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# Transmission capacity allocation in zonal electricity markets

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University of Cambridge. January 7<sup>th</sup>-11<sup>th</sup>, 2019.



# Outline

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1. Introduction
2. Day-ahead zonal electricity market models
3. Policy analysis using 4-node, 3-zone network
4. Cutting-plane algorithms for robust day-ahead zonal electricity markets
5. Simulation results for the Central Western European network
6. Conclusions

▷ Introduction

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Day-ahead electricity  
market models

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Cutting-plane  
algorithms for robust  
day-ahead electricity  
markets

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Simulation results  
for the Central  
Western European  
network

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Conclusions

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Appendix

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# Introduction

# Zonal electricity markets

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- European electricity market organized as a zonal market, [EC 714/2009](#)
- Two types of export/import limits:
  - Limits on the bilateral exchange between neighbouring zones, Available-Transfer-Capacity Market Coupling (**ATCMC**)
  - Limits on the net position configuration of zones, Flow-Based Market Coupling (**FBMC**)
- Both methodologies should be **N-1 robust** (*Critical Branches/Critical Outages*), [Amprion et al. \(2017\)](#)
- FBMC used to clear day-ahead electricity market at the Central Western European system since May 2015
- Other markets might implement FBMC in the near future (e.g. Nord Pool, [Energinet et al. \(2017\)](#))

# Flow-Based Market Coupling (FBMC)

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- Preferred methodology for electricity market operations of the EC, [EU 2015/1222](#): “... a method that takes into account that electricity can **flow via different paths** and optimizes the available capacity in **highly interdependent grids** ...”
- Increases in day-ahead market welfare of **95M€/year** with respect to ATCMC, [Amprion et al. \(2013\)](#)
- Congestion management and balancing costs not included in studies. They amounted to **945M€** in 2015, [ENTSO-E \(2015\)](#).
- Questions:
  - Do FBMC or ATCMC correctly account for physical flows? Why do they/do they not?
  - Does FBMC significantly improve the overall welfare of the market with respect to ATCMC?

# Literature

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- [Jensen et al. \(2017\)](#), and many references therein, study ATCMC and conclude that its performance is significantly **worse than that of a nodal market**
- [Waniek et al. \(2009\)](#), [Waniek et al. \(2010\)](#) study the accuracy of the approximation of flows in FBMC at cross-border lines
- [Marien et al. \(2013\)](#) study how **discretionary aggregation parameters** (for export/import limits) affect the outcome of FBMC
- [Van den Bergh et al. \(2015\)](#) summarize the concepts and methodology used for FBMC at the Central Western European system
- [Dierstein \(2017\)](#) analyzes the impacts of **discretionary aggregation parameters** on welfare, exchanges, prices and counter-trading costs

# Contributions

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1. We propose a new framework for modelling ATCMC and FBMC in which we **derive export/import limitations directly from the physics of the real network.**  
(the proposed models **do not depend** on discretionary aggregation parameters)
2. We present cutting-plane algorithms to systematically account for the **N-1 security criterion on day-ahead markets.**
3. We perform **numerical simulations** using an industrial-scale instance of the Central Western European system considering **100 years of operating conditions.**
  - Vast similarities between ATCMC and FBMC in all aspects.
  - **Zonal market designs fail at allocating transmission capacity and are outperformed by a nodal market.**

Introduction

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Day-ahead  
electricity market  
▷ models

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# Day-ahead electricity market models



# Assumptions

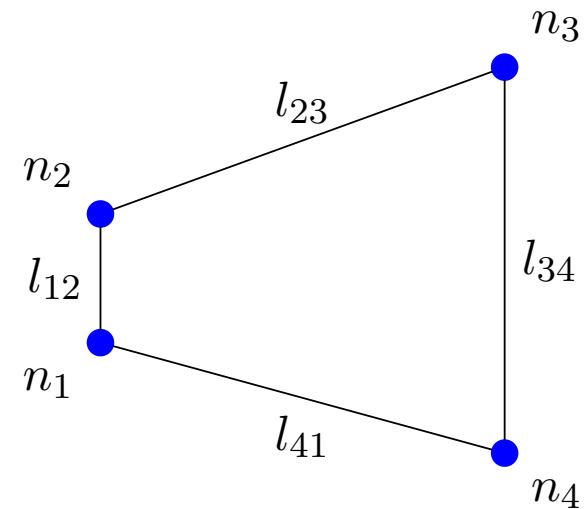
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1. All energy trades take place at the day-ahead auction
2. Bids are price-quantity pairs, associated with a node/zone
3. Participants bid truthfully
4. System operator knows:
  - Topology of the network
  - Susceptance and thermal limits of lines
  - Installed production capacity at each node
5. Consumers have an infinite valuation (only for simplicity)

# Nodal electricity market

$\min$  production cost  
bids,  
flows

s.t. fractional bids  
net production =  
outgoing flows, at each node  
line thermal limits  
power-angle constraints

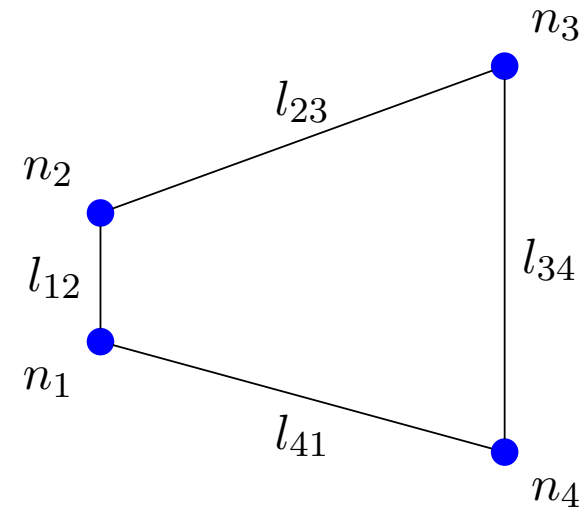


# Nodal electricity market

$$\begin{aligned}
 \min_{v, f, \theta} \quad & \sum_{g \in G} P_g Q_g v_g \\
 \text{s.t.} \quad & 0 \leq v_g \leq 1 \quad \forall g \in G \\
 & \sum_{g \in G(n)} Q_g v_g - Q_n = \\
 & \sum_{l \in L(n, \cdot)} f_l - \sum_{l \in L(\cdot, n)} f_l \quad \forall n \in N \quad [\rho_n] \\
 & -F_l \leq f_l \leq F_l \quad \forall l \in L \\
 & f_l = B_l (\theta_{m(l)} - \theta_{n(l)}) \quad \forall l \in L
 \end{aligned}$$

$P, Q$ : price and quantity

$F_l, B_l$ : capacity and susceptance line  $l$



$$G = \{1, 2, 3, 4\},$$

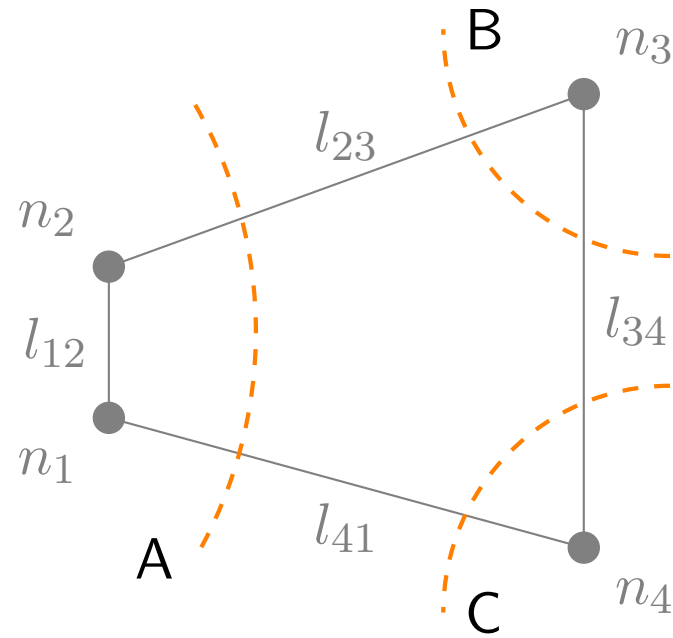
$$G(n_1) = \{1\}, \dots$$

$$N = \{n_1, n_2, n_3, n_4\}$$

$$L = \{l_{12}, l_{23}, l_{34}, l_{41}\},$$

$$L(n_1, n_2) = \{l_{12}\}, \dots$$

# Zonal network organization

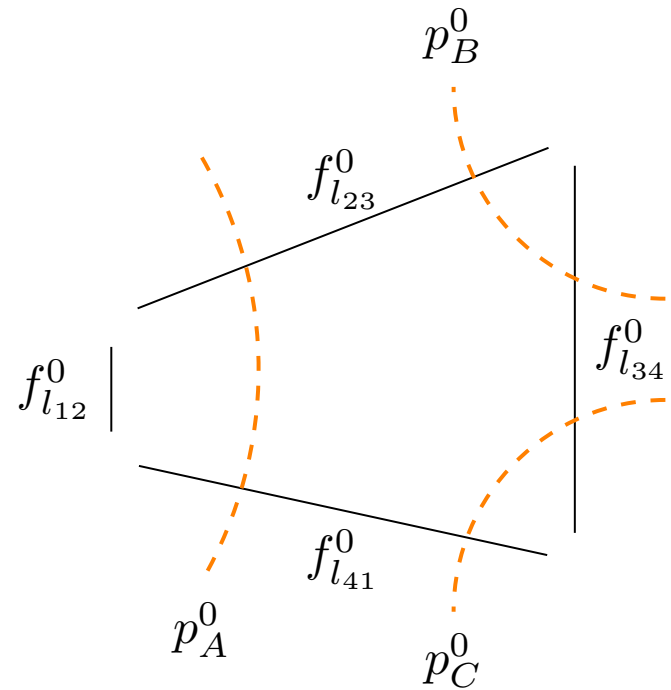


$$G = \{1, 2, 3, 4\}, G(A) = \{1, 2\}, \dots$$

$$N = \{n_1, n_2, n_3, n_4\}, N(A) = \{n_1, n_2\}, \dots$$

# Flow-Based Market Coupling with Approximation (FBMC-A)

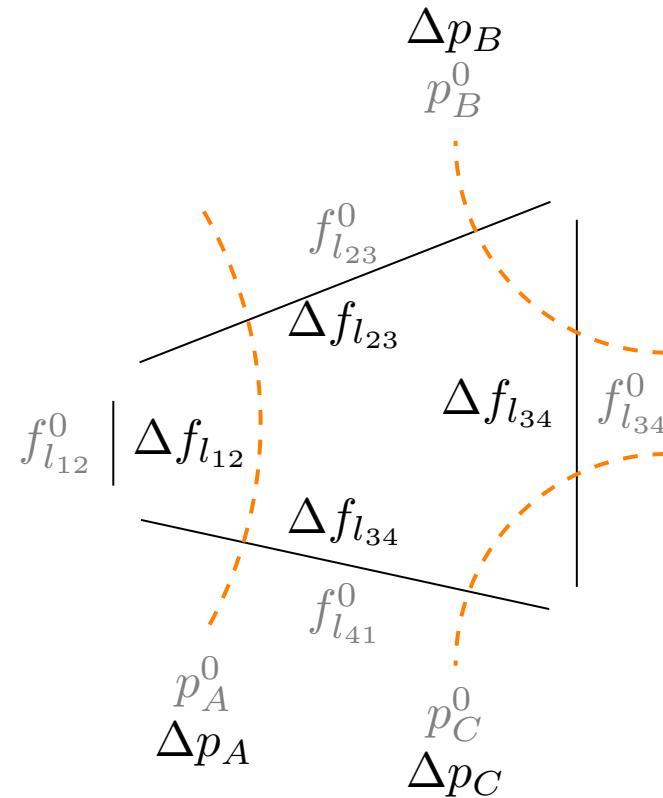
1. Select a base case  $(p^0, f^0)$  (net positions, flows on branches)



# Flow-Based Market Coupling with Approximation (FBMC-A)

1. Select a base case  $(p^0, f^0)$  (net positions, flows on branches)
2. Compute zone-to-line Power-Transfer-Distribution-Factors,  $PTDF_{l,z}$ , so that

$$\Delta f_l \approx \sum_{z \in Z} PTDF_{l,z} \Delta p_z$$



# Flow-Based Market Coupling with Approximation (FBMC-A)

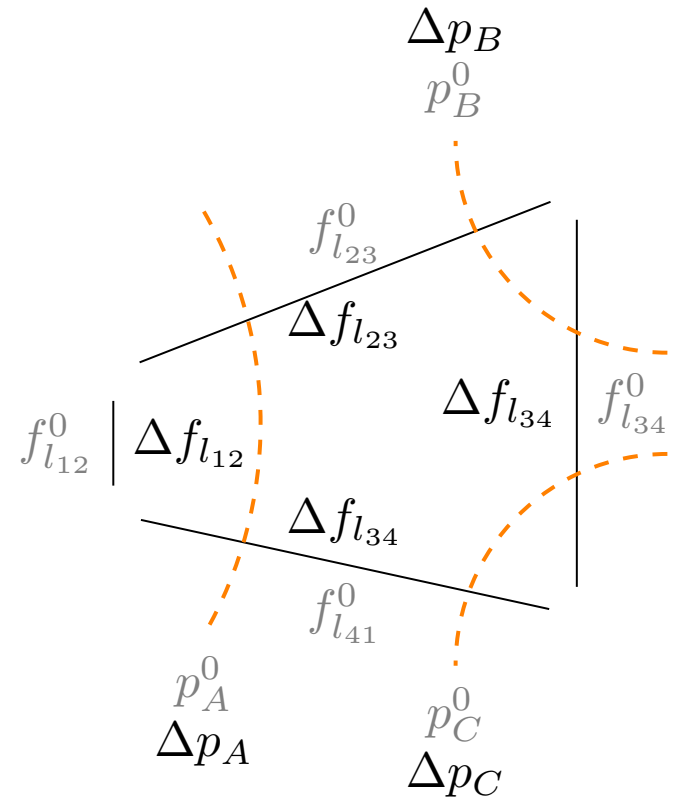
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$$\Delta f_l \approx \sum_{z \in Z} PTDF_{l,z} \Delta p_z$$

3. Define **flow-based domain**:

$$\mathcal{P}^{FB-A} := \left\{ p \in \mathbb{R}^{|Z|} \mid \sum_{z \in Z} p_z = 0, \right.$$

$$\left. -F_l \leq \sum_{z \in Z} PTDF_{l,z} (p_z - p_z^0) + f_l^0 \leq F_l \quad \forall l \in L \right\}$$



# Flow-Based Market Coupling with Approximation (FBMC-A)

4. Clear day-ahead market by solving:

$$\begin{aligned} \min_{v,p} \quad & \sum_{g \in G} P_g Q_g v_g \\ \text{s.t.} \quad & 0 \leq v_g \leq 1 \quad \forall g \in G \\ & \sum_{g \in G(z)} Q_g v_g - \sum_{n \in N(z)} Q_n = p_z \quad \forall z \in Z \quad [\rho_z] \\ & \sum_{z \in Z} p_z = 0 \\ & -F_l \leq \sum_{z \in Z} PTDF_{l,z} (p_z - p_z^0) + f_l^0 \leq F_l \quad \forall l \in L \end{aligned}$$

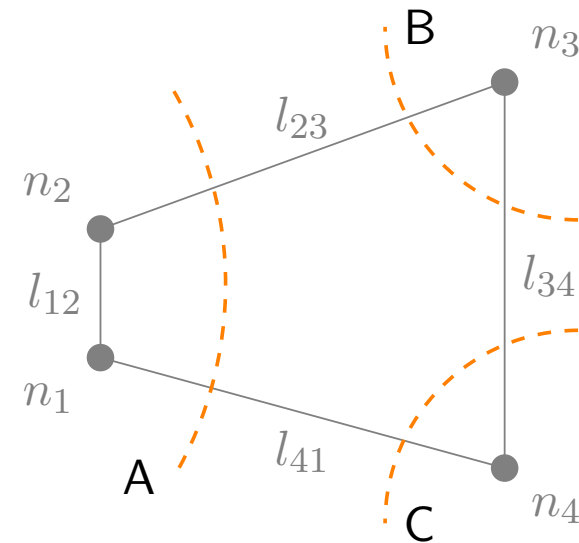
- Circular definitions: base case  $(p^0, f^0)$ , market clearing point
- Discretionary parameters: zone-to-line PTDF (among others)



# Zonal electricity market

$$\begin{aligned}
 \min_{v,p} \quad & \sum_{g \in G} P_g Q_g v_g \\
 \text{s.t.} \quad & 0 \leq v_g \leq 1 \quad \forall g \in G \\
 & \sum_{g \in G(z)} Q_g v_g - \sum_{n \in N(z)} Q_n = \\
 & \quad p_z \quad \forall z \in Z \quad [\rho_z] \\
 & p \in \mathcal{P}
 \end{aligned}$$

- $\mathcal{P}$  should include all feasible cross-border trades, [EC 714/2009](#), Annex I, Art. 1.1
- $\mathcal{P}$  should not include configurations that can harm security, [EC 714/2009](#), Annex I, Art. 1.7



$$G = \{1, 2, 3, 4\},$$

$$G(A) = \{1, 2\}, \dots$$

$$N = \{n_1, n_2, n_3, n_4\},$$

$$N(A) = \{n_1, n_2\}, \dots$$

# Deriving $\mathcal{P}$ directly from physics: an example

Physics:

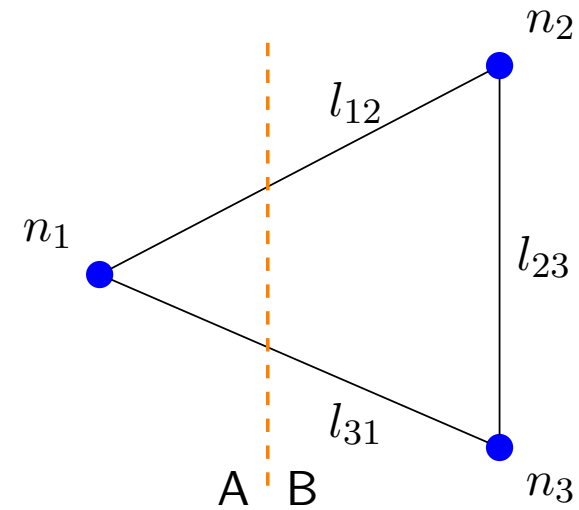
$$r_1 + r_2 + r_3 = 0$$

$$-100 \leq r_1 \leq 100$$

$$-100 \leq r_2 \leq 100$$

$$-100 \leq r_3 \leq -50$$

$$-25 \leq f_{12} = 1/3 r_1 - 1/3 r_2 \leq 25$$



Zonal net positions:

$$p_A = r_1$$

$$p_B = r_2 + r_3$$

$$G = \{1, 2, 3\}$$

$$Q_1 = 200, Q_2 = 200, Q_3 = 50$$

$$N = \{n_1, n_2, n_3\}$$

$$L = \{l_{12}, l_{23}, l_{31}\}, F_{12} = 25$$

100MW demand per node

# Deriving $\mathcal{P}$ directly from physics: an example

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Are these zonal net positions feasible?

$$p_A = 0 \quad p_B = 0$$

$$p_A = 200 \quad p_B = -200$$

$$p_A = -100 \quad p_B = 100$$

$$p_A = 50 \quad p_B = -50$$

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$$p_A = 0 \quad p_B = 0 \quad \text{Yes}$$

$$p_A = 200 \quad p_B = -200$$

$$p_A = -100 \quad p_B = 100$$

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$$p_A = 50 \quad p_B = -50 \quad \text{Yes}$$

Zonal net positions:

$$p_A = r_1$$

$$p_B = r_2 + r_3$$

True net position feasible set  $\mathcal{P}$ :

$$p_A + p_B = 0$$

$$-12.5 \leq p_A \leq 87.5$$

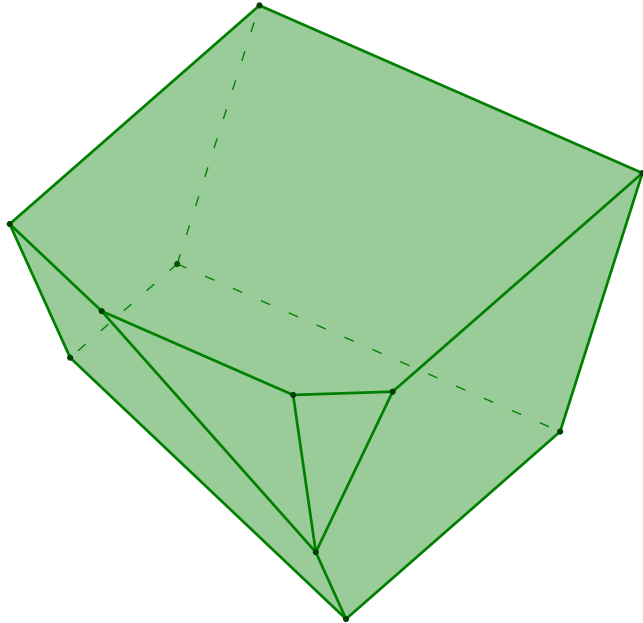


# Flow-Based Market Coupling with Exact Projection (FBMC-EP)

space of nodal injections



space of zonal net positions



$$\mathcal{R} := \{r \in \mathbb{R}^{|N|} \mid r \text{ is feasible for the real network}\}$$

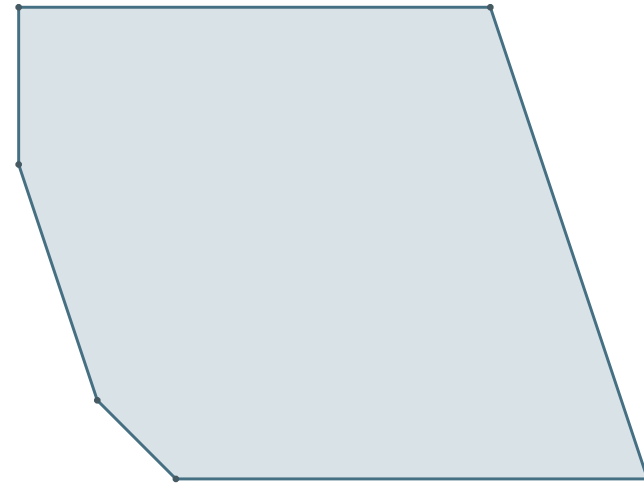
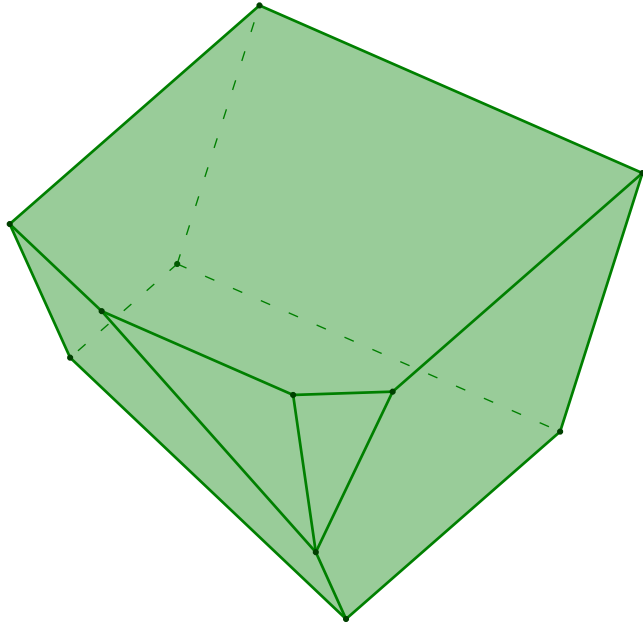
4-node, 3-zone network:  $p_A = r_1 + r_2$ ,  $p_B = r_3$ ,  $p_C = r_4$

# Flow-Based Market Coupling with Exact Projection (FBMC-EP)

space of nodal injections

→

space of zonal net positions



$$\mathcal{R} := \{r \in \mathbb{R}^{|N|} \mid r \text{ is feasible for the real network}\}$$

$$\mathcal{P}^{FB-EP} := \left\{ p \in \mathbb{R}^{|Z|} \mid \exists r \in \mathcal{R} : \right. \\ \left. p_z = \sum_{n \in N(z)} r_n \quad \forall z \in Z \right\}$$

4-node, 3-zone network:  $p_A = r_1 + r_2$ ,  $p_B = r_3$ ,  $p_C = r_4$

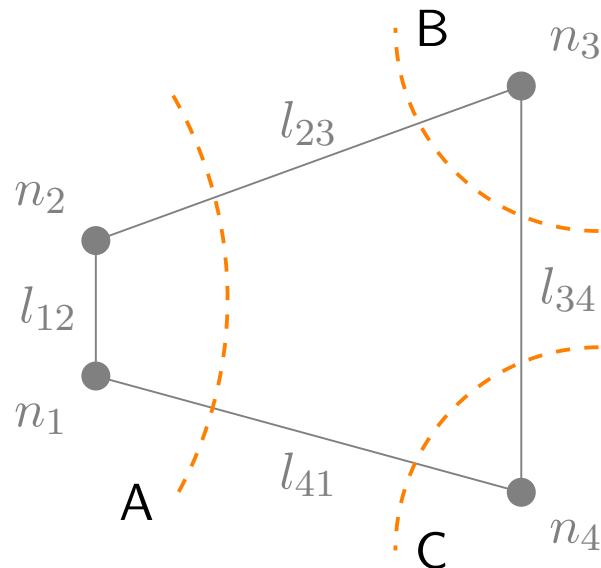
# Flow-Based Market Coupling with Exact Projection (FBMC-EP)

$$\mathcal{P}^{FB-EP} = \left\{ \begin{aligned} & \mathbf{p} \in \mathbb{R}^{|Z|} \left| \exists (\bar{v}, f, \theta) \in [0, 1]^{|G|} \times \mathbb{R}^{|L|} \times \mathbb{R}^{|N|} : \right. \\ & \sum_{g \in G(z)} Q_g \bar{v}_g - \mathbf{p}_z = \sum_{n \in N(z)} Q_n \quad \forall z \in Z, \\ & \sum_{g \in G(n)} Q_g \bar{v}_g - \sum_{l \in L(n, \cdot)} f_l + \sum_{l \in L(\cdot, n)} f_l = Q_n \quad \forall n \in N, \\ & \left. -F_l \leq f_l \leq F_l, f_l = B_l (\theta_{m(l)} - \theta_{n(l)}) \quad \forall l \in L \right\} \end{aligned}$$

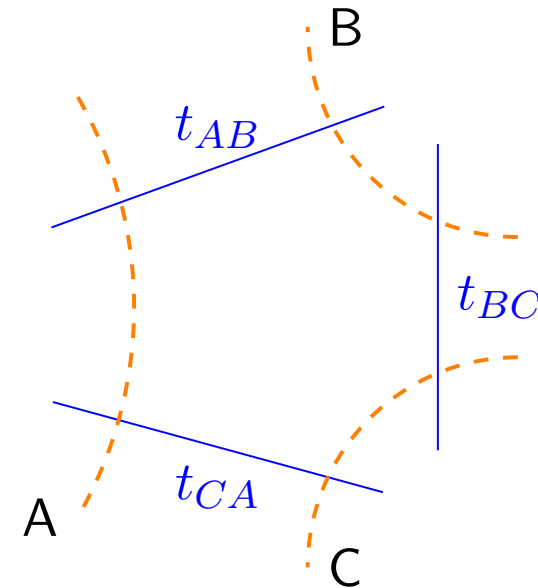
- $\mathcal{P}^{FB-EP}$  **allows for all trades that are feasible** with respect to the real network and **bans only trades that can be proven to be infeasible** for the real network
- $\mathcal{P}^{FB-A}$  provides no guarantees: might ban feasible trades and, also, allow infeasible trades

# Available-Transfer-Capacity Market Coupling (ATCMC)

net positions of zones  $Z$



flow on interconnectors  
between zones  $T$



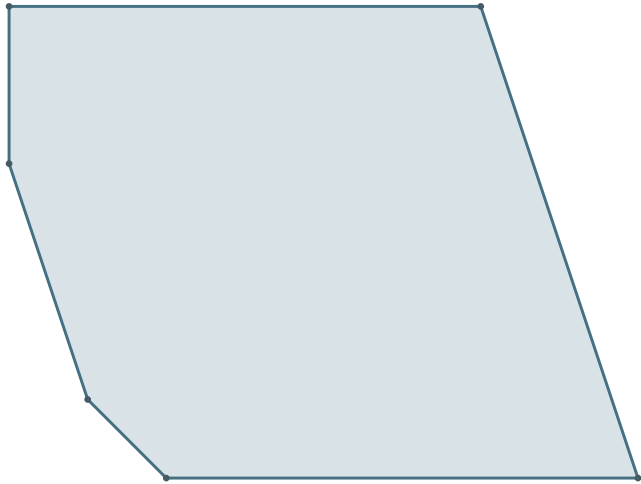
- ATCMC clears day-ahead market over the network  $\mathcal{G}(Z, T)$
- Available-transfer-capacities (ATCs):  $ATC_t^- \leq e_t \leq ATC_t^+$
- How to compute ATCs?

# Available-Transfer-Capacity Market Coupling (ATCMC)

space of zonal net  
positions



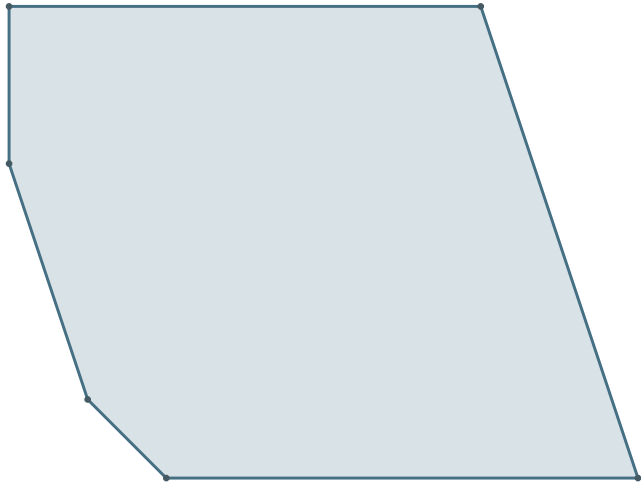
space of exchanges



$\mathcal{P}^{FB-EP}$

# Available-Transfer-Capacity Market Coupling (ATCMC)

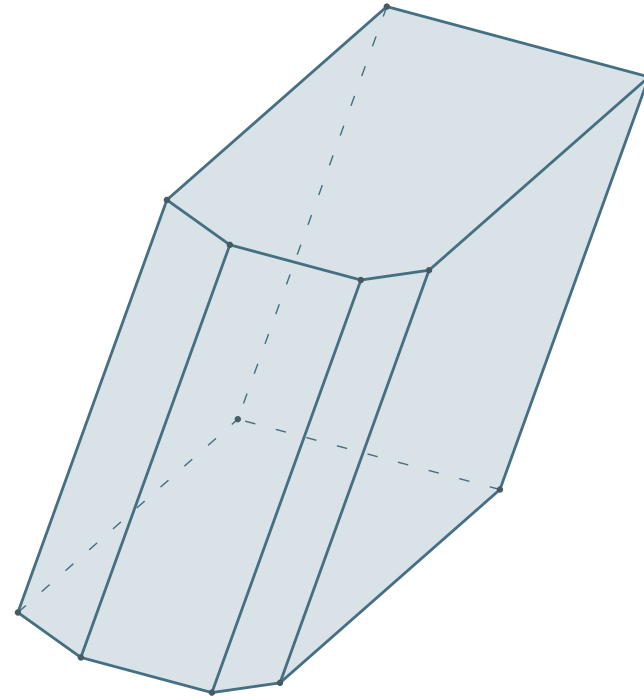
space of zonal net positions



$\mathcal{P}^{FB-EP}$

→

space of exchanges



$$\mathcal{E} := \left\{ e \in \mathbb{R}^{|T|} \mid - \sum_{l \in L(t)} F_l \leq e_t \leq \sum_{l \in L(t)} F_l \quad \forall t \in T, \right. \\ \left. \exists p \in \mathcal{P} : p_z = \sum_{t \in T(z, \cdot)} e_t - \sum_{t \in T(\cdot, z)} e_t \quad \forall z \in Z \right\}$$

# Maximum-volume Available-Transfer-Capacities (ATCs)

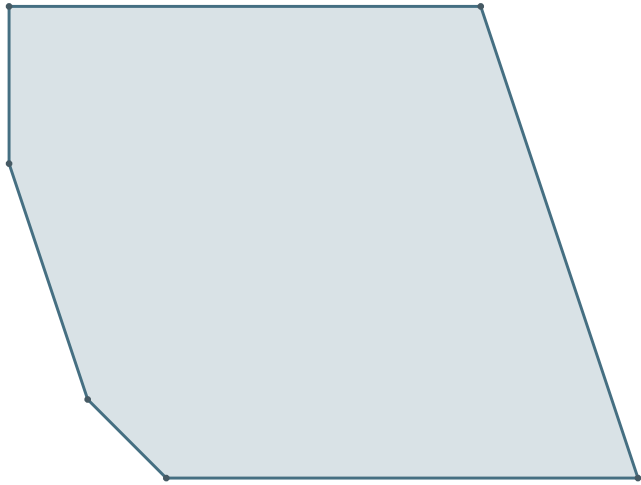
- ATC limits,  $ATC_t^- \leq e_t \leq ATC_t^+ \forall t \in T$ , are a box in the space of exchanges
- Compute ATCs as a maximum volume box inside  $\mathcal{E}$ ,

$$\begin{aligned} \max_{ATC} \quad & \prod_{t \in T} (ATC_t^- + ATC_t^+) \\ \text{s.t.} \quad & [-ATC^-, ATC^+] \subseteq \mathcal{E} \end{aligned}$$

- $\mathcal{E}^{ATC} := [-ATC^{-,*}, ATC^{+,*}]$  allows for the **largest subset of bilateral exchanges** that can be accommodated using a box and it **bans** all trades that would result in **infeasible zonal net positions**
- Implemented methodology for computing ATCs provides no guarantees

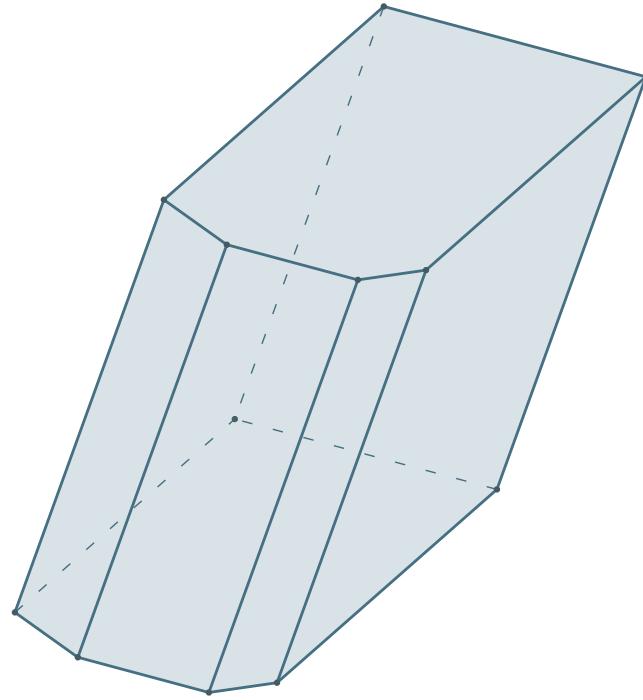
# Maximum-volume Available-Transfer-Capacities (ATCs)

space of zonal net  
positions



$\mathcal{P}$

space of exchanges

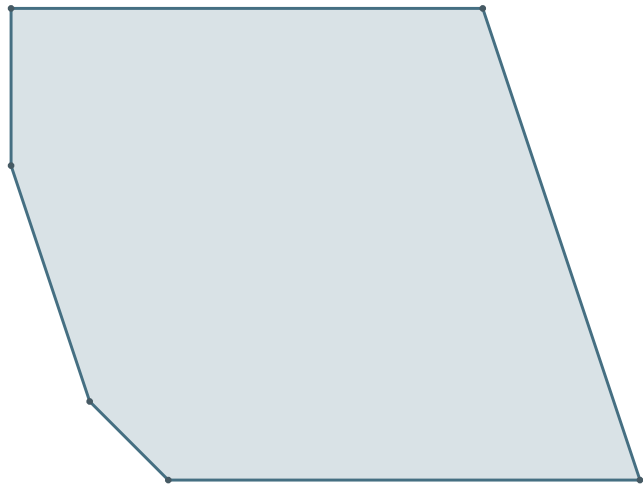


$\mathcal{E}$



# Maximum-volume Available-Transfer-Capacities (ATCs)

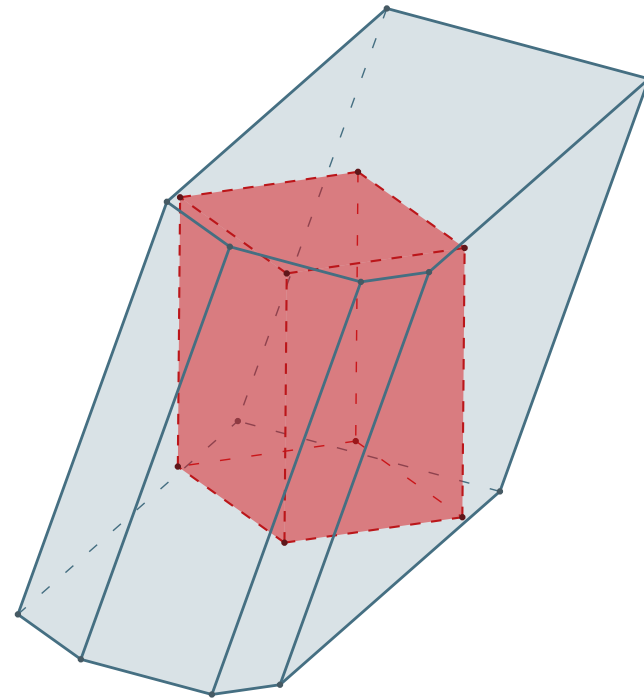
space of zonal net  
positions



$\mathcal{P}$



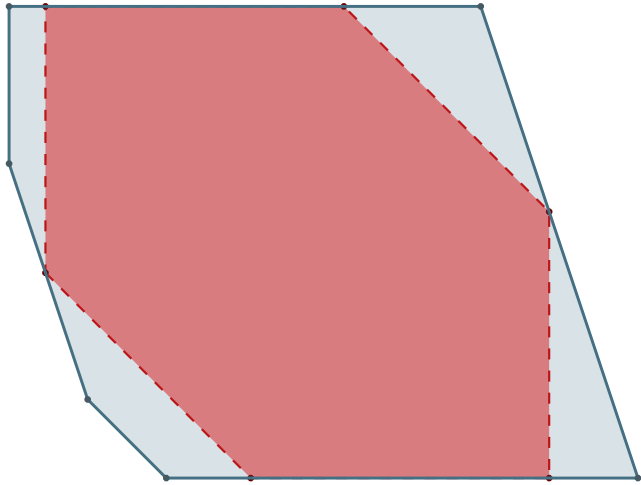
space of exchanges



$$\mathcal{E}, \mathcal{E}^{ATC} = [-ATC^{-,*}, ATC^{+,*}]$$

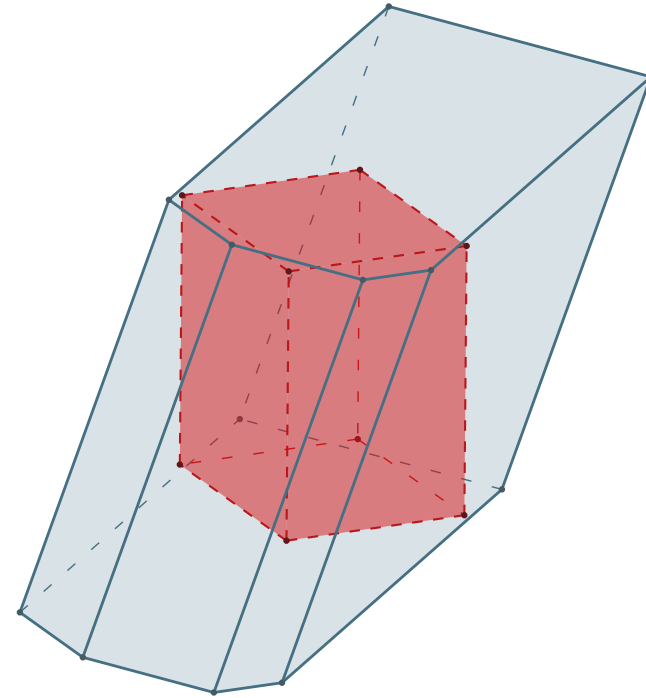
# Maximum-volume Available-Transfer-Capacities (ATCs)

space of zonal net positions



$\mathcal{P}$

space of exchanges



$$\mathcal{P}^{ATC} := \left\{ p \in \mathbb{R}^{|Z|} \mid \exists e \in \mathcal{E}^{ATC} : \right.$$

$$\left. p_z = \sum_{t \in T(z, \cdot)} e_t - \sum_{t \in T(\cdot, z)} e_t \quad \forall z \in Z \right\}$$

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Introduction

Day-ahead electricity  
market models

Cutting-plane  
algorithms for robust  
day-ahead electricity  
markets

Simulation results  
for the Central  
Western European  
network

Conclusions

Appendix

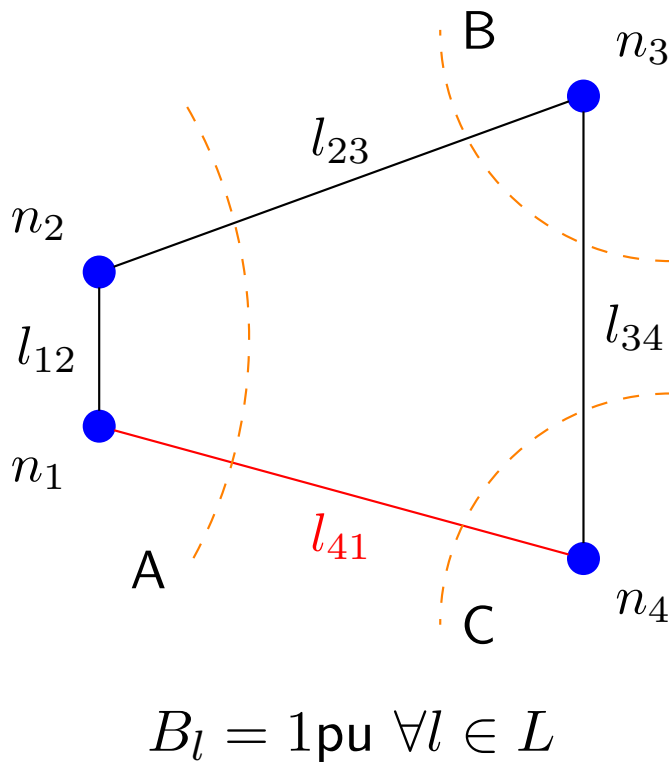
# Policy analysis using 4-node, 3-zone network

## Recap: 4 policies

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- **LMP**: Minimizes cost over nodal model
- **FBMC-A**: Minimizes cost over zonal model with  $\mathcal{P} = \mathcal{P}^{FB-A}$
- **FBMC-EP**: Minimizes cost over zonal model with  $\mathcal{P} = \mathcal{P}^{FB-EP}$
- **ATCMC**: Minimizes cost over zonal model with  $\mathcal{P} = \mathcal{P}^{ATC}$

# Inter-zonal congestion case study



Generators			
$g$	$n(g)$	$Q_g$ [MW]	$P_g$ [\$/MWh]
1	$n_1$	500	8
2	$n_2$	200	45
3	$n_3$	300	18
4	$n_4$	500	200

Consumers	
$n$	$Q_n$ [MW]
$n_2$	300
$n_4$	300

$$F_{l_{41}} = 100 \text{ MW}, F_l = +\infty \forall l \in L \setminus \{l_{41}\}$$

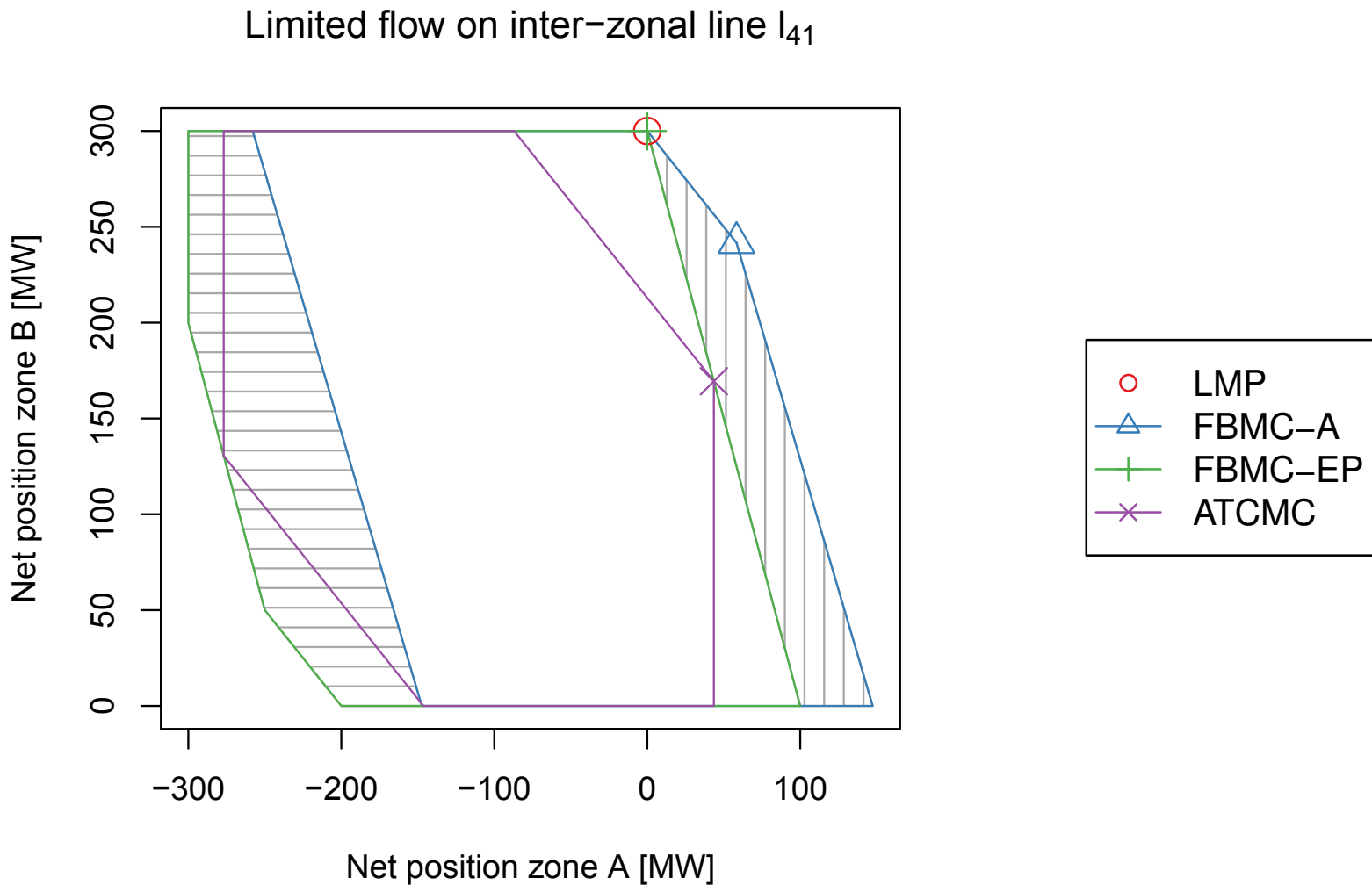
# Inter-zonal congestion: results

Summary of clearing quantities and prices for a case of inter-zonal congestion ( $l_{41}$  limited to 100MW)

Policy	Total cost [\$]	$\rho$ [\$/MWh]	Abs. error flow approx. [MW]	Overload $l_{41}$ [MW]
LMP	15 200	(8, 45, 82, 119)	0	0
FBMC-A	7 217	(8, 18, 200)	475	79
FBMC-EP	7 800	(8, 18, 23)	300	50
ATCMC	23 208	(8, 18, 200)	—	50

- Cleared quantities on all zonal markets overload line  $l_{41}$ : **failure to account for inter-zonal congestion**
- Flow approximation error and overload larger in FBMC-A than in FBMC-EP
- Cost of ATCMC larger than cost of LMP, ATCMC clears at an infeasible point

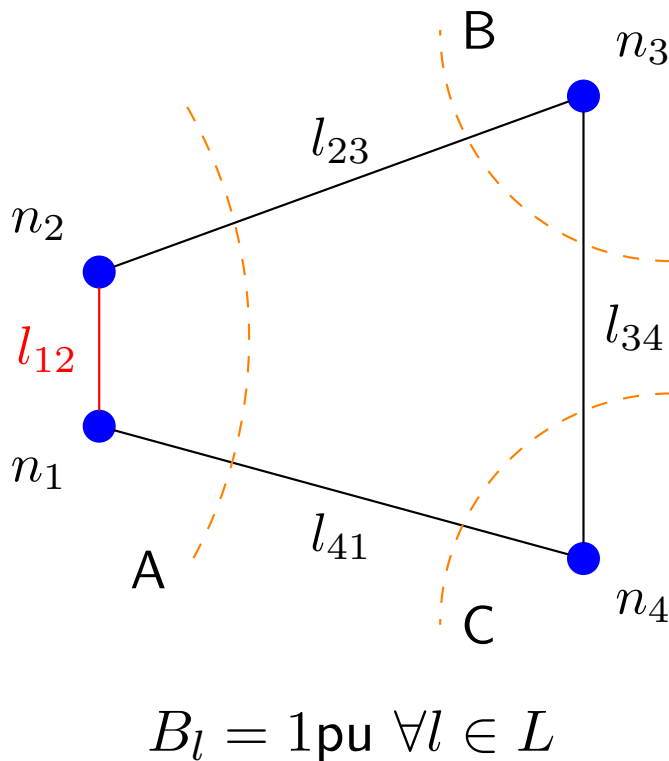
# Inter-zonal congestion: space of zonal net positions



Vertically shaded area:  $\mathcal{P}^{FB-A} - \mathcal{P}^{FB-EP}$

Horizontally shaded area:  $\mathcal{P}^{FB-EP} - \mathcal{P}^{FB-A}$

# Intra-zonal congestion case study



Generators			
$g$	$n(g)$	$Q_g$ [MW]	$P_g$ [\$/MWh]
1	$n_1$	500	8
2	$n_2$	200	45
3	$n_3$	300	18
4	$n_4$	500	200

Consumers	
$n$	$Q_n$ [MW]
$n_2$	300
$n_4$	300

$$F_{l_{12}} = 100 \text{ MW}, F_l = +\infty \forall l \in L \setminus \{l_{12}\}$$



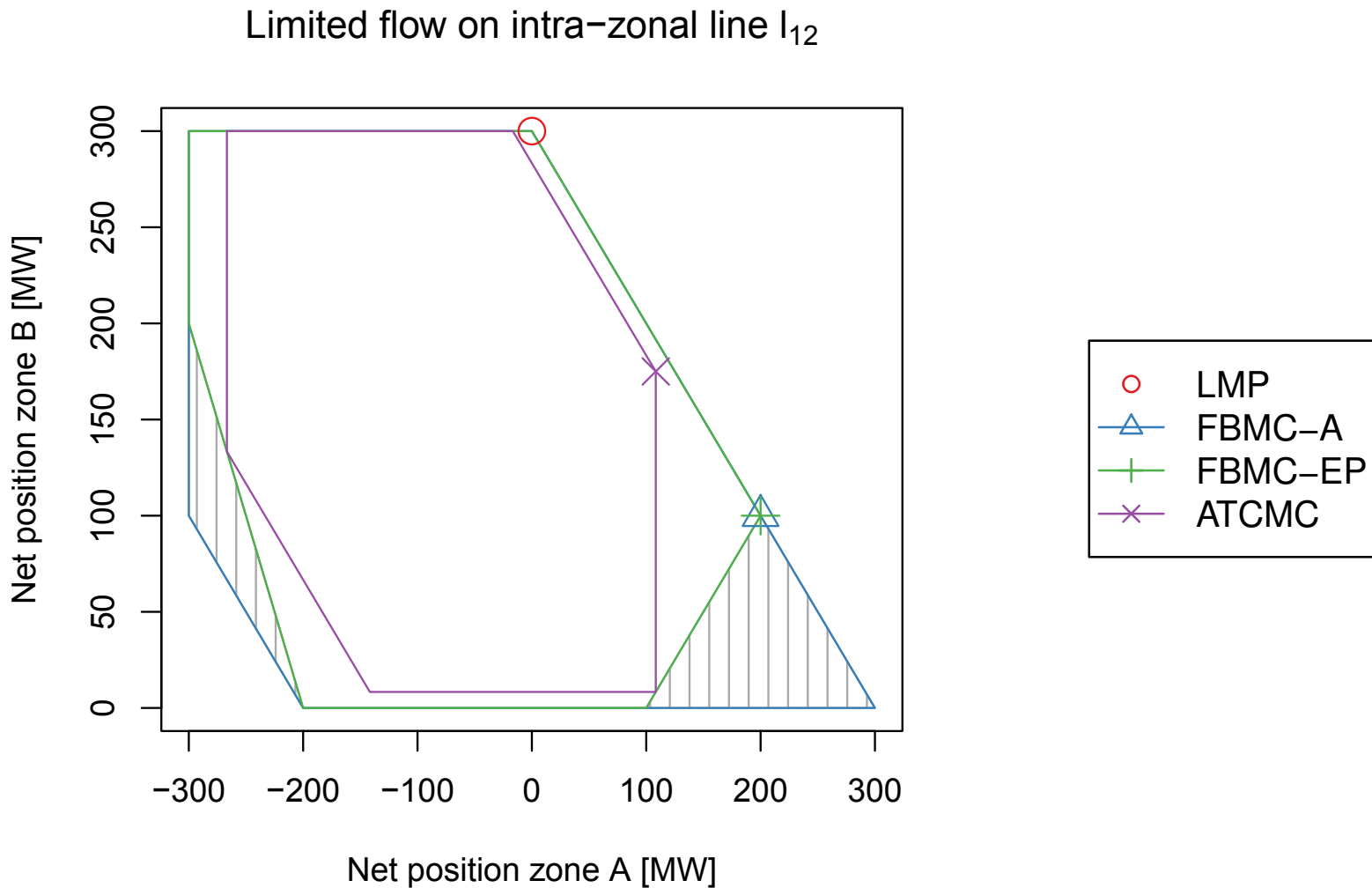
# Intra-zonal congestion: results

Summary of clearing quantities and prices for a case of intra-zonal congestion ( $l_{12}$  limited to 100MW)

Policy	Total cost [\$]	$\rho$ [\$/MWh]	Abs. error flow approx. [MW]	Overload $l_{12}$ [MW]
LMP	10 267	(8, 45, 32.7, 20.3)	0	0
FBMC-A	5 800	(18, 18, 200)	536	150
FBMC-EP	5 800	(18, 18, 18)	300	150
ATCMC	9 750	(8, 18, 200)	—	108

- Cleared quantities on all zonal markets overload line  $l_{12}$ : **failure to account for intra-zonal congestion**
- Flow approximation error larger in FBMC-A than in FBMC-EP

# Intra-zonal congestion: space of zonal net positions



Vertically shaded area:  $\mathcal{P}^{FB-A} - \mathcal{P}^{FB-EP}$

# Discussion

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- FBMC-A, FBMC-EP, ATCMC led to clearing quantities that would overload the transmission system
- FBMC-A can be cleared with infeasible net positions and can prevent feasible trades from being accepted  
In direct conflict with [EC 714/2009](#), Annex I, Art. 1.1 and 1.7
- **All zonal market designs suffer the same problem** for inter- and intra-zonal transmission capacity allocation: losing track of nodal injections leads to **inaccurate physical flow estimations**
- Given these results, in what follows, we only use FBMC-EP for modelling flow-based market coupling

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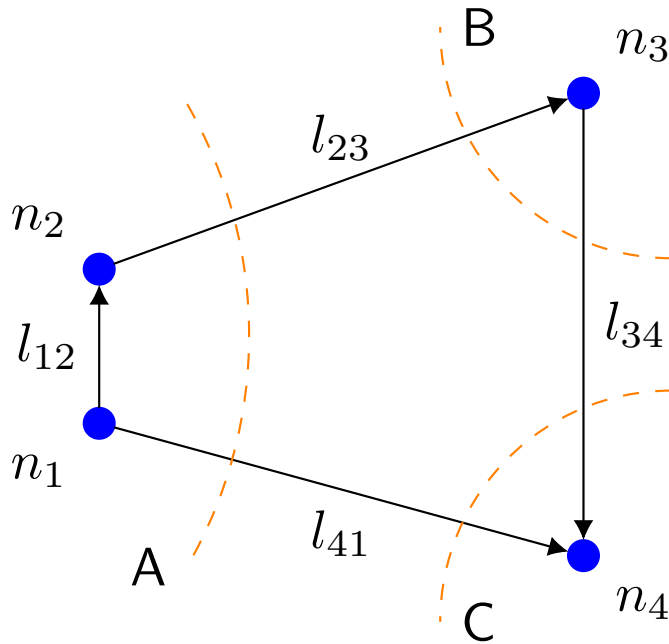
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# Cutting-plane algorithms for robust day-ahead electricity markets

# Nodal N-1 security criterion

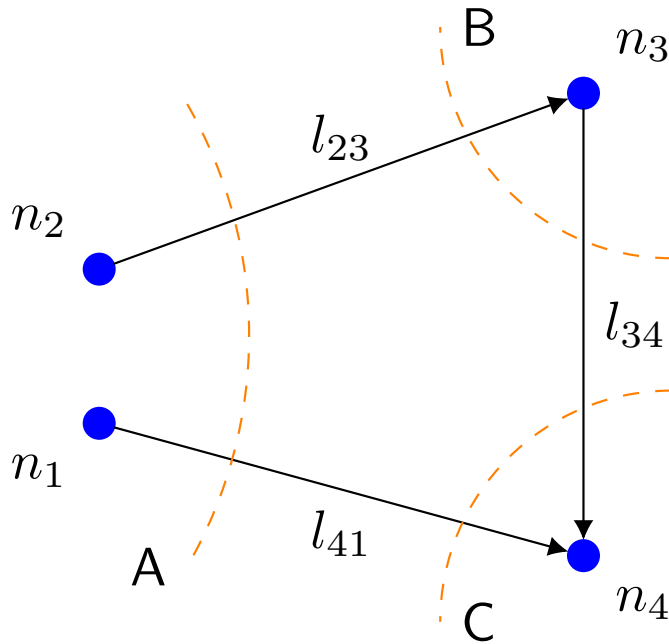
- **Nodal net injections**,  $r_n = \sum_{g \in G(n)} Q_g v_g - Q_n$ , should be feasible even if a single transmission element is not available



Generators			
$g$	$n(g)$	$Q_g$ [MW]	$P_g$ [\$/MWh]
1	$n_1$	500	8
2	$n_2$	200	45
3	$n_3$	300	18
4	$n_4$	500	200

# Nodal N-1 security criterion

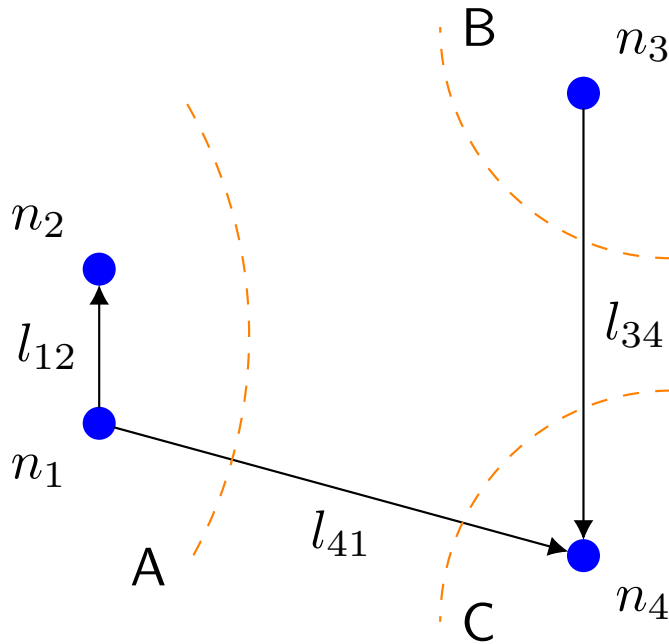
- **Nodal net injections**,  $r_n = \sum_{g \in G(n)} Q_g v_g - Q_n$ , should be feasible even if a single transmission element is not available



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# Nodal N-1 security criterion

- **Nodal net injections**,  $r_n = \sum_{g \in G(n)} Q_g v_g - Q_n$ , should be feasible even if a single transmission element is not available



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$g$	$n(g)$	$Q_g$ [MW]	$P_g$ [\$/MWh]
1	$n_1$	500	8
2	$n_2$	200	45
3	$n_3$	300	18
4	$n_4$	500	200

# Nodal day-ahead market clearing problem with N-1 security

$$\begin{aligned}
 \min_{v,r} \quad & \sum_{g \in G} P_g Q_g v_g \\
 \text{s.t.} \quad & 0 \leq v_g \leq 1 \quad \forall g \in G \\
 & \sum_{g \in G(n)} Q_g v_g - Q_n = r_n \quad \forall n \in N \quad [\rho_n] \\
 & r \in \mathcal{R}_{N-1}
 \end{aligned}$$

where  $\mathcal{R}_{N-1} = \bigcap_{\substack{u \in \{0,1\}^{|L|} \\ \|u\|_1 \leq 1}} \mathcal{R}(u)$  and

$$\begin{aligned}
 \mathcal{R}(u) = \left\{ r \in \mathbb{R}^{|N|} \mid \exists (f, \theta) \in \mathbb{R}^{|L|} \times \mathbb{R}^{|N|} : \right. \\
 r_n = \sum_{l \in L(n, \cdot)} f_l - \sum_{l \in L(\cdot, n)} f_l \quad \forall n \in N, \\
 \left. -F_l \leq f_l \leq F_l, \quad f_l = B_l(1 - u_l) (\theta_{m(l)} - \theta_{n(l)}) \quad \forall l \in L \right\}
 \end{aligned}$$



# Cutting-plane algorithm for FBMC under N-1 security

- $\mathcal{R}_{N-1}$  is a **convex polytope**  $\rightarrow$  we can describe it as  $\bar{V}r \leq \bar{W}$ , for certain  $\bar{V} \in \mathbb{R}^{M \times |N|}$  and  $\bar{W} \in \mathbb{R}^{|N|}$
  - **MCO**( $V, W$ ) (Market Clearing Oracle): Clears day-ahead nodal market using  $Vr \leq W$  as a description of  $\mathcal{R}_{N-1}$
  - **IO**( $r$ ) (Injection Oracle): Checks if  $r \in \mathcal{R}_{N-1}$ , returning a separating hyperplane  $v^\top r \leq w$  if  $r \notin \mathcal{R}_{N-1}$
- 1: Initialize  $V := 0_{1, |N|}, W := 0, \text{inclusion} := \text{FALSE}$
  - 2: **while** !inclusion **do**
  - 3:   Call **MCO**( $V, W$ )  $\rightarrow r$
  - 4:   Call **IO**( $r$ )  $\rightarrow \text{inclusion}, (v, w)$
  - 5:    $V := [V^\top \ v]^\top, W := [W^\top \ w]^\top$
  - 6: **end while**
  - 7: Terminate: inner model of **MCO**( $V, W$ ) gives the optimal clearing.

# Injection oracle (IO)

- Recall  $\mathcal{R}_{N-1} = \bigcap_{\substack{u \in \{0,1\}^{|L|} \\ \|u\|_1 \leq 1}} \mathcal{R}(u)$
- Generating Benders' cuts independently for every  $u \in \{0,1\}^{|L|}$ ,  $\|u\|_1 \leq 1$  can be very expensive
- A better approach: use a **distance function**  $d$  between the query point  $r$  and the set  $\mathcal{R}_{N-1}$

$$d(r, \mathcal{R}_{N-1}) := \max_{\substack{u \in \{0,1\}^{|L|} \\ \|u\|_1 \leq 1}} \min_{\bar{r} \in \mathcal{R}(u)} \|r - \bar{r}\|_1$$

which can be evaluated by solving a single-level MILP

- Cutting-plane algorithm converges finitely
- Algorithm inspired by original work by [Street et al. \(2014\)](#) on security constrained unit commitment

# Zonal day-ahead market clearing problem with N-1 security

- **Zonal net positions** should be feasible even if a single transmission element is not available

$$\begin{aligned} \min_{v,p} \quad & \sum_{g \in G} P_g Q_g v_g \\ \text{s.t.} \quad & 0 \leq v_g \leq 1 \quad \forall g \in G \\ & \sum_{g \in G(z)} Q_g v_g - \sum_{n \in N(z)} Q_n = p_z \quad \forall z \in Z \quad [\rho_z] \\ & p \in \mathcal{P} \end{aligned}$$

- **FBMC**:  $\mathcal{P} \equiv \mathcal{P}_{N-1}^{FB-EP} := \{p \in \mathbb{R}^{|Z|} \mid \text{there exists a feasible } (\bar{v}, f, \theta) \text{ for each single-element unavailability scenario}\}$
- **ATCMC**:  $\mathcal{P} \equiv$  projection of ATC box, computed as maximum-volume box inside  $\mathcal{E}_{N-1}$  (defined in an analogous fashion to  $\mathcal{E}$ )

# Set of feasible net positions under N-1 security

- Define  $\mathcal{P}_{N-1}^{FB-EP} := \bigcap_{\substack{u \in \{0,1\}^{|L|} \\ \|u\|_1 \leq 1}} \mathcal{P}^{FB-EP}(u)$ , where

$$\mathcal{P}^{FB-EP}(u) = \left\{ p \in \mathbb{R}^{|Z|} \mid \begin{aligned} &\exists (\bar{v}, f, \theta) \in [0, 1]^{|G|} \times \mathbb{R}^{|N|} \times \mathbb{R}^{|L|} : \\ &\sum_{g \in G(z)} Q_g \bar{v}_g - \sum_{n \in N(z)} Q_n = p_z \quad \forall z \in Z, \\ &\sum_{i \in G(n)} Q_i \bar{v}_i - Q_n = \sum_{l \in L(n, \cdot)} f_l - \sum_{l \in L(\cdot, n)} f_l \quad \forall n \in N, \\ &-F_l \leq f_l \leq F_l, \quad f_l = B_l(1 - u_l) (\theta_{m(l)} - \theta_{n(l)}) \quad \forall l \in L \end{aligned} \right\}$$

- $\mathcal{P}_{N-1}^{FB-EP}$  is a **convex polytope**  $\rightarrow$  we can clear FBMC using an analogous cutting-plane approach to that used for LMP
- Computing max-volume ATCs: similar approach, but separating hyperplanes guarantee box inclusion instead of point inclusion

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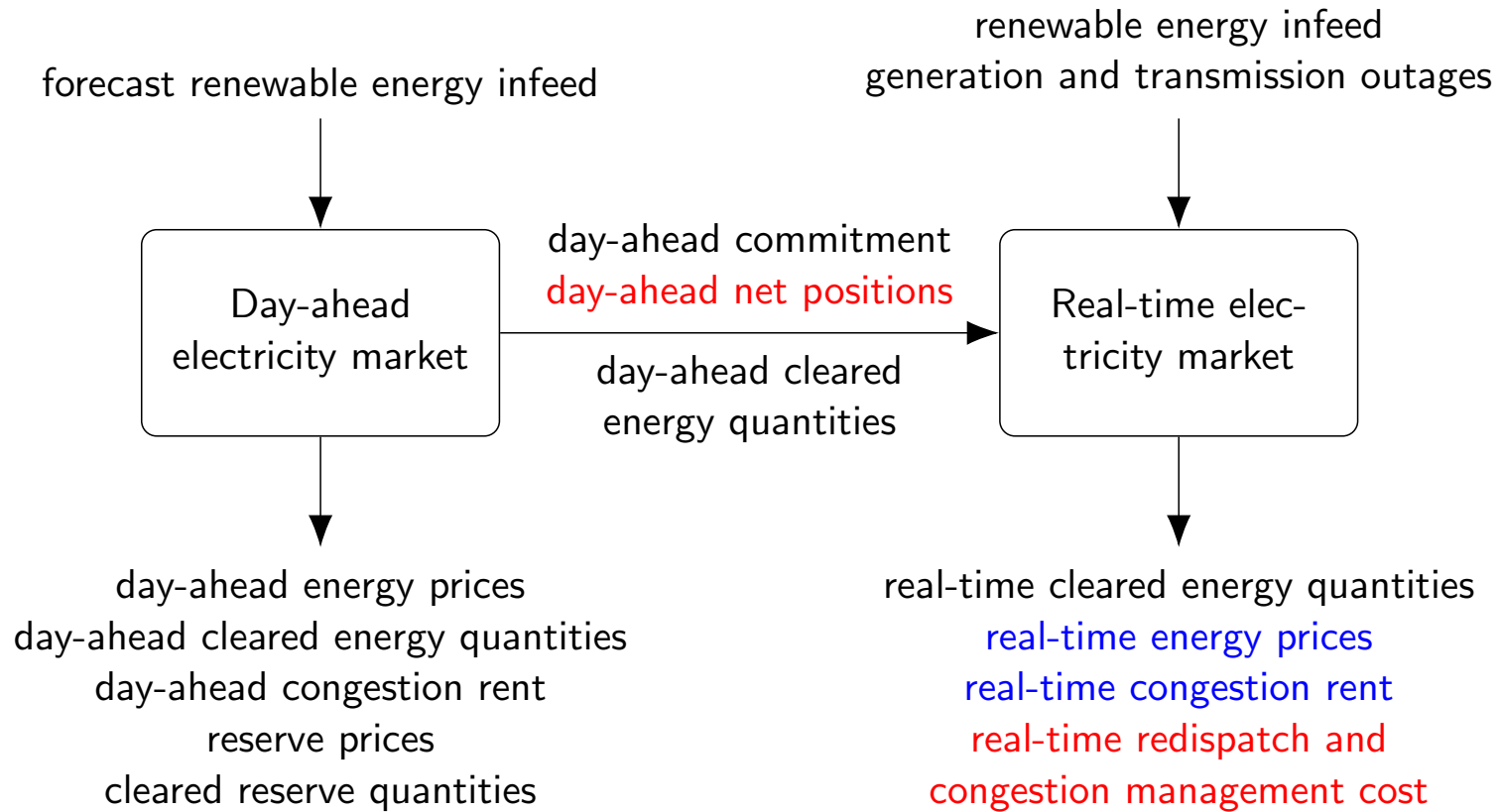
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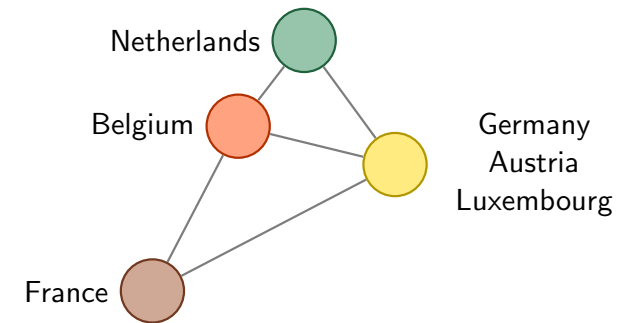
# Simulation results for the Central Western European network

# Two-settlement system



- Commitment refers to  $\{0, 1\}$  (on-off) decisions → European rules for integer pricing, [Madani and Van Vyve \(2015\)](#)

# Central Western European network



- 632 buses, 945 branches (3 491 individual circuits), 346 slow thermal generators (154GW), 301 fast thermal generators (89GW) and 1 312 renewable generators (149GW)
- 768 typical snapshots  $\times$  1 150 random uncertainty realizations  $\rightarrow$   $\sim$ 100 years of operation

# Implementation and deployment on HPC infrastructure

- Implementation in Julia/JuMP, using Gurobi, Ipopt compiled with HSL and Xpress as mathematical programming solvers
- High-performance computing (HPC) deployment using Julia's built-in parallel computing capabilities
- Total simulation time:  $\sim 7\,579$  CPU-hours

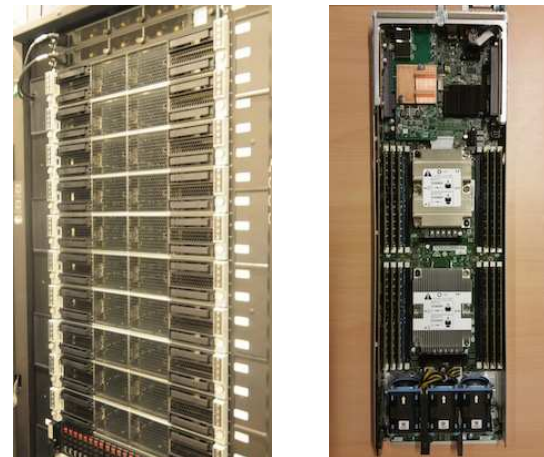
## Cab

Lawrence Livermore National  
Laboratory



## Lemaitre3

Consortium des Équipements de  
Calcul Intensif





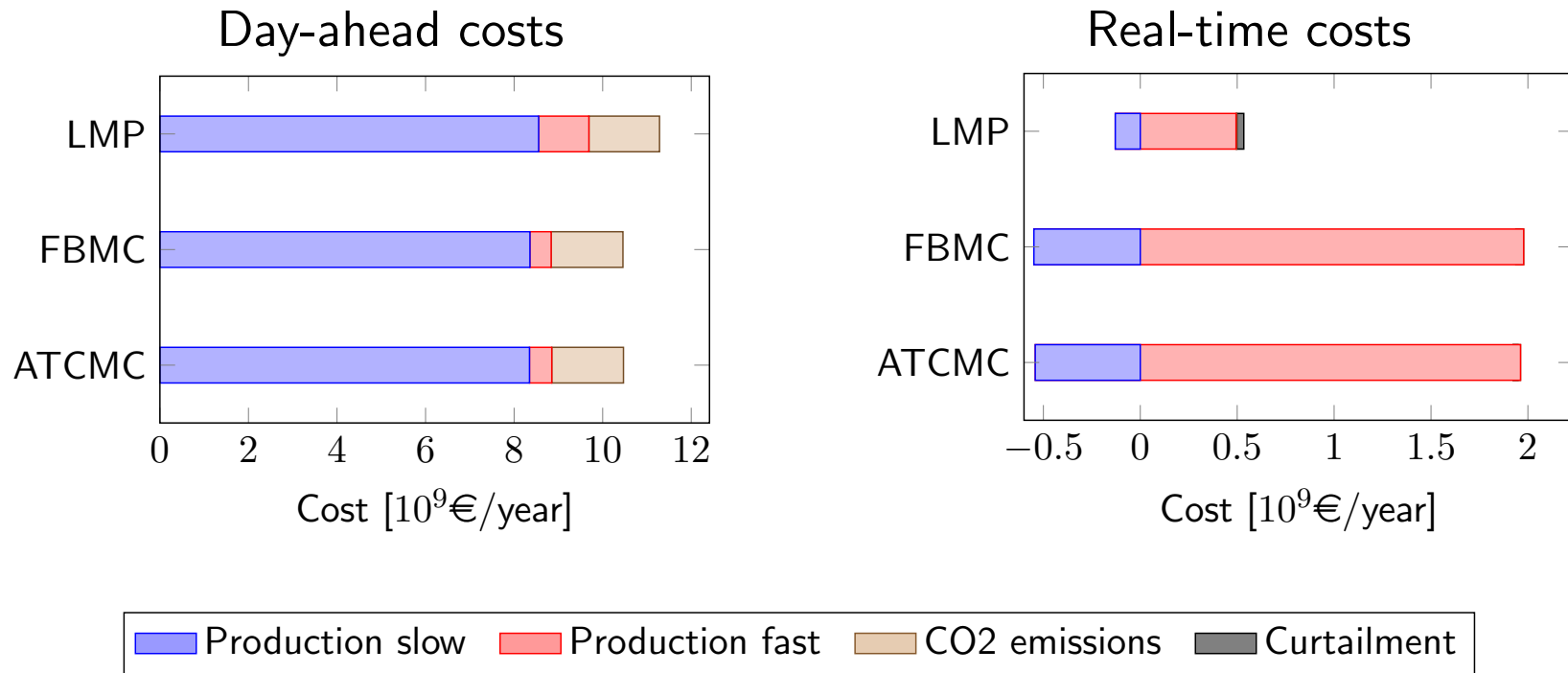
# CWE results: total cost performance

Total costs and efficiency of different policies

Policy	Day-ahead [M€/year]	Real-time [M€/year]	Total [M€/year]	Efficiency losses
PF	–	11 476	11 476	-2.90%
LMP	11 284	534	11 818	–
FBMC	10 458	1 963	12 420	5.09%
ATCMC	10 470	1 949	12 419	5.08%

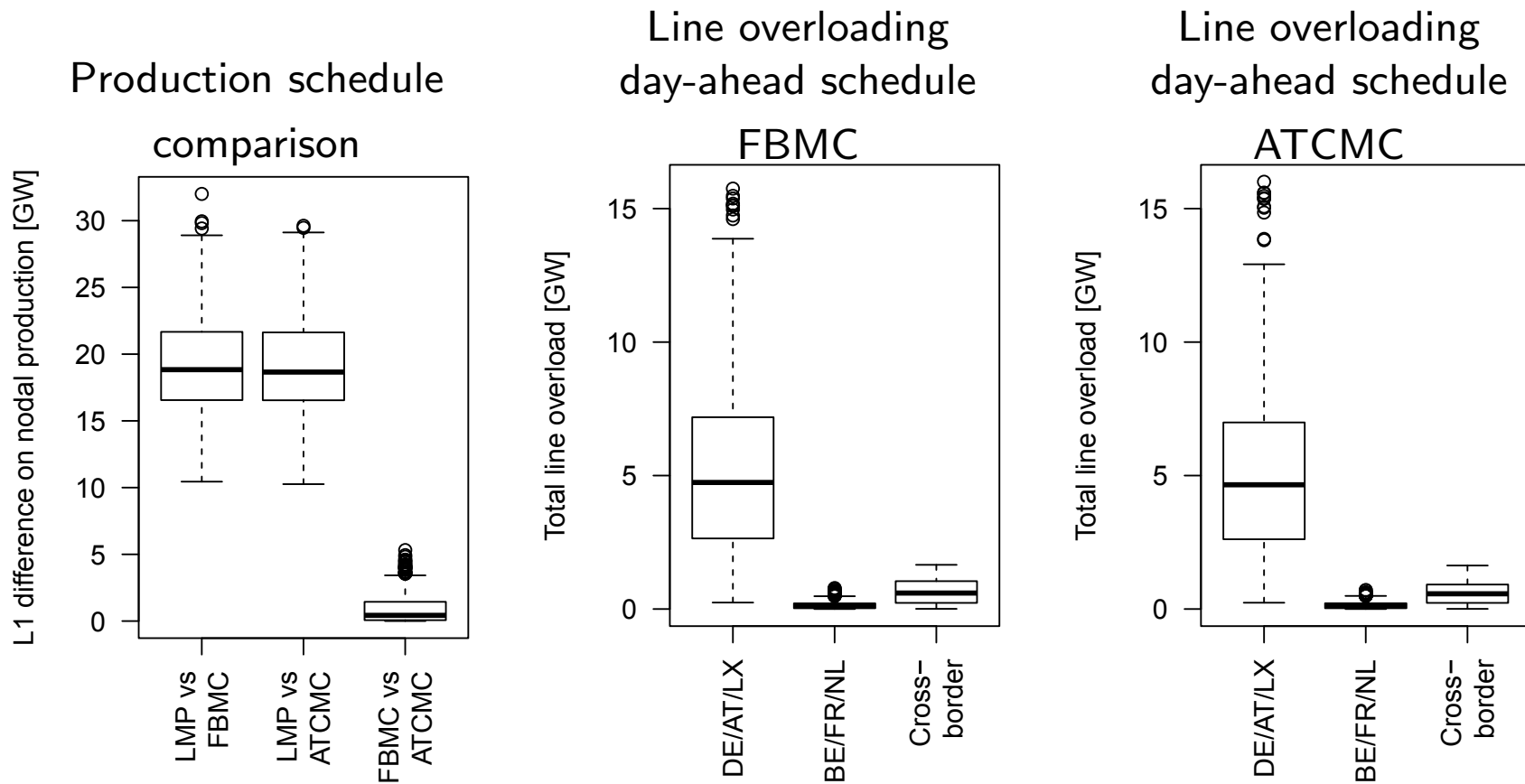
- PF: Perfect Foresight benchmark
- LMP schedules out-of-merit production to meet N-1 security, leading to inefficiencies with respect to PF
- Efficiency losses of zonal markets with respect to LMP amount to about 5.1% of total costs, ~600M€/year
- FBMC and ATCMC do not present a significant difference

# CWE results: costs composition



- Day-ahead costs larger for LMP, real-time costs much larger for FBMC and ATCMC than LMP
- Zonal policies: re-dispatching slow (cheap) generators down and re-dispatching fast (expensive) generators up in real time

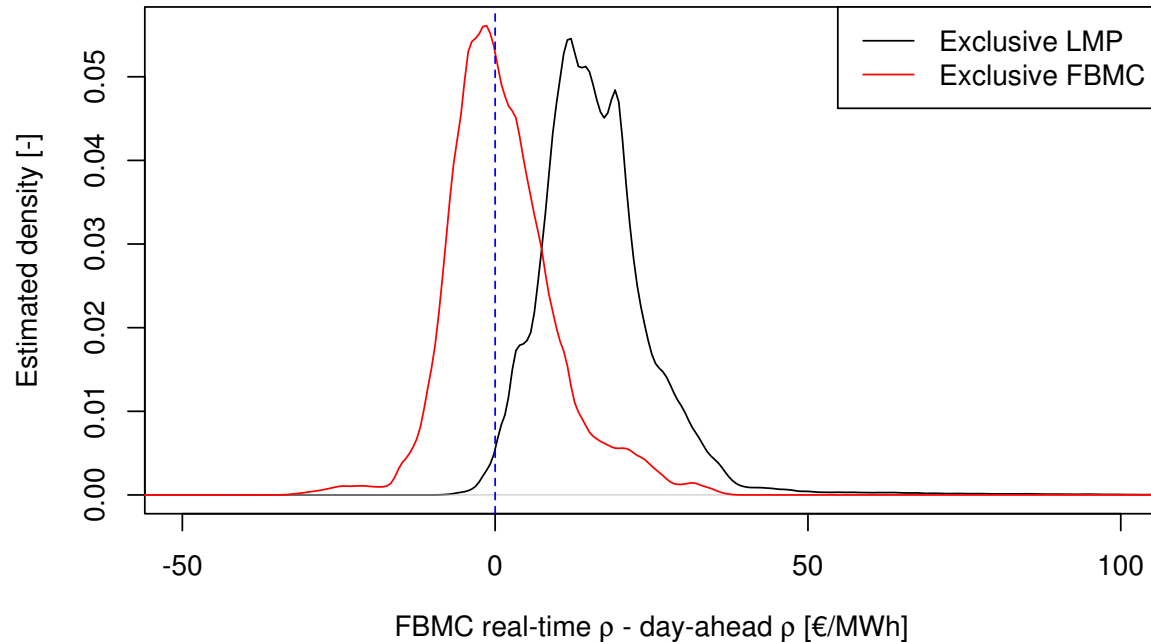
# CWE results: production schedules and line overloading



- Large differences** between day-ahead schedules of **nodal** and **zonal** policies, **small differences** between **FBMC** and **ATCMC**
- Zonal policies lead to decisions **overloading inter- and intra-zonal transmission lines**

# CWE results: difference on commitment decisions

Price change in FBMC at nodes where  $UC^{FBMC} \neq UC^{LMP}$



- “Exclusive LMP”: weighted average change  $\rho_n^{RT} - \rho_{z(n)}^{DA}$  over all nodes where LMP committed slow units and FBMC did not commit units. “Exclusive FBMC” series computed analogously.
- LMP commits capacity where it is needed. FBMC suffers from suboptimal commitment.

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- **New framework for modelling zonal electricity markets:**
  - Projecting network constraints onto space of exports/imports
  - Free from discretionary parameters (base case, flow approximation, etc.)
- ATCMC and FBMC fail at allocating inter- and intra-zonal transmission capacity
- CWE: **ATCMC and FBMC do not present significant performance differences**
- CWE: **Nodal design outperforms ATCMC and FBMC** by **~600M€/year**

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# Thank you

## Contact:

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# Future extensions

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- Application of the developed framework to answer policy questions in European electricity markets via simulation
- Extensions of the proposed framework:
  - Pricing AC power flow constraints implicitly on active power
  - TSO-DSO coordination

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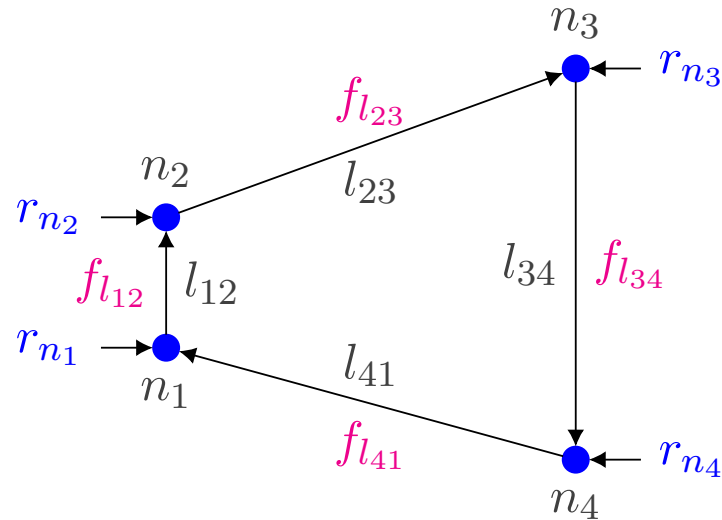
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# Electricity markets 101

# Transportation networks



- Paths are decision variables
- Must respect nodal balance constraints ( $\forall n \in N$ )

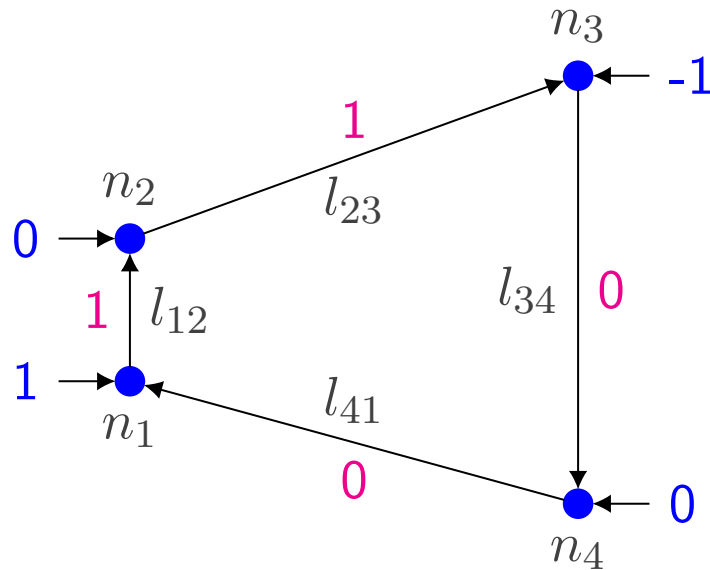
$$r_n = \sum_{l \in L(n, \cdot)} f_l - \sum_{l \in L(\cdot, n)} f_l$$

- Must respect capacities of branches ( $\forall l \in L$ ):

$$-F_l \leq f_l \leq F_l$$

- Note:  $\sum_{n \in N} r_n = 0$

# Transportation networks



- Paths are decision variables
- Must respect nodal balance constraints ( $\forall n \in N$ )

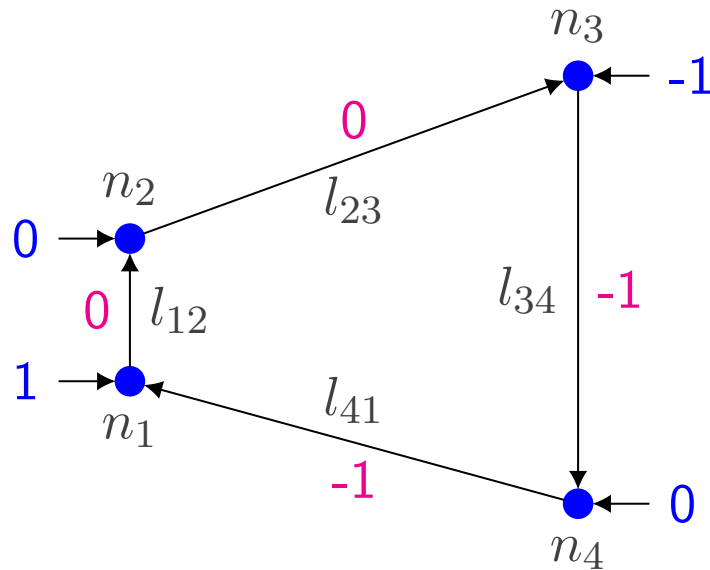
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# Transportation networks



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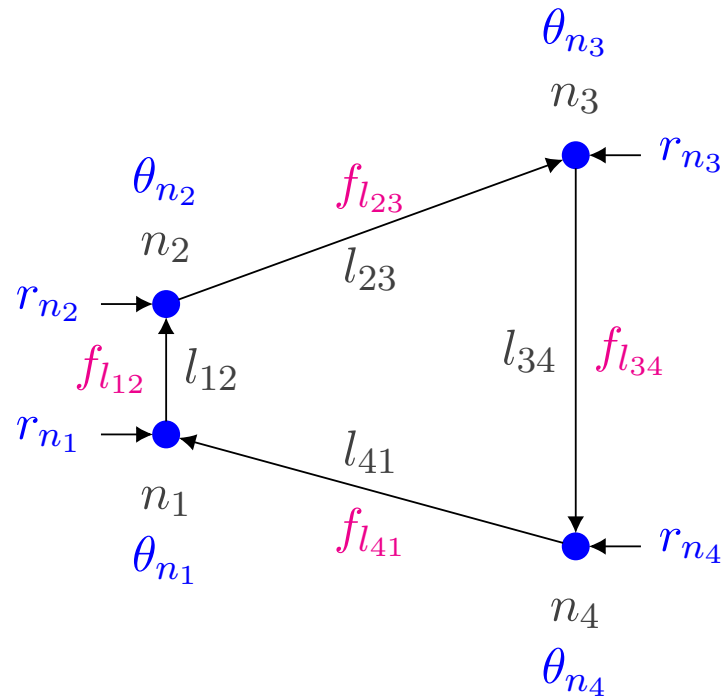
$$r_n = \sum_{l \in L(n, \cdot)} f_l - \sum_{l \in L(\cdot, n)} f_l$$

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- Note:  $\sum_{n \in N} r_n = 0$

# Electrical networks



- **Flow paths are implied by injections**

- Must respect nodal balance constraints ( $\forall n \in N$ )

$$r_n = \sum_{l \in L(n, \cdot)} f_l - \sum_{l \in L(\cdot, n)} f_l$$

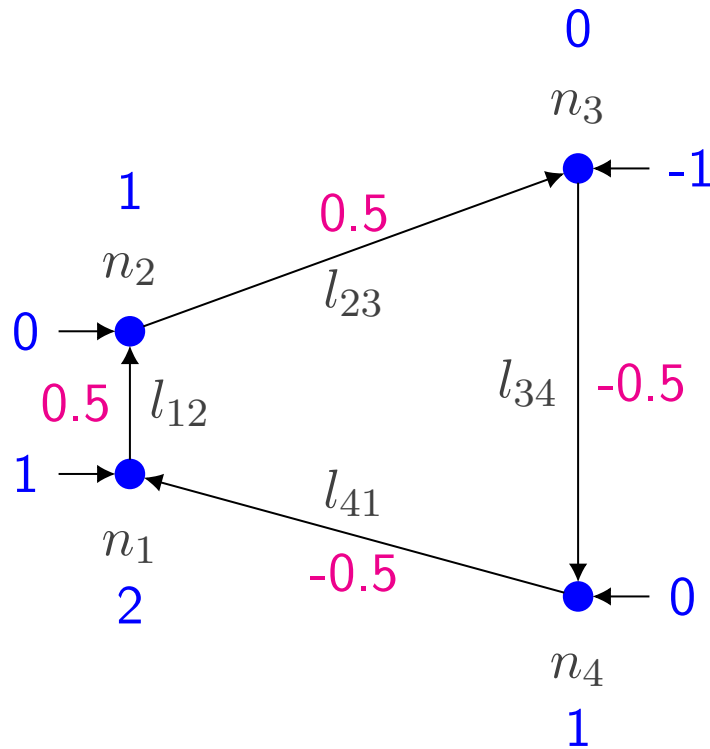
- Must respect capacities of branches ( $\forall l \in L$ ):

$$-F_l \leq f_l \leq F_l$$

- Must respect power-angle constraints ( $\forall l \in L$ , branch  $l$  from  $m(l)$  to  $n(l)$ ):

$$f_l = B_l (\theta_{m(l)} - \theta_{n(l)})$$

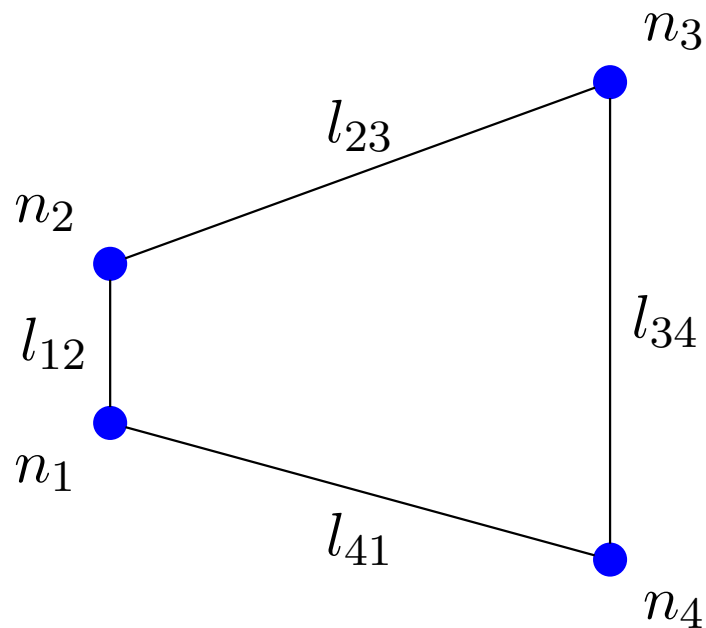
# Electrical networks



$$B_{l_{12}} = B_{l_{23}} = B_{l_{34}} = B_{l_{41}} = 0.5$$

- **Flow paths are implied by injections**
- Must respect nodal balance constraints ( $\forall n \in N$ )
 
$$r_n = \sum_{l \in L(n, \cdot)} f_l - \sum_{l \in L(\cdot, n)} f_l$$
- Must respect capacities of branches ( $\forall l \in L$ ):
 
$$-F_l \leq f_l \leq F_l$$
- Must respect power-angle constraints ( $\forall l \in L$ , branch  $l$  from  $m(l)$  to  $n(l)$ ):
 
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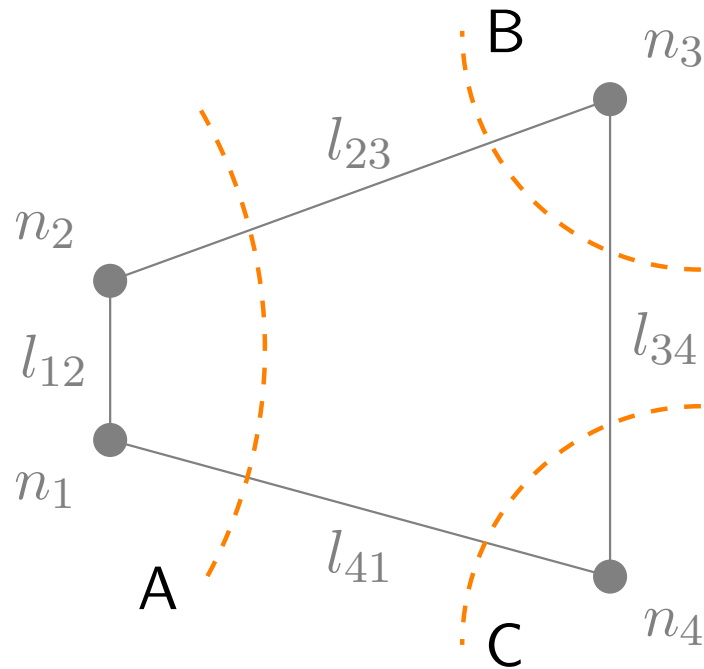
# Nodal markets: trading power over an electrical network



- Participants can **trade freely** when located at the **same node**
- Participants in **different nodes** can trade up to **congestion** of lines
- Congestion: power flow equations, thermal limits of transmission lines, among others (**physics**)



# Zonal markets: trading power over an aggregated network



- Participants **trade freely** within each **bidding zone**
- Participants in **different zones** can trade up to certain **export/import limits**
- Limits: **aggregation** of power flow equations and others