \in T} TC_y v_{yt} \\
& + \zeta + \frac{1}{1-a} \sum_{s \in S} \pi_s (Q(w, \lambda_s) - \sum_{t \in T} \lambda_{st} \bar{b}_t - \zeta)^+ \\
& \text{s.t. } (11), (12), (13), (14), (15), (16), \\
& (17), (18), (19), (20), (21), (22), (23), \qquad (26)
\end{aligned}$$

where the second-stage cost given first-stage decision w and price realization λ_s is computed by solving the following linear program:

[5] Analogous constraints to Eqs. (16), (17), (20), (21) that account for the state of the unit in the end of the previous day are included in the model but not presented here.

[6] By comparison, in self-dispatch the units not only fix the state they will operate in during each hour but also their hourly output.

[7] The incorporation of a risk-seeking measure in our model would induce load serving entities to self-commit units above day-ahead market schedules in order to capture potentially high prices while incurring fixed costs of transitioning to and operating in higher states. This conduct is illegal in certain markets, including California [17], page 52. Since our paper focuses on the management of generation resources by load serving entities rather than speculation by a merchant generator, we conduct our analysis by using a risk measure that encapsulates risk averse as well as risk neutral behavior.

[8] Virtual bidding was enacted in California in February 2011. The average day-ahead hub price in California was 28.3$/MWh in 2012, the average real-time hub price was 28.7$/MWh.

$$Q(w,\lambda_s) = \min \sum_{t \in T} \lambda_{st} b_t + \sum_{x \in X, t \in T} VOM_x p_{xt} + \sum_{x \in X, m \in 1 \ldots M-1, t \in T} HR_{x,m+1} \cdot F \cdot p_{xmt}$$
$$\text{s.t. } (4),(5),(6),(7),(8),(9),(10),(23). \quad (27)$$

Note that the market procurement b_t, total production p_t, production per state p_{xt} and production per segment p_{xmt} are now contingent on scenario s since they are second-stage decisions. Demand uncertainty can be modeled by introducing scenario-dependent demand D_{st} in Eq. (4). The notation $Q(w,\lambda_s)$ indicates that first-stage decisions are fixed in the constraints of the model. When the self-commitment problem is solved over a large number of scenarios, it is impossible to solve the problem in extended form. In Sect. IV we present a decomposition algorithm for solving the problem.

C. Evaluation Model

A generator will respond to real-time profit opportunities by changing its dispatch only if the real-time prices can yield increased profit. In order to compute the real-time profit opportunity of generators we solve the following problem:

$$\begin{aligned} &\min \sum_{t \in T} \bar{\lambda}_t \bar{b}_t - \sum_{t \in T} (\lambda_t^{RU} \bar{r}_t^+ + \lambda_t^{RD} \bar{r}_t^-) \\ &+ \sum_{t \in T} \lambda_{st}(b_t - \bar{b}_t) + \sum_{x \in X, t \in T} VOM_x p_{xt} \\ &+ \sum_{x \in X, m \in 1 \ldots M-1, t \in T} HR_{x,m+1} \cdot F \cdot p_{xmt} \\ &+ \sum_{x \in X, t \in T} (OC_x + F \cdot HR_{x1} \cdot BP_{x1}) u_{xt} \\ &+ \sum_{y \in Y, t \in T} TC_y v_{yt} \\ &\text{s.t. } (4),(5),(6),(7),(8),(9),(10),(23) \\ &u_{xt} = u^\star_{xt}, v_{yt} = v^\star_{yt}, u_t = u^\star_t, v_t = v^\star_t, \\ &x \in X, y \in Y, t \in T \qquad (28) \end{aligned}$$

The payoff of the generator consists of day-ahead energy and ancillary services market revenues (the first two terms of the objective function), real-time charges for deviations from the day-ahead position $\bar{b}_t$ (the third term), and generator costs (all remaining terms). We ignore real-time ancillary services in order to simplify the presentation of the model. The incorporation of real-time ancillary services can be accomplished by modifying the objective function and constraints of the self-commitment and evaluation model without complicating the computational approach presented in the next section.

Eqs. (28) fix the commitment and transition decisions to their day-ahead values. This implies that only the production of units can be rescheduled in the real-time market. When day-ahead commitment decisions $(u^\star_{xt}, v^\star_{yt})$ are fixed to the optimal solution of the day-ahead market model we obtain the payoff C_s^M of bidding in the market, whereas when they are fixed to the optimal solution of the self-commitment model we obtain the payoff C_s^{SC} of self-commitment. The real-time payoff will be at least as favorable as the day-ahead payoff, since the day-ahead solution is feasible for the evaluation model. The goal of self-commitment is to improve on this payoff by adapting it to the risk preferences of the supplier, using a deeper time horizon and using a more detailed model for the multi-stage generating unit. This profit opportunity comes at the cost of assuming the risk of the real-time market.

In order to quantify the benefit of self-commitment, we compute a distribution of costs over a set of price samples, O. This set of price samples is different from the set of scenarios S that are used as input for the stochastic self-commitment model (see Fig. 1). We then obtain a vector of sample costs $C^{SC} = (C_s^{SC}, s \in O)$, $C^M = (C_s^M, s \in O)$ and apply the $CVaR_a$ operator:

$$R^M = CVaR_a(C^M) \quad (29)$$
$$R^{SC} = CVaR_a(C^{SC}) \quad (30)$$

Obtaining the cost distribution of self-commitment, C^{SC}, requires the solution of a stochastic program. In the following section we describe a decomposition algorithm for tackling the self-commitment problem. The computation of $CVaR_a$ given a distribution of costs in Eqs. (29), (30) is immediate: in the case of a discrete set of outcomes the $CVaR_a$ operator simply averages the $(1-a)\%$ worse outcomes. The generator then decides to self-commit if $R^{SC} \leq R^M$ and to participate in the day-ahead market otherwise.

The decision-making strategy described in Fig. 1 assumes that generators self-commit after the day-ahead market closes. There is also the option of self-committing before the day-ahead market closes, which we do not analyze in this paper. Following the California MRTU market design for multi-stage units [1], §6.6.2.1, we assume that in case the day-ahead market commits a unit above its specified self-commitment level, then the market rewards the unit as if the unit had not self-committed in the first place. In the case where the day-ahead market would commit the unit at a certain state, whereas the generator would find it advantageous to self-commit the unit at a higher state, there is an inherent bias towards real-time prices that are higher than day-ahead prices. In these cases, we assume that generators would prefer to self-commit after the day-ahead market closes, in order to avoid the loss of buying back their day-ahead position in the real-time market had they self-committed before day-ahead market closure. This assumption justifies the decision strategy presented in Fig. 1. By considering self-commitment after the clearing of the day-ahead market we can also compute uplift payments independently from the self-commitment decision, which implies that we can easily incorporate uplift payments without complicating the model.

IV. Solution Methodology

In this section we present a decomposition algorithm for solving the problem in Sect. III-B, which cannot be solved in extended form when a large number of scenarios is accounted for. We first use elementary arguments to prove the convexity of the value function in the first-stage decisions and compute the subgradient of the value function with respect to first-stage decisions. We then use these results to apply Benders' decomposition on the problem.

The self-commitment problem has the following form:

$$\min c^T w + \zeta + \frac{1}{1-a}\sum_{s\in S}\pi_s(Q(w,\lambda_s)-\zeta)^+ \quad (31)$$
$$\text{s.t. } w \in W, \quad (32)$$

where W is a set of polyhedral and integer constraints, w is a set of first-stage decision variables and c is a vector of cost coefficients. The second-stage cost for a given realization is given by

$$(P2_s): Q(w,\lambda_s) = \min \lambda_s^T z \quad (33)$$
$$\text{s.t. } Aw + Bz = h \quad (34)$$
$$z \geq 0 \quad (35)$$

where z are second-stage continuous decision variables, A, B are matrices of appropriate dimension, and λ_s and h are vectors of appropriate dimension.

Proposition 1. *The value function $V(w,\zeta) = \sum_{s\in S}\pi_s(Q(w,\lambda_s)-\zeta)^+$ is a convex function of (w,ζ).*

Proof: According to theorem 2, paragraph 3.1 of [19], we have that $Q(w,\lambda_s)$ is a convex function of w. We get convexity of the value function from the fact that the non-negative sum of convex functions, $Q(w,\lambda_s)-\zeta$, is convex; the composition of convex functions, $(Q(w,\lambda_s)-\zeta)^+$, is convex; and the expectation operator preserves convexity. ■

Proposition 2. *The subgradient of $V(w,\zeta)$ at (w,ζ) is given by*

$$\partial V(w,\zeta) = \sum_{s\in S}\pi_s 1_s \begin{bmatrix} -\sigma_s^T A \\ -1 \end{bmatrix} \quad (36)$$

where $1_s = 1_{Q(w,\lambda_s)\geq\zeta}$ and σ_s are the dual optimal multipliers of the coupling constraints in Eq. (34).

Proof: It suffices to show that for any $(w',\zeta') \neq (w,\zeta)$

$$(Q(w',\lambda_s)-\zeta')^+ \geq$$
$$1_s[(Q(w,\lambda_s)-\zeta)^+ - \sigma_s^T A(w'-w) - (\zeta'-\zeta)]$$

Suppose $Q(w,\lambda_s)\geq\zeta$. For any $(w',\zeta') \neq (w,\zeta)$ we have that

$$1_s[(Q(w,\lambda_s)-\zeta)^+ - \sigma_s^T A(w'-w) - (\zeta'-\zeta)] =$$
$$Q(w,\lambda_s) - \sigma_s^T A(w'-w) - \zeta'$$
$$\leq Q(w',\lambda_s) - \zeta'$$
$$\leq (Q(w',\lambda_s) - \zeta')^+$$

The first inequality follows from the fact that $-\sigma_s^T A$ is a subgradient of $Q(w,\lambda_s)$ at w [19]. On the other hand, if $Q(w,\lambda_s) < \zeta$ we have

$$1_s[(Q(w,\lambda_s)-\zeta)^+ - \sigma_s^T A(w'-w) - (\zeta'-\zeta)] = 0$$
$$\leq (Q(w',\lambda_s)-\zeta')^+.$$

■

We can now apply Benders' decomposition to the problem. We formulate a relaxation of the first-stage problem by introducing an auxiliary variable θ:

$$(P1): \min c^T w + \zeta + \frac{1}{1-a}\theta \quad (37)$$
$$\text{s.t. } \theta \geq D^l \begin{bmatrix} w \\ \zeta \end{bmatrix} + d^l, 1 \leq l \leq k \quad (38)$$
$$w \in W, \theta \geq 0, \zeta \geq \zeta_{LB} \quad (39)$$

where ζ_{LB} is a lower bound on the value at risk, which is easily computable from problem data and prevents unboundedness of $(P1)$. Using the previous results, we propose the following algorithm:

Step 0: Set $k = 1$. Initialize $\hat{\theta}^1 = -\infty$, and $(\hat{w}^1, \hat{\zeta}^1)$. Go to step 2.

Step 1: Solve $(P1)$. Set $(\hat{w}^k, \hat{\zeta}^k)$ equal to the optimal first-stage solution. Go to step 2.

Step 2: For all $s \in S$, solve $(P2_s)$ using $\hat{w}^k$ as input. Set $\hat{\sigma}_s^k$ equal to the dual optimal multipliers of the coupling constraints in Eq. (34). Set $1_s^k = 1_{(\hat{\sigma}_s^k)^T(h-A\hat{w}^k)\geq\hat{\zeta}^k}$. Go to step 3.

Step 3: Set

$$D^k = \sum_{s\in S}\pi_s 1_s^k(-(\hat{\sigma}_s^k)^T A, -1) \quad (40)$$
$$d^k = \sum_{s\in S}\pi_s 1_s^k(\hat{\sigma}_s^k)^T h \quad (41)$$

If $\hat{\theta}^k = \sum_{s\in S}\pi_s 1_s^k((\hat{\sigma}_s^k)^T(h - A\hat{w}^k) - \hat{\zeta}^k)$ then exit with $(\hat{w}^k, \hat{\zeta}^k)$ as the optimal solution. Otherwise, set $k = k+1$ and go to step 1.

Note that, following Rockafellar and Uryasev [15] and Ehrenmann and Smeers [20], we are considering $CVaR$ as the objective in our analysis. A convex combination of expectation and $CVaR$ [14] also yields a coherent risk measure. Such an objective would in no way complicate the application of the methodology or decomposition presented in the paper. The methodology and computational approach is not confined to combined cycle units. The model can be applied to other types of units, provided no integer decisions are involved in the second stage. Simpler technologies, for example thermal generators with on-off unit commitment decisions, can be represented with fewer binary decision variables, and probably less computational effort.

V. CASE STUDY

We present results from a case study performed on a combined cycle unit with three combustion turbines. We use the technical and cost characteristics of units ALAMIT3 - ALAMIT5 of the WECC 240 bus model [21], assuming that the three generators are connected in a 3×1 configuration. The minimum up and down times of the units have been set equal to 4 hours, and the overall unit up/down times have been set equal to 6 hours. The heat rate curve of the unit is shown in Table I.

We run our case study for a 48-hour horizon. We analyze two seasons, Spring and Summer, for two typical

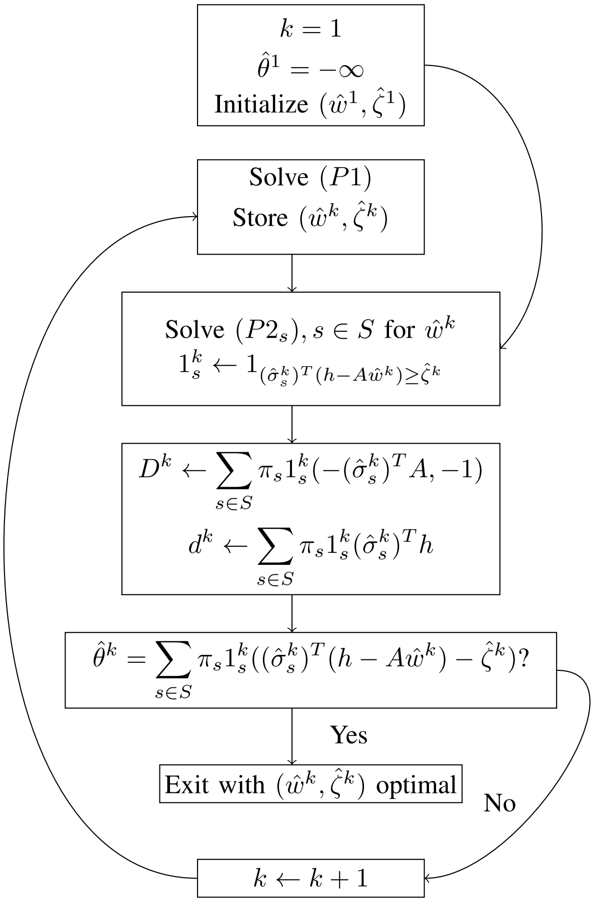

Fig. 4. The flow diagram of the Benders decomposition algorithm.

TABLE I
INCREMENTAL HEAT RATE CURVE OF THE COMBINED CYCLE UNIT USED IN THE CASE STUDY. BREAKPOINTS ARE IN MW AND HEAT RATES ARE IN MMBTU/MWH.

	Seg. 1	Seg. 2	Seg. 3	Seg. 4	Seg. 5	Seg. 6
$BP_{1\times1,\star}$	15	72.2	129.4	186.6	243.8	301
$BP_{2\times1,\star}$	316	373.2	430.4	487.6	544.8	602
$BP_{3\times1,\star}$	624.5	710.14	795.78	881.42	967.06	1052.7
$HR_{1\times1,\star}$	9.04	8.55	8.88	9.21	9.54	9.87
$HR_{2\times1,\star}$	9.25	8.75	9.08	9.42	9.76	10.09
$HR_{3\times1,\star}$	8.87	8.39	8.71	9.04	9.36	9.68

planning horizons, Weekday-Weekend (e.g. Friday-Saturday) and Weekday-Weekday (e.g. Monday-Tuesday). We use 2012 real-time and day-ahead electricity price data and day-ahead ancillary services price data for the NP15 hub of the California market, which is publicly available at the California ISO website. We use the price data in order to calibrate a second-order autoregressive model that can be used for generating a large number of Monte Carlo simulation samples. The order of the autoregressive model is chosen in order to minimize the mean absolute error between the model and the observed data. Natural gas prices are set equal to 3.11\$/MMBtu, based on the annual average day-ahead prices at the PG&E Citygate hub.

The model that we have presented assumes that generators can ramp down if market prices are not favorable. Certain markets, including California, allow units to ramp down in

TABLE III
THE UNIT COMMITMENT SCHEDULE (MW) FOR RISK-NEUTRAL SELF-COMMITMENT ($a = 0$) VERSUS DAY-AHEAD MARKET COMMITMENT FOR SUMMER WEEKDAY-WEEKDAY.

Hours	1-21	22 - 28	29 - 32	33-36	37 - 47	48
Self-Commit	1053	1053	1053	1053	1053	0
Market	1053	0	301	602	1053	0

real time but prohibit units from withholding capacity below the level that is committed by the day-ahead market in order to prevent gaming. This can be easily captured in our model by requiring that the state of the unit in real time cannot be lower than the day-ahead commitment u_{xt}^{DA}:

$$u_{3\times1,t} \geq u_{3\times1,t}^{DA}, u_{2\times1,t} + u_{3\times1,t} \geq u_{2\times1,t}^{DA},$$
$$u_{1\times1,t} + u_{2\times1,t} + u_{3\times1,t} \geq u_{1\times1,t}^{DA}, t \in T \quad (42)$$

The following results have been obtained by enforcing this constraint on the self-commitment model of Section III-B.

A. Impact of Risk Aversion

The decomposition algorithm of Sect. IV is applied to a stochastic program with $|S| = 100$ scenarios. The proposed algorithm is capable of tackling problems with thousands of scenarios, however as we explain in Section V-D the incremental benefit of the increased number of scenarios is negligible relative to the increase in run time. For this reason we proceed with $|S| = 100$ scenarios for the rest of the paper. The unit is assumed to be in 3×1 configuration in the end of the previous day. In order to investigate the impact of risk aversion on the model, we run each problem against a $CVaR_a$ risk criterion with $a = 0$, 0.25, 0.50, 0.75. Note that $a = 0$ corresponds to a risk-neutral decision maker.

We use $|O| = 10,000$ samples in the Monte Carlo simulation in order to attain results with high confidence. The 95% confidence intervals of the risk-adjusted profits of self-commitment versus bidding in the day-ahead market are shown in Table II. The day-ahead market commitment produces the same average profit for every level a of risk aversion, however the payoff of the profit distribution has a different value depending on the level of risk aversion, which is the result shown in the table.

The superior performance of self-commitment can be understood by comparing the unit commitment schedules of self-commitment versus the market for Summer Weekday-Weekday, which is shown in Table III. The market shuts the unit down in hour 22 and starts it up again in hour 29. This results in startup costs and lost profits in the second day due to the fact that the unit needs 8 hours in order to transition back to 3×1 mode. Instead, the self-commitment model keeps the unit in 3×1 mode throughout the entire horizon. The unit commitment schedule for Summer Weekday-Weekday for the risk-averse cases $(a = 0.25, 0.50, 0.75)$ follows a similar trend: the unit is kept at 3×1 or a lower mode during hours 22-36, but does not shut down.

We observe that the incentive for generators to self-commit is enhanced by less risk aversion. This is due to the fact

TABLE II
95% CONFIDENCE INTERVALS OF RISK-ADJUSTED PROFITS (IN $\$ \cdot 10^3$ OVER THE 48-HOUR HORIZON) OF SELF-COMMITMENT VERSUS DAY-AHEAD MARKET COMMITMENT FOR REFERENCE AND VOLATILE PRICES. DAY TYPES ARE AS FOLLOWS: (I) SPRING WEEKDAY-WEEKEND, (II) SPRING WEEKDAY-WEEKDAY, (III) SUMMER WEEKDAY-WEEKEND, (IV) SUMMER WEEKDAY-WEEKDAY.

		Reference prices				Volatile prices			
		$a = 0$	0.25	0.50	0.75	$a = 0$	0.25	0.50	0.75
(I)	Self-Commit	59.7-64.5	0	0	0	88.2-100.1	23.8-26.7	0	0
	Market	0	0	0	0	0	0	0	0
(II)	Self-Commit	60.0-64.4	4.7-6.4	0	0	106.3-113.5	24.0-26.8	0	0
	Market	0	0	0	0	0	0	0	0
(III)	Self-Commit	357.4-360.4	334.9-335.9	324.8-325.7	315.8-317.2	402.2-411.7	349.0-350.5	332.0-333.3	317.2-319.3
	Market	350.4-352.6	327.7-328.2	320.8-321.1	317.4-317.6	379.2-382.7	342.7-343.7	330.4-330.8	323.2-323.5
(IV)	Self-Commit	414.9-420.9	375.8-376.7	366.2-367.1	359.4-359.6	451.7-460.9	389.2-390.6	372.0-372.5	365.0-365.3
	Market	390.5-392.6	369.2-369.7	362.8-363.0	359.4-359.6	417.9-421.1	383.6-384.5	372.0-372.5	365.0-365.3

that increased risk aversion reduces the differences of near-optimal unit commitment schedules. The results of Table II demonstrate that self-commitment can deliver consistent benefits relative to the market due to the fact that it is adaptive to the risk preferences of the generator. Self-commitment can limit the room for economic signals from the real-time market. This price distortion is mitigated by the fact that the market schedule is fairly close to the optimal self-commitment schedule. In equilibrium, self-commitment generates the same schedule as the day-ahead market. The computation of such an equilibrium is an interesting question for future research. We revisit this question using our model in Section V-C.

B. Impact of Price Volatility

In order to investigate the impact of price volatility we run the same model against a set of price scenarios whose spread around the average hourly value is 150% the spread of the original price data. The results are shown in the right side of Table II. We observe that the increased price volatility increases the benefits of self-commitment in the risk-neutral case. This may be somewhat counterintuitive since higher price volatility implies higher risk which would make the day-ahead market more desirable. On the other hand, higher volatility also increases the value of explicitly accounting for uncertainty in a stochastic self-commitment model, which drives the result. In contrast, the benefits of self-commitment are less pronounced for the risk-averse cases $(a = 0.25, 0.50, 0.75)$. In the case of $a = 0.25$ for Summer Weekday-Weekend, the market schedule slightly outperforms self-commitment due to the large number of scenarios that are neglected by the CVaR risk criterion.

The increase in expected profits can be understood by examining the distribution of profits for Summer Weekday-Weekend for both the case of reference (non-volatile) prices as well as volatile prices, shown in Fig. 5. For the case of market bidding, a significant portion of the distribution is concentrated around \$320,000, which is the day-ahead profit. This is due to the fact that for the given commitment, a large number of realizations cannot make a profit in real time and only accrue the day-ahead market profit. It is also intuitive that the spread of the profit distribution should increase in the positive direction as price volatility increases. This stems from the fact that high revenue outcomes under reference prices result in even higher revenues under volatile prices.

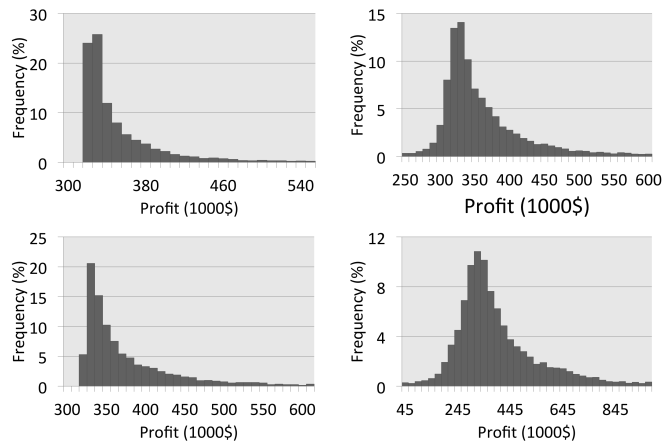

Fig. 5. The distribution of profits for Summer Weekday-Weekend for market bidding with reference prices (upper left), risk-neutral self-commitment with reference prices (upper right), market bidding with volatile prices (lower left) and risk-neutral self-commitment with volatile prices (lower right).

Instead, low revenue outcomes will result in a profit no less than the profit accrued in the day-ahead market, which results in a significant concentration of mass around the day-ahead profit. In the case of self-commitment, increased price volatility increases the positive spread of the profit distribution in the lower right panel, but leaves the negative spread of the distribution nearly unaffected. The positive bias resulting from increased price volatility can be understood by the fact that periods of low prices correspond to low generator output, with a minor influence on revenue, whereas periods of high prices correspond to periods of high output with a major influence on revenue. We therefore observe that flexible combined cycle units utilize their capability to ramp up and down rapidly in order to increase the value of their optionality in conditions of increased price volatility. As renewable resources are increasingly integrated in the system, resulting in increased real-time price volatility, this enhances the incentive to self-commit for risk-neutral market participants. Dispatch constraints (ramp rate limits and minimum run levels) and fixed production and transition costs imply that the shift to self-commitment introduces the risk of profits dropping below day-ahead levels, which we observe in the left and right lower panels. With increasing risk-aversion $(a = 0.25, 0.50, 0.75)$, price volatility makes self-commitment less valuable relative to maintaining the day-ahead market schedule. Although the

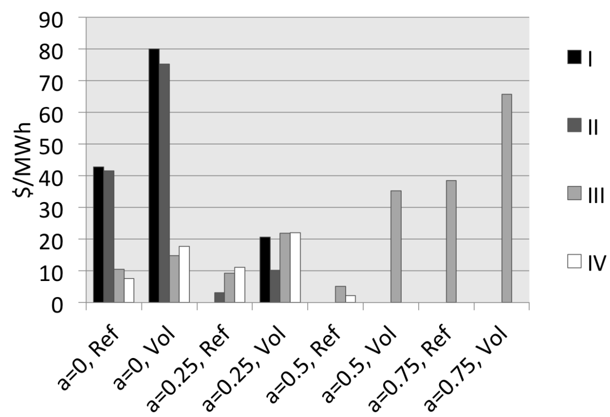

Fig. 6. Real-time price reduction (in $/MWh) that results in a day-ahead unit commitment decision that is identical to the optimal self-commitment decision. Day types are as follows: (I) Spring Weekday-Weekend, (II) Spring Weekday-Weekday, (III) Summer Weekday-Weekend, (IV) Summer Weekday-Weekday.

present analysis has focused on the impact of price volatility, the decrease in average energy prices that results from large-scale renewable energy integration can also be analyzed using the model presented in this paper.

C. Feedback of Real-Time Prices on Self-Commitment

The self-commitment of units increases real-time supply and may depress real-time electricity prices. Prices used as the reference for self-commitment may be prices generated under mild self-commitment of units. However, once units are self-committed this will drive prices down. One approach for analyzing how real-time price feeds back into the decision of self-commitment requires the development of an equilibrium model. However, there are discrete choices in the model regarding the commitment of states and transitions of multi-stage units. Given the negative result on the existence of competitive market-clearing prices in systems with discrete decisions [22], [16], it is unclear how one might construct an equilibrium model for analyzing this feedback.

In order to provide insight into this feedback effect, we can use the model presented in this paper in order to compute the reduction in real-time prices that renders the unit indifferent between self-commitment and bidding in the market, i.e. results in a day-ahead unit commitment decision that is identical to the optimal self-commitment decision. The results are presented in Fig. 6. For cases where the self-commitment and day-ahead schedule differ, we find that the price decrease of real-time prices that would render units indifferent between self-commitment and the day-ahead market schedule range between 2.2-80$/MWh. We observe that volatility increases the required reduction in real-time prices that would render units indifferent between self-commitment and the day-ahead schedule. Instead, lower risk aversion does not imply that a higher price separation is needed in order to render the units indifferent.

Arbitrage between the day-ahead and real-time market limits the average price differences between these markets. This observation is confirmed by an examination of historical price data from 2009-2012 in the CAISO NP15 hub (annual difference of day-ahead and real-time prices between -2.37$/MWh and +0.19$/MWh), the ISO New England Internal hub (annual difference between -0.66$/MWh and -0.01$/MWh), the PJM Dominion hub (annual difference between -0.42$/MWh and +0.59$/MWh), the New York ISO Capital hub (annual difference between +0.77$/MWh and +1.43$/MWh), and the MISO Consumer Energy hub (annual difference between +0.40$/MWh and +1.05$/MWh). None of these price differences are within the range of price differences required in Fig. 6.

D. Running Time and Size of the Scenario Set

A large scenario set better represents uncertainty at the cost of requiring longer running time. We investigate this tradeoff in Table IV, where we present the running time, number of cuts and 95% confidence intervals of profit for the case of reference prices with $|S| = 100$ and $|S| = 1000$ scenarios. In most cases both models result in the same self-commitment decision. In cases where the two models differ (e.g. Spring Weekday-Weekend for $a = 0.25$), the increased benefits in moving from 100 to 1000 scenarios appear negligible compared to the required increase in running time. For this reason we use $|S| = 100$ scenarios in the rest of the paper.

The problems are solved using CPLEX 12.5.0.0 on a Macbook (2.4 GHz Intel Core i5, 8GB 1333 MHz DDR3). The algorithm is terminated in 100 iterations with the best solution found in case a certificate of optimality is not furnished. Summer weekdays and weekends require fewer iterations since the search space of the algorithm is smaller due to the constraints of Eqs. (42). The algorithm is initialized with the market schedule.

VI. Conclusions and Future Research

We have presented a risk-averse stochastic model for self-committing combined cycle units under real-time electricity price uncertainty. We have used the model in order to investigate the impact of risk aversion and price volatility on the incentive for generators to self-commit their units.

The advantage of a stochastic self-commitment model in terms of accounting for uncertainty, risk aversion, and a longer horizon can yield consistent benefits relative to the day-ahead market. This can increase the benefits of self-commitment and may limit the room for economic signals in the market. The benefit of self-commitment is reduced by increasing risk aversion. Price volatility increases the benefit of self-commitment in the risk-neutral case. This may appear counter-intuitive since the day-ahead market hedges against risk, which is greater in conditions of higher price volatility. The result is driven by the fact that self-commitment explicitly accounts for the increased uncertainty resulting from higher price volatility and arrives at more profitable unit commitment schedules due to the dominant role of high-price periods on the profit distribution. Instead, for greater risk aversion the marginal benefits of self-commitment relative to the day-ahead market schedule decrease with higher price volatility.

TABLE IV
RUNNING TIMES, NUMBER OF CUTS, AND 95% PROFIT CONFIDENCE INTERVALS FOR 100 AND 1000 SCENARIOS. DAY TYPES ARE AS FOLLOWS: (I) SPRING WEEKDAY-WEEKEND, (II) SPRING WEEKDAY-WEEKDAY, (III) SUMMER WEEKDAY-WEEKEND, (IV) SUMMER WEEKDAY-WEEKDAY.

Day	α	$\lvert S\rvert$	Time (sec)	Cuts	Profit ($\$ \cdot 10^3$)
I	0	100	537	100	59.7 - 64.5
I	0	1000	2679	100	59.7 - 64.5
I	0.25	100	588	100	0
I	0.25	1000	2901	100	4.2 - 6.0
I	0.5	100	499	100	0
I	0.5	1000	2522	100	0
I	0.75	100	469	100	0
I	0.75	1000	2343	100	0
II	0	100	532	100	60.0 - 64.4
II	0	1000	2875	100	60.0 - 64.4
II	0.25	100	465	100	4.7 - 6.4
II	0.25	1000	3058	100	4.2 - 6.0
II	0.5	100	387	100	0
II	0.5	1000	2582	100	0
II	0.75	100	456	100	0
II	0.75	1000	2593	100	0
III	0	100	229	69	357.4 - 360.4
III	0	1000	2637	100	361.4 - 367.7
III	0.25	100	243	79	334.9 - 335.9
III	0.25	1000	1979	69	334.9 - 335.9
III	0.5	100	190	74	324.8 - 325.7
III	0.5	1000	1526	66	324.8 - 325.7
III	0.75	100	240	93	315.8 - 317.2
III	0.75	1000	2112	86	317.4 - 317.6
IV	0	100	162	65	414.9 - 420.9
IV	0	1000	1534	32	413.3 - 419.4
IV	0.25	100	159	67	375.8 - 376.7
IV	0.25	1000	2045	80	375.8 - 376.7
IV	0.5	100	203	74	366.2 - 367.1
IV	0.5	1000	1844	14	366.2 - 367.1
IV	0.75	100	242	87	359.4 - 359.6
IV	0.75	1000	2591	100	359.4 - 359.6

Self-commitment can limit the room for economic signals in the market. This effect is mitigated by the fact that the market schedule is fairly close to the optimal self-commitment schedule. In equilibrium, self-commitment generates the same schedule as the day-ahead market, although modeling this equilibrium can prove challenging.

Flexible ramp products are currently under consideration in various markets, including the California and Midwest ISO, in order to contribute towards the increased ramping requirements caused by renewable energy integration. This ancillary service is expected to influence the benefits of self-commitment and is worth exploring in future research.

APPENDIX

A. Nomenclature

Sets

S: set of scenarios for stochastic optimization

O: set of samples for Monte Carlo simulation

$T = \{1, \ldots, N\}$: set of time periods

X: set of states

Y: set of transitions

Decision variables

b_t: amount of power bought in period t (sold if negative)

$p_{xt}, r^+_{xt}, r^-_{xt}, u_{xt}$: production, regulation up, regulation down and indicator for commitment of unit in state x, period t

v_{yt}: indicator for transition of unit over y in period t

p_{xmt}: incremental production for unit in segment $m+1$ of state x in period t

u_t, v_t: indicator for commitment, startup of unit in period t

Parameters

N: number of periods in horizon

M: number of segments in incremental heat rate curve

π_s: probability of scenario s

UT, DT: minimum up and down time of unit

UT_x, DT_x: minimum up and down time of state x

HR_{xm}, BP_{xm}: incremental heat rate/breakpoint in segment m of state x

$VOM_x, R^+_x, R^-_x, RU_x, RD_x$: variable O&M cost, ramp up rate, ramp down rate, regulation up limit, and regulation down limit of state x

TC_y: transition cost of MSG unit for transition y

$F(y), T(y)$: set of states that y transitions from/to

D_t: demand in period t

$\bar{\lambda}_t, \lambda^{RU}_t, \lambda^{RD}_t$: day-ahead energy price, regulation up price, and regulation down in period t

λ_{st}: real-time price in sample / scenario s, period t

$\bar{b}_t, \bar{r}^+_t, \bar{r}^-_t$: cleared day-ahead market energy procurement, regulation up and regulation down

B. Conditional Value at Risk

This section is based on Rockafellar and Uryasev [15]. Consider a decision vector w within a decision set W and a random vector λ that obeys a probability measure $\mathbb{P}$ that is independent of w. For each w, we denote by $\Pi(w, \cdot)$ the cumulative distribution function of a cost $Q(w, \lambda)$ which depends on both the decision vector w as well as the random vector λ:

$$\Pi(w, \zeta) = \mathbb{P}[\lambda | Q(w, \lambda) \leq \zeta], \tag{43}$$

where we assume that $Q(w, \lambda)$ is continuous in w and measurable in λ, with $\mathbb{E}[|Q(w, \lambda)|] < \infty$ for each $w \in W$.

Definition 1. *The a-value at risk associated with a decision w is the value*

$$\zeta_a(w) = \min\{\zeta | \Pi(w, \zeta) \geq a\}. \tag{44}$$

Definition 2. *The a-conditional value at risk of the cost associated with a decision w is*

$$CVaR_a(w) = \mathbb{E}_{\Pi_\alpha}[Q(w, \lambda)] \tag{45}$$

where the expectation is taken with respect to the following measure:

$$\Pi_a(w, \zeta) \quad = \quad \begin{cases} 0, & \zeta < \zeta_a(w) \\ \frac{\Pi(w,\zeta)-a}{1-a}, & \zeta \geq \zeta_a(w) \end{cases}$$

The level of a controls the level of risk aversion. Effectively, the decision maker ignores the a% most favorable outcomes, as if they had never occurred, and only accounts for the payoff in the $(1-a)$% worst cases. As a increases, more favorable outcomes are accounted for and the lottery is evaluated more favorably by the decision maker. In the case where $a = 0$, all outcomes are accounted for in the assessment of the lottery, which corresponds to risk-neutral behavior.

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