A Mixed Integer Second Order Cone Program for Transmission-Distribution System Co-Optimization

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***Abstract*—Integrating renewable energy in the electricity mix raises several challenges for transmission and distribution system operators. One main challenge relates to the provision of balancing services from distribution system resources, which requires that the constraints of the distribution network be carefully accounted for when deciding on the dispatch of distribution system resources. In this paper, we present a mixed-integer second order cone program for solving a real-time dispatch problem where transmission and distribution are modeled in an integrated fashion. The model offers promising perspectives when tested on real instances of the Italian, Danish and Spanish systems.**

I. Introduction

The integration of renewable energy sources presents novel challenges which will need to be tackled in order to achieve the ambitious and immediate renewable energy integration goals that have been set by policy makers worldwide. One major challenge relates to the fact that distribution networks (DN), with increasing amounts of distributed renewable resources, are no longer only consuming power generated by the transmission network (TN), but also producing power through distributed renewable resources (such as solar panels), and also hosting direct and indirect storage (e.g. in the form of deferrable demand or electric vehicles). According to the present paradigm, which is rapidly changing, one system operator is in charge of the high-voltage transmission network, the Transmission System Operator (TSO), whereas the distribution network typically absorbs the production. Distribution system loads are aggregated at a transmission bus and the optimization of power generation is performed at the transmission level.

With the integration of distributed renewable energy sources and distributed sources of flexibility, there is an increasing need for accounting for the low and medium-voltage distribution network in short-term operations. By ignoring the short-term operating constraints of distribution networks, which has been the predominant paradigm to date, the system is limited in the extent to which it can absorb distributed renewable energy connected to the distribution network. Because of the uncertain nature of renewable energy sources and flexibility mechanisms like demand-side management, the Distribution System Operator (DSO) will need to assume a more active role in operating the network. In the coming years, the objective will be to maintain the existing quality of service in the supply of power while utilizing renewable resources to the greatest possible extent [1].

This paradigm shift challenges the conventional approach towards interfacing TSO and DSO operations, and requires increased coordination in order to operate the system efficiently and securely. TSO-DSO coordination is currently receiving an increased amount of attention by practitioners and the research community [2], [3], [4]. Rather than examining coordination schemes for harmonizing TSO-DSO operations, this paper will consider a centralized version of transmission and distribution network operations. Indeed, even if it could be unrealistic to consider one operator handling the complete transmission and distribution network, this would provide a benchmark for examining the extent to which alternative TSO-DSO coordination schemes can be compared.

The model that we investigate is inspired by the EU SmartNet project on transmission and distribution operational coordination [5]. The data used in the case study is also based on SmartNet. Following the problem posed in SmartNet, we are not investigating the commitment of reserves in a coordinated fashion, but rather the activation of reserves for the purpose of balancing, assuming that these reserves have already been committed. The broader problem of committing reserves is treated by Ntakou et al. [6] and falls out of the scope of this paper.

We are specifically interested in the activation of reserves in the real-time market, where the dispatch decisions can be seen as adjustments on a predefined dispatch that results from earlier processes. In real time, the goal is to deploy reserves that have been cleared in a previous reserve capacity market (day-ahead for example) and to make sure that renewable energy resources are balanced properly.

Optimizing dispatch decisions in real time while accounting for the complete transmission and distribution network is challenging for various reasons. The distribution network requires a more precise representation of non-linearities than the transmission network and the power flow equations cannot be as simplified as is common practice for transmission networks. The simplification that is performed in transmission networks can be justified by the minor role of losses, the reduced significance of reactive power flows, the less significant role of voltage constraints, and a number of other technical factors.

In distribution networks, these approximations are no longer acceptable and a more precise representation of the power flow equations is needed [6].

In order to account for the non-linearity of power flows, in this paper, we resort to the Second Order Cone (SOC) relaxation introduced by Jabr [7] and used in [8], [9], [10]. In the distribution system markets considered in this paper, the production or consumption bids that are bid into the market can be associated to specific features such as temporal linking, an exclusive choice of bids, a minimum duration for accepted bids, and other non-convex constraints. This will have two effects: (i) it will necessitate the introduction of binary variables in the market clearing problem, and (ii) the temporal linking implies that we will need to consider the problem over a certain time horizon. Concerning the first item, we present in this paper the market clearing model developed by SmartNet [11] for the detailed description of the bid constraints. Regarding the inter-temporal linking, we will make the common assumption employed in various systems that the real-time market is cleared every 15 minutes. We will limit the horizon of the optimization to 1 hour, resulting in a time horizon of at most 4 time steps. We will therefore assume an observable deterministic output from renewable energy sources and distributed loads, since forecasts on very short time frames tend to be quite precise.

The objective of our work is to provide a Mixed Integer Second Order Cone (MISOC) representation of the transmission and distribution real-time dispatch problem and show preliminary results on realistic instances of the Italian, Danish and Spanish networks. The novelty of the work is on (i) the detailed formulation of a coordinated transmission and distribution auction (Sections II and III), (ii) the presentation of results on three case studies of realistic scale, and (iii) the comparison of MISOC against linear approximations of the power flow equations as well as a comparison of CPLEX and Gurobi to solve this kind of problems (Section IV).

We will present the general assumptions that are employed in our paper in section II. We will then develop the real-time dispatch problem in section III. We will present the results on the realistic test cases considered in section IV before concluding in section V.

II. General assumptions

A. Topology of the network

We adopt the following assumptions about the network, following [6]:

- The transmission network includes high-voltage producers (conventional generators), industrial consumers and large renewable energy resources such as utility-scale wind and solar power. The transmission network is assumed to be meshed. We use the direct current approximation of the power flow equations which is a common assumption when considering high-voltage networks.
- The distribution network hosts low-voltage renewable energy sources such as solar panels, flexible direct and indirect storage such as electric vehicles, and residential loads. We assume that the network is radial and use the second order cone relaxation of Jabr for representing power flows in the distribution network. This relaxation is proven to be exact under mild assumptions that we do not satisfy in general in this paper. Nevertheless, experimental evidence and theoretical analysis suggests that the second order cone relaxation provides high quality results for radial networks [9], [12], even if the required assumptions for exactness are not met.

We consider a single transmission network connecting to several distribution networks, in line with the typical layout of a national T&D network. Transmission networks and distribution networks are connected through interface nodes (denoted by N_∞), with one interface node corresponding to each distribution network. Interface nodes are assumed to belong to distribution networks. We assume that aggregations of producers and consumers at each node of the distribution network are represented by a single marginal supply function that represents the marginal cost of reserve activation.

B. Bid structure

The market clearing model presented in this paper is inspired by the products that are available in the Central Western European (CWE) day-ahead energy market. The first unit that we will use in order to define a complete bid is the segment bid. A segment bid (or S-bid) is characterized by a minimum and maximum quantity of real power and a certain marginal cost. Note that we allow consumption and production bids, so we have no assumptions on the signs of the minimum and maximum quantity. A Q-bid links several segment bids. We make explicit the relationships between segment bids when defining bid constraints in section III. We also link Q-bids over time and we refer to such bids as Qt-bids. We assume that a bid is associated with a certain node i, at a certain moment t.

Then, a segment bid (i, t, qt, q, s) is defined by the following 5 fields [11]:

1) a node $i \in N$,
2) a time-step $t \in T$,
3) a Qt field (or Qt-bid) qt,
4) a Q field (or Q-bid) q, and
5) a segment (or S-bid) s.

To this segment bid is associated the Q-bid (i, t, qt, q) which is associated to the Qt-bid (i, t, qt). To make the explanations more concrete, the reader can refer to the example on Fig. 1. Each bid can be rejected, partially accepted or totally accepted by an operator maximizing the welfare (or minimizing the total activation cost) over the transmission and distribution network. Each S-bid sb is associated with a cost $c_{sb}(x_{sb}) = a_{sb}(P_{sb}x_{sb})^2 + b_{sb}P_{sb}x_{sb} + c_{sb}$. Here, x_{sb} is the fraction of acceptance of the bid and P_{sb} is the difference between the maximum and minimum quantity of the bid. We define the notation that we will require for defining the RTDP in the next section.

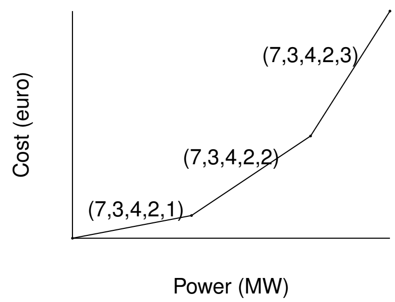

Fig. 1. Example of a bid. $i = 7$ stands for the node, $t = 3$ for the time-step, $qt = 4$ for the Qt-field, $q = 2$ for the Q-field. There are 3 S-bids (7,3,4,2,1), (7,3,4,2,2), (7,3,4,2,3) associated to the Q-bid (7,3,4,2). The Q-bid (7,3,4,2) is part of the Qt-bid (7,3,4).

NOTATION

The following notation is used:

Sets

- $N = (TN) \cup (N_\infty) \cup (\bigcup_{k \in N_\infty} DN_k)$. The network consists of $|N|$ nodes, and each node either belongs to the transmission network (TN), is an interface node, or belongs to the distribution network. The number of interface nodes determines the number of distribution networks, which is why $DN = (\bigcup_{k \in N_\infty} DN_k)$.
- N_∞ represents the set of interface nodes and $|N_\infty|$ designates the number of distribution networks.
- L represents the set of lines of the transmission network linking nodes $(n, m) \in (TN \cup N_\infty)^2$.
- DN also denotes the set of distribution lines. Indeed, we assume that the distribution network is radial, and we associate each node to the line linking it to its ancestor.
- $T = \{1, \ldots, t_{final}\}$ denotes the time horizon of the market clearing problem, with discrete time-steps.
- SB / QB / QtB denotes the set of segment bids, Q-bids, and Qt-bids.
- $ExQB$ / $ExQtB$ denotes the set of exclusive options for Q-bids and Qt-bids. An element $exqb \in ExQB$ / $exqtb \in ExQtB$ is a set of a subsets of QB / QtB in which only one of the Q-bids / Qt-bids can be accepted.
- MDP corresponds to a set of minimum duration pairs. This set determines pairs of Q-bids for which the first one should be activated if the second has been activated.

Parameters

- $\underline{z}$ / $\overline{z}$ denotes the lower/upper bound of a certain variable z (for example, power generation capacity or voltage limits).
- B_l denotes the susceptance of transmission line $l \in L$.
- ΔP_n / ΔQ_n denotes the real/reactive power demand at node $n \in N$.
- R_i / X_i /G_i / B_i denote the resistance / reactance / shunt conductance / shunt susceptance of distribution line $i \in DN$.
- S_l represents the power limit of line $l \in L \cup DN$.
- A_i/C_i denotes the unique ancestor / the children of node $i \in DN$.
- P_{sb} corresponds to the real power quantity of segment bid $sb \in SB$.
- $QP_{i,t}$ represents the limit on apparent power injection.

Variables

- p_n / q_n denote the balancing real / reactive power production at node $n \in N$ (naturally, $q_n = 0$ if $n \in TN \cup N_\infty$).
- θ_n denotes the bus angle of transmission bus $n \in TN \cup N_\infty$.
- f_l / f_i^p / f_i^q denote the flow of power of transmission line $l \in L$ / real / reactive flow of power of distribution line $i \in DN$.
- v_i denotes the voltage magnitude squared at distribution node $i \in DN$.
- l_i denotes the current magnitude squared of distribution line $i \in DN$.
- x_{sb} corresponds to the fraction of quantity activation of segment bid $sb \in SB$.
- s_{sb} denotes the activation of segment bid $sb \in SB$.
- q_{qb} represents the activation of Q-bid $qb \in QB$.
- qt_{qtb} represents the activation of Qt-bid $qtb \in QtB$.
- α_{qb} / ω_{qb} denote the beginning / end of activation of bid $qb \in QB$.
- a_{sb}, b_{sb}, c_{sb} are the parameters of the welfare function of a segment bid $sb \in SB$.

III. THE REAL-TIME DISPATCH PROBLEM

In the following subsections, we describe the coordinated real-time market model. This model includes the transmission network constraints, the distribution network constraints, the interconnection between them, the bid constraints and the objective function. Since the only inter-temporal constraints arise from the bids, we drop the t index on the variables and parameters when modeling the network.

A. Detailed modeling

1) Direct current power flow transmission constraints: We model the transmission network through the B-θ formulation [13].

$$f_l = B_l(\theta_n - \theta_m), \ \forall l = (n, m) \in L \tag{1}$$

$$p_n + \sum_{l=(m,n)} f_l - \sum_{l=(n,m)} f_l = \Delta P_n, \ \forall n \in TN \tag{2}$$

$$-S_l \le f_l \le S_l, \ \forall l \in L \tag{3}$$

$$\underline{p_n} \le p_n \le \overline{p_n}, \ \forall n \in TN \tag{4}$$

(1) is the B-θ representation of flows, (2) accounts for power balance in the transmission network, and (3), (4) correspond to line and generation limits.

2) SOCP power flow distribution constraints: For the distribution network, we use the branch flow model and relax it through a second order cone relaxation [9].

$$p_i + \sum_{j \in C_i} (f_j^p - l_j R_j) - f_i^p + G_i v_i = \Delta P_i, \ \forall i \in DN \tag{5}$$

$$q_i + \sum_{j \in C_i} (f_j^q - l_j X_j) - f_i^q - B_i v_i = \Delta Q_i,\ \forall i \in DN \quad (6)$$

$$v_i - v_{A_i} = 2(R_i f_i^p + X_i f_i^q) - l_i(R_i^2 + X_i^2), \forall i \in DN \quad (7)$$

$$(f_i^p)^2 + (f_i^q)^2 \le S_i^2,\ \forall i \in DN \quad (8)$$

$$(f_i^p - l_i R_i)^2 + (f_i^q - l_i X_i)^2 \le S_i^2,\ \forall i \in DN \quad (9)$$

$$(f_i^p)^2 + (f_i^q)^2 \le v_i l_i,\ \forall i \in DN \quad (10)$$

$$p_i^2 + q_i^2 \le (QP_i)^2,\ \forall i \in DN, \quad (11)$$

$$\underline{p_i} \le p_i \le \overline{p_i},\ i \in DN \quad (12)$$

$$\underline{q_i} \le q_i \le \overline{q_i},\ i \in DN \quad (13)$$

$$0 \le \underline{v_i} \le v_i \le \overline{v_i},\ i \in DN \quad (14)$$

$$l_i \ge 0,\ i \in DN \quad (15)$$

(5), (6) are the real and reactive power balance constraints. (7) shows voltage constraints. Constraints (8), (9) limit the apparent power on distribution lines. (10) is the SOC relaxation of the non convex equality constraint linking flows, current and voltage. (11) represents limits on apparent power injections. (12)-(15) correspond to box constraints on the variables that we introduce in the distribution network.

3) Flow consistency T & D interconnection constraints: The transmission and distribution network interact through the following interconnection constraints:

$$p_k + \sum_{l=(m,k)} f_l - \sum_{l=(k,m)} f_l = \Delta P_k - \sum_{j \in C_k} (f_j^p - l_j R_j),\ \forall k \in N_\infty \quad (16)$$

$$\underline{p_k} \le p_k \le \overline{p_k},\ \forall k \in N_\infty \quad (17)$$

Constraint (16) ensures that the power that is originating from the transmission network flows into the distribution network through an interface node. (17) imposes capacity constraints on the power generation at the interface.

4) Bid constraints: Bids are associated with specific attributes that provide a rich set of options for distributed resources to represent complex operating constraints for their assets. We describe each constraint after presenting it. A detailed description of each constraint is provided in the SmartNet project documentation [11].

$$p_{i,t} = \sum_{sb=(i,t,qt,q,s)} P_{sb} x_{sb},\ \forall i \in N,\ t \in T \quad (18)$$

$$\underline{x}_{sb} s_{sb} \le x_{sb} \le s_{sb} \overline{x}_{sb},\ \forall sb \in SB \quad (19)$$

$$s_{(i,t,qt,q,s)} \le q_{(i,t,qt,q)},\ \forall (i,t,qt,q,s) \in SB \quad (20)$$

$$q_{(i,t,qt,q)} \le qt_{(i,t,qt)},\ \forall (i,t,qt,q) \in QB, (i,t,qt) \in QtB \quad (21)$$

$$qt_{(i,t,qt)} \le \sum_{qb=(i,t,qt,q)} q_{qb},\ \forall (qt,i,t) \in QtB \quad (22)$$

$$q_{(i,t,qt,q)} - q_{(i,t-1,qt,q)} - \alpha_{(i,t,qt,q)} + \omega_{(i,tq,t,q)} = 0,\ \forall (i,t,qt,q), (i,t-1,qt,q) \in QB \quad (23)$$

$$\alpha_{qb} + \omega_{qb} \le 1,\ \forall qb \in QB \quad (24)$$

$$q_{(i,t,qt,q)} \ge \alpha_{(i,\tau,qt,q)}, \forall ((i,t,qt,q),(i,\tau,qt,q)) \in MDP \quad (25)$$

$$\sum_{qb \in exqb} q_{qb} \le 1,\ \forall exqb \in ExQB \quad (26)$$

$$\sum_{qtb \in exqtb} qt_{qtb} \le 1,\ \forall exqtb \in ExQtB \quad (27)$$

$$\underline{RP}_{i,t} \le p_{i,t+1} - p_{i,t} \le \overline{RP}_{i,t},\ \forall i \in N, t \in T - \{t_{final}\} \quad (28)$$

$$0 \le x_{sb} \le 1,\ sb \in SB \quad (29)$$

$$s \in \{0,1\}^{|SB|}, q \in \{0,1\}^{|QB|}, qt \in \{0,1\}^{|QtB|}, \alpha \in \{0,1\}^{|QB|}, \omega \in \{0,1\}^{|QB|} \quad (30)$$

(18) describes how bids impact the net injection of real power. (19) defines the activation of a segment bid. (20) imposes that a segment-bid is activated only if the associated Q-bid is also activated. The same holds with a Q-bid and the associated Qt-bid in (21). (22) ensures that a Qt-bid is activated if at least one of the associated Q-bids is also activated. (23) defines that two consecutive Q-bids of a Qt-bid are linked: the Q-bid at t can only be activated if the one at $t-1$ has also been activated. Constraint (24) imposes the fact that a bid cannot be starting and ending at the same time. (25) is ensuring that if a bid is activated, it remains active for a minimum amount of time. (26), (27) indicate that certain Q-bids or Qt-bids should be activated only if others are not (i.e. an exclusive choice has to be made). (28) are ramp constraints on real power outputs. (29) and (30) denote that x should be fractional as opposed to the other bid-related variables which are binary variables.

5) The complete model: The problem that we wish to solve is to maximize welfare:

$$\max\ f(x) = \sum_{sb \in SB} \big(a_{sb}(P_{sb} x_{sb})^2 + b_{sb} P_{sb} x_{sb} + c_{sb}\big)$$
$$s.t. \qquad (1) - (30)$$

This problem is a large-scale MISOC for networks of realistic size, such as the ones treated in this paper.

B. The approximation considered

Since distribution systems may host thousands to millions of resources [6], it is natural to consider linear approximations in order to achieve scalability in the resulting dispatch optimization problem. We have four conic constraints appearing in the preceding model of the distribution network: (8)-(11). We will consider two types of simplifications:

1) Direct current (DC) approximation in the distribution network [13]. In this case, $R_i = X_i = G_i = B_i = 0, \forall i \in DN$ and we neglect reactive power, current or voltage (the variables $Q_i, f_i^q, l_i, v_i \forall i \in DN$ are no longer present in the problem). In other words, we use the DC approximation of power flow equations in both the TN and the DN.
2) Ben-Tal approximation. This is a linearization of the conic constraints (8)-(11) introduced by Ben-Tal et al. [14] in order to replace them by a set of linear constraints.

   We can cast constraints (8)-(11) in the following format:

$$\sqrt{x^2 + y^2} \le z$$

Note that z should be positive, which is the case for (8)-(11). In Ben-Tal's approximation, this constraint is approximated and replaced by the following set of constraints by introducing ξ and η variables:

$$\xi^0 \geq |x| \tag{31}$$

$$\eta^0 \geq |y| \tag{32}$$

$$\xi^j = \cos(\frac{\pi}{2^{j+1}})\xi^{j-1} + \sin(\frac{\pi}{2^{j+1}})\eta^{j-1}, \quad j = 1, \ldots, \nu \tag{33}$$

$$\nu^j \geq |-\sin(\frac{\pi}{2^{j+1}})\xi^{j-1} + \cos(\frac{\pi}{2^{j+1}})\eta^{j-1}|, \quad j = 1, \ldots, \nu \tag{34}$$

$$\xi^\nu \leq z \tag{35}$$

$$\eta^\nu \leq \tan(\frac{\pi}{2^{\nu+1}})\xi^\nu \tag{36}$$

One conic constraint is then replaced by $2(\nu + 1)$ variables and $2(\nu + 2)$ constraints. With $\nu = 6$, the approximation is proven to guarantee a $O(2e^{-4})$ tightness, meaning that the original conic constraint would be violated at most by $O(2e^{-4})$ [14]. We will keep $\nu = 6$ for the numerical experimentation.

For the remainder of the paper, we will refer to SOCP as the original model, DC when we use the DC approximation on the complete network and Ben-Tal when we approximate the conic constraints of the DN with Ben-Tal's approximation.

IV. Numerical experiment

We perform experiments on the Italian, Danish and Spanish network data provided by SmartNet [11]. We compare the SOC formulation presented in this paper to the linear approximations, DC and Ben-Tal. We perform experiments using two commercial solvers, CPLEX (version 12.8) and Gurobi (version 8.0).

A. The data

We use two different topologies of the Italian network: `693` and `652`; and one of the Danish network (`401`) and the Spanish network (`301`). `693_T00` is a toy example which is derived from the topology of `652` whereas `652_T66`, `401_T00` and `301_T00` are real country-scale instances. Detailed information on the size of each network is presented in TABLE I. In TABLE I, we report the number of transmission nodes, the number of distribution nodes, the number of distribution networks, the number of time-steps, the total number of bids, and the nominal voltage range of the nodes in the TN and in the DN.

B. Comparison of the formulations and the solvers

We test the different instances with different solvers for the different formulations and test cases in TABLE II. Note that it is normal to have negative objective values since we are maximizing the welfare and that most of the bids have a negative marginal cost in terms of welfare.

Concerning the solvers, from TABLE II we observe that CPLEX handles the linearization of the problem better that

TABLE I
OVERVIEW OF THE ITALIAN (`693_T00`, `652_T66`), DANISH (`401_T00`) AND SPANISH (`301_T00`) DATA USED IN THE NUMERICAL EXPERIMENTS.

Test Case	`693_T00`	`652_T66`	`401_T00`	`301_T00`
# Tr Nodes	27	3,648	144	1,537
# Dist Nodes	175	2,410	3,046	2,799
# DNs	4	638	138	464
# Times	4	3	4	4
# S-bids	1,667	26,578	37,926	45,037
TN V range (kV)	15 - 400	2 - 400	10 - 410	1 - 400
DN V range (kV)	15 - 21	15 - 22	10.5 - 22	15 - 22

Gurobi. Gurobi, on the other hand, appears to be more effective on the SOCP formulation of the problem. That being said, by using the appropriate solver, we can solve the problem in the order of magnitude of one second for `693_T00` and dozens of seconds for `652_T66` and hundreds of seconds for `401_T66` and `301_T66` (even less for the DC). Concerning the quality of the resulting solution, we observe that the DC approximation may be far from optimal and tends to underestimate or overestimate the number of bids that should be accepted. The Ben-Tal formulation provides almost as good solutions as SOCP, but still might deviate with respect to the number of bids that should be accepted. Since Ben-Tal is an approximation of SOCP and seems to be harder to handle for the solver, we do not consider it further for solving the problem at hand. We also observe that solving SOCP is not too time consuming. Given these experimental observations, we will only consider SOCP and solve the problem with Gurobi as a solver.

C. Results

The results of our numerical experiments on the test cases with the SOCP formulation are presented in TABLE III. In discussing these results, it is important to keep in mind that dispatchers in real-time operations require updated decisions every 15 minutes. Interestingly, for the three real instances, we manage to obtain a solution in in the time frame of an imbalance interval. Nevertheless, the solution that we obtain is not feasible, in the sense of satisfying the non-linear non-convex power flow equations. Recall that the SOC relaxation arises from constraint (10), where we should have an equality if we want to obtain a physically implementable solution. Even if we cannot measure the distance to a physically implementable solution, we report the SOCP gap in TABLE III (second to last line). The SOCP gap remains reasonably small for the two first cases and the last one. On the contrary, the large SOCP gap for `401_T00` might suggest that the obtained dispatch is far from being feasible. That being said, in view of the short solve time of the MISOC that we are solving in this paper, we can envision using the solution of this model as a warm start for a nonlinear solver or any other method that could provide a physically implementable solution (i.e. a solution satisfying (1)-(30) with (10) being an equality). This extension will be explored in future research.

TABLE II
PERFORMANCE OF THE DIFFERENT SOLVERS AND FORMULATIONS ON THE 4 INSTANCES CONSIDERED. IN THE SECOND COLUMN, THE LINES REPRESENT THE SOLVE TIME, THE OBJECTIVE VALUE, THE PERCENTAGE OF REAL POWER LOST IN THE NETWORK AND THE NUMBER OF ACCEPTED BIDS.

		DC		Ben-Tal		SOCP	
Test Case		Gurobi	CPLEX	Gurobi	CPLEX	Gurobi	CPLEX
693_T00	Time (s)	0.58	0.61	7.4	16	1.5	5.5
	Obj (€)	$-7.8e^3$	$-7.8e^3$	$-7.5e^3$	$-7.5e^3$	$-7.5e^3$	$-7.5e^3$
	Losses	0%	0%	1.64%	1.64%	1.64%	1.64%
	# S-bids	100	99	110	115	113	113
652_T66	Time (s)	105	24.7	$3.54e^3$	320	68.5	$3.6e^3*$
	Obj (€)	$-7.8e^3$	$-7.8e^3$	$-7.5e^3$	$-7.5e^3$	$-7.5e3$	$-7.5e^3$
	Losses	0%	0%	0.37%	0.37%	0.37%	0.37%
	# S-bids	10,935	10,935	10,935	10,935	10,935	10,935
401_T00	Time (s)	75.1	48.1	$3.6e^3*$	$1.05e^3$	421	$3.6e^3$
	Obj (€)	$-4.176e^4$	$-4.175e^4$	$-4.258e^4$	$-4.259e^4$	$-4.258e^4$	-
	Losses	0%	0%	1.58%	1.56%	1.57%	-
	# S-bids	3,328	3,348	3,148	3,157	3,132	-
301_T00	Time (s)	142	44.9	$3.6e^3*$	916	377	$3.6e^3*$
	Obj (€)	$2.925e^3$	$-2.925e^3$	-	$-2.958e^3$	$-2.925e^3$	-
	Losses	0%	0%	-	0.53%	0.53%	-
	# S-bids	374	457	-	510	345	-

The star index shows that the run time exceeded 1h. In addition, if no feasible solution is returned, the other fields of the table are not filled.

TABLE III
RESULTS ON THE TEST CASES THAT WE ANALYZE.

Test Case	693_T00	652_T66	401_T00	301_T00
# Var	$2.16e^4$	$4.53e^5$	$4.35e^5$	$4.90e^5$
# Bin	$4.22e^3$	$6.49e^4$	$1.49e^5$	$1.57e^56$
# Constr	$3.08e^4$	$5.60e^5$	$6.21e^5$	$6.65e^5$
# SOC	$2.21e^3$	$3.06e^4$	$3.49e^4$	$3.22e^4$
Objective (€)	$-7,51e^3$	$8.33e^3$	$-4.26e^4$	$-2.96e^3$
Gap	0.89	0.66	$3.93e^3$	$4.5e^{-5}$
Time (s)	1.52	68.5	421	377

V. CONCLUSION

Throughout this paper, we provide a detailed description of how transmission and distribution operations can be optimized simultaneously in real time. The model leads to an MISOC of large scale if we are to consider real-world instances of the problem. We consider approximations of the problem using a DC approximation and the Ben-Tal formulation. We observe that the MISOC tackled with the appropriate solver provides the best trade-off between quality of the solution and execution time. Preliminary results show that this type of problem can be solved efficiently and fit the time limit requirement of the real-time market for the Italian, Danish and Spanish test cases considered in this paper.

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REFERENCES

[1] D. Callaway and I. Hiskens. Achieving controllability of electric loads. *Proceedings of the IEEE*, 99(1):184–199, 2011.

[2] I. Mezghani, A. Papavasiliou, and H. Le Cadre. A generalized nash equilibrium analysis of electric power transmission-distribution coordination. In *Proceedings of the Ninth International Conference on Future Energy Systems*, pages 526–531. ACM, 2018.

[3] European Distribution System Operators for Smart Grids. Flexibility: The role of dsos in tomorrow's electricity market. technical report. Technical report, 2014.

[4] F. Kunz and A. Zerrahn. The benefit of coordinating congestion management in germany. In *European Energy Market (EEM), 2013 10th International Conference on the*, pages 1–8. IEEE, 2013.

[5] (2015, jun.) The Smartnet website. [Online] http://smartnet-project.eu/.

[6] M. Caramanis, E. Ntakou, W. Hogan, A. Chakrabortty, and J. Schoene. Co-optimization of power and reserves in dynamic t&d power markets with nondispatchable renewable generation and distributed energy resources. *Proceedings of the IEEE*, 104(4):807–836, 2016.

[7] R. A. Jabr. Radial distribution load flow using conic programming. *IEEE transactions on power systems*, 21(3):1458–1459, 2006.

[8] Q. Peng and S. H. Low. Distributed algorithm for optimal power flow on a radial network. In *Decision and Control (CDC), 2014 IEEE 53rd Annual Conference on*, pages 167–172. IEEE, 2014.

[9] M. Farivar and S. Low. Branch flow model: Relaxations and convexification: Part i. *IEEE Transactions on Power Systems*, 28(3):2554–2564, 2013.

[10] N. Li, L. Chen, and S. Low. Exact convex relaxation of opf for radial networks using branch flow model. In *Smart Grid Communications, 2012 IEEE Third International Conference on*, pages 7–12. IEEE, 2012.

[11] A. Ashouri, P. Sels, G. Leclercq, O. Devolder, Frederik G., and R. Dhulst. Network and market models: preliminary report. Technical report, 2017. [Online] http://smartnet-project.eu/wp-content/uploads/2016/03/D2.4_Preliminary.pdf.

[12] B. Kocuk, S. Dey, and A. Sun. Strong socp relaxations for the optimal power flow problem. *Operations Research*, 64(6):1177–1196, 2016.

[13] F. Schweppe, M. Caramanis, R. Tabors, and R. Bohn. *Spot Pricing of Electricity*. Springer Science & Business Media, 2013.

[14] A. Ben-Tal and A. Nemirovski. On polyhedral approximations of the second-order cone. *Mathematics of Operations Research*, 26(2):193–205, 2001.