A Bi-Level Optimization Formulation of Priority Service Pricing

Yuting Mou, **Anthony Papavasiliou**, Philippe Chevalier

anthony.papavasiliou@uclouvain.be

Center for Operations Research and Econometrics
Université catholique de Louvain

INFORMS Annual Meeting 2018

Outline

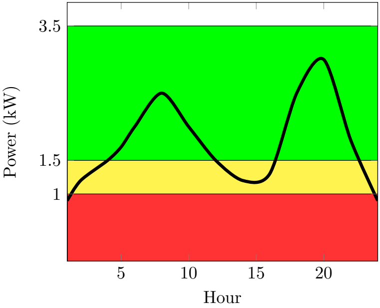
Outline

Motivation

- With the increasing integration of renewable production, the engagement of demand response becomes essential
- We investigate *priority service pricing* as a paradigm for DR aggregator business models
- Based on priority service pricing, the concept of **ColorPower** is proposed for residential demand response aggregation

ColorPower

The basic service offered by ColorPower are strips of power with differentiated priority



- Strips of higher priority correspond to higher price (simple)
- Control is behind the meter and consumers self-select how to allocate individual devices to strips (non-intrusive)

Motivation

- Textbook priority service pricing theory [Chao & Wilson, 1987] relies on stringent assumptions (convex costs/constraints)
- If assumptions are not satisfied, the allocation of consumers in the designed menu may degenerate
- Profits of the company cannot be guaranteed

- Our **goal**: reformulate priority service pricing (which is a Stackelberg game) as an MIP
- Impose a constraint on profits explicitly
- Generalize setting relative to [Chao & Wilson, 1987] $\Rightarrow$ couple demand response with unit commitment

Motivation

- Textbook priority service pricing theory [Chao & Wilson, 1987] relies on stringent assumptions (convex costs/constraints)
- If assumptions are not satisfied, the allocation of consumers in the designed menu may degenerate
- Profits of the company cannot be guaranteed

- Our **goal**: reformulate priority service pricing (which is a Stackelberg game) as an MIP
- Impose a constraint on profits explicitly
- Generalize setting relative to [Chao & Wilson, 1987] $\Rightarrow$ couple demand response with unit commitment

Outline

Utility and Consumer Models

- Stackelberg game
  - Leader: utility
  - Follower: consumer
- Utility
  - Offers a price menu of reliability-price pairs $\{r_i, \pi_i\}$
  - Subject to profit target
  - Goal: maximize welfare
- Consumer
  - Characterized by valuation V_l (private information)
  - Subscribes to a profile Θ_t, i.e., $\bar{D}_{l,t} = \bar{D}_l \cdot \Theta_t$, where $\bar{D}_l$ is average power demand
  - Consumer subscribes to the menu to maximize its net surplus

The Bi-Level Model

$$
\begin{aligned}
(Consumer\ l)\ \ \max\ \ & Benefits(l) - Payment(l) \\
s.t.\ \ & \text{bounds of subscription quantity}
\end{aligned}
$$

$$
\begin{aligned}
(Utility)\ \ \max\ \ & ConsumerBenefits - CompanyCosts \\
s.t.\ \ & \text{unit commitment} \\
& \text{energy dynamics of pumped hydro units} \\
& \text{supply and demand balance} \\
& \text{company profits constraint} \\
& \text{reliability constraint}
\end{aligned}
$$

Outline

The Consumer Model

Given multiple options in the price menu, π_i and r_i, consumers subscribe to the price menu based on V_l and $\bar{D}_l$ so as to maximize their net benefits:

$$(CP_l): \quad \max_{s_{l,i}} \quad \sum_{t \in T} \sum_{i \in I} (V_l \cdot r_i \cdot s_{l,i} \cdot \Theta_t - s_{l,i} \cdot \pi_i) \tag{1}$$

$$(\gamma_l): \quad \sum_{i \in I} s_{l,i} \leq \bar{D}_l \tag{2}$$

$$s_{l,i} \geq 0, i \in I \tag{3}$$

- In choosing option i, a consumer of type l procures a *profile* Θ_t.
- The variable $s_{l,i}$ indicates the amount of power that consumer l allocates to option i.

The Consumer Model

Given multiple options in the price menu, π_i and r_i, consumers subscribe to the price menu based on V_l and $\bar{D}_l$ so as to maximize their net benefits:

$$(CP_l): \max_{s_{l,i}} \quad \sum_{t \in T} \sum_{i \in I} (V_l \cdot r_i \cdot s_{l,i} \cdot \Theta_t - s_{l,i} \cdot \pi_i) \tag{1}$$

$$(\gamma_l): \quad \sum_{i \in I} s_{l,i} \leq \bar{D}_l \tag{2}$$

$$s_{l,i} \geq 0, i \in I \tag{3}$$

- In choosing option i, a consumer of type l procures a *profile* Θ_t.
- The variable $s_{l,i}$ indicates the amount of power that consumer l allocates to option i.

The Consumer Model (cont'd)

Proposition

There exists $\tilde{\mathbf{s}}_l = (\tilde{s}_{l,i}, i \in I)$ with $\tilde{s}_{l,i} \in \{0, \bar{D}_l\}$ which attains the optimal objective function value. Proof

The above proposition implies that $s_{l,i}$ can be expressed as $s_{l,i} = \bar{D}_l \cdot \mu_{l,i}$, where $\mu_{l,i} \in \{0,1\}$ are binary variables and $\sum_{i \in I} \mu_{l,i} = 1,\ l \in L$.

Optimality Conditions

$\forall l \in L$, the optimality conditions for the consumer problem include primal feasibility, dual feasibility and zero dual gap, which are guaranteed by:

$$
\begin{aligned}
& s_{l,i} = \bar{D}_l \cdot \mu_{l,i},\ \ \mu_{l,i} \in \{0,1\}, i \in I \\
& \gamma_l \geq r_i \cdot V_l - \pi_i, i \in I \\
& \gamma_l \geq 0 \\
& \gamma_l \leq V_l \cdot \sum_{i \in I} r_i \cdot \mu_{l,i} \ - \sum_{i \in I} \pi_i \cdot \mu_{l,i}
\end{aligned}
$$

Optimality Conditions

$\forall l \in L$, the optimality conditions for the consumer problem include primal feasibility, dual feasibility and zero dual gap, which are guaranteed by:

$$s_{l,i} = \bar{D}_l \cdot \mu_{l,i},\ \ \mu_{l,i} \in \{0,1\}, i \in I$$
$$\gamma_l \geq r_i \cdot V_l - \pi_i, i \in I$$
$$\gamma_l \geq 0$$
$$\gamma_l \leq V_l \cdot \sum_{i \in I} r_i \cdot \mu_{l,i} - \sum_{i} \pi_i \cdot \mu_{l,i}$$

Optimality Conditions

$\forall l \in L$, the optimality conditions for the consumer problem include primal feasibility, dual feasibility and zero dual gap, which are guaranteed by:

$$s_{l,i} = \bar{D}_l \cdot \mu_{l,i}, \;\; \mu_{l,i} \in \{0,1\}, i \in I$$
$$\gamma_l \geq r_i \cdot V_l - \pi_i, i \in I$$
$$\gamma_l \geq 0$$
$$\gamma_l \leq V_l \cdot \sum_{i \in I} r_i \cdot \mu_{l,i} - \sum_{i} \pi_i \cdot \mu_{l,i}$$

The product of a continuous variable and a binary variable can be represented by McCormick Envelopes .

Outline

The Utility Model

The utility cannot access the type/valuation of individuals, but instead the distribution of types ⇒ (i) valuation V_i and (ii) total demand D_i, $i \in I$

$$\max \sum_{t \in T} \Big(\sum_{i \in I} V_i \cdot d_{i,t} - h_t(\mathbf{m}, \mathbf{o}, \mathbf{p}) \Big) \tag{4}$$

$$f_g(\mathbf{m}, \mathbf{n}, \mathbf{o}, \mathbf{p}) \leq 0, g \in G \tag{5}$$

$$|T| \cdot \sum_{i \in I} s_i \cdot \pi_i - \sum_{t \in T} h_t(\mathbf{m}, \mathbf{o}, \mathbf{p}) = \Pi_\star \tag{6}$$

$$\sum_{i \in I} d_{i,t} = \sum_{g \in G} p_{g,t}, t \in T \tag{7}$$

$$d_{i,t} \leq s_i \cdot \Theta_t, i \in I, t \in T \tag{8}$$

$$s_i = \sum_{l \in L} s^\star_{l,i}, i \in I, \;\; s_i \leq D_i, i \in I \tag{9}$$

$$\sum_{t \in T} r_i \cdot s_i \cdot \Theta_t = \sum_{t \in T} d_{i,t}, i \in I \tag{10}$$

$$s_i, d_{i,t}, p_{g,t} \geq 0, i \in I, g \in G, t \in T \tag{11}$$

$$m_{g,t}, n_{g,t}, o_{g,t} \in \{0, 1\}, g \in G, t \in T \tag{12}$$

Outline

The Bi-Level Model as an MIP

$$\min \quad \sum_{t \in T} \Big(h_t(\mathbf{m}, \mathbf{o}, \mathbf{p}) - \sum_{i \in I} V_i \cdot d_{i,t} \Big) \tag{13}$$

$$f_g(\mathbf{m}, \mathbf{n}, \mathbf{o}, \mathbf{p}) \le 0, g \in G \tag{14}$$

$$\sum_{i \in I} d_{i,t} = \sum_{g \in G} p_{g,t} \tag{15}$$

$$d_{i,t} \le \sum_{l} \bar{D}_l \cdot \bar{\mu}_{l,i} \cdot \Theta_t, i \in I, t \in T \tag{16}$$

$$c_t = h_t(\mathbf{m}, \mathbf{o}, \mathbf{p}), t \in T \tag{17}$$

$$|T| \cdot \sum_{i \in I} \sum_{l \in L} \bar{D}_l \cdot y_{l,i} - \sum_{t \in T} c_t = \Pi_\star \tag{18}$$

$$|T| \cdot \sum_{l \in L} w_{l,i} \cdot \bar{D}_l = \sum_{t \in T} d_{i,t}, i \in I \tag{19}$$

$$y_{l,i} \le \Pi^+ \cdot \bar{\mu}_{l,i}, y_{l,i} \le \pi_i, y_{l,i} \ge \Pi^+ \cdot \bar{\mu}_{l,i} + \pi_i - \Pi^+ \tag{20}$$

$$w_{l,i} \le \bar{\mu}_{l,i}, w_{l,i} \le r_i, w_{l,i} \ge \bar{\mu}_{l,i} + r_i - 1 \tag{21}$$

$$\gamma_l \ge r_i \cdot V_l - \pi_i, l \in L, i \in I \tag{22}$$

$$\gamma_l \le \sum_{i \in I} w_{l,i} \cdot V_l - \sum_{i \in I} y_{l,i}, l \in L \tag{23}$$

Outline

Price Menu Comparison

Textbook			Bi-Level	
Target reliability [%]	Realized reliability[%]	Price [€/MWh]	Reliability [%]	Price [€/MWh]
20.2	0.3	0	4.8	0
95.2	94.6	46.5	94.7	55.4
98.3	98	52.8	98	62.1
99.8	99.7	57.6	99.7	67.9
100	100	58.6	100	68.9

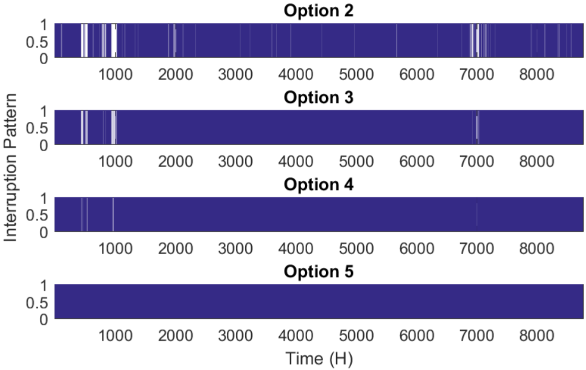
- The prices in the menu of the bi-level model higher, this allows us to achieve the profit target
- There is no reliability deviation in the bi-level model menu

Welfare Comparison

	Social Welfare (M €)	Consumer Benefits (M €)	Consumer Net Benefits (M €)	Company Profits (M €)
Flat Tariff	5698	6878	5295	402
BiLevel	5760	6954	5357	402
Real-Time Pricing	5769	6982	5506	263

- Priority service pricing with 5 options achieves 86.9% of the welfare gain of real-time pricing
- The Bi-Level model hits the profit target exactly

Interruption Patterns



- A continuous interruption of 63 hours is possible

Outline

Conclusions:

- Priority service pricing with 5 options captures approximately 90% of the efficiency gains of real-time pricing
- We contribute to priority service theory by modeling the Stackelberg game as an MIP, which allows us to override certain stringent assumptions of the textbook theory
- Our model allows us to determine the interruption patterns in a unit commitment model, and gain insights regarding the precise definition of reliability

Future Work:

- Design the menu considering capacity expansion [Joskow & Tirole, 2007] $\Rightarrow$ guarantee that the promised reliability can be delivered on a daily/weekly/monthly basis
- Apply the idea to design a capacity and energy based tariff [Chao, 1986]

Conclusions:

- Priority service pricing with 5 options captures approximately 90% of the efficiency gains of real-time pricing
- We contribute to priority service theory by modeling the Stackelberg game as an MIP, which allows us to override certain stringent assumptions of the textbook theory
- Our model allows us to determine the interruption patterns in a unit commitment model, and gain insights regarding the precise definition of reliability

Future Work:

- Design the menu considering capacity expansion [Joskow & Tirole, 2007] $\Rightarrow$ guarantee that the promised reliability can be delivered on a daily/weekly/monthly basis
- Apply the idea to design a capacity and energy based tariff [Chao, 1986]

Thank you!

Questions?

Demand Functions

Consider a fixed residential electricity tariff V^r and assume an affine demand function with an elasticity e at the historical observed quantity and price, then the demand function at hour t is given as

$$\begin{aligned} \frac{d_t^r(v) - D_t^r}{D_t^r} &= e \cdot \frac{v - V^r}{V^r}, \\ \Longrightarrow d_t^r(v) &= \frac{D_t^r \cdot e}{V^r} \cdot v + D_t^r(1-e) \\ &= D_t^r \cdot \big(\frac{e}{V^r} \cdot v + (1-e)\big) \end{aligned}$$

The intercept is calculated as

$$V_{\max} = \frac{e-1}{e} \cdot V^r.$$

Demand Functions

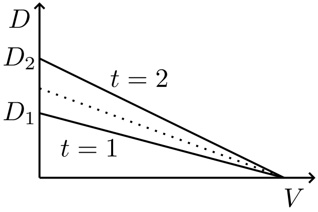
Consider a fixed residential electricity tariff V^r and assume an affine demand function with an elasticity e at the historical observed quantity and price, then the demand function at hour t is given as

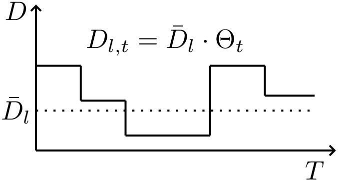
$$\begin{aligned} \frac{d_t^r(v) - D_t^r}{D_t^r} &= e \cdot \frac{v - V^r}{V^r}, \\ \Longrightarrow d_t^r(v) &= \frac{D_t^r \cdot e}{V^r} \cdot v + D_t^r(1-e) \\ &= D_t^r \cdot \big(\frac{e}{V^r} \cdot v + (1-e)\big) \end{aligned}$$

The intercept is calculated as

$$V_{\max} = \frac{e-1}{e} \cdot V^r.$$

Representation of Consumer Types





- Hourly linear demand functions are calibrated based on historical data.
- Each individual residential consumer of type l is associated with valuation V_l.
- V_l indicates the priority order in which this consumer gets supplied.
- **Synchronization** assumption: all consumers follow the same profile Θ_t.

Proof of Proposition 1

The KKT conditions of (CP_l) are given by

$$0 \leq s_{l,i} \perp -r_i \cdot V_l + \pi_i + \gamma_l \geq 0 \tag{24}$$

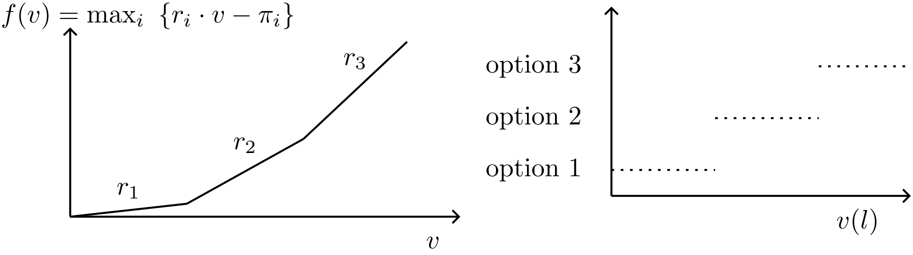
$$0 \leq \gamma_l \perp \bar{D}_l - \sum_i s_{l,i} \geq 0 \tag{25}$$

There are two cases to be considered: Case 1: If $\bar{D}_l - \sum_i s^\star_{l,i} > 0$, then $\gamma_l = 0$, which implies that consumer l gets zero benefits at the optimal solution, so $\tilde{s}_{l,i} = 0$ for all $i \in I$ is optimal.
Case 2: If $\bar{D}_l - \sum_{i \in I} s^\star_{l,i} = 0$, then it suffices to show that if two options are 'active' (in the sense that $s > 0$) then they have an equal payoff, and can therefore be equivalently replaced by a single option. Applying this argument for all options that are active gives the desired conclusion. Consider any two options i and j for which $s^\star_{l,i} > 0$ and $s^\star_{l,j} > 0$. Then $-r_i \cdot V_l + \pi_i + \gamma_l = 0$ and $-r_j \cdot V_l + \pi_j + \gamma_l = 0$, and substituting out γ_l, we have $r_i \cdot V_l - \pi_i = r_j \cdot V_l - \pi_j$.
Back to Proposition.

Proof of Proposition 2

Assume the options in the menu are ordered in the following way, $r_1 < r_2 ... < r_I$, then it holds that $\pi_1 < \pi_2 ... < \pi_I$, so that no option is dominated by others.
Consumer surplus is expressed as $f(v) = \max_i \{r_i \cdot v - \pi_i\}$, so $f(v)$ is a piece-wise convex function of v [Pointwise maximum - Boyd & Vandenberghe 2004].
Since each piece of $f(v)$ is increasing, $f(v)$ must be increasing as well and the slop of each piece is increasing, which corresponds to r_i, so each piece of $f(v)$ is ordered from 1 to I. (Figures on the next slide.)
Back to Proposition.

Illustration of Valid Cuts



McCormick Envelopes

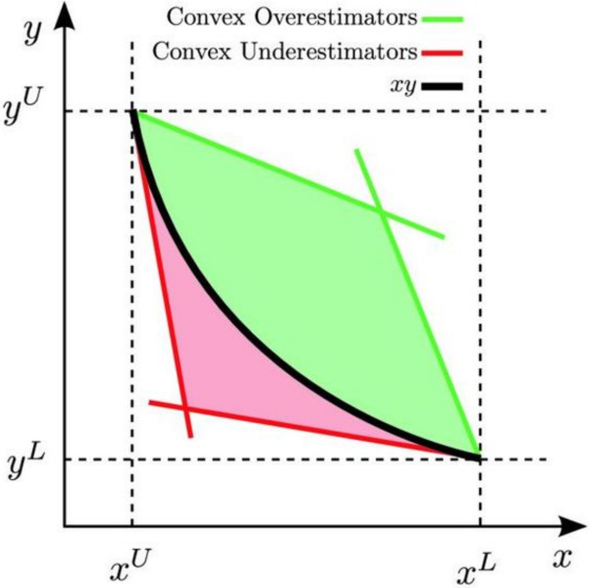
Using McCormick envelopes to represent binary-continuous products: $\pi_i \cdot \mu_{l,i}$ is represented by $y_{l,i}$

$$y_{l,i} \leq \Pi^+ \cdot \mu_{l,i},\ y_{l,i} \geq 0,\ y_{l,i} \leq \pi_i,\ y_{l,i} \geq \Pi^+ \cdot \mu_{l,i} + \pi_i - \Pi^+ \tag{26}$$

$r_i \cdot \mu_{l,i}$ is represented by $w_{l,i}$

$$w_{l,i} \leq \mu_{l,i},\ w_{l,i} \geq 0,\ w_{l,i} \leq r_i,\ w_{l,i} \geq \mu_{l,i} + r_i - 1 \tag{27}$$

Back


Convex Overestimators
Convex Underestimators
xy
y
y^U
y^L
x^U
x^L
x

System Settings

- We present a case study for the Belgian system.
- The entire conventional generator fleet consists of 55 units, the installed capacity of each technology follows the projected capacity of year 2050.
- Renewable production and import profiles are fixed.
- Pumped hydro resources have a roundtrip efficiency of 76.5%.
- Hourly demand functions are calibrated.
- Consider a horizon of one year and resolution of one hour.
- The model is implemented in Julia and run on a server with two Intel Xeon 2.66GHz 6-core CPUs and 48GB RAM.

Decomposition Using ADMM

Scheme

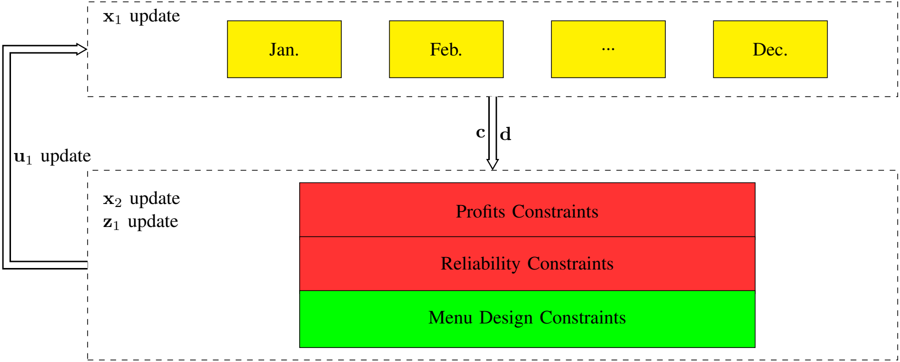


Figure: The application of ADMM as a heuristic for decomposing the bi-level menu design problem.

Decomposition Using ADMM

Notations

- $\mathcal{C}_1$: the set of constraints (14)-(17) that relate to the unit commitment.
- $\mathcal{C}_2$: the set of constraints (20) - (23) that relate to the menu design.
- $\mathcal{C}_3$: the set of constraints (18) - (19) which create a coupling between the unit commitment and the menu design parts of the problem
- $\mathbf{x}_1$: the set of variables that appear $\mathcal{C}_1$ *or* $\mathcal{C}_3$.
- $\mathbf{x}_2$: the set of variables that appear $\mathcal{C}_2$ *or* $\mathcal{C}_3$.

Decomposition Using ADMM

Reformulation

The bi-level problem can then be written in stylized form as follows:

$$\min f_1(\mathbf{x}_1) + f_2(\mathbf{x}_2) \tag{28}$$

$$\mathbf{x}_1 \in \mathcal{C}_1 \cap \mathcal{C}_3 \tag{29}$$

$$\mathbf{x}_2 \in \mathcal{C}_2 \cap \mathcal{C}_3 \tag{30}$$

In order to bring the problem to a form which is amenable to the application of the ADMM algorithm, the original problem can be rewritten as follows:

$$\min_{\mathbf{x}_1 \in \mathcal{C}_1, \mathbf{x}_2 \in \mathcal{C}_2, \mathbf{z}_1, \mathbf{z}_2} \quad f_1(\mathbf{x}_1) + f_2(\mathbf{x}_2) + g(\mathbf{z}_1, \mathbf{z}_2) \tag{31}$$

$$A_1\mathbf{x}_1 - \mathbf{z}_1 = 0 \tag{32}$$

$$A_2\mathbf{x}_2 - \mathbf{z}_2 = 0 \tag{33}$$

where g is the indicator function of $\mathcal{C}_3$.

Decomposition Using ADMM

Iterations

The ADMM iterations can then be expressed as follows:

$$
\begin{aligned}
\mathbf{x}_1^{k+1} &:= \arg\min_{\mathbf{x}_1 \in \mathcal{C}_1} \left(f_1(\mathbf{x}_1) + (\rho/2)\|A_1\mathbf{x}_1 - \mathbf{z}_1^k + \mathbf{u}_1^k\|_2^2 \right) \\
\mathbf{x}_2^{k+1} &:= \arg\min_{\mathbf{x}_2 \in \mathcal{C}_2} \left(f_2(\mathbf{x}_2) + (\rho/2)\|A_2\mathbf{x}_2 - \mathbf{z}_2^k + \mathbf{u}_2^k\|_2^2 \right) \\
\mathbf{z}_1^{k+1} &:= \Pi_{\mathcal{C}_3}(A_1\mathbf{x}_1^{k+1} + \mathbf{u}_1^k) \\
\mathbf{z}_2^{k+1} &:= \Pi_{\mathcal{C}_3}(A_2\mathbf{x}_2^{k+1} + \mathbf{u}_2^k) \\
\mathbf{u}_1^{k+1} &:= \mathbf{u}_1^k + A_1\mathbf{x}_1^{k+1} - \mathbf{z}_1^{k+1} \\
\mathbf{u}_2^{k+1} &:= \mathbf{u}_2^k + A_2\mathbf{x}_2^{k+1} - \mathbf{z}_2^{k+1}
\end{aligned}
$$

where $\Pi_{\mathcal{C}}$ is the projection operator on the set $\mathcal{C}$ and $(\mathbf{u}_1, \mathbf{u}_2)$ are the scaled dual variables.

Decomposition Using ADMM

Iterations

Moreover, in our problem $f_2(\mathbf{x}_2) = 0$. The solution process can then be simplified by dropping the $\mathbf{z}_2$ variables. We then update $\mathbf{x}_2$ by looking for a feasible solution in the set of constraints $\mathcal{C}_2$ and $\mathcal{C}_3$. More specifically, we implement the following algorithm:

$$\begin{aligned}
\mathbf{x}_1^{k+1} &:= \arg\min_{\mathbf{x}_1 \in \mathcal{C}_1} \left(f_1(\mathbf{x}_1) + (\rho/2)\|A_1\mathbf{x}_1 - \mathbf{z}_1^k + \mathbf{u}_1^k\|_2^2 \right) \\
\mathbf{x}_2^{k+1} &\in \mathcal{C}_2 \cap \mathcal{C}_3 \\
\mathbf{z}_1^{k+1} &:= \Pi_{\mathcal{C}_3}(A_1\mathbf{x}_1^{k+1} + \mathbf{u}_1^k) \\
\mathbf{u}_1^{k+1} &:= \mathbf{u}_1^k + A_1\mathbf{x}_1^{k+1} - \mathbf{z}_1^{k+1}
\end{aligned}$$