

An ADMM-based Method for Computing Risk-Averse Equilibrium in Capacity Markets

Hanspeter Höschle, Hélène Le Cadre, Yves Smeers, Anthony Papavasiliou, Ronnie Belmans

Abstract—Uncertainty in electricity markets introduces risk for investors. High fixed cost and increased dependency on infrequent and uncertain price spikes characterize investments. The risk-averse behavior of investors might lead to poor decision-making and undermines generation adequacy. Electricity market models rarely treat the interaction of market design and risk aversion. The representation of capacity mechanisms in modeling approaches focusing on risk aversion is limited. Our contribution addresses two problems. First, we propose a stochastic market equilibrium model. Investors are represented as risk-averse agents. The Conditional Value-at-Risk is used as risk measure. Second, we propose an algorithm based on the Alternating Direction Method of Multipliers to compute a risk-averse equilibrium. We benchmark our approach with a state-of-the-art solver relying on a Mixed Complementarity Problem reformulation. We show that for larger case studies our proposed approach is preferable. The algorithm converges in all cases while conventional solvers fail to compute a risk-averse equilibrium. The methodology is transferable to other risk-averse equilibrium models. With reference to capacity markets, we conclude that they are more beneficial in a risk-averse market. Capacity markets result in lower total cost, while avoiding expected energy not served. This statement still holds with increased price caps in energy-only markets.

Index Terms—Alternating Direction Method of Multipliers, Capacity Mechanisms, Market Equilibrium, Power System Economics, Power System Planning, Risk Analysis

NOMENCLATURE

A. Sets

\mathcal{T}	Set of time steps
\mathcal{S}	Set of scenarios
\mathcal{A}	Set of all agents
\mathcal{N}	Set of generators
\mathcal{X}_i	Set of strategies for generator i
\mathcal{X}_c	Set of strategies for consumer c
\mathcal{X}_p	Set of strategies for price-setting agent p
\mathcal{X}	Set of all combinations of strategies

B. Parameters

$W_{s,t}$	Weighting factor of each time step	-
P_s	Probability of scenario	-
$\bar{\lambda}_s, \bar{\lambda}_s^c$	Price caps on markets	€/MW, €/MWh
$D_{s,t}^e$	Demand on energy-based market	MWh/h
$\lambda_s^{\#}$	Target price on capacity market	€/MW
$D_s^{\#}$	Target demand on capacity market	MW
$\underline{D}_s^c, \bar{D}_s^c$	Minimum and maximum capacity demand	MW
C_i^g	Variable cost of generation	€/MWh
C_i^{inv}	Annualized fixed cost	€/MW
$A_{i,s,t}$	Underlying profile of availability	-
R_i	Ramping rate of technology	-
CR_i^r	Credits for RES target	-
CR_i^c	Credits for capacity market	-
β_i	Probability level in (0,1) for agent i	-
γ_i	Weighting of objective function	-

ρ	Penalty factor used in the algorithm	-
ψ, ψ	Stopping criteria based on primal and dual residuals	-

C. Strategies of agents

χ_i	Strategy $(x_{i,s,t}^e, x_{i,s}^r, x_{i,s}^c, c_i)$ of generator i	
$x_{i,s,t}^e$	Energy output of generator i	MWh
$x_{i,s}^r$	RES certificates of generator i	MWh
$x_{i,s}^c$	Available capacity of generator i	MW
c_i	Installed capacity of generator i	MW
χ_c	Strategy $(x_{c,s,t}^e, x_{c,s}^r, x_{c,s}^{\#}, x_{c,s}^c)$ of consumer c	
$x_{c,s,t}^e$	Energy non-served	MWh
$x_{c,s}^r$	Gap RES certificates	MWh
$x_{c,s}^{\#}$	Capacity demand	MW
$x_{c,s}^c$	Unserved capacity demand	MW
λ_p	Prices $(\lambda_{s,t}^e, \lambda_s^r, \lambda_s^c)$ set by price-setting agent p	
$\lambda_{s,t}^e$	Hourly price for energy output	€/MWh
λ_s^r	Annual price for RES certificates	€/MWh
λ_s^c	Annual price for capacity	€/MW

D. Auxiliary

$u_{i,s}$	Utility of each agent i for scenario s	
α_i	Approximation of Value-at-Risk (endogenous) for β_i	€
$\text{CVaR}_i(\cdot)$	Conditional Value-at-Risk for β_i	€
$\Pi_i(\cdot)$	Utility function of each agent i	€
$\pi_i(\cdot)$	Profit function of generator i	€

I. INTRODUCTION

INVESTMENTS in generation technologies in the power sector are taken with expectations about future revenues to recover initial capital expenditures. This affects conventional technologies with varying ratios of fixed and variable costs, as well as emerging Renewable Energy Sources (RES) with high fixed and low variable costs [1]. This implies that each investment is assessed thoroughly for uncertainties and risks.

Uncertainties and the resulting risks in the power sector have increased in both kind and extent. First, the development of decentralized generation and demand response have introduced a major uncertainty about the development of residual demand. This uncertainty comprises both peak load levels and consumption patterns. The uncertainty is further amplified by the intermittent injection of RES. Second, operational cost of generation depends on the underlying prices for resources such as natural gas, subsidy schemes and the prices of CO₂ emissions. This dependency is transferred into the cost of generation and consequently manifested in variable prices of electric energy. Finally, increasing uncertainties about energy policy-making and regulation introduce major risks on investment in generation technologies. Ever-changing market designs, support mechanisms, and regulations for market participation reduce confidence in the generators' long-term profitability.

With the objective of restoring the confidence of investors and provide stable long-term market signals, capacity mechanisms (CMs) have been implemented in order to complement energy-based markets. By remunerating firm capacity, they should provide an adequate long-term price signal [2]. The goal is to ensure generation adequacy by setting a minimum capacity demand [3]. A discussion of currently implemented CMs together with a detailed description is provided in [4]. CMs interact with markets for electric energy, markets for flexibility or ancillary services, and support schemes for RES. A detailed study of the economic results is provided in [5].

Investors in the power sector are typically risk-averse i.e., they have a negative evaluation of risk. In other words, the investors attribute higher importance to less favorable future outlooks. The combination of high fixed costs and the dependency on occasional peak pricing are perceived as a hurdle for new investments.

In order to capture these risks in the changing power sector, long-term models including investment decisions require two main adaptations. On the one hand, the models should capture uncertainties by means of multiple scenarios varying the parameters related to the sources of uncertainty. On the other hand, high temporal resolution is necessary to capture uncertainties related to variability and occasional scarcity. The risk aversion of investors should be incorporated and modeled by risk measures altering the objective of the investors.

A. Risk aversion in capacity expansion models

Investment decisions must be taken accounting for multiple more or less probable future outlooks. Investors create scenarios to capture both the most probable future outcomes and to account for less probable outcomes but with the most extreme results. This is the typical way of quantifying risk as the combination of probabilities and revenues for a set of scenarios. Risk-neutral behavior describes decision-making based on expected profits, hence, taking all scenarios into account based on their given probabilities [6].

Because of the changing market circumstances and the nature of the capital-intensive investment, it is valid to assume that investment decisions are taken in a risk-averse manner. In other words, investors assign higher weights to the scenarios with worse profits. Naturally, this makes the investment decisions differ from the risk-neutral decision. One extreme example is the worst-case approach in which only the scenario with the worst profit is the baseline for decision-making. In general, risk measures describe the way in which weights are assigned to given scenarios [7].

Considerable effort has been made in the description of different risk measures and their application in various industries [7], [8], [9]. One popular risk measure for representing risk-averse behavior is the Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR_{*i*}), also Average Value-at-Risk, introduced in Rockafellar and Uryasev [8]. Let β_i be a probability level in (0,1) for each agent that describes the risk aversion. The VaR is defined as a percentile for β_i of the profit distribution of scenarios. The CVaR_{*i*} describes the average of the scenario profits equal or worse to the VaR. As a result,

in contrast to the VaR, the CVaR_{*i*} also contains information about the distribution of the scenarios with the worst profits. An extensive list of risk measures and their implications is given, for example, in [7]. The methodology that we propose in this paper could be extended to a larger class of risk measures next to CVaR_{*i*}.

In Section II, we position our market equilibrium model in market models with different capacity mechanisms and highlight our contribution with respect to the literature. As a starting point, we use the approach of incorporating risk measures in equilibrium models introduced in [9]. The authors apply the CVaR_{*i*} in an equilibrium model with investment decisions. The model is solved by a reformulation as a Mixed Complementarity Problem (MCP). This approach is used for benchmarking our proposed methodology.

B. Contributions

Our contribution focuses on two missing elements in the literature that were identified. First, we assess the impact of capacity mechanisms on risk-averse market participants. In order to do so, risk measures are introduced to the modeling of market participants' decision-making. The initial purpose of risk reduction through capacity mechanisms can only be assessed to a limited extent in a risk-free context. Detailed capacity expansion models with capacity mechanisms [10], [11] do not include risk measures yet. Existing stochastic capacity expansion planning models in the literature [9], [12] under-represent the characteristics of capacity mechanisms by adding a simplified capacity constraint. A combination of both, however only incorporating cases with two technologies, can be found in [13]. We contribute to the existing literature on capacity expansion planning by proposing a model formulation that closes this gap. The proposed model accounts for a representation of a capacity mechanism. In particular, a centralized capacity market with a downward-sloped demand curve is introduced. At the same time, the proposed model takes into account risk-averse behavior of multiple agents by using the risk measure CVaR_{*i*}.

Second, the introduction of risk measures in market equilibrium models introduces non-convexity as described in Ralph and Smeers [14] and consequently new challenges to the solution techniques. As stated in [9], [15], [16], state-of-the-art solvers based on MCP reformulation [17] are not necessarily able to find an equilibrium for large-scale capacity expansion problems including endogenous risk measures. As a result of these solver problems, the numerical examples are very limited. Our contribution proposes an algorithm inspired by Alternating Direction Method of Multipliers (ADMM) in form of the optimal exchange as described in [18]. Our approach for equilibrium models brings advantages in computing a Nash Equilibrium (NE) for settings with risk-averse agents based on CVaR_{*i*}. At the same time, the findings offer hope that it is also applicable for computing a NE with other risk measures. The methodology enables us to simulate larger case studies in terms of scenarios, temporal resolution, and number of risk-averse market participants thanks to the proposed iterative updates of the agent's decisions and the market prices.

In Section II, we describe the capacity expansion planning model in a risk-averse setting as an equilibrium problem which has a NE as solution concept. Thereafter, Section III describes the proposed methodology based on ADMM that we use to compute an equilibrium. Section IV applies the methodology.

II. CAPACITY EXPANSION PLANNING AS A NON-COOPERATIVE GAME

We describe a capacity planning model that involves price-taking agents without market power including generators and a consumer. Additionally, a price-setting market agent is included. The problem is formalized mathematically as a non-cooperative game in strategic form. We detail the utility functions of all the agents involved. The generator problem is presented in a risk-averse setting.

A. Solution concepts

A non-cooperative game with market clearing conditions is formulated as presented in Fig. 1. The set of agents is defined as $\mathcal{A} := (G_i)_{i \in \mathcal{N}} \cup \{c\} \cup \{p\}$ (finite number of generators $(G_i)_{i \in \mathcal{N}}$, one consumer c as an aggregation of a multiplicity of atomic consumers, and a price-setting agent p). Our goal is to compute a NE as solution of the non-cooperative game. We assume that all agents' strategies belong to a bounded set, as classical in Kakutani's fixed point theorem that is used to prove the existence of a NE. Arrow and Debreu [19] introduced the game approach to general equilibrium. An alternative presentation is given in Mas Collet *et al.* [20]. We apply the same reasoning in the simpler context of partial equilibrium problem as developed in de Maere d'Aertryke and Smeers [21], for both a NE and Generalized Nash Equilibrium (GNE), and Ralph and Smeers [14].

For all agent $i \in \mathcal{A}$, we denote \mathcal{X}_i its set of strategies. We let $\mathcal{X} := \times_{i \in \mathcal{A}} \mathcal{X}_i$ denote the set of all possible combinations or profiles of strategies that may be chosen by the agents in \mathcal{A} , when each agent $i \in \mathcal{A}$ chooses one of its strategies in \mathcal{X}_i . We now introduce the utility function of all the agents involved in the game: for any agent $i \in \mathcal{A}$ we define its utility function $\Pi_i : \mathcal{X} \rightarrow \mathbb{R}$. This setting gives rise to a non-cooperative game for the price-taking agents $\Gamma := (\mathcal{A}, \mathcal{X}, (\Pi_i)_{i \in \mathcal{A}})$ that we formulate in strategic form, where each agent maximizes selfishly its objective Π_i . We let χ_{-i} be the vector containing the strategies of all the other agents in \mathcal{A} than i . Formally, given the strategies of all the other agents in \mathcal{A} , χ_{-i} , each agent $i \in \mathcal{A}$ solves independently and simultaneously:

$$\max_{\chi_i \in \mathcal{X}_i} \Pi_i(\chi_i, \chi_{-i}). \quad (1)$$

The associated solution concept is that of a NE [22], [23]: a strategy profile $\chi^* \in \mathcal{X}$ is a NE if, and only if, $\Pi_i(\chi^*) \geq \Pi_i(\chi_i, \chi_{-i}^*), \forall i \in \mathcal{A}, \forall \chi_i \in \mathcal{X}_i$. Under the strategy bounded assumption, it is possible to prove the existence of a NE.

We represent three markets, an hourly market for energy, an annual market for availability (capacity market) and an annual market capturing the policy target for RES. The latter is organized as a market for RES certificates forming a lower bound for green energy. This market-based price for these certificates

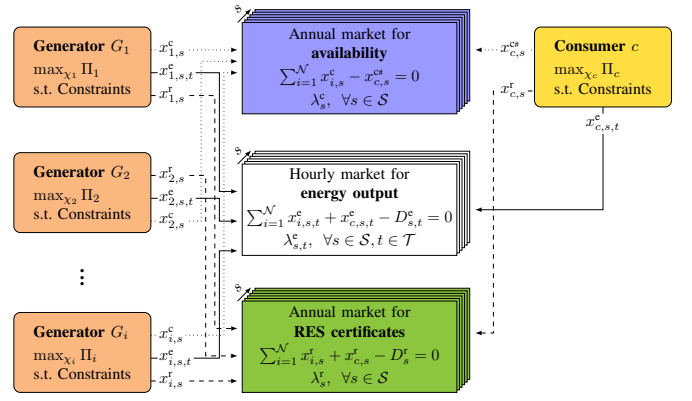


Fig. 1. Graphical representation of model setup with 3 markets (rectangles) for energy output, availability and RES certificates. Market participants/agents (rounded corners) are shown with their decision variables.

forms an additional revenue stream for RES (see Fig. 1). The price-setting agent, p , sets the prices for energy, availability and RES certificates, $\chi_p = \lambda_p = (\lambda_{s,t}^e, \lambda_s^c, \lambda_s^r) \in \mathcal{X}_p$. The price-setting agent could also be modeled as three independent price-setters for each market respectively. Because its utility functions and constraints for the different markets are separable, this formulation would yield the same NE. Each generator $i \in \mathcal{N}$ decides the offered market volumes and the installed capacity, $\chi_i = (x_{i,s,t}^e, x_{i,s}^c, x_{i,s}^r, c_i) \in \mathcal{X}_i$. The consumer c decides on non-served energy, capacity demand, non-served capacity, and the missing RES certificates, $\chi_c = (x_{c,s,t}^e, x_{c,s}^c, x_{c,s}^r, c_c) \in \mathcal{X}_c$.

In this paper, we assume that the generators do not anticipate the impact of their investment decisions on the market clearing prices. In this approach, the market operator moves simultaneously with the generators. It has a NE as solution concept.

An alternative approach assumes that the generators anticipate the impact of their investments on the prices set by the market operator. This formulation is a multi-leader (generators, consumer), one follower (market operator) Stackelberg game, that can be formulated as a bilevel optimization problem [24]. The equilibrium of such games are characterized as Subgame Perfect Nash Equilibrium (SPNE) [23] and traditionally computed using backward induction. Each leader solves independently and simultaneously an optimization problem formulated as an Mathematical Program with Equilibrium Constraints (MPEC), in which the optimality conditions for the market operator's program are the constraints shared by all leaders. The equilibrium problem among the above MPECs represents a Generalized Nash Equilibrium (GNE) game [25], that may have zero, a unique or a multiplicity of equilibria [26]. The comparison of both concepts could be an interesting future research direction.

B. Risk-averse generators using risk measure CVaR_i

In the risk-averse case, each generator $i \in \mathcal{N}$ has the utility function $\Pi_i(\chi_i, \lambda_p)$, formally given by (2a). Note that in the proposed model formulation, the utility function does not depend on the decision of the other generators and consumer, only on the price-setting agent, λ_p . The objective

is to maximize the weighted (γ_i) sum of the expected profit and the risk measure CVaR_i with individual probability level β_i , which is based on the profit as well. A $\gamma_i > 0$ also ensures that no scenario is valued with an endogenous probability of 0. For $\gamma_i=1$, the model describes the decision of a risk-neutral generator.

Each generator $i \in \mathcal{N}$ decides on the installed capacity c_i that is valid in all scenarios. The expected profit is the profit per scenario $\pi_{i,s}$ weighted with each scenario's probability P_s . The profit originates from the revenues achieved over the different markets. The revenues are based on the offered energy output $x_{i,s,t}^e$, offered RES certificates $x_{i,s}^r$ and offered availability $x_{i,s}^c$ multiplied with the respective prices.

The costs depend on the variable cost of generation C_i^g and the investment cost C_i^{inv} . Each generator $i \in \mathcal{N}$ solves the following optimization problem to determine its optimal investment and offered values:

$$\max_{\chi_i} \Pi_i(\chi_i, \lambda_p) = \gamma_i \cdot \sum_{s=1}^S P_s \cdot \pi_{i,s}(\chi_i, \lambda_p) + (1 - \gamma_i) \cdot \text{CVaR}_i(\chi_i, \lambda_p), \quad (2a)$$

$$\text{s.t. } x_{i,s,t}^e \leq A_{i,s,t} \cdot c_i, \quad \forall s \in \mathcal{S}, t \in \mathcal{T}, \quad (\mu_{i,s,t}) \quad (2b)$$

$$x_{i,s,t+1}^e \leq x_{i,s,t}^e + R_i \cdot c_i, \quad \forall s \in \mathcal{S}, t \in \mathcal{T}, \quad (\rho_{i,s,t}^{\text{up}}) \quad (2c)$$

$$x_{i,s,t+1}^e \geq x_{i,s,t}^e - R_i \cdot c_i, \quad \forall s \in \mathcal{S}, t \in \mathcal{T}, \quad (\rho_{i,s,t}^{\text{dn}}) \quad (2d)$$

$$x_{i,s}^c \leq CR_i^c \cdot c_i, \quad \forall s \in \mathcal{S}, \quad (\mu_{i,s}^c) \quad (2e)$$

$$x_{i,s}^r \leq CR_i^r \cdot \sum_{t=1}^T x_{i,s,t}^e \cdot W_{s,t}, \quad \forall s \in \mathcal{S}, \quad (\mu_{i,s}^r) \quad (2f)$$

$$x_{i,s,t}^e, x_{i,s}^c, x_{i,s}^r, c_i \geq 0, \quad \forall s \in \mathcal{S}, t \in \mathcal{T}. \quad (2g)$$

The CVaR_i formulation follows the work of Ehrenmann and Smeers [9] and is based on the methodology presented in [6], [8]¹. The constraints limit the set of strategies and represent technical limitations for the offered volumes. For each time step, the offered energy is limited by the installed capacity (2b). The change of offered energy from one time step to the next is limited by the ramping capabilities depending on the ramp rate R_i . This holds for increased (2c) and decreased (2d) energy output. Similarly, the offered availability is limited by the de-rated (with factor CR_i^c) installed capacity (2e). Finally, the offered RES certificates are limited by total energy output (2f), de-rated by the factor CR_i^r . In case of one scenario with probability $P_s=1$, the model becomes the risk-free deterministic model as presented in [10]. The installed capacity and offered market volumes only take positive values (2g).

C. Consumer c

Consumer c maximizes the expected consumer surplus $\Pi_c(\chi_c, \lambda_p)$ given by the consumer surplus across the three markets, formally described by the utility function (3a). We assume that the demand for energy and RES certificates is inelastic. The demand for RES certificates D_s^r is set exogenously as a share of the total inelastic energy demand $D_{s,t}^e$. Hence, in case of insufficient supply the price reaches the price cap

¹For readability, the reformulation of the CVaR_i is only presented in the electronic appendix, and the formulation is synthesized by the auxiliary term $\text{CVaR}_i(\chi_i, \lambda_p)$. The complete linearized model formulation of the CVaR_i risk measure is provided in an electronic appendix provided at http://esat.kuleuven.be/~hhoschle/paper_admin/

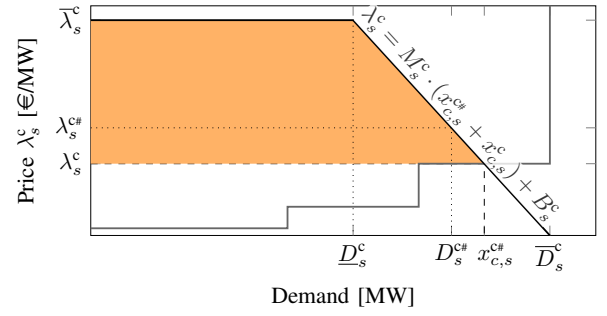


Fig. 2. Demand curve for capacity-based market with a constant slope M_s^c and symmetrical bounds for minimum and maximum capacity demand.

$\bar{\lambda}^c, \bar{\lambda}^r$. Consequently, on the markets for RES and energy, the consumer surplus is given by the served demand multiplied by the difference of price cap and market clearing price (see first and second row of utility function).

The capacity demand is modeled as being elastic, similar to the current capacity markets in Great Britain [27] with the simplification that the slope is constant. The sloped part of the demand curve is described by a linear expression (3b) given the slope, M_s^c , and y-intercept, B_s^c . The consumer surplus can be calculated from the served capacity, $x_{c,s}^{\#}$, not-served capacity, $x_{c,s}^c$, and the price, λ_s^c . As an example, the surplus is visualized in Fig. 2.

An adaptation of the model to handle changing slopes is presented in [5]. The same approach can also be used for the demand for energy, e.g. [28], or RES certificates. Given the assumptions, the optimization problem of the consumer is as follows:

$$\max_{\chi_c} \Pi_c(\chi_c, \lambda_p) = \sum_{s=1}^S P_s \left[(\bar{\lambda}^r - \lambda_s^r) \cdot (D_s^r - x_{c,s}^r) + \sum_{t=1}^T W_{s,t} \cdot (\bar{\lambda}^e - \lambda_{s,t}^e) \cdot (D_{s,t}^e - x_{c,s,t}^e) \right] + (\bar{\lambda}^c - \lambda_s^c) \cdot \underline{D}_s^c + 1/2 \cdot (\bar{\lambda}^c - \lambda_s^c) \cdot (x_{c,s}^{\#} - \underline{D}_s^c) \quad (3a)$$

$$\text{s.t. } x_{c,s}^{\#} + x_{c,s}^c = \lambda_s^c / M_s^c - B_s^c / M_s^c, \quad \forall s \in \mathcal{S}, \quad (\omega_s^c) \quad (3b)$$

$$x_{c,s,t}^e, x_{c,s}^{\#}, x_{c,s}^c, x_{c,s}^r \geq 0, \quad \forall s \in \mathcal{S}, t \in \mathcal{T}. \quad (3c)$$

D. Price-setting agent p

The price-setting agent, p , sets the prices, λ_p , on the three markets given the volumes of the generators, χ_i , and the consumer, χ_c . Its objective is to minimize the excess demand², formally given by the utility function (4a).

The brackets contain the market clearing conditions for each market. First, the RES demand, D_s^r , is equal to the RES certificates of all generators plus not-served RES certificates, $x_{c,s}^r$. Second, the energy demand, $D_{s,t}^e$, is equal to the offered energy of all generators plus energy not-served, $x_{c,s,t}^e$. Finally, the resulting capacity demand, as defined by the consumer's demand curve, must be equal to the offered capacity of all generators. For all market clearings, either the excess demand or the price of a market clearing is 0.

The prices, chosen by the price-setting agent, are limited by the markets' respective price caps (4b)-(4d). Similarly, price

²The price-setting agent, p , does not have a decision variable that spans the scenarios. Consequently, the concept of risk aversion is not applicable.

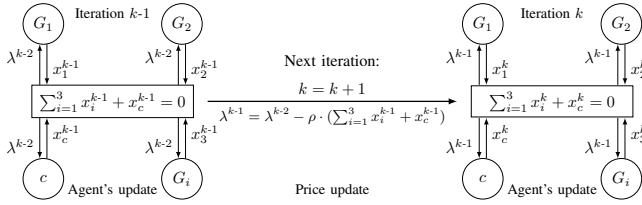


Fig. 3. Iterations of decentralized update process. This process of optimal exchange ADMM is also described as a form of “tâtonnement”, “trial and error” or price adjustment process [18].

floors are possible. We assume a lower limit of 0 for all prices. Formally, the optimization problem reads as follows:

$$\begin{aligned} \max_{\lambda_p} \Pi_p(\lambda_p, \chi_i, \chi_c) = & - \sum_{s=1}^S \left(\sum_{i=1}^N x_{i,s}^r + x_{c,s}^r - D_s^r \right) \cdot \lambda_s^r \\ & - \sum_{s=1}^S \sum_{t=1}^T \left(\sum_{i=1}^N x_{i,s,t}^e + x_{c,s,t}^e - D_{s,t}^e \right) \cdot \lambda_{s,t}^e \\ & - \sum_{s=1}^S \left(\sum_{i=1}^N x_{i,s}^c - x_{c,s}^{\#} \right) \cdot \lambda_s^c \end{aligned} \quad (4a)$$

$$\text{s.t. } 0 \leq \lambda_{s,t}^e \leq \bar{\lambda}^e, \quad \forall s \in \mathcal{S}, t \in \mathcal{T}, \quad (4b)$$

$$0 \leq \lambda_s^c \leq \bar{\lambda}^c, \quad \forall s \in \mathcal{S}, \quad (4c)$$

$$0 \leq \lambda_s^r \leq \bar{\lambda}^r, \quad \forall s \in \mathcal{S}. \quad (4d)$$

III. METHODOLOGY

In this section, we link the non-cooperative game formulation, Γ , from Section II, and an algorithmic approach to compute a risk-averse equilibrium. We propose an algorithm inspired by ADMM to compute the NE and show the relationship to MCP reformulation. We describe the necessary steps to ensure that the proposed algorithm converges to a NE.

A. ADMM to compute an equilibrium

We propose an algorithm inspired by ADMM to compute an equilibrium of the non-cooperative game. Typically, ADMM is used to solve optimization problems. ADMM offers benefits if a problem is separable in local optimization subproblems. ADMM is widely used in decentralized optimization and sees increased applications in machine learning, image processing, and decentralized network operation, such as electricity distribution systems or sensor networks [29], [30].

ADMM is known for its good convergence for both convex and also non-convex optimization. Boyd *et al.* [18] provide a convergence proof for convex problems, and recently, more papers with extended convergence proofs for other classes of optimization problems are available [31].

The non-cooperative game, Γ , can be interpreted as a specific sharing problem, namely an optimal exchange, wherein equality constraints that combine the otherwise separable decision variables x_i are matched on the market clearing conditions (Fig. 1) with the dual variables λ .

Equally to an application of ADMM for distributed optimization, the mechanism of iterative update steps for each agent and consequently of the price is used to converge towards an equilibrium of the game. The exchange of information by the agents, i.e., operation decisions and market prices between the update steps is shown in Fig. 3.

Provided our proposed algorithm converges, no agent has an incentive to deviate from its decision and the market clearing conditions are satisfied. Hence, an equilibrium is reached. Note that there is no guarantee on the uniqueness of the found equilibrium. Whereas in our simulations the proposed algorithm always converges to an equilibrium, the PATH solver based on MCP reformulation can become instable. This is due to the non-convexity introduced by endogenous risk assessment of the agents as described in [9], [32].

The MCP reformulation of market equilibrium problems is a well known approach for solving equilibrium problems. It is for example applied in [9], [10], [12] or described in details in [33]. By making use of the Karush-Kuhn-Tucker (KKT) conditions, a set of complementarity conditions can be derived. The combined set of complementarity constraints of all agents and the market clearing conditions are solved as a square system using dedicated state-of-the-art solvers, e.g. the PATH solver [17].

B. ADMM-based approach for the equilibrium problem

In order to use the proposed ADMM-based algorithm for computing an equilibrium, the KKT-conditions of the market equilibrium are used for modifying the ADMM's update steps. The resulting optimality conditions of the update steps satisfy the KKT-conditions of the market equilibrium. If the ADMM-based algorithm converges using the same optimality conditions, the obtained result can be interpreted as the coinciding equilibrium. This transfer step from equilibrium problem to the ADMM-based algorithm for distributed optimization is ensured by the specification of the augmented Lagrangian $L_{\rho,i}$, $L_{\rho,c}$. It is adapted such that the optimality conditions of minimizing the unaugmented Lagrangian $L_{0,i}$, $L_{0,c}$ match the equilibrium problem's KKT-condition.

During the update step, the updated decision variables χ_i^{k+1} are obtained by minimizing the augmented Lagrangian. For the optimal exchange with a sharing constraint as described in Section III-A, the augmented Lagrangian function $L_{\rho,i}$ for each agent is as follows at each iteration $k+1$, $k \in \mathbb{N}^*$ [18]:

$$\begin{aligned} \chi_i^{k+1} = \operatorname{argmin}_{\chi_i \in \mathcal{X}_i} L_{\rho,i}(\chi_i, \lambda^k) = & f_i(\chi_i, \lambda^k) + \underbrace{\lambda^k \cdot \chi_i}_{1^{\text{st}} \text{ penalty term}} + \underbrace{\rho/2 \cdot \|\chi_i - (\chi_i^k - \bar{\chi}^k)\|_2^2}_{2^{\text{nd}} \text{ penalty term}}. \end{aligned} \quad (5)$$

The first penalty term is the multiplication of the sharing constraint's dual variable and the respective decision variable. The second penalty term is an expression of the impact of the decision variable on the remaining imbalance of the sharing constraint weighted with a penalty factor $\rho > 0$. The augmented Lagrangian $L_{\rho,i}$ in iteration $k+1$ given the prices λ^k is minimized using a quadratic solver.

When the optimum is reached, the second penalty term becomes zero. To make sure that our proposed algorithm converges to the same solution as we would expect from the equilibrium problem, we ensure that the optimality conditions of each agent's update step are the same as the respective KKT conditions of an MCP reformulation. Hence, we modify the original agent's objective function, Π_i , (2a) such that

the optimality conditions of the unaugmented Lagrangian $L_{0,i}, \forall i \in \mathcal{N}$ (6) coincide with the respective KKT conditions. Analogously, this is done for the objective of the consumer c (3a) and $L_{0,c}$ ³.

1) *Agent update*: For the agents' update step, the unaugmented Lagrangian for the risk-averse generator looks as follows:

$$\begin{aligned} L_{0,i} := & \gamma \cdot \left(C_i^{\text{inv}} \cdot c_i + \sum_{s=1}^S P_s \sum_{t=1}^T W_{s,t} \cdot C_i^g \cdot x_{i,s,t}^e \right) \\ & - (1 - \gamma) \cdot \text{CVaR}_i(\chi_i, \lambda), \\ & - \gamma \sum_{s=1}^S P_s \left(\sum_{t=1}^T W_{s,t} \cdot \lambda_{s,t}^e \cdot x_{i,s,t}^e \right. \\ & \quad \left. + \underbrace{\lambda_s^c \cdot x_{i,s}^c + \lambda_s^r \cdot x_{i,s}^r}_{\text{adapted 1st penalty term}} \right) \quad (6) \\ \text{s.t.} \quad & \text{constraints (2b)-(2g)}. \end{aligned}$$

The unaugmented Lagrangian, $L_{0,i}$, (6) can be read as the investment cost and cost of generation (first row), the unchanged CVaR_i expression (second row), and the first penalty term (third and fourth row). The penalty term represents the revenues in the same way as they are part of the agents' profit, $\pi_{i,s}$, in the utility function (2a). Compared to the original ADMM algorithm, the first penalty term is adapted. It is scaled by the weighting γ , the exogenous probabilities P_s for each scenario and the weight $W_{s,t}$ of each time step. The modified objective function represents the weighted (γ) sum of expected costs and the weighted CVaR_i .

As a result, the optimality conditions of the update step coincide with the KKT conditions of the equilibrium problem. Consequently, an equilibrium found by our proposed algorithm coincides with an equilibrium computed with the MCP reformulation³. As example, the optimality conditions resulting from the unaugmented Lagrangian, $L_{0,i}$, (6) and the utility function, Π_i , (2a) are compared for the offered energy, $x_{i,s,t}^e$. For readability, we assume that the constraints, (2b)-(2g), are summarized by $g(\chi_i) \geq 0$ and that μ is the associated dual variable. The optimality condition, (7), can be interpreted as an expression for which prices, energy output ($x_{i,s,t}^e > 0$) is justified. In other words, a generator only offers energy, if, for a time step t in a scenario s , the energy price, $\lambda_{s,t}^e$, at least covers the variable costs, C_i^g . This is weighted with the exogenous probability, P_s , and the endogenous valuation of each scenario, i.e., the risk-adjusted probabilities, $q_{i,s}$. They describe each generator's weighted valuation of each scenario [8]. Formally, this yields the following comparison:

$$\begin{aligned} 0 & \leq \frac{\partial L_{0,i}(\chi_i, \lambda_p)}{\partial x_{i,s,t}^e} + \mu \cdot \frac{\partial g(\chi_i)}{\partial x_{i,s,t}^e} \\ \Leftrightarrow 0 & \leq -\frac{\partial \Pi_i(\chi_i, \lambda_p)}{\partial x_{i,s,t}^e} + \mu \cdot \frac{\partial g(\chi_i)}{\partial x_{i,s,t}^e} \\ \Leftrightarrow 0 & \leq W_{s,t} \cdot (\gamma_i \cdot P_s + q_{i,s}) \cdot (C_i^g - \lambda_{s,t}^e) + \mu \cdot \frac{\partial g(\chi_i)}{\partial x_{i,s,t}^e} \\ & \perp x_{i,s,t}^e \geq 0, \quad \forall i \in \mathcal{N}, s \in \mathcal{S}, t \in \mathcal{T}. \quad (7) \end{aligned}$$

³The Lagrangian and full sets of KKT conditions for generators and consumer are provided in an electronic appendix provided at http://esat.kuleuven.be/~hshoschle/paper_admm/

2) *Price update*: The update of the prices is performed based on the remaining imbalance in the respective market clearing conditions found after each iteration k . As an example, the price for energy, $\lambda_{s,t}^{e,k+1}$, in the consecutive iteration, $k+1$, is reduced if there is excess supply ($x_{i,s,t}^e + x_{c,s,t}^e > D_{s,t}^e$), and vice versa (8a). This is done accordingly for the capacity market (8b), and the RES target (8c). The price update uses the remaining imbalance and the regularization terms borrowed from the ADMM. It is restricted by the penalty factor, ρ .

This update step emulates the utility function of the price-setting agents. However, instead of obtaining the market prices as result of the price setter's optimization problem, the prices are found borrowing the iterative update step. The formal description of the price updates is as follows:

$$\lambda_{s,t}^{e,k+1} = \lambda_{s,t}^{e,k} - \rho/2 \cdot \left(\sum_{i=1}^{\mathcal{N}} x_{i,s,t}^e + x_{c,s,t}^e - D_{s,t}^e \right), \quad (8a)$$

$$\lambda_s^{c,k+1} = \lambda_s^{c,k} - \rho/2 \cdot \left(\sum_{i=1}^{\mathcal{N}} x_{i,s}^c + x_{c,s}^c \right), \quad (8b)$$

$$\lambda_s^{r,k+1} = \lambda_s^{r,k} - \rho/2 \cdot \left(\sum_{i=1}^{\mathcal{N}} x_{i,s}^r + x_{c,s}^r - D_s^r \right). \quad (8c)$$

Experimentally, we found that a constant penalty factor $\rho=1.1$ provides reliable and stable convergence towards. A discussion on the choice of penalty parameters can be found in [18].

3) *Stopping criteria*: The iterative process is controlled by means of two stopping criteria. We introduce two stopping criteria for the primal and dual residual, ψ and ψ . The algorithm stops if the primal and dual stopping criteria simultaneously come under a threshold ϵ . The threshold ϵ is chosen based on the number of agents, scenarios and time steps as described in [29]. Moreover, it is parameterized with a parameter τ to control the algorithm based on the desired precision: $\epsilon = \tau \cdot \sqrt{|\mathcal{A}| \cdot |\mathcal{S}| \cdot |\mathcal{T}|}$.

For each market clearing condition, the primal residual r^{k+1} is the remaining imbalance for each market clearing condition in each scenario and time step if applicable (9a)-(9c). In case an equilibrium is obtained, the imbalances on all market clearing conditions converge to zero.

Consequently, we define a primal stopping criterion ψ^{k+1} as the sum of the primal residuals normalized by an l_2 -norm (9d). This approach follows Boyd *et al.* [18]:

$$r_{s,t}^{e,k+1} = \sum_{i=1}^{\mathcal{N}} x_{i,s,t}^{e,k+1} + x_{c,s,t}^{e,k+1} - D_{s,t}^e, \quad \forall s \in \mathcal{S}, t \in \mathcal{T}, \quad (9a)$$

$$r_s^{c,k+1} = \sum_{i=1}^{\mathcal{N}} x_{i,s}^{c,k+1} - x_{c,s}^{c,k+1}, \quad \forall s \in \mathcal{S}, \quad (9b)$$

$$r_s^{r,k+1} = \sum_{i=1}^{\mathcal{N}} x_{i,s}^{r,k+1} + x_{c,s}^{r,k+1} - D_s^r, \quad \forall s \in \mathcal{S}, \quad (9c)$$

$$\psi^{k+1} = \|r_{s,t}^{e,k+1}\|_2 + \|r_s^{c,k+1}\|_2 + \|r_s^{r,k+1}\|_2. \quad (9d)$$

For each decision variable of each agent that is part of a market clearing condition, a dual residual s^k is defined. It is a measure for the change of the decision variable from the previous iteration to the current iteration (10a)-(10c).

The change is defined as the difference between the decision variable in iteration k and $k+1$. The change is corrected by the average value of the decision variables for all agents. Hence,

the residual is also linked to the change of the other agents. This is valued with the penalty factor, ρ , in order to link the change in decision variables to the change of prices. In case an equilibrium is obtained, the agents do not have an incentive to deviate. Consequently, the change of each decision variable for all agent also converges zero.

Analogously to the primal stopping criteria, we define the dual stopping criterion $\tilde{\psi}^{k+1}$ as the sum of the dual residuals normalized by an l_2 -norm (10d).

$$s_{i,s,t}^{e,k+1} = \rho((x_{i,s,t}^{e,k+1} - \bar{x}_{s,t}^{e,k+1}) - (x_{i,s,t}^{e,k} - \bar{x}_{s,t}^{e,k})), \quad \forall i \in \mathcal{A}, s \in \mathcal{S}, t \in \mathcal{T}, \quad (10a)$$

$$s_{i,s}^{c,k+1} = \rho((x_{i,s}^{c,k+1} - \bar{x}_s^{c,k+1}) - (x_{i,s}^{c,k} - \bar{x}_s^{c,k})), \quad \forall i \in \mathcal{A}, s \in \mathcal{S}, \quad (10b)$$

$$s_{i,s}^{r,k+1} = \rho((x_{i,s}^{r,k+1} - \bar{x}_s^{r,k+1}) - (x_{i,s}^{r,k} - \bar{x}_s^{r,k})), \quad \forall i \in \mathcal{A}, s \in \mathcal{S}, \quad (10c)$$

$$\tilde{\psi}^{k+1} = \|s_{i,s,t}^{e,k+1}\|_2 + \|s_{i,s}^{c,k+1}\|_2 + \|s_{i,s}^{r,k+1}\|_2. \quad (10d)$$

We can show with our simulations for small problems that if the PATH solver finds an equilibrium, our proposed methodology yields the same result. However, the experiments show that the PATH solver fails to return a solution for larger problems in the risk-averse environment. This is also described in [15]. In fact, the solving process is terminated after several restarts by the solver returning an error. In contrast, the proposed methodology reliably finds solutions for the given set of experiments.

IV. CASE STUDY: IMPACT OF CAPACITY MECHANISMS ON RISK-AVERSE INVESTORS

In this section, the application of the proposed methodology is illustrated using a stylized case study. In the first part, we benchmark the proposed algorithm with the classical MCP reformulation. The comparison uses indicators such as computation time, scalability and convergence⁴. In the second part, we use the model to illustrate the mutual impact of a capacity market and risk aversion based on changing installed capacities and expected consumer cost.

A. Input data and scenarios

The case study compares two different market settings. First, it includes a setting with only a market for energy demand and RES certificates, in the remainder referred to as the energy-only case. Second, we consider a market setting that additionally includes a market for availability referred to as capacity market which is modeled as described in Section II.

The differences in the scenarios, hence the uncertainty, originate from the different underlying profiles for load, wind and solar power. We use load, solar and wind (onshore) profiles for 2013, 2014 and 2015 based on Belgian data [34]. The scenarios are composed by combining the profiles to a total of $|\mathcal{S}|=27$ scenarios. For all model runs, each scenario has equal probability $P_s=1/|\mathcal{S}|$. In order to test the scalability of the algorithm, the number of scenarios is varied between 1 and 27. Respectively, the profiles and probabilities are adjusted.

⁴All computations are executed on an Intel i7 Quad Core at 2.7Ghz and 16GB RAM using Julia 0.5 including Complementarity and JuMP, and the PATH 4.7 solver.

TABLE I
INPUT PARAMETERS PER TECHNOLOGY $i \in \mathcal{N}$ IN CASE STUDY

Type $i \in \mathcal{N}$	C_i^g [€/MWh]	C_i^{inv} [€/MW,year]	R_i [%/h]	CR_i^e [-]	CR_i^c [-]
<i>Base</i>	36	138 000	50	0.0	1.0
<i>Mid</i>	53	82 000	80	0.0	1.0
<i>Peak</i>	76	59 000	100	0.0	1.0
<i>PV</i>	0	110 000	-	1.0	0.0
<i>Wind</i>	0	76 500	-	1.0	0.0

For each scenario, 5 or 10 representative days with associated weights are selected. The selection is based on [35] and optimizes the representation of a full year with reduced profiles. Each day is split into hourly time steps resulting in a total of $|\mathcal{T}|=120$ respectively 240 time steps. Within each representative day, the weight of each hour is equal.

In both settings, the energy market has a price cap of $\bar{\lambda}^e=3000$ €/MWh. The target for the RES certificates is set to 20% of the total energy demand. In scenarios with a capacity market, the target capacity price, $\lambda_s^{c\#}=0.5 \cdot C_{Peak}^{inv}$, and target capacity demand $D_s^{c\#}$, equal to the peak demand of the respective scenario, determines the demand curve. The minimum and maximum capacity demand $\underline{D}_s^c, \bar{D}_s^c$ are set symmetrically at 97% and 103% of $D_s^{c\#}$ (Fig. 2).

The generators are grouped by technologies. Three conventional (*Base*, *Mid*, *Peak*) and two renewable (*PV*, *Wind*) technologies are introduced. Table I lists the economic (fixed and variable cost) and technological (ramping) parameters of the different technologies. The de-rating factors (CR_i^e, CR_i^c) limits the participation in the markets for availability and RES certificates for each technology, e.g. only renewables may offer RES certificates.

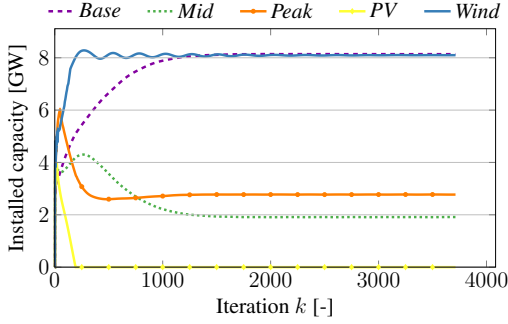
All conventional resources are assumed to be risk-averse and parameterized with the same $\beta = \beta_i, \forall i \in \mathcal{N}$. A sensitivity analysis on β is executed in order to highlight the impact of the risk aversion. The value of β varies between 1, i.e. risk-neutral and 0, i.e. taking only the worst-case scenario into account. In the case study, all generators have the same risk aversion, although their risk exposure is different due to different technical availability and proportion of fixed and variable cost.

B. Convergence and computation time

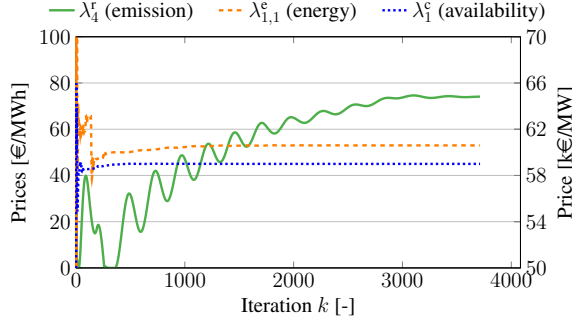
A set of model runs both for a risk-neutral and risk-averse setting is computed to compare the proposed methodology and the existing PATH solver using an MCP reformulation. Looking at the convergence behavior, Fig. 4 and Fig. 5 illustrate the behavior of the algorithm for a risk-averse case of 27 scenarios.

Fig. 4a shows that the installed capacities for all technologies converge to a stable level already after relatively few iterations. The remainder of the iterations, before the stopping criteria are reached, are spent on reducing the imbalance to a minimum by adapting prices (Fig. 4b).

A scaling of the balancing constraints can further improve the convergence behavior. In this case study, the balancing



(a) Installed capacities



(b) Prices

Fig. 4. Convergence of the installed capacities and selected prices for 27 scenarios and 5 days in risk-averse setting before reaching stopping criteria.

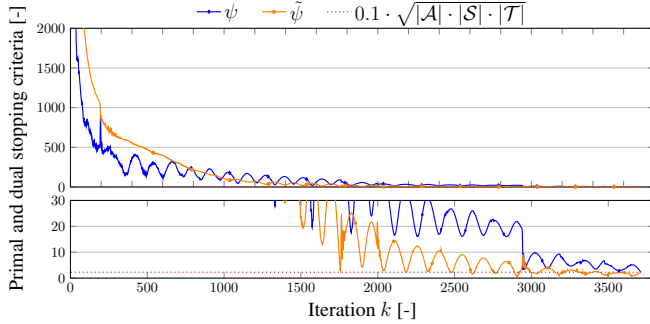


Fig. 5. Primal ψ and dual stop criteria $\tilde{\psi}$ for iteration k with stopping criteria for 27 scenarios, 240 time steps (5 days) in the risk-averse setting.

constraint for the RES certificates is scaled due to the summation of the RES target over the time period. It is scaled by $1/1752$ based on 8760 hours and the RES target of 20%. This scaling aligns all market clearing conditions to the same magnitude and improves the price update step.

Fig. 5 shows the development of the norms for the primal ψ and dual $\tilde{\psi}$ residuals over the iterations. The graph presents the price adaptation process and the reaction of agents to the updated prices leading to an oscillating convergence with continuously decreasing amplitudes. The algorithm stops if both curves are simultaneously below the threshold ϵ .

Besides the fact that the algorithm reliably converges to a solution, we also observe improvements in terms of computation time for larger case studies, i.e. including more scenarios or representative days. Table II shows a comparison. For a limited number of scenarios, the state-of-the-art PATH solver

TABLE II
COMPARISON OF COMPUTATION TIME (IN MIN) FOR PATH AND PROPOSED APPROACHED IN RISK-NEUTRAL (RN) AND RISK-AVERSE (RA) SETTINGS.

Number Scenarios			1	2	3	6	9	12	18	27
PATH	5d	m	0.45	2.43	7.11	25.03	57.59	14.67	99.36	326.14
	10d	m	1.14	6.90	21.89	15.81	42.27	119.74	367.87	*
Own approach	5d	rn	2.66	2.84	24.11	7.77	7.73	40.74	19.40	47.98
		ra	2.81	29.26	12.97	14.00	17.97	30.10	20.11	118.93
	10d	rn	4.10	6.19	7.97	43.93	52.80	59.35	61.56	110.27
		ra	3.97	11.34	10.07	48.16	53.65	71.03	65.07	84.01

*: No solution, time limit reached after 720 min.

outperforms our implementation. The reduction in computation time with more scenarios is an expected outcome of a decomposition algorithm. In fact, future work could include further decomposition of the individual agent's update step based on the scenarios. The impact of risk-averse agents compared to risk-neutral agents in terms of computation time is minor.

C. Mutual impact of risk aversion and capacity mechanisms

Focusing on the model outcome of the case study, we can point to the positive effect of capacity markets in a risk-averse setting. We examine the impact of risk aversion on the risk-adjusted expected cost and the installed capacities with decreasing β . As a reminder, we distinguish an energy-only market (EOM) setting and a setting with a capacity market.

The risk-adjusted expected cost represents all costs accruing to the consumer in the three markets plus the costs for EENS. The EENS is valued with a moderate value-of-lost load of 3000 €/MWh, which is equal to the price cap for energy. We compare settings with and without a capacity market including 27 scenarios and 10 representative days. All objectives are weighted with $\gamma=0.5$.

Assuming unchanged model parameters, e.g., load, variable costs, etc., and that the capacity demand curve is parameterized properly, we study the impact of different market parameters that are commonly linked to the discussion of capacity mechanisms. We discuss our findings along the following three changing market parameters:

- 1) Impact of market design: EOM or capacity market
- 2) Impact of increasing risk aversion in market
- 3) Impact of a higher price cap for the energy-based market

We support this discussion by using three figures (Fig. 6a, 6b, and 7). In all figures, the x-axis displays the assumed level of risk aversion in the market reaching from risk-neutral ($\beta=1$) to a very high level of risk aversion ($\beta=0.1$). The y-axis shows the total (Fig. 7) and the relative (Fig. 6) risk-adjusted expected cost (lines) and the change of the generation mix (bars) relative to the risk-neutral case. For each generator, the change in percentage is calculated based on the total installed capacity and maximum EENS in the risk-neutral case. The relative total installed capacities are depicted in the stacked bars on the left.

1) *Energy-only market or capacity market:* Fig. 6 shows the increasing expected consumer cost (dashed line) in a more risk-averse context. With the given parameters for the capacity demand curve, the capacity market shows a more beneficial

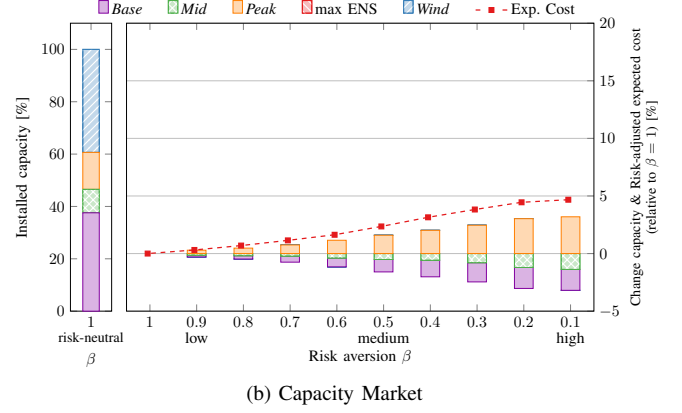
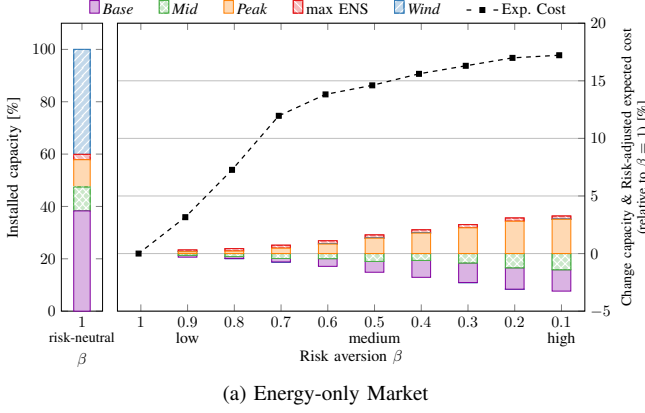


Fig. 6. Installed capacity per generator with increasing risk aversion (decreasing β) relative to the risk-neutral ($\beta=1$) scenario. The risk-adjusted expected cost for consumers aggregates all expenses of the 3 combined markets plus the EENS, valued at 3000 €/MWh.

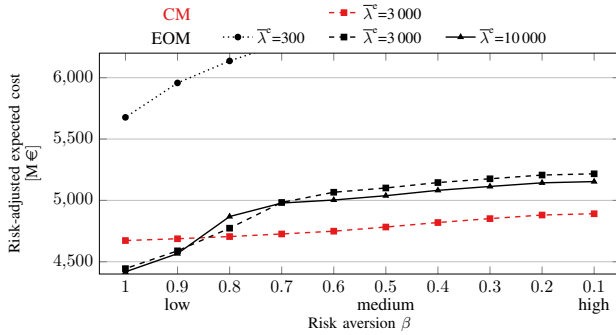


Fig. 7. The effect of the energy-based price cap $\bar{\lambda}^e$ on the risk-adjusted expected cost under different levels of risk aversion. EENS is valued at 10 000 €/MWh.

outcome for risk-averse markets. For the EOM (Fig. 6a) the results show an increase in the EENS. Starting from a risk-neutral case with EENS due to the price cap, the EENS further increases, which can be explained by the overall decrease of installed capacities. With a capacity market, EENS is avoided for all levels of risk aversion (Fig. 6b). The investment signals from the capacity market remain sufficient even if the market participants become more risk-averse. This can be explained by the additional revenue stream from a capacity market that is available in all scenarios (while price spikes only occur in some scenarios with scarcity). The impact of the difference in scenarios is thus reduced and a more stable investment signal is provided.

2) *Impact of risk aversion:* In the same line, the impact of risk aversion on the risk-adjusted expected cost can be analyzed. By comparing the dashed black and red lines in Fig. 6 and Fig. 7, we conclude that a market with a capacity market is more resilient to increased risk aversion than an energy-only market. The cost increase with more risk aversion has two origins. First, we observe a shift in the generation mix from *Base* and *Mid* towards *Peak* leading to increasing operating costs leading consequently to higher prices per energy. It is *Base* and *Mid* capacity that leaves the market, as their risk exposure is higher than for *Peak* because of the underlying cost structure for fixed and variable costs. The

resulting gap is partly filled by the *Peak* generator. For the *Peak* generator, we observe smaller changes, which can be explained by the variation in the amount of hours and levels of scarcity with increasing risk aversion. Depending on the reaction of the competitive generators, it is favorable to increase the capacity. The capacity of *Wind* is not affected by the risk aversion despite the changing behavior of the other generators. This is because the emission target is not affected and the prices λ_s^r reach for all β a sufficient level. This also holds for the setting of a capacity market.

For the capacity market (Fig. 6a), the total installed capacity does not decrease with increasing risk aversion. The unchanged capacity demand curve leads to a full replacement of the *Base* and *Mid* capacity by *Peak* capacity with lower fixed costs. Note that the change for *Base* and *Mid* capacity is nearly the same in both market settings. Thus, the capacity market has no direct impact on decision of *Base* and *Mid*. The cost difference is therefore linked to the difference in costs for installing *Peak* capacity, which is used to a limited extent and the costs associated with EENS. This forms additional cost for the consumers. Already at a low level of risk-aversion, the capacity market outperforms the EOM (Fig. 7). While risk aversion in the case of an EOM increases the risk-adjusted expected cost by up to 17.8%, with a capacity market the cost increases is only 4.69% (Fig. 6).

3) *Impact of a higher energy-based price cap:* Often higher price caps are argued to overcome the problem of inadequate investments in an EOM. Fig. 7 shows the impact of different price caps, $\bar{\lambda}^e$, given a risk-averse market context. In order to compare the results, the EENS is valued uniformly at 10 000 €/MWh for all tested price cap levels. A low price cap ($\bar{\lambda}^e=300$ €/MWh) leads to extremely high cost due to very high volumes of EENS (cropped dotted line). On the other extreme, a higher price cap does not have the same impact as changing from an EOM to a capacity market. On the contrary, once a sufficient high price cap is set, e.g. $\bar{\lambda}^e=3\,000$ €/MWh, a further increase to $\bar{\lambda}^e=10\,000$ €/MWh does not significantly improve the situation in terms of risk-adjusted expected cost. The reason is that in contrast to leveling revenues across all scenarios in a capacity market, increased price caps only affect outcomes with scarcity and high prices. In a risk-

averse market, the market participants value those scenarios lower. We conclude that for addressing investment signals in a risk-averse market, capacity markets are better suited than an increase of the price cap for energy. The positive effect of a capacity market on risk-averse behavior through providing stable revenues cannot be achieved by increasing scarcity pricing of high residual demand.

V. CONCLUSION

Uncertainties about demand levels, revenues on electricity markets, and market design create major risks for investment decisions. Risk aversion in capital-intensive investment might lead to inadequate investments and might undermine generation adequacy in the long-term. Electricity market models need to capture the interaction of market design and risk aversion in order to assess those effects.

Our paper contributes to the literature in two respects. First, the proposed methodology based on ADMM in the context of market equilibrium models with risk-averse generation investment extends the current applications of ADMM. A non-cooperative game is introduced incorporating markets for energy output, RES certificates and availability. At the same time, risk aversion is modeled by means of the Conditional Value-at-Risk (CVaR) as the risk measure.

In this context, we show that our algorithm is suitable for computing a risk-averse equilibrium of a non-cooperative game under non-restrictive assumptions. The computed solution coincides with the MCP reformulation. Additionally, the algorithm reliably converges to a solution whereas the solver based on an MCP reformulation fails to compute an equilibrium for larger models with risk-averse agents. Moreover, the ADMM implicitly incorporates decomposition, which decreases the computation time for larger case studies.

Consequently, the proposed methodology is not limited to non-cooperative games in capacity expansion planning but can also be applied to other equilibrium models, especially including endogenous risk measures.

Second, the case study clearly indicates that incorporating risk measures in the decision-making of investors in the context of capacity mechanisms reveals important insights. The results show that with increasing risk aversion paired with higher dependency on peak and scarcity pricing, capacity markets yield lower total cost at even lower levels of expected energy not served. The investment signals from the capacity market remain sufficient even if the market participants become more risk-averse. The positive effect of a capacity market on risk-averse behavior through providing stable revenues cannot be achieved by increasing the price cap in order to have scarcity pricing of high residual demand.

The case study provides insights for a specific capacity mechanism. We would like to highlight that the proposed methodology enables the research of cases with more detailed representation of capacity mechanisms as presented in [5]. These case studies could provide more insights on other changing market parameters. Because of the expected increasing shares of RES, further research is necessary on the participation of RES in capacity mechanisms and risk-averse

investment decision-making. Additionally, capacity mechanisms introduced in a market zone will have implications for interconnected market zones. An extension of the proposed models towards multi-zonal market settings as presented in [10] could provide valuable insights. These research questions on capacity mechanisms combined with risk-averse behavior are crucial in order to understand the impact of such a complementary market mechanism.

The level of decomposition in the proposed algorithm can be further enhanced by decomposing the individual agent's update step along the scenarios. This could be achieved by using, e.g., progressive hedging [36]. Such a decomposition could enable modelers to extend the number of scenarios or to consider higher temporal resolution.

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Hanspeter Höschle received the Diplom degree in industrial engineering and management in 2012 from the Karlsruhe Institute of Technology, Germany. Since 2012, he has been working as a Ph.D. researcher at the KU Leuven, Belgium, on power system economics and the modeling of electricity markets, in particular capacity mechanisms. He holds a Ph.D. fellowship of the Research Foundation - Flanders (FWO) and the Flemish Institute for Technological Research (VITO).



Hélène Le Cadre holds a Magistère in mathematics from Ecole Normale Supérieure (ENS) Cachan and Rennes 1 university, France. She graduated from Mines-Télécom Atlantique, with a speciality in applied mathematics and information science. She received the Ph.D. degree in applied mathematics from the same university. She worked as a research fellow at Ecole des Mines de Paris (Mines Paris-Tech) and Ecole Nationale Supérieure des Techniques Avancées (ENSTA ParisTech), France. She is currently working as a researcher at VITO /

EnergyVille, in Belgium. Her research interests include game theory, statistical machine learning, performance analysis and operations research. She has been involved in European projects (SmartNet, ELVIRE, ETICS) dealing with applications of these techniques to the design of electricity markets and communication networks, as well as in fundamental research projects (PGMO) in operations research and optimization.



Yves Smeers is Professor Emeritus at Ecole Polytechnique de Louvain, (Université catholique de Louvain) and a researcher at the Center for Operations research and Econometrics of that University. He spent his career in energy modelling, working successively on global energy models, sector gas and energy models and market simulation of the restructured sectors. His current interest is on risk averse stochastic equilibrium models for the analysis of market design, coordination and investment. He holds degrees in engineering and economics from the universities of Liege and Louvain in Belgium and a MS in Industrial Administration and a PhD in Operations Research from Carnegie Mellon.



Anthony Papavasiliou (M'06) received the B.S. degree in electrical and computer engineering from the National Technical University of Athens, Greece, and the Ph.D. degree from the Department of Industrial Engineering and Operations Research (IEOR) at the University of California at Berkeley, Berkeley, CA, USA. He holds the ENGIE Chair at the Université catholique de Louvain, Louvain-la-Neuve, Belgium, and is a faculty member of the Center for Operations Research and Econometrics. His research is focused on power system operations, applications

of mathematical programming in energy systems, and energy policy. He has held consulting and internship positions at the Belgian regulatory commission of electricity and gas, N-SIDE, Pacific Gas and Electric, Quantil, Sun Run, the United States Federal Energy Regulatory Commission, the Palo Alto Research Center and the E3MLab of the National Technical University of Athens.



Ronnie Belmans (S'77-M'84-S'89-F'05) received his MSc degree in electrical engineering in 1979 and a PhD degree in 1984, both from KU Leuven, Belgium. In 1989 he added a Special Doctorate from KU Leuven and in 1993 a 'Habilitation', from the RWTH, Aachen, Germany. Currently, Ronnie Belmans is full professor at KU Leuven, teaching techno-economical aspects of power systems, electrical energy and regulatory affairs, among others. His research interests include smart grids, security of energy supply and the techno-economic aspects

of the liberalization of the electricity market. He is vice president of the KU Leuven Energy Institute as well as co-founder and CEO of EnergyVille, a research collaboration in Genk specializing in energy in smart cities and buildings, in cooperation with VITO, imec and UHasselt. He is chairman of the board of directors of the VREG, the Flemish regulator for electricity & gas markets and is honorary chairman of the board of directors of ELIA, the Belgian transmission system operator.