

A Stochastic Unit Commitment Model for Integrating Renewable Supply and Demand Response

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Abstract—We present a stochastic unit commitment model for assessing the reserve requirements resulting from the large-scale integration of renewable energy sources and deferrable demand in power systems. We present three alternative demand response paradigms for assessing the benefits of demand flexibility in absorbing the uncertainty and variability associated with renewable supply: centralized co-optimization of generation and demand by the system operator, demand bids and coupling renewable resources with deferrable loads. We present simulation results for a model of the Western Interconnection.

I. INTRODUCTION

This paper focuses on assessing the benefits of demand-side flexibility on absorbing the variability and uncertainty of renewable supply. In the paper we consider three fundamental approaches for modeling flexible demand. At the fully centralized end, we consider the case where the system operator centrally co-optimizes the dispatch of demand-side resources, renewable supplies and generators. This is unrealistic in practice as the system operator operates the system at a bulk scale and cannot enforce control on the system down to the retail level. In addition, the optimization problem at hand is too complex to solve. Nevertheless, this ideal model provides a benchmark for the potential benefits of demand flexibility. Sioshansi [1] considers this model in a deterministic setting. We extend this approach to account for the uncertainty introduced by renewable energy supply. A fully decentralized approach for coordinating demand response that we also consider in this paper is to establish real-time pricing at the retail level. This possibility was introduced by Schweppe et al. [2] and is discussed by Borenstein et al. [3]. The common approach of reasoning about real-time pricing in the power system economics literature is the use of decremental demand bids. Sioshansi and Short [4] use this approach in the context of a unit commitment model. Borenstein and Holland [5] and Joskow and Tirole [6], [7] also use this approach for analyzing retail pricing. However, there is strong institutional opposition to this approach as it exposes retail consumers to the volatility of electricity prices. In addition, real-time prices

often fail to convey the economic value of demand response due to the non-convex operating costs of system operations. This effect has been reported by Sioshansi [4]. Moreover, demand-side bidding fails to capture the cross-elasticity of deferrable demand over time. An intermediate approach for integrating demand response that we consider in this paper is coupling the operations of renewable resources with deferrable demand. The motivation of coupling renewable generation with deferrable demand is to create a net resource that appears “behind the meter” as a virtual power plant from the point of view of the system operator.

In order to accurately assess the impacts of renewable energy and demand response integration on power system operations it is necessary to represent the balancing operations of the remaining grid by using a unit commitment model of the daily scheduling and dispatch procedure performed by the system operator. In this paper, a stochastic formulation of the unit commitment model is used in order to quantify the level of reserves that are required in order to integrate renewable resources reliably and the contribution of demand response in mitigating these requirements. The fact that a unit commitment model can accurately represent the balancing operations of the system has resulted in numerous renewable integration studies based on unit commitment, including Ruiz et al. [8], Sioshansi and Short [4], Wang et al. [9], Contantinescu et al. [10], Tuohy et al. [11], Morales et al. [12], Bouffard et al. [13], Papavasiliou et al. [14] and Papavasiliou and Oren [15].

Despite the fact that stochastic unit commitment is appropriate for quantifying the impacts of renewable energy and demand response integration, the model introduces challenges in terms of representing uncertainty and solving the resulting large-scale mixed integer linear program. Dupacova et al. [16] pioneered scenario selection and scenario reduction algorithms motivated by stability results on the optimal values of stochastic programs with respect to perturbations in probability measures. Faster variants of these algorithms were presented by Heitsch et al. [17] and their effectiveness in the stochastic unit commitment problem was demonstrated by Gröwe-Kuska

et al. [18]. As a result of this work, this class of algorithms was subsequently adopted in the stochastic unit commitment literature. Among the wind integration studies referenced above, the algorithms of Heitsch et al. [17] are used by Tuohy et al. [11] and Morales et al. [12]. Although these algorithms can be applied in a straightforward fashion for the case of renewable integration studies without transmission constraints, they may underestimate the severity of certain scenarios [14]. Furthermore, it is not clear how they can be used for selecting and weighing scenarios of multi-area renewable production multiplexed with composite element (generator and transmission line) outages [15]. In order to address these challenges, Papavasiliou and Oren [15] propose a scenario selection and weighing algorithm inspired by important sampling that is also used in this paper. According to the proposed algorithm, scenarios are selected according to their effect on the expected cost and weighed such that their selection does not bias the objective function of the stochastic unit commitment formulation. In order to address the size of the resulting large-scale mixed integer linear program, the decomposable structure of the problem can be exploited. Decomposition algorithms based on Lagrangian relaxation for the stochastic unit commitment problem were pioneered by Takriti et al. [19]. Alternative relaxations were subsequently presented by Carpentier et al. [20] and Nowak and Römisich [21]. Shiina and Birge [22] presented an alternative decomposition approach for solving the stochastic unit commitment problem using column decomposition. In Papavasiliou et al. [14] the authors present a dual decomposition algorithm for solving the problem that is also used in this paper.

The remaining paper is organized as follows. In Section II we provide an overview of the components of our model. In Section III we describe in detail the demand flexibility models that we consider in our analysis. Results from a test case of the Western Interconnection are presented in Section IV. In Section V we discuss the conclusions of our work.

II. MODEL OVERVIEW

The modeling approach adopted in this paper follows a two-stage stochastic formulation proposed by Ruiz et al. [23]. Generators are partitioned in a set of slow resources that need to be committed in the day-ahead time frame and a set of fast generators that can be committed and dispatched in real time, after uncertainty in the system has been revealed. In Fig. 1 we present a diagram for integrating demand response models with the unit commitment and real-time dispatch model. The *decision support* module is a stochastic unit commitment model that determines the day-ahead unit commitment schedule of slow generation resources, while accounting for the randomness of renewable supply and firm (inflexible) demand. Given the day-ahead schedule determined by the stochastic unit commitment model, we evaluate the performance of various demand response strategies in the economic dispatch phase, represented by the *evaluation* module.

A. Statistical Models

The analysis is driven by uncertainty in renewable supply, firm (inflexible) demand, and real-time prices. We use a second order autoregressive time series model for representing wind speed. A static power curve is used for converting wind speed to wind power production. Our calibration and simulation methodology follows Brown et al. [24], Torres et al. [25] and Callaway [26]. The calibration and simulation procedure follows the steps outlined in Papavasiliou and Oren [15]. The fit of the wind model to the available data is presented in Papavasiliou and Oren [15]. Firm demand is also modeled as a second order autoregressive process, assume to be independent of renewable production.

B. Stochastic Unit Commitment

In order to determine the day-ahead reserves that are committed by the system operator in order to accommodate the simultaneous integration of renewable supply and deferrable demand, we formulate a unit commitment model that assumes that the system operator co-optimizes the dispatch of flexible loads and generation resources. The model follows the formulation in Papavasiliou et al. [14]. An integral constraint can be introduced to the stochastic unit commitment model that represents the need to supply a total of R units of energy to deferrable loads within the horizon T :

$$\sum_{t \in T} e_{st} = R, s \in S, \quad (1)$$

where S is the set of scenarios and e_{st} is the amount of energy supplied to deferrable loads in scenario s , period t .

III. DEMAND FLEXIBILITY

As we discuss in Section I, we consider three fundamental approaches for modeling demand flexibility. In a fully decentralized approach, price-responsive loads bid valuations and demand flexibility is introduced in the objective function of the problem. Decision variables for such price-responsive loads are denoted by d_{lt} . In a fully centralized approach, demand flexibility can be accounted for explicitly by the system operator and is introduced in the problem through constraints rather than through the objective function. In this case the system operator controls loads directly through decision variable e_t while respecting their operating constraints. Coupling represents an intermediate approach where deferrable loads coordinate their consumption with renewable suppliers in order to appear “behind the meter” from the point of view of the system operator.

A. Centralized Load Control

In the centralized load control approach we assume that the system operator co-optimizes the dispatch of flexible loads and generation resources. The formulation of the centralized load control model is obtained from the economic dispatch model by enforcing $d_{lt} = 0$ for all loads l and periods t . The net demand D_{st} in the market clearing constraint represents the difference of random firm demand and random renewable supply.

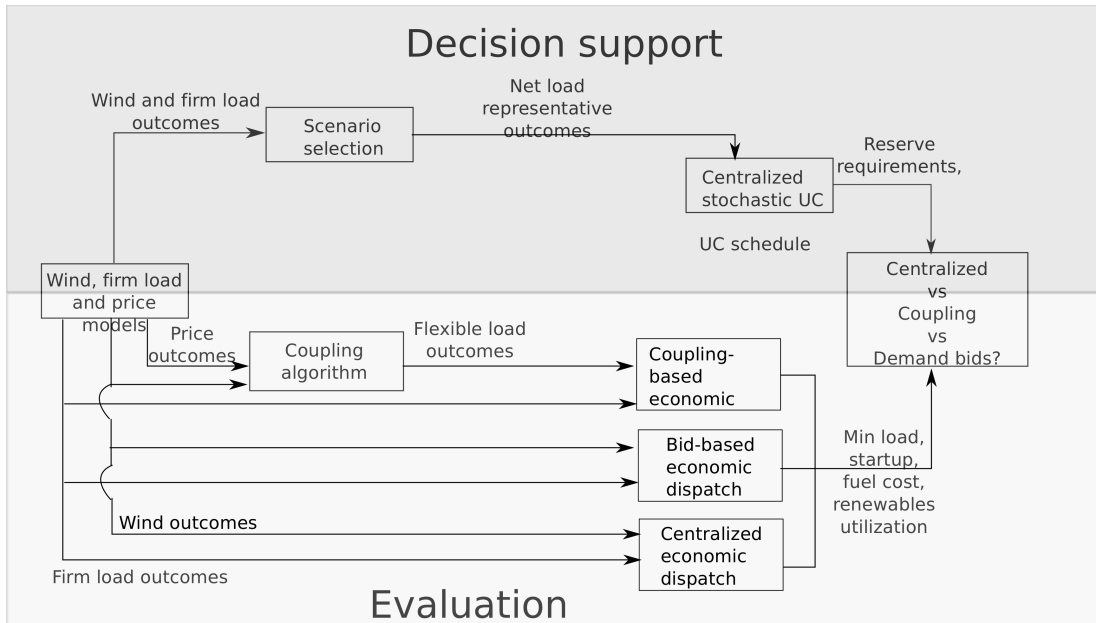


Fig. 1. Overview of the model.

B. Demand Bids

The demand model that we present in this section is based on Borenstein and Holland [5] and Joskow and Tirole [6], [7]. We assume a linear demand function that consists of a fraction α of inflexible consumers who face a fixed retail price λ^R , and a fraction $1 - \alpha$ of price-responsive consumers who face the real-time price of electricity λ_t . The demand function $D_t(\cdot)$ for each period can therefore be expressed as:

$$D_t(\lambda_t; \omega) = a_t(\omega) - \alpha b \lambda^R - (1 - \alpha) b \lambda_t, \quad (2)$$

where ω represents an element of the sample space that determines the realized inflexible demand, $a_t(\omega)$ is the intercept and b is the slope of the demand function. Note that we assume a common slope for all time periods and a time-varying stochastic intercept that depends on the realization of inflexible demand. The calibration of the demand functions is described in detail in Papavasiliou [27]. The resulting economic dispatch model is obtained by enforcing $e_t = 0$ in the economic dispatch model. Again, net demand in the market clearing constraint represents the difference of firm demand and renewable supply.

C. Coupling

In this model an aggregator contractually owns the output from a large group of renewable generation assets. The aggregator enters into a contractual agreement to supply deferrable loads. Loads specify their energy demand in the form of requests for a certain amount of energy over a fixed time window. The aggregator can control the loads directly and uses renewable power from its assets as the primary energy source for satisfying deferrable demand. In the case of renewable supply shortage, the aggregator can resort (to a limited extent)

to the real-time market for procuring power at the prevailing real-time price. The aggregator compensates deferrable loads at a rate ρ for each unit of unserved energy. Any excess renewable power is supplied to the system. The setup is similar to dynamic scheduling [28], whereby demand and supply resources from different control areas pair their schedules in order to produce a zero net output to the remaining system. Such scheduling is currently implemented in the ERCOT market. The model is described in detail in Papavasiliou and Oren [29].

IV. RESULTS

A. Preliminaries

We present results for a model of the Western Electricity Coordinating Council (WECC), also used in other studies [30], [14], [15]. The model consists of 124 generators and we do not account for transmission constraints. Details about the system can be found in Papavasiliou et al. [14]. We consider three wind/demand response integration studies that are summarized in Table I. For each level of wind integration, we assume a demand response integration level that is approximately one-for-one in terms of energy demand and capacity. We assume that deferrable requests span over 24 hours. We consider 6 levels of power supply for the control problem. The penalty of unserved energy is $\rho = 5000$ \$/MWh. We use 12 scenarios for the formulation of the stochastic unit commitment model. The wind data that is used for the calibration of the statistical models is based on the National Renewable Energy Laboratory (NREL) 2006 Western Wind and Solar Integration Study [31]. The moderate and deep wind integration studies correspond to the 2012 and 2020 wind integration targets of California. Further details about the wind production data can

TABLE I
KEY PARAMETERS OF THE DEMAND RESPONSE CASE STUDY.

	No Wind	Moderate	Deep
Wind capacity (MW)	0	6688	14143
DR Capacity C (MW)	0	5000	10000
Daily wind energy (MWh)	0	46485	95414
Daily DR energy R (MWh)	0	40000	80000
Flexible/firm demand (%)	0	6.1	12.2

be found in Papavasiliou and Oren [15]. In order to reduce the computational requirements of the model we focus on eight representative day types instead of simulating an entire year of operations for the system.

B. Costs, Load Loss, Capacity Requirements and Spillage

In table II we present the operating costs and daily load losses for the case with no wind and no demand response in the system. The operating costs do not include the cost of lost load. Note that for the average demand of the system under consideration, the 1-day-in-10-years reliability criterion requires daily load shed of no more than 179 MWh. This can be used as a benchmark against which we can compare the extent to which each demand response mechanism is acceptable from a reliability perspective.

In Tables III, V we present the daily operating cost of each policy for the moderate and deep integration cases respectively. The column with bold figures, that corresponds to centralized load dispatch by the system operator, contains absolute cost values. Cost figures corresponding to the other policies are relative to the centralized operating costs. The row with total costs weighs the cost of each day type with its relative frequency in the year in order to yield annual results. The last row shows the relative performance of centralized control with respect to the other policies, normalized by the cost of centralized control. Note that the operating costs of price-based demand response outperform those of coupling. This can be attributed to the diversification effect of including flexible demand in the market. The “cost of anarchy” that results from using price signals in order to control load response, rather than centralized control, ranges from 2.43% - 6.88% for the case of demand-side bidding and 3.06% - 8.38% in the case of coupling. Although demand bids result in lower operating costs, demand-side bidding results in load shedding that is 3.4 times greater than the 1-day-in-10-years criterion for the moderate integration case and 6.8 times greater for the deep integration case. Coupling results in the operation of the system within reliability limits as we note in Tables IV, VI.

In Table VII we present a breakdown of operating costs by type for each of the policies that we consider for each integration level. We note that price response and coupling result in cost increases in all cost categories. As Sioshansi [1] argues, the marginal cost signal itself does not necessarily induce efficient load response due to the fact that it fails to capture the non-convex operating costs of the system. The observation of Sioshansi is also supported by our results.

In Table VIII we present the amount of capacity that is committed by each policy as well as the amount of renew-

TABLE II
DAILY COST OF OPERATIONS AND LOAD SHEDDING FOR EACH DAY TYPE FOR THE DEMAND RESPONSE STUDY - NO WIND.

	Daily Cost (\$)	Shed (MWh)
WinterWD	7,390,206	0.001
SpringWD	7,145,737	4.317
SummerWD	13,684,880	30.869
FallWD	9,589,506	0
WinterWE	6,079,003	0.001
SpringWE	5,855,883	0
SummerWE	11,839,573	0
FallWE	7,868,146	154.285
Total	9,012,031	17.301

TABLE III
DAILY COST OF OPERATIONS FOR EACH DAY TYPE FOR THE DEMAND RESPONSE STUDY - MODERATE INTEGRATION.

	Cost (\$)	Δ Cost (\$)	Δ Cost (\$)
	Centralized	Coupled	Decoupled
WinterWD	7,320,620	256,740	300,051
SpringWD	6,408,355	172,006	139,589
SummerWD	13,625,136	155,096	219,124
FallWD	9,640,017	316,089	157,159
WinterWE	5,890,755	300,701	246,408
SpringWE	3,637,240	707,223	244,353
SummerWE	11,739,177	176,230	234,101
FallWE	7,735,502	277,817	189,465
Total	8,677,857	265,128	211,010
relative (%)		3.06	2.43

TABLE IV
DAILY LOAD LOSS FOR EACH DAY TYPE FOR THE DEMAND RESPONSE STUDY - MODERATE INTEGRATION.

	Shed (MWh)	Shed (MWh)	Shed (MWh)
	Centralized	Coupled	Decoupled
WinterWD	0	0	177.257
SpringWD	1.532	1.869	701.828
SummerWD	3.617	4.346	821.719
FallWD	1.661	1.661	799.323
WinterWE	0	0	642.105
SpringWE	0	0.249	453.791
SummerWE	0.059	1.100	215.816
FallWE	6.792	10.005	976.766
Total	1.705	2.217	609.914

TABLE V
DAILY COST OF OPERATIONS FOR EACH DAY TYPE FOR THE DEMAND RESPONSE STUDY - DEEP INTEGRATION.

	Cost (\$)	Δ Cost (\$)	Δ Cost (\$)
	Centralized	Coupled	Decoupled
WinterWD	6,656,665	633,164	556,775
SpringWD	5,692,860	978,182	572,465
SummerWD	13,661,862	505,869	835,609
FallWD	9,321,281	772,659	404,523
WinterWE	5,220,109	711,882	616,931
SpringWE	4,251,600	910,253	576,010
SummerWE	12,136,223	329,929	472,930
FallWE	7,930,823	700,205	515,431
Total	8,419,322	705,497	578,909
relative (%)		8.38	6.88

able supply spillage. Capacity requirements do not change significantly for each integration study, which suggests that the additional deferrable demand can be fully absorbed by the installed renewable capacity. Wind spillage is negligible across all cases.

TABLE VI
DAILY LOAD LOSS FOR EACH DAY TYPE FOR THE DEMAND RESPONSE
STUDY - DEEP INTEGRATION.

	Shed (MWh) Centralized	Shed (MWh) Coupled	Shed (MWh) Decoupled
WinterWD	0.001	8.290	552.769
SpringWD	0	351.782	1382.459
SummerWD	0.001	36.643	1952.332
FallWD	33.660	143.629	1210.443
WinterWE	0	0	929.960
SpringWE	0	32.601	1008.222
SummerWE	2.081	58.725	1157.565
FallWE	57.005	132.134	1260.137
Total	10.231	112.452	1221.492

TABLE VII
BREAKDOWN OF DAILY OPERATING COSTS FOR EACH DEMAND RESPONSE
POLICY FOR EACH INTEGRATION LEVEL (\$).

	Min load	Fuel	Startup	Total
No wind	1,382,156	7,549,491	80,384	9,098,537
Centralized Moderate	1,246,552	7,364,815	66,489	8,677,857
Bids Moderate	1,317,383	7,471,363	100,123	8,888,866
Coupled Moderate	1,330,130	7,532,898	79,958	8,942,958
Centralized Deep	1,194,606	7,174,611	50,105	8,419,322
Bids Deep	1,360,543	7,494,472	143,217	8,998,232
Coupled Deep	1,432,948	7,592,595	99,276	9,124,819

TABLE VIII
CAPACITY REQUIREMENTS AND WIND POWER SPILLAGE FOR EACH
DEMAND RESPONSE POLICY.

	Capacity (MW)	Spillage (MWh)
No wind	26,123	N/A
Moderate	26,254	0
Deep	26,789	2

V. CONCLUSIONS

In this paper we present a stochastic unit commitment model that accounts for renewable energy and demand response integration. We compare three demand response paradigms: centralized load dispatch, demand-side bidding and coupling. We analyze the case of no wind in the network, as well as cases of wind integration that correspond to the 2012 and 2020 wind integration targets of California, with a corresponding one-for-one increase in flexible demand. Our analysis is performed on a model of the Western Electricity Coordinating Council that consists of 124 generators. We find that the “cost of anarchy” incurred by decentralizing demand response ranges between 3.06% - 8.38% for the case of coupling. Price response results in a cost increase ranging between 2.43% - 6.88% of the cost corresponding to centralized load dispatch. However, price response fails to capture the cross-elasticity of demand across time periods, resulting in excessive lost load. For the case studies that we consider, the additional integration of deferrable demand imposes no additional capacity requirements to the system.

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