Market-Based Control Mechanisms
for Electric Power Demand Response

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Abstract—We propose a settlement mechanism for optimally scheduling real time electricity consumption which is suitable for an automated demand response control system. Our proposed settlement mechanism, supply function bidding, is interpreted as a Newton algorithm for optimization problems with decomposable structure, and it is shown to satisfy the second fundamental theorem of welfare economics for the case of affine supply function bids. We simulate the behavior of our proposed control mechanism for the case of demand response via room temperature control, and we demonstrate how a suboptimal control policy can have adverse impacts both in terms of system performance but also in terms of economic incentives.

I. Introduction

The reliable operation of power systems requires a continual balancing of electricity supply and demand. As system operators around the United States are integrating renewable energy sources at an unprecedented scale, this balancing task is becoming more challenging than ever. In contrast to traditional practices of the past where supply resources were scheduled to track the inelastic demand of electricity consumers, in future power grids demand side resources are expected to share an active role in balancing the grid and to adjust their energy consumption to the unpredictable and volatile energy supply of renewable resources. The necessary technological transformation of power systems which can enable this new paradigm is underway, with significant expansions in the measurement, control and communications infrastructure of existing systems.

As demand side resources become active participants in modern power systems, the development of decentralized, market-based mechanisms which will enable a fair allocation of supply resources is necessitated. In this paper we propose a market based mechanism for scheduling short term energy consumption which is suited for the provision of balancing services at an aggregate level. Ancillary services, the balancing services which are provided to system operators for ensuring reliable system operations at the sub-hourly and minute-by-minute time frame, include regulation, load following, spinning and non-spinning reserve as well as replacement reserve. These services have been traditionally provided by flexible generators which are capable of adjusting their output rapidly in response to unanticipated imbalances between supply and
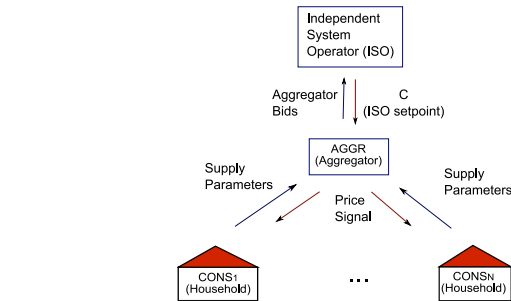

Fig. 1.   Demand response aggregation scenario.

demand. In modern power systems, the provision of these services from aggregations of electricity consumers who can adjust their electricity demand is becoming an attractive alternative [1].

A. System Design

The system which we consider is shown in Fig. 1. An aggregator who coordinates the electricity consumption of a large group of flexible electricity consumers commits to deliver a certain capacity, c, of ancillary services to the system operator by bidding in the ancillary services market. This entitles the system operator to broadcast set-points of power consumption within a band of c units of power for the duration of the provision of the ancillary services. The aggregator commits to adjust the aggregate energy consumption within its service area such that the difference between a baseline energy consumption level and actual aggregate consumption in the service area closely follows the set-points of the system operator. The objective of the aggregator is to allocate energy rationing responsibilities among individual consumers such that aggregate disutility over the entire horizon is minimized.

Due to the decentralized nature of the planning problem, we propose a market-based control mechanism for allocating power to individual consumers whereby the aggregator uses a supply function auction mechanism to price demand response. In particular, prior to the demand response event the aggregator broadcasts a price signal, λ, for each interval of the event. Consumers respond by bidding a parameter for

each time interval which provides a compact description of their supply function at the neighborhood of the broadcast price. The process iterates until price converges, in advance of the demand response event. Loads then commit to dispatch according to their bids for the entire duration of the event, and the aggregator rewards loads according to the converged market clearing prices.

The process which we propose is entirely analogous to the function of day-ahead electricity markets for the determination of generation dispatch schedules [2], with the aggregator reproducing the function of an auctioneer, the system operator set-points representing inelastic demand and loads acting as suppliers of negawatts[1]. An instantiation of the proposed system would require a control center at the aggregator site which communicates to processors embedded in load switches at consumer locations. These processors can sustain a model of consumer utility, and can be calibrated by users according to their demand response experience.

The justification for using a market-based control mechanism for implementing demand response is decentralization. In our proposed system, a large resource allocation problem is decomposed to individual subproblems which simplifies the solution of the problem. Arriving at an optimal consumer schedule rapidly is essential since ancillary services are often cleared at frequent time intervals in real time markets, leaving little time for the determination of an optimal schedule. Moreover, in our choice of design each participating load retains privacy of its personal information and only has access to the information which is required for the clearance of the auction, as opposed to other market-based control architectures [3] in which information flows between nodes of the network. In contrast to supply function bidding auctions in day-ahead electricity markets, we use parameter bids to describe supply functions at the neighborhood of the prevailing price rather than entire supply function bids. Since demand response may involve large numbers of participating loads, this design aims at minimizing the amount of information that flows between the aggregator and consumers at any given time in order to prevent congestion in communication channels.

We will denote $U_i(x)$ as the disutility that loads incur by deviating from their baseline energy consumption, where x denotes the amount of energy rationing, or equivalently the amount of negawatts which are supplied. Consumer supply functions are denoted by $x^\star(\lambda)$ and map price, λ, to the optimal supply $x^\star$. That is, $x_i^\star(\lambda)$ solves

$$\min_{x_i} U_i(x_i) - \lambda x_i. \tag{1}$$

Supply functions are approximated by parameterized functions, $x_i(p_i, \lambda)$, which map a parameter bid p_i and the market price, λ, to a quantity of supply, x_i. For example, in affine bids agent i bids a pair of parameters $p_i = (\alpha_i, \beta_i)$ which respectively define the intercept and slope of a linear approximation of the true supply function. Based on the bids of all

suppliers, the aggregator sets the market clearing price λ so as to equate supply with demand.

B. Literature review

There is a substantial literature on market based control architectures, how these relate to decomposition methods in optimization and applications of these control architectures in complex systems. Decomposition methods are general methods for efficiently solving classes of problems with a decomposable structure, whereby a problem is cast in the form of a master problem and a group of small subproblems which can be solved more efficiently than the original problem. Boyd [4] classifies decomposition methods in primal and dual methods. In primal decomposition methods, primal variables are used for communication between the master and subproblems, as opposed to dual methods where Lagrange multipliers are used for this purpose. Our work falls in the latter category, though other authors have used primal methods for similar applications as we discuss below.

Algorithms often admit an economic interpretation, which in turn suggests mechanisms for organizing complex systems such that a goal is attained at minimal cost. Scarf [5] draws such an analogy for the case of the simplex method, where simplex iterations are interpreted as the iterated search for a set of commodity prices which equilibrate an economic system by providing zero profts to agents who participate in the market, and negative profts for all other agents.The economic interpretation of decomposition algorithms arises by identifying the primal problem as the task of an auctioneer who strives to maximize social benefit, and the subproblems as the profit maximization tasks of self-interested agents. This is emphasized in the work by Kelly et al. [6], who apply both primal and dual decomposition methods for the problem of optimizing the allocation of network bandwidth to routes in capacity constrained communication networks. In their work, they partition the original problem to a class of subproblems in which agents choose payments such that they maximize personal benefits, and a problem of a network administrator who chooses link prices such that a specific objective function, proportional fairness, is maximized. We similarly decompose our original optimization problem to the subproblems of profit maximizing loads and the master problem of the aggregator, who seeks to clear the market while minimizing payments to loads.

Once a problem has been decomposed into its component subproblems, a specific choice of decomposition algorithm for solving the problem may suggest an economic mechanism, or bargaining game, which can be used for arriving to the optimal solution, with each iteration of the algorithm corresponding to a round of bargaining. A standard example is the interpretation of the subgradient algorithm as a tatonnement process. In our paper we derive an interpretation of the Newton algorithm as a supply function bidding process. This interpretation is then used for the design of the control mechanism which we propose.

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[1]The term negawatts refers to energy rationing, the demand-side equivalent of megawatts, the power supplied by generators.

The superior convergence speed of second order algorithms such as the Newton method suggests that this may be a preferable approach for organizing a system which needs to be scheduled under tight time constraints. Therefore, our motivation for proposing supply function bidding for demand response contrasts to the original game theoretic motivation for supply function bidding as a generalization of Cournot and Bertrand competition. Klemperer and Meyer [7] argue how supply function bidding can increase the flexibility of agent strategies and allow them to respond well in the presence of uncertain demand in oligopolistic settings, whereas our motivation for proposing supply function bidding in this paper is computational.

The fact that decomposition algorithms converge suggests that establishing a market and letting agents work towards the maximization of their personal benefit results in an optimal organization of the entire system, which is essentially the conclusion of the second fundamental theorem of welfare economics. In the paper we provide an equilibrium proof for our proposed mechanism, which is inspired by the paper of Johari and Tsitsiklis [8] who prove the second fundamental theorem of welfare economics for a specific type of supply function bids.

Newton algorithms such as the one which we present have been employed in the literature for various applications. Ygge and Ackermanns [9] also consider a decomposition scheme for demand response, and Kurose and Simha [3] consider a decomposition scheme for optimal sharing of files in an interconnected computer system. Although these papers implement decomposition algorithms, they do not propose specific market mechanisms. Closest to our work is that of Galiana [10], who presents a dual decomposition algorithm for the commitment of electric power generation units in the day ahead electricity market. Galiana applies the Newton algorithm to the dual problem of the unit commitment problem, he provides an interpretation of dual decomposition as a market mechanism for unit commitment in a day ahead market consisting of profit maximizing generation units, profit maximizing distribution companies and a transmission operator who seeks to maximize merchandising surplus, and he presents an equilibrium proof. However, unlike our proof which relies on information which becomes available during the bargaining iterations, his proof requires that the transmission operator has access to historical data of the transmission system. The accuracy of this data is questionable and can undermine the efficiency of the resulting equilibrium.

II. ANALYSIS

A. Consumer, aggregator and system optimization

We are seeking to optimally allocate energy rationing responsibilities to participating loads in a demand response system, such that total disutility is minimized while meeting the specified set-points of the system operator. The problem can be formulated as follows:

$$(\text{SYST}) \quad \begin{aligned} \min_{x_1 \ldots x_{N_C}} \quad & \sum_{i=1}^{N_C} U_i(x_i) \\ \text{s.t.} \quad & \sum_{i=1}^{N_C} x_i = c. \end{aligned} \quad (2)$$

By taking the Lagrangian of (SYST), the problem decomposes to the profit maximization problem of individual agents who seek to optimally trade off revenues with costs,

$$(\text{CONS-}i) \quad \min_{p_i} \quad U_i\big(x_i(p_i, \lambda)\big) - \lambda' x_i(p_i, \lambda), \quad (3)$$

and an expenditure minimization problem for the aggregator who seeks to achieve the desired aggregate response at minimum payment,

$$(\text{AGGR}) \quad \begin{aligned} \min_{\lambda} \quad & c'\lambda \\ \text{s.t.} \quad & \sum_{i=1}^{N_C} x_i(p_i, \lambda) = c. \end{aligned} \quad (4)$$

Note that in a multi-period problem, c, λ and x_i can represent time-varying vectors. The cost U_i can then capture the total cost over the entire demand response horizon.

B. Equilibrium

We now prove the second theorem of welfare economics for supply function bidding, under certain assumptions on the cost functions of agents. We define a competitive equilibrium as a combination of prices λ^* and parameters p^* such that the market clears, no agent has an incentive to deviate and the aggregator minimizes payments:

$$
\begin{aligned}
&(\lambda, p_i) \in \arg\max\{\lambda' x_i(p_i, \lambda) - U_i(x_i(p_i, \lambda))\} \quad \forall i \\
&\lambda^* \in \arg\min c'\lambda^* \\
&\sum_{i=1}^{N_C} x_i(p_i^*, \lambda^*) = c
\end{aligned} \quad (5)
$$

We will consider the case of affine supply function bids, $x_i(p_i, \lambda) = a_i + B_i \lambda$.

Proposition 1: Suppose U_i, $i = 1, ..., N_C$, are continuously differentiable, coercive[2], convex functions. Then there exists a pair λ^*, p^* of equilibrium prices and supply function parameters.

   Proof:
The Lagrangian of the (SYST) problem is

$$L(x, \lambda) = \sum_{i=1}^{N_C} U_i(x_i) + \lambda'\left(c - \sum_{i=1}^{N_C} x_i\right). \quad (6)$$

The feasible region defined by the market clearing constraint of (SYST) is a closed set and the objective function is coercive, therefore the Weierstrass theorem applies and an optimal solution x^* exists. Moreover, by the convexity of U_i

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[2]A function f is coercive if $\lim_{|x| \to \infty} f(x) = \infty$.

proposition 5.2.1 of [11] applies and therefore there exists a geometric multiplier λ^* such that

$$x^* \in \arg\min_x L(x, \lambda^*).$$

The Lagrangian decomposes into terms $U_i(x_i) - \lambda^{*'}x_i$ therefore $x_i^* \in \arg\max\{\lambda^{*'}x_i - U_i(x_i)\}$ where x_i^* is the component of x^* corresponding to agent i. Therefore, an equilibrium exists because we can always choose α_i^* and B_i^* such that $x_i^* = \alpha_i^* + B_i^*\lambda^*$.

The optimization problem of the aggregator is

$$\begin{aligned} \min_\lambda \quad & c'\lambda \\ \text{s.t.} \quad & \sum_{i=1}^{N_C} \alpha_i^* + \sum_{i=1}^{N_C} B_i^*\lambda = c \end{aligned} \qquad (7)$$

This problem has a feasible solution, namely λ^*. In order to guarantee that this is also an optimal solution it is sufficient that $\sum_i B_i^*$ be nonsingular. If this is indeed the case then we have found our equilibrium pair of prices and parameters. If this is not the case then we can perturb $B_i^{**} = B_i^* + \epsilon$, where ϵ is an appropriately chosen matrix, such that $\sum_i B_i^* + \epsilon$ becomes non-singular and we can then set $\alpha_i^{**} = \alpha_i^* + B_i^*\lambda^* - B_i^{**}\lambda^*$. ∎

We next provide a standard economic interpretation of the subgradient method as a tatonemment process, as well as an intepretation of the Newton algorithm as a supply function bidding auction. These interpretations suggest market mechanisms for attaining the efficient equilibrium which was shown to exist in this section.

C. Tatonnement and the subgradient method

We now apply the steepest descent algorithm on the dual function of the problem (SYST) and obtain the familiar mechanism of Walrasian tatonnement. The dual function of (SYST) is

$$q(\lambda) = \sum_{i=1}^{N_C} \min_{x_i} \{U_i(x_i) - \lambda x_i\} + \lambda c. \qquad (8)$$

Denote $x_i^*(\lambda)$ as the optimal response of consumer i to price λ. Then $c - \sum_{i=1}^{N_C} x_i^*(\lambda)$ is a subgradient of the dual function at point λ. The dual variable is updated according to:

$$\lambda^{k+1} = \lambda^k + \gamma^k (c - \sum_{i=1}^{N_C} x_i^\star(\lambda^k)), \qquad (9)$$

where $\gamma^k > 0$ is the iteration step-size at step k. This iteration can be interpreted as follows: at stage k of the bargaining process the aggregator posts a price. Consumers respond with the quantity that they would be willing to supply at that price. In the next iteration $k + 1$ the aggregator adjusts the price in order to more closely approximate the supply-demand equality constraint. If there is excess supply, $c - \sum_{i=1}^{N_C} x_i^\star(\lambda) < 0$, then price decreases, whereas if there is inadequate supply, $c - \sum_{i=1}^{N_C} x_i^\star(\lambda) > 0$, price increases.

D. Supply function bidding and the Newton method

We now apply the Newton method to the Lagrangian function of (SYST) and we obtain an affine supply function bidding mechanism. We assume that the cost functions $U_i(x)$ are strictly convex. We denote by $x_i^*(\lambda)$ the optimal response to price λ and $\lambda^*(x_i)$ as its inverse. The existence of $\lambda^*(x)$ is guaranteed by the first order conditions for optimality: $\lambda - \nabla U_i(x) = 0$. The existence of its inverse in an open neighborhood about the optimal solution (λ^*, x^*) is guaranteed by the strict convexity of the cost functions, which ensures that the implicit function theorem applies for the first order optimality conditions $\lambda - \nabla U_i(x) = 0$.

The Newton iterations solve the following system:

$$\nabla^2 L(x^k, \lambda^k)(\Delta x^k, \Delta \lambda^k) = -\nabla L(x^k, \lambda^k), \qquad (10)$$

where $L(x, \lambda)$ is the Lagrangian of (SYST), given in equation 6.

Proposition 2: Suppose $x_i^*(\lambda)$ are concave functions. Then $(\lambda^{k+1}, x_i^{k+1})$ lies on the hyperplane $H_i(\lambda, x)$ which supports $x_i^*(\lambda)$ at the point $(\lambda^*(x_i^k), x_i^k)$.

Proof: The Newton algorithm applied to the Lagrangian yields

$$\nabla^2 U_i(x_i^k)(x_i^{k+1} - x_i^k) - (\lambda^{k+1} - \lambda^k) = -(\nabla U_i(x_i^k) - \lambda^k). \qquad (11)$$

By the implicit function theorem we have

$$\begin{aligned} \nabla x_i^*(\lambda^*(x_i^k)) &= \nabla^2 U_i(x_i^*(\lambda^*(x_i^k)))^{-1} \\ &= \nabla^2 U_i(x_i^k)^{-1} \end{aligned} \qquad (12)$$

since $x_i^*(\lambda^*(x_i^k)) = x_i^k$. Therefore, the hypograph of $x_i^*(\lambda)$ is supported at $\lambda^*(x_i^k)$ by the vector $(-\nabla x_i^*(\lambda^*(x_i^k)), 1) = (-\nabla^2 U_i(x_i^k)^{-1}, 1)$. Therefore we have the following expression for $H_i(\lambda, x)$:

$$H_i(\lambda, x) = (-\nabla^2 U_i(x_i^k)^{-1}, 1) \begin{pmatrix} \lambda - \nabla U_i(x_i^k) \\ x - x_i^k \end{pmatrix} \qquad (13)$$

By substituting λ^{k+1} from equation 11 in the latter equation we get $H_i(\lambda^{k+1}, x^{k+1}) = 0$ and the result follows. ∎

Proposition 2 suggests a supply function bidding mechanism for arriving to the economic equilibrium of the system. In particular, the proposition reassures us that if loads announce an affine supply function which supports the hypograph of the true supply function at $(\lambda^k, x_i^*(\lambda^k))$, and if the auctioneer clears the market as if loads were to adhere to their affine bids, then this is equivalent to applying the Newton algorithm on the Lagrangian of (SYST), and this mechanism is therefore guaranteed to converge to the optimal solution. The hyperplane which supports the hypograph of the true supply function at $(\lambda^k, x_i^*(\lambda^k))$ is determined by the following affine supply function bid:

$$\begin{aligned}
x_i^*(\alpha_i^{k+1}, B_i^{k+1}, \lambda) &= \alpha_i^{k+1} + B_i^{k+1}\lambda, \text{ where} \\
\alpha_i^{k+1} &= x^k - \nabla^2 U_i(x^k)^{-1} \nabla U_i(x_i^k), \\
B_i^{k+1} &= \nabla^2 U_i(x_i^k)^{-1},
\end{aligned} \tag{14}$$

and the price update which clears the market is given by the following equation:

$$\lambda^{k+1} = (\sum_{i=1}^{N_C} B_i^{k+1})^{-1}(c - \sum_{i=1}^{N_C} \alpha_i^{k+1}). \tag{15}$$

A graphical explanation of the algorithm which we described is presented in Fig. 2. Note that the information which is communicated to the aggregator can equivalently be expressed in terms of the first and second order information of the agent cost functions $\nabla U_i(x_i^k)$, $\nabla^2 U_i(x_i^k)$. Moreover, note that this is a truly decentralized mechanism, since each agent has access to its individual information, and this decomposition is not evident from applying the standard Newton iterations of Eq. 10. Also note that both the aggregator and the consumers have, at iteration k, all the necessary information which is required to update their announcements at the following iteration. Moreover, from the equilibrium propositions which we proved above we conclude that all market agents are incented to implement this iterative search mechanism, since it is guaranteed to converge to the competitive equilibrium (λ^*, x^*). Moreover, this equilibrium solution is efficient as it also solves (SYST).

This mechanism represents a second order analogue of the Walrasian auction derived from the interpretation of the subgradient method. In both cases the auctioneer talks first by announcing a price. Agents respond to the price by optimizing their quantity response to the prevailing price. In the Walrasian tatonnement process this response consists of a single piece of information, a zeroth order approximation of $x^*(\lambda)$ which is essentially the most accurate way for consumers to describe their supply function in the neighborhood of the prevailing quantity, whereas in the supply function bidding case the response is the most accurate first order approximation of the supply function in the neighborhood of the prevailing quantity. In both auctions the auctioneer then updates the price signal by guessing the agents' supply functions in the neighborhood of the existing iteration given their announced bids. This bargaining process repeats until the sequence of broadcast prices converges. Once the price converges the market clears and loads are bound to supply the promised response.

III. SIMULATIONS

We present an application of our methodology for the case of air conditioning load response. We describe the model of customer utilities, and we derive our algorithm explicitly for this multi-period model. We compare the convergence speed of the tatonnement and supply function bidding mechanisms, and we present a greedy mechanism which results in adverse economic incentives.
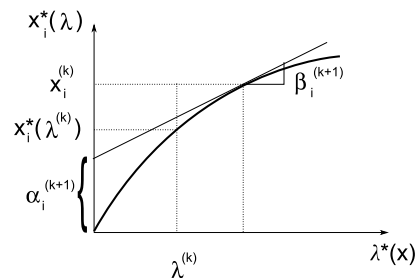

Fig. 2. Graphical interpretation of supply function bidding mechanism.

A. Model description

In order to describe the utility function U_i of each customer, we first define a linearized thermodynamic model for the temperature response of their homes:

$$T_{i,t+1} = T_{i,t} + a_i(T_{\text{out}} - T_{i,t}) + b_i(x_{i,t} - E_{i,\text{b}}), \tag{16}$$

where $a_i > 0$ and $b_i > 0$ are coefficients of the thermodynamic model, T_{out} is the outside temperature, and $E_{i,\text{b}}$ is the baseline power consumption of home i, i.e. the level of power consumption which would keep room temperature at the preferred level T_i^{des}. The temperature dynamics imply that room temperature increases in proportion to the difference between room temperature and outside temperature, and temperature decreases in proportion to the power which is consumed for air conditioning.

The baseline power consumption $E_{i,\text{b}}$ can be expressed as a function of the model parameters. In this case we would have $T_{i,t+1} = T_{i,t} = T_i^{\text{des}}$, and $x_{i,t} = 0$. From (16) we obtain the following expression:

$$E_{i,\text{b}} = \frac{a_i}{b_i}(T_{\text{out}} - T_i^{\text{des}}). \tag{17}$$

Substituting back to equation (16) we obtain the following recursive expression for temperature dynamics:

$$T_{i,t+1} = (1 - a_i)T_{i,t} + a_i T_i^{\text{des}} + b_i x_{i,t}. \tag{18}$$

We iterate Eq. (18) and solve for the temperatures $T_{i,t}$ as a function of $x_{i,0}...x_{i,t-1}$:

$$\begin{aligned}
T_{i,t+1} &= (1 - a_i)^{t+1}T_{i,0} + \sum_{j=0}^{t} a_i(1 - a_i)^j T_i^{\text{des}} \\
&+ \sum_{j=0}^{t} b_i(1 - a_i)^{t-j} x_{i,j}.
\end{aligned} \tag{19}$$

Utility functions describe the comfort of each customer in terms of a numeraire good which is assumed to be money. Equivalently these functions can describe the discomfort of each customer and can be interpreted as cost functions which we seek to minimize. Here we assume that the utility functions are convex functions which penalize deviations from the desired temperature T_i^{des}, and that customer discomfort is additive in time. The level of discomfort for each period is:

$$\tilde{U}_{i,t}(T_{i,t}) = c_i |T_{i,t} - T_i^{\text{des}}|^{\rho_i} \tag{20}$$

where $c_i > 0$ and $\rho_i > 0$ are parameters of the disutility function. Using Eq. (19), we can express the utility

functions in terms of $x_{i,0} \ldots x_{i,t-1}$: $U_{i,t}(x_{i,0} \ldots x_{i,t-1}) = \check{U}_{i,t}\big(T_{i,t}(x_{i,1} \ldots x_{i,t-1})\big)$. The utility functions are strictly convex for $\rho > 1$.

We use a time step of one minute for the simulations. We consider a system of 5 users who track a set-point of 1 kW for a horizon of 120 minutes. In order to generate sample values T_i^{des} (K) for the preferred temperature of each user, we sample a $\mathcal{N}(73, 1)$ distribution. Similarly, we generate T_0 (K) from a $\mathcal{N}(73, 1)$ distribution, a_i (min^{-1}) from a $\mathcal{N}(0.05, 0.01)$ distribution and b_i (K(kW min)$^{-1}$) from a $\mathcal{N}(0.5, 0.01)$ distribution. We have chosen the parameter values of the thermodynamic model so that room temperature increases by one degree if air conditioning stays off for one minute with an outside temperature difference of 20 degrees, and so that running an AC in the room for one minute can bring the temperature down by one degree. We use a common exponent ρ for the utility function of each user, and the coefficient c_i of the utility function is chosen so that changing temperature by 5 degrees towards a favorable value for the duration of one hour has an incremental value of \$6.

B. Algorithm description

We clarify how the algorithms of sections II-C, II-D apply for this multi-period example and we motivate certain issues that arise in the multi-period case. Since the algorithms which we describe are executed prior to the initiation of the demand response event, the aggregator must provide a complete vector $c = (c_t)$, $t = \{0, ..., N_T - 1\}$, of set-points, where N_T is the horizon of the event. In this paper we assume that the aggregator has advance knowledge of these set-points or can estimate them accurately, however in future work we wish to explore the possibility for adapting our settlement mechanisms to a real-time setting where the set-points c_t are first revealed and the aggregator then runs an auction.

For the tatonnement mechanism, at iteration k the aggregator announces a vector of prices $\lambda = (\lambda_t)$, where λ_t is the reward for period t. Each participating load i then determines its optimal response response vector $x_i^* = (x_{i,t}^*)$ by solving:

$$\min_{x_i} U_{i,0}(T_{i,0}) + \sum_{t=0}^{N_T-1} \left\{ U_{i,t+1}\big(x_{i,0} \ldots x_{i,t}\big) - \lambda_t\, x_{i,t} \right\}. \quad (21)$$

The aggregator then updates the price for each period, according to:

$$\lambda_t^{k+1} = \lambda_t^k + \gamma^k \big(c_t - \sum_{i=1}^{N_C} x_{i,0}^\star(\lambda)\big) \quad (22)$$

where γ^k is the step size at iteration k, and the process iterates until price converges.

The supply function bidding mechanism requires the gradient and Hessian of consumer utility functions at iteration k, where

$$\nabla U_i(x_i^k) = \begin{bmatrix} \sum_{t=0}^{N_T-1} \dfrac{\partial U_{i,t+1}(x_{i,0}^k \ldots x_{i,t}^k)}{\partial x_{i,0}} \\ \vdots \\ \dfrac{\partial U_{i,N_T}(x_{i,0}^k \ldots x_{i,N_T-1}^k)}{\partial x_{i,N_T-1}} \end{bmatrix} \quad (23)$$

is an $N_T \times 1$ vector with $x_i^k = a_i^k + B_i^k \lambda^k$, and

$$\nabla^2 U_i(x_i^k) =$$
$$\begin{bmatrix} \sum_{t=0}^{N_T-1} \dfrac{\partial^2 U_{i,t+1}(x_{i,0}^k \ldots x_{i,t}^k)}{\partial x_{i,0}^2} & \cdots & \dfrac{\partial^2 U_{i,N_T}(x_{i,0}^k \ldots x_{i,N_T-1}^k)}{\partial x_{i,0}\partial x_{i,N_T-1}} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial^2 U_{i,N_T}(x_{i,0}^k \ldots x_{i,N_T-1}^k)}{\partial x_{i,N_T-1}\partial x_{i,0}} & \cdots & \dfrac{\partial^2 U_{i,N_T}(x_{i,0}^k \ldots x_{i,N_T-1}^k)}{\partial x_{i,N_T-1}^2} \end{bmatrix} \quad (24)$$

is an $N_T \times N_T$ matrix. The supply function parameters and price are then updated according to Eqs. 14 and 15 respectively.

The implementation of the tatonnement mechanism would require that the problem of Eq. 21 be solved at the load location, which places a significant computational requirement for the processors at the nodes of the network. On the other hand, only N_T parameters are communicated to the aggregator at each iteration. In contrast, the supply function bidding iterations of Eq. 14 require simpler calculations at the nodes of the network, however the amount of information which is communicated to the aggregator at each iteration is substantially higher: N_T parameters for a_i and $N_T \times N_T$ parameters for B_i. This is the inevitable consequence of the fact that the Newton method requires more detailed information about the supply function of the users.

C. Dynamics of demand response

We now describe how the system evolves in a demand response event for the choice of parameter values which we described in section III-A. The first frame of Fig. 3 shows the deviation of temperatures from preferred levels. The system enters a steady state, in which each customer is subject to a specific deviation from preferred temperature. It is easy to show that for each setpoint c announced by the system operator consumers respond, in steady state, with a unique combination of temperature deviations from their preferred setpoints. On average, users deviate 0.53 to 1.33 degrees from their preferred temperatures. Due to the market clearing equality constraint, the system cannot enter its steady state immediately. As a result, temperatures follow an optimal transient trajectory and gradually merge to their steady state values.

The second frame shows the amount of energy rationing for each consumer. The level of rationing varies between customers, with stringent users as well as users with worse thermodynamic characteristics in their homes undertaking a smaller responsibility. In the first period there is an apparent impulse in the applied control, as users rush to enter their optimal temperature trajectories, shown in the previous figure.
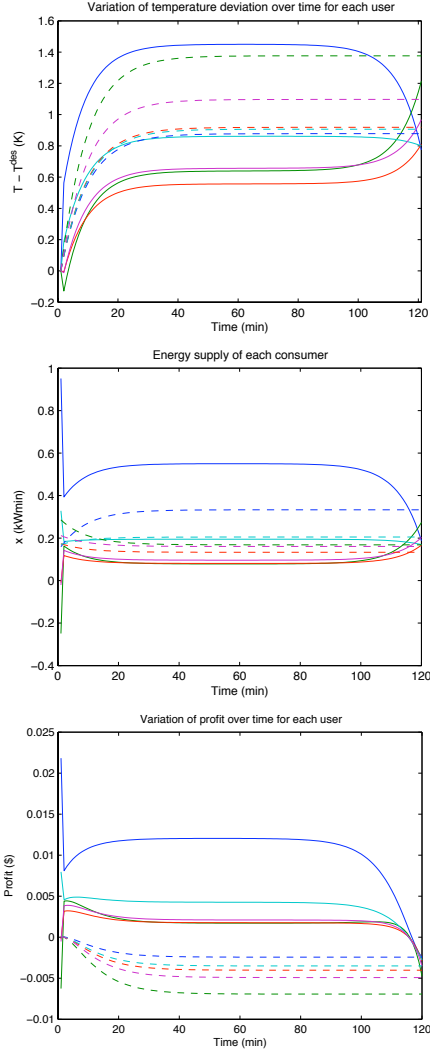
Fig. 3. Demand response dynamics ($\rho = 2$).

The third frame shows the profitability of each consumer. Consumers make average profits ranging from \$0.0018 to \$0.0105 per minute. Profits become negative towards the end of the event due to the fact that there are few periods left in the horizon and users are not accounting for future discomfort due to temperature deviations so they are providing supply curves which are tolerant, thus driving the market price down. This could be prevented by imposing terminal conditions on the room temperatures. Terminal conditions could also prevent the well-known rebound effect of demand response. The total revenue of all consumers for demand response in this example amounts to \$4.78.

D. Greedy mechanism

As we described in section III-B, the iterative equations require an advance knowledge of the time varying set-point c. In addition, the supply function mechanism places excessive requirements on communication channels for large horizons, since the amount of information which is communicated is
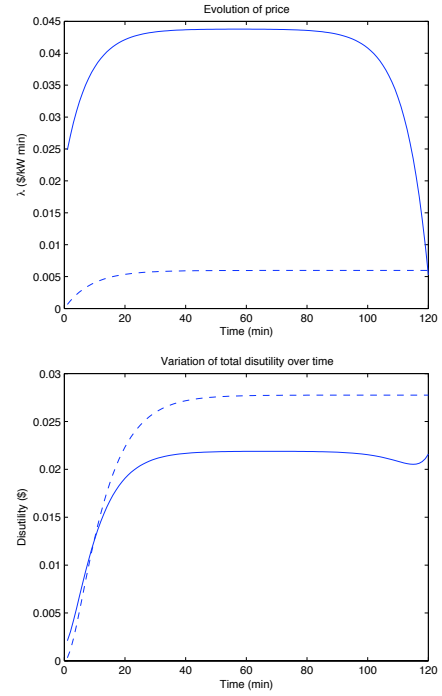

Fig. 4. Price and utility dynamics ($\rho = 2$).

$\mathcal{O}(N_T^2)$. In this section we consider an alternative, heuristic iteration, where loads solve their problem for a single period lookahead. i.e. they implement a greedy algorithm without accounting for the impact of present period actions on future disutility. Although this approach does not require a-priori knowledge of set-points and reduces the amount of data which is communicated, the greedy mechanism is expected to lead to suboptimal performance. In addition, we demonstrate that this approach has an adverse impact on the economic profits of loads by removing their incentive to participate in the demand response event.

The results of the greedy mechanism are overlaid on those of the optimal solution in Fig. 3 with broken lines. In the first and second frame we observe that the greedy mechanism also enters a steady state, however, the temperature and control trajectories differ markedly from those of the optimal solution.

The third frame of Fig. 3 shows that consumers receive negative profits when they implement the greedy mechanism. This can be attributed to low market clearing prices, as we can see in the first frame of Fig. 4. Although we calculated that the aggregator paid \$4.78 to loads in the optimal response, the aggregator pays merely \$0.67 when the greedy mechanism is implemented. The low market clearing prices result from the greedy bidding behavior of the market participants. Consumers provide bids which do not account for the impact of their present actions on their future comfort, therefore their bids underestimate the true cost of demand response. Consequently, the resulting market clearing prices are not sufficient to compensate customers for the future discomfort that their present actions impose. From the second frame of Fig. 4 we observe
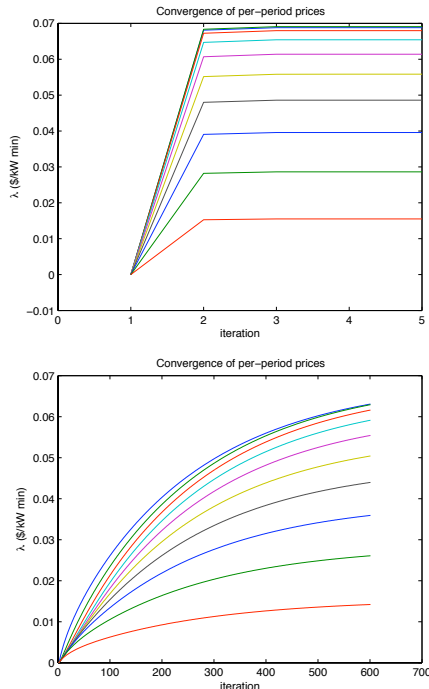
Fig. 5. Convergence results for logarithmic disutility function with $N_T = 10$, $N_C = 10$, $c = 1$ kW.

that the system performs suboptimally, although the greedy mechanism intially outperforms the optimal solution.

E. Covergence speed

In this section we compare the convergence speed of the tatonnement mechanism and the supply function bidding mechanism. We test the two mechanisms for the utility function of Eq. 20 with exponent $\rho = 2.5$. In order to compare the robustness of our comparison, we also test an alternative, logarithmic utility function:

$$\tilde{U}_{i,t}(T_{i,t}) = -\log(T_{\max} - T_{i,t}) - \log(T_{i,t} - T_{\min}), \quad (25)$$

where $T_{\min} = 63, T_{\max} = 83$. The convergence of the two mechanisms is shown in figure for the logarithmic utility function. The results for the exponential utility function are nearly identical. In both cases the Newton based method converges in three steps, whereas the subgradient method, implemented with a constant step size, requires a few hundred steps before approaching the optimal solution. Moreover, the subgradient method requires manual tuning of the step size. The Newton method is clearly preferable in terms of computation speed. The tradeoff, as we discussed in section III-B, is the additional burden which is placed on communication channels, as well as the fact that it requires a more detailed knowledge of consumer utility functions.

In order to motivate the applicability of our method in a real-world setting we present the solution times of the proposed mechanisms in table I for large-scale systems for a horizon

	$N_C = 100$	$N_C = 1000$	$N_C = 10000$
Tatonnement	115.90	1087.3	12897
Supply function	1.43	29.57	1645.7

of 60 minutes. We are using the logarithmic utility function, with 600 iterations for the tatonnement mechanism, and 5 iterations for the supply function mechanism. The computational requirements of both mechanisms are $\mathcal{O}(N_T^3)$ for parallel operation. The results of table I do not account for the parallel nature of the system, and are therefore $\mathcal{O}(N_C N_T^3)$, and also do not account for communicaiton delays, which especially impact the tatonnement mechanism. Communication delays and differences in computation times among local processors motivate future work on whether the proposed algorithms converge in an asynchronous setting.

IV. CONCLUSION

We propose a market-based control mechanism for implementing demand response, which relies on affine supply function bidding. We show that the mechanism results in an economic equilibrium, and we interpret the mechanism as a dual decomposition algorithm. We compare our mechanism to a tatonnement mechanism and explain the tradeoffs in terms of computational requirements, communication requirements and convergence speed. We apply our methodology for the case of air conditioning demand response, and we point out that heuristic mechanisms may have adverse impacts on both system performance as well as economic incentives.

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