

Large-Scale Integration of Deferrable Demand and Renewable Energy in Power Systems

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Outline

- 1 Introduction
- 2 Methodology
 - Unit Commitment Model
 - Decomposition and Scenario Selection
 - Wind Model
- 3 Results
- 4 Conclusions and Perspectives

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EU Parliament hears Helios Project

In Athens News

EU parliamentarians will hear any update on the Helios Project over the funding of Project Helios, the largest wind farm in Europe. The report is presented by Minister Vassilis Rapanos and the European Commission.

Rapanos said the Helios Project is the largest wind farm in Europe and Greece's largest energy project, as it is the first large-scale wind farm in the country. The project aims to generate 1,000 MW of electricity, which will be fed into the Greek power grid. The project is expected to begin in 2012 and to be completed in 2014.

*The Helios Project is the largest wind farm in Europe and Greece's largest energy project, as it is the first large-scale wind farm in the country. The project aims to generate 1,000 MW of electricity, which will be fed into the Greek power grid. The project is expected to begin in 2012 and to be completed in 2014.

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Germany: Nuclear power plants to close by 2022

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Germany saw mass anti-nuclear protests in the wake of the Fukushima disaster

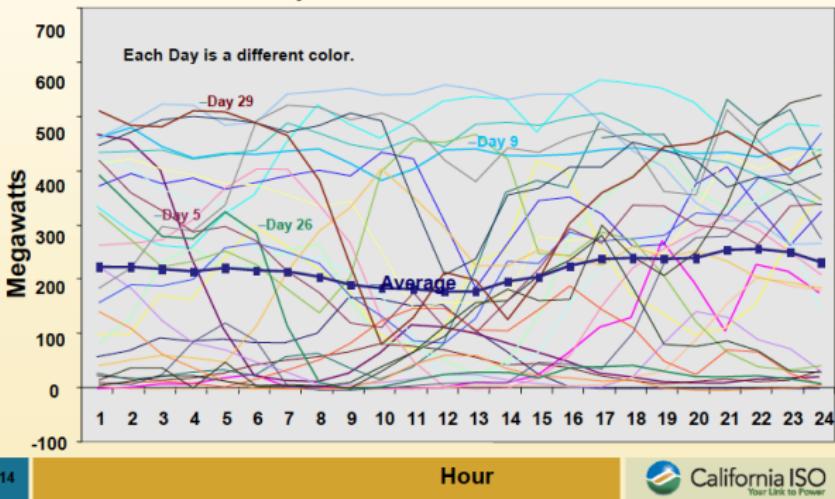
Germany's coalition government has announced a reversal of policy that will see all the country's nuclear power plants phased out by 2022.

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Uncertainty

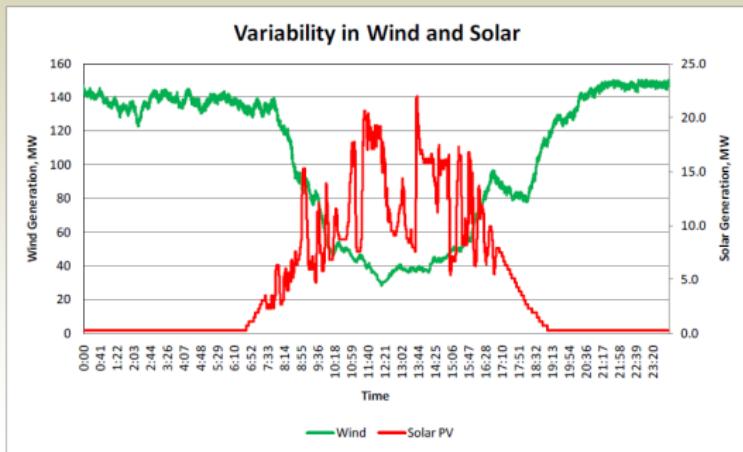
Tehachapi Wind Generation in April – 2005

Could you predict the energy production for this wind park
either day-ahead or 5 hours in advance?



Variability

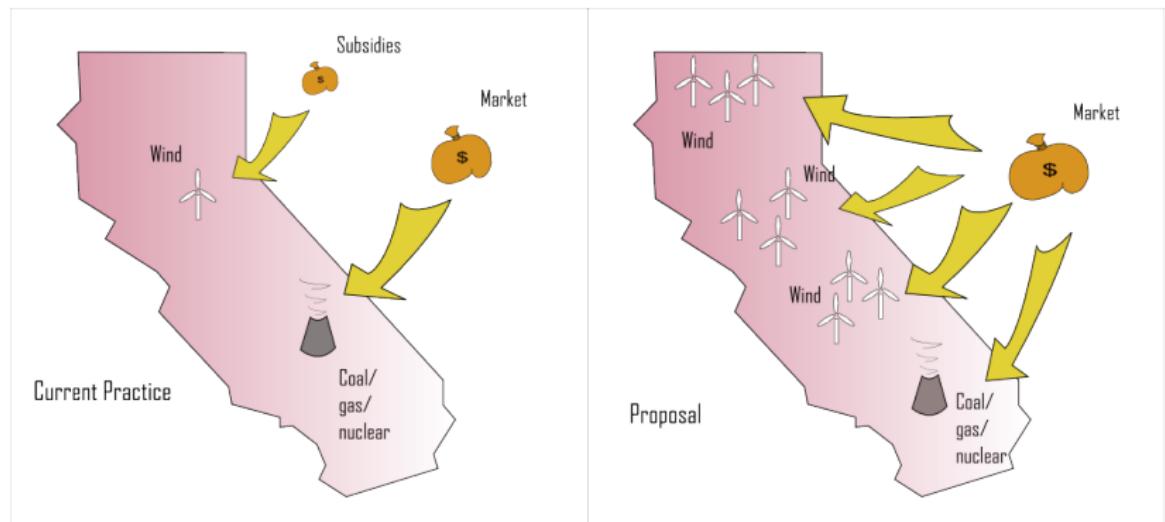
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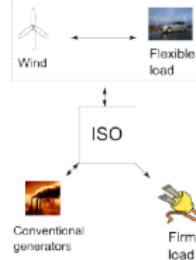
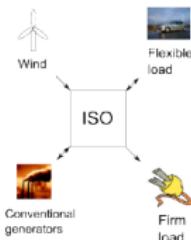
A 150 MW wind plant and a 24 MW solar resource

Slide 5

A Vision for Renewable Energy



Load Flexibility



System operator control

Centralized

Coupling renewables with deferrable demand

Price-responsive demand

Decentralized

Research Objective

Want to quantify:

- Renewable energy utilization
- Cost of unit commitment and economic dispatch
- Capital investment in generation capacity

Stochastic unit commitment an appropriate model:

- Quantifies renewable energy utilization (decision variable)
- Quantifies operating costs (objective function)
- Endogenously determines reserves (as opposed to ad hoc rules)

Unit Commitment

- Objective: $\min \sum_{g,t} (K_g u_{gt} + S_g v_{gt} + C_g p_{gt})$
- Load balance: $\sum_{g \in G} p_{gt} = D_t, \forall t$
- Min / max capacity limits: $P_g^- u_{gt} \leq p_{gt} \leq P_g^+ u_{gt}, \forall g, t$
- Ramping limits: $-R_g^- \leq p_{gst} - p_{gs,t-1} \leq R_g^+, \forall g, t$
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- Transmission line thermal constraints
- Kirchhoff voltage/current laws

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The Real Thing

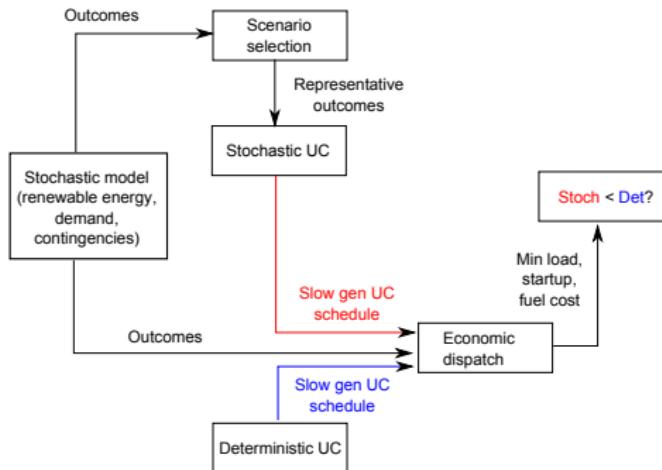


- 1,210 generators, 3 part offers (startup, no load, 10 segment incremental energy offer curve)
- 10,000 - Demand bids – fixed or price sensitive
- 50,000 - Virtual bids / offers
- 8,700 - eligible bid/offer nodes (pricing nodes)
- 6,125 - monitored transmission elements
- 10,000 - transmission contingencies modeled

Relevant Literature

- Wind integration studies based on stochastic unit commitment: (Bouffard and Galiana, 2008), (Wang, Shahidehpour et al., 2008), (**Ruiz, Philbrick et al., 2009**), (Tuohy, Meibom et al., 2009), (Morales, Conejo et al., 2009), (Constantinescu, Zavala et al., 2011)
- Wind / price-responsive demand study based on deterministic unit commitment: (Sioshansi and Short, 2009)
- Deferrable demand response study based on deterministic unit commitment: (Sioshansi, 2011)

Validation Process



Unit Commitment and Economic Dispatch

- Deterministic model (Sioshansi and Short, 2009)
 - ➊ Reserve requirements

$$\sum_{g \in G} s_{gt} + \sum_{g \in G_f} f_{gt} \geq T_t^{\text{req}}, \sum_{g \in G_f} f_{gt} \geq F_t^{\text{req}}, t \in T$$

- ➋ Import constraints

$$\sum_{l \in IG_j} \gamma_{jl} e_{lt} \leq IC_j, j \in \mathcal{IG}, t \in T$$

- Slow generator schedules are fixed in economic dispatch model: $w_{gt} = w_{gt}^*, g \in G_s$

Two-Stage Stochastic Unit Commitment

- 1 In the first stage we commit slow generators:

$u_{gst} = w_{gt}, v_{gst} = z_{gt}, g \in G_s, s \in S, t \in T$ (corresponds to day-ahead market)

- 2 Uncertainty is revealed: net demand D_{nst} , line availability B_{ls} , generator availability P_{gs}^+, P_{gs}^-
- 3 Fast generator commitment and production schedules are second stage decisions: $u_{gst}, g \in G_f$ and $p_{gst}, g \in G_f \cup G_s$ (corresponds to real-time market)
- 4 Objective:

$$\min \sum_{g \in G} \sum_{s \in S} \sum_{t \in T} \pi_s (K_g u_{gst} + S_g v_{gst} + C_g p_{gst})$$

Lagrangian Decomposition Algorithm

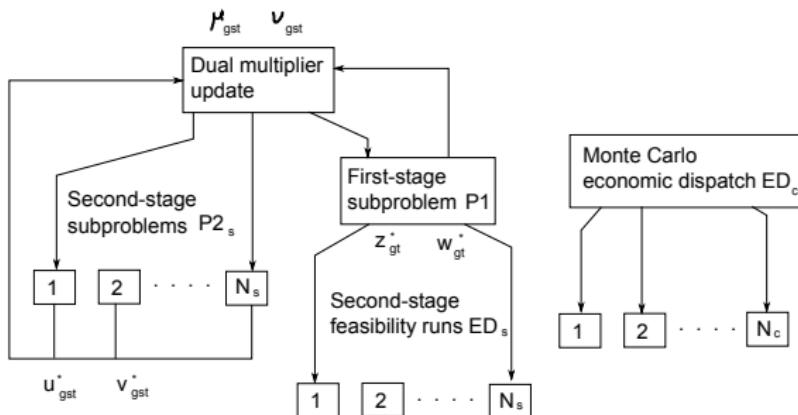
- Past work: (Takriti et al., 1996), (Carpentier et al., 1996), (Nowak and Römisch, 2000), (Shiina and Birge, 2004)
- Key idea: relax non-anticipativity constraints on both unit commitment and startup variables
 - ➊ Balance size of subproblems
 - ➋ Obtain lower and upper bounds at each iteration

Lagrangian:

$$\begin{aligned}\mathcal{L} = & \sum_{g \in G} \sum_{s \in S} \sum_{t \in T} \pi_s (K_g u_{gst} + S_g v_{gst} + C_g p_{gst}) \\ & + \sum_{g \in G_s} \sum_{s \in S} \sum_{t \in T} \pi_s (\mu_{gst}(u_{gst} - w_{gt}) + \nu_{gst}(v_{gst} - z_{gt}))\end{aligned}$$

Parallelization

- Second-stage subproblems, second-stage feasibility runs and economic dispatch simulations can be parallelized
- Implemented in MPI, CPLEX



Scenario Selection for Wind Uncertainty and Contingencies

- Past work: (Gröwe-Kuska et al., 2002), (Dupacova et al., 2003), (Heitsch and Römisch, 2003), (Morales et al., 2009)
- Scenario selection algorithm inspired by importance sampling

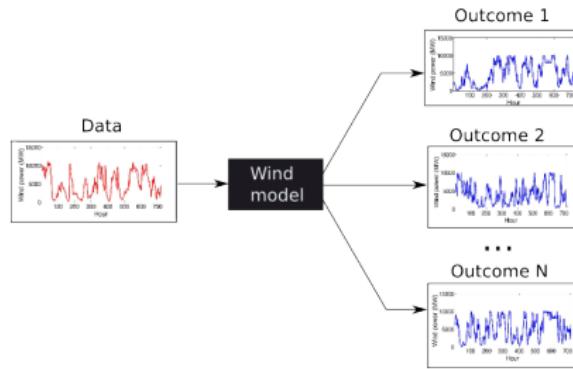
- ① Generate a sample set $\Omega_S \subset \Omega$, where $M = |\Omega_S|$ is adequately large. Calculate the cost $C_D(\omega)$ of each sample $\omega \in \Omega_S$ against the best deterministic unit commitment

$$\text{policy and the average cost } \bar{C} = \sum_{i=1}^M \frac{C_D(\omega_i)}{M}.$$

- ② Choose N scenarios from Ω_S , where the probability of picking a scenario ω is $C_D(\omega)/\bar{C}$.
- ③ Set $\pi_s = C_D(\omega)^{-1}$ for all $\omega^s \in \hat{\Omega}$.

Wind Model Data Source

- 2 wind integration cases: moderate (7.1% energy integration, 2012), deep (14% energy integration, 2020)
- California ISO interconnection queue lists locations of planned wind power installations
- NREL Western Wind and Solar Interconnection Study archives wind speed - wind power for Western US



Calibration

- Relevant literature: (Brown et al, 1984), (Torres et al., 2005), (Morales et al, 2010)
- Calibration steps

- ➊ Remove systematic effects:

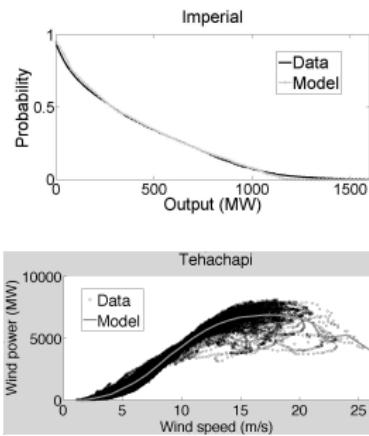
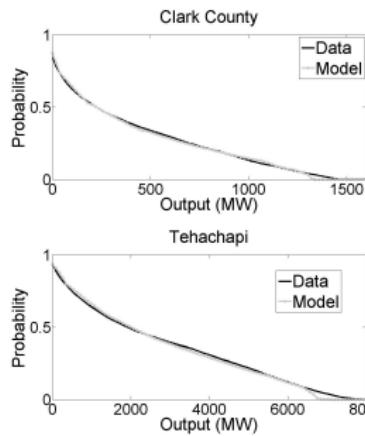
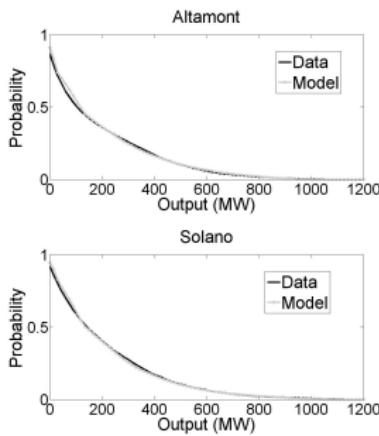
$$y_{kt}^S = \frac{y_{kt} - \hat{\mu}_{kmt}}{\hat{\sigma}_{kmt}}.$$

- ➋ Transform data to obtain a Gaussian distribution:

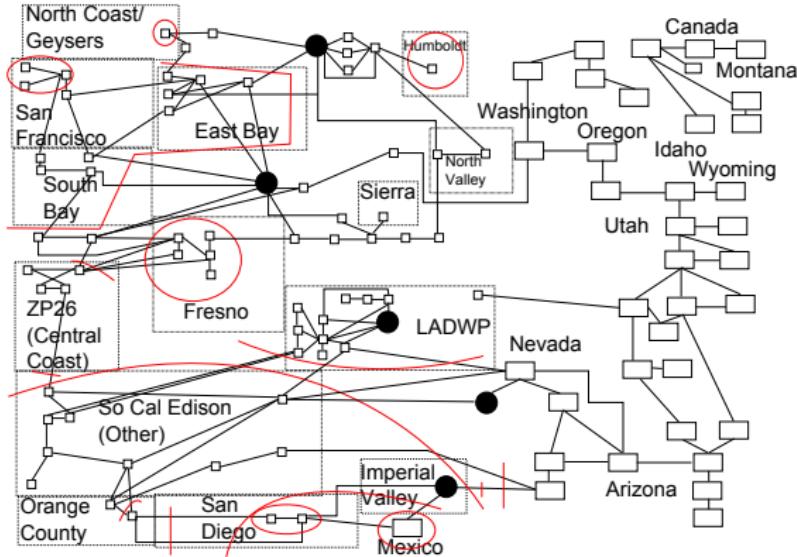
$$y_{kt}^{GS} = N^{-1}(\hat{F}_k(y_{kt}^S)).$$

- ➌ Estimate the autoregressive parameters $\hat{\phi}_{kj}$ and covariance matrix $\hat{\Sigma}$ using Yule-Walker equations.

Data Fit



WECC Model



Model Summary

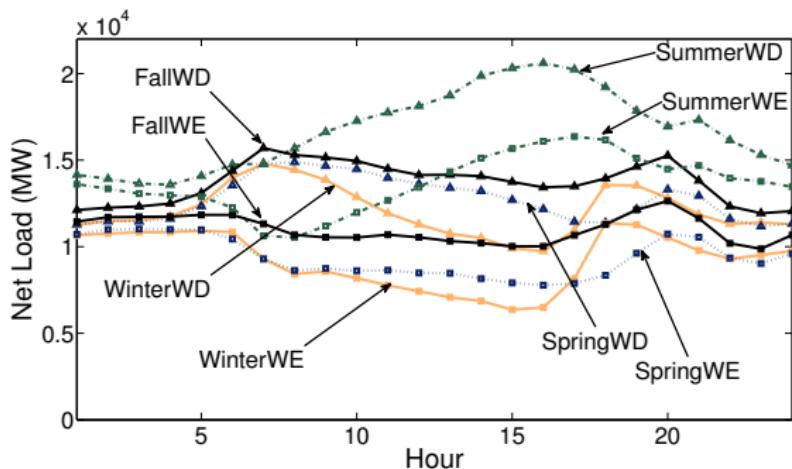
- 124 units (82 fast, 42 slow)
- 53665 MW power plant capacity
- 225 buses
- 375 transmission lines
- 42 scenarios
- Four studies
 - Deep (14% energy integration) without transmission constraints, contingencies
 - With transmission constraints, contingencies:
 - No wind
 - Moderate (7.1% energy integration, 2012)
 - Deep (14% energy integration, 2020)

Competing Reserve Rules

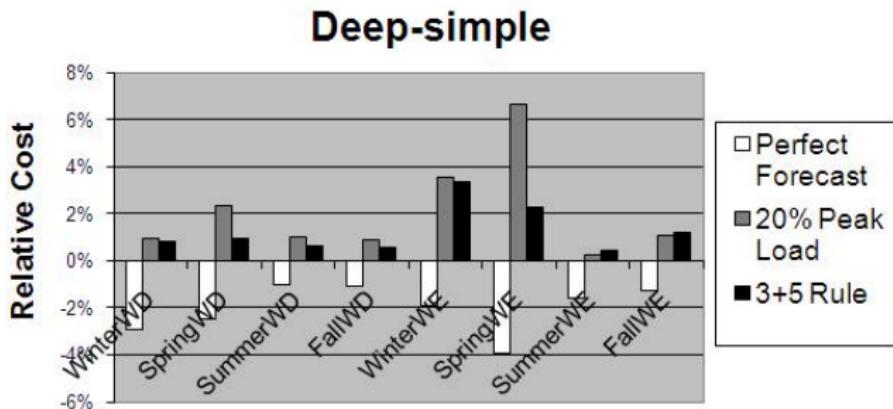
- Perfect foresight: anticipates outcomes in advance
- Percent-Of-Peak-Load rule: commit total reserve T_{req} at least $x\%$ of peak load, $F_{\text{req}} = 0.5 T_{\text{req}}$
- 3+5 rule: commit fast reserve F_{req} at least 3% of hourly forecast load plus 5% of hourly forecast wind, $T_{\text{req}} = 2F_{\text{req}}$

Day Types

- 8 day types considered, one for each season, one for weekdays/weekends
- Day types weighted according to frequency of occurrence

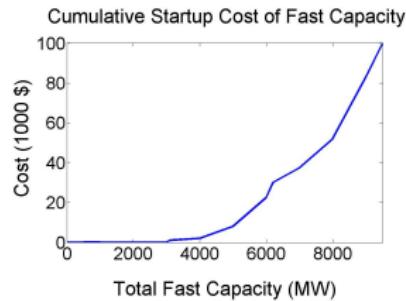
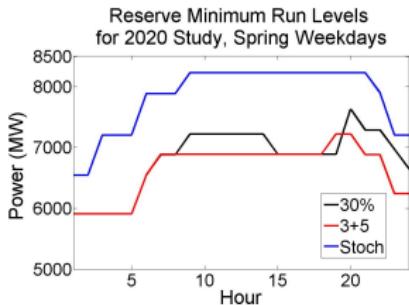


Policy Comparison - Deep Integration, No Transmission, No Contingencies

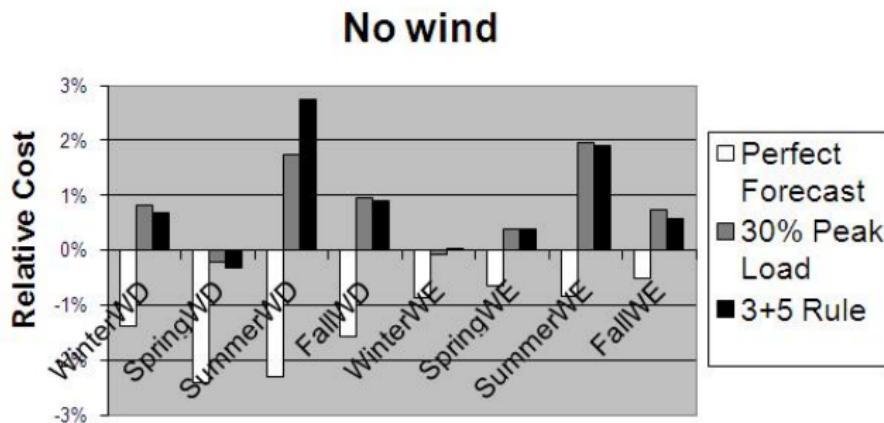


Explanation of SUC Superior Performance

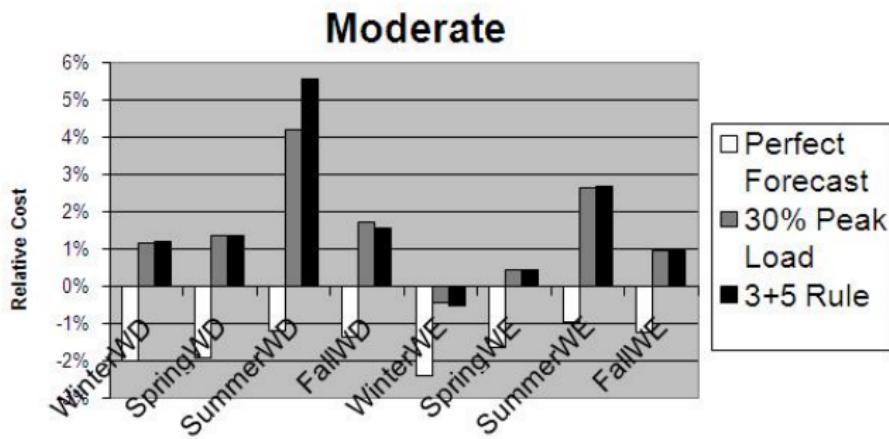
- When reserve constraints are binding, deterministic policy overcommits.
- When reserve constraints are not binding, deterministic policy underestimates value of protecting against adverse wind outcomes.



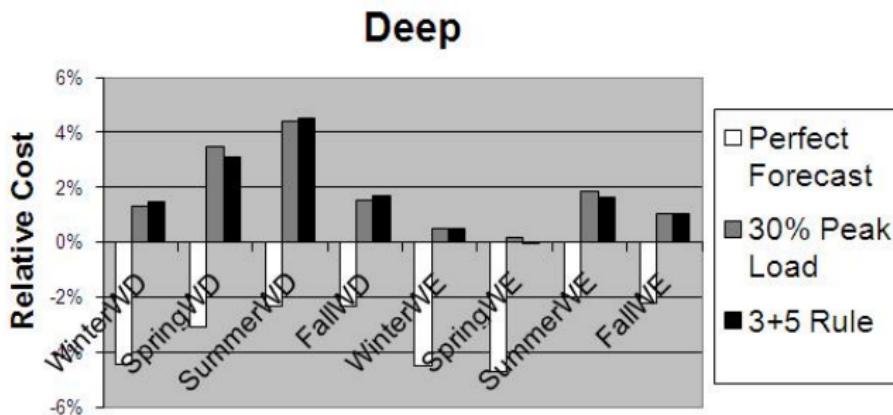
Policy Comparison - No Wind Integration



Policy Comparison - Moderate Integration



Policy Comparison - Deep Integration



Summary

	Deep-S	No Wind	Moderate	Deep
RE daily waste (MWh)	100	0	890	2,186
Cost (\$M)	5.012	11.508	9.363	7.481
Capacity (MW)	20,744	26,377	26,068	26,068
Daily savings (\$)	38,628	104,321	198,199	188,735
Forecast gains (%)	32.4	35.4	41.9	46.7

Demand Response Results

	Daily Cost (\$)	Daily Load Shed (MWh)
No wind	9,012,031	17.301
Centralized Moderate	8,677,857	1.705
Bids Moderate	211,010	609.914
Coupled Moderate	265,128	2.217
Centralized Deep	8,419,322	10.231
Bids Deep	578,909	1221.492
Coupled Deep	705,497	112.452

Running Times

- CPLEX 11.0.0
- DELL Poweredge 1850 servers (Intel Xeon 3.4 GHz, 1GB RAM)
- $(P_1), (P_{2s})$ run for 120 iterations, (ED_s) run for last 40 iterations
- Average running time of 43776 seconds on single machine
- Average MIP gap of 1.39%

Conclusions

- **Consistent performance of scenario selection:**
Stochastic unit commitment policy yields 32.4% - 46.7% of potential benefits of perfect foresight over various types of uncertainty
- **Transmission constraints and contingencies strongly influence results - need for advanced optimization**
 - Overestimation of capacity credit from 1.2% of installed wind capacity to 39.8% for deep integration
 - Underestimation of daily operating costs from 7.481 \$M to 5.102 \$M for deep integration
- **First steps towards integrating deferrable demand models with renewable supply uncertainty:** Deferrable demand imposes no additional capacity requirements, coupling results in 3.06% - 8.38% operating cost increase

Perspectives

- Parallelization in high performance computing facility of Lawrence Livermore National Labs
 - Sensitivity of optimal solution and objective function value on number of selected scenarios
 - **Scoping study** for industry
- Expansion of model for transmission / generation expansion planning
- Further refinement of demand response model

References

- ① A. Papavasiliou, S. S. Oren, R. P. O'Neill, *Reserve Requirements for Wind Power Integration: A Stochastic Programming Framework*, accepted at IEEE Transactions on Power Systems.
- ② A. Papavasiliou, S. S. Oren, *Multi-Area Stochastic Unit Commitment for High Wind Penetration in a Transmission Constrained Network*, submitted to Operations Research, 2011 ENRE INFORMS student travel scholarship runner-up.
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Thank you

Questions?

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Demand Response Study

	Zero	Moderate	Deep
Wind capacity (MW)	0	6,688	14,143
DR capacity (MW)	0	5,000	10,000
Daily wind energy (MWh)	0	46,485	95,414
Daily DR energy (MWh)	0	40,000	80,000
DR/firm energy (%)	0	6.1	12.2

Centralized Load Dispatch

- Stochastic unit commitment with additional constraint:

$$\sum_{t=1}^N p_{gst} = R$$

- Assumptions of centralized load control:

- Central co-optimization of generation and demand (computationally prohibitive)
 - Perfect monitoring and control of demand

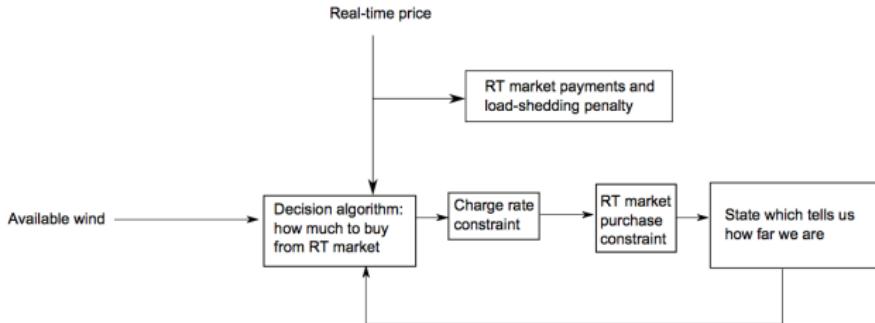
- Centralized load control represents an idealization that can be used for:

- Quantifying the cost of decentralizing demand response
 - Estimating the capacity savings of deferrable demand

Demand Bids

- Based on retail consumer model of (Borenstein and Holland, 2005), (Joskow and Tirole, 2005), (Joskow and Tirole, 2006)
- State contingent demand functions used in economic dispatch $D_t(\lambda_t; \omega) = a_t(\omega) - \alpha b \lambda^R - (1 - \alpha) b \lambda_t$
- Note that the demand function model has to:
 - Be comparable to the deferrable demand model in terms of total demand R
 - Be consistent with the observed inflexible demand in the system

Coupling



$$\min_{\mu_t(x_t)} \mathbb{E} \left[\sum_{t=1}^{N-1} \lambda_t (\mu_t(x_t) - s_t)^+ \Delta t + \rho r_N \right]$$

$$\mu_t(x) \leq C, (\mu_t(x) - s_t)^+ \leq M_t, r_{t+1} = r_t - u_t$$

Integrating Demand Response in Stochastic Unit Commitment

