# A Comparison of Priority Service versus Real-Time Pricing for Enabling Residential Demand Response

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***Abstract*—The unprecedented growth of renewable energy has led to various challenges in power system operations. Demand response can provide additional flexibility to the system in order to balance the effects of the massive integration of renewable resources. This paper focuses on the comparison of two approaches for enabling demand response, real-time pricing and priority service pricing. This analysis is centered on the impact to consumers by assessing the effects of these schemes on their comfort and bill. We use a simple example of a single household with one appliance. The example provides an end-to-end illustration of (i) how to design a priority service menu from a time series of real-time prices, (ii) how a household selects options from this menu, (iii) how devices in the household are dispatched by a home energy router, and (iv) what consumer welfare losses are relative to the golden standard of real-time pricing.**

## I. Introduction

The recent large-scale integration of renewable resources in electric power systems has resulted in various challenges in power system operations, due to the unpredictable, highly variable and non-controllable fluctuation of these resources. This has accordingly increased the need for the incorporation of flexibility into the system. Nowadays, there exists a large amount of unused flexible resources connected to the low-voltage distribution grid in the form of flexible residential and commercial demand, which can be exploited efficiently in order to break the barriers that are bounding the growth of renewable energy integration.

Demand response paradigms for efficient dispatch of residential load, analyzed extensively in the literature, include real-time pricing (RTP) and direct load control (DLC). The real-time pricing approach considers consumers as real-time participants into the electricity market that react instantaneously to prices, for example, by reducing consumption during periods of peak demand [1]. However, if the price is too variable, this scheme places excessive risk on consumers [2]. On the contrary, direct load control considers an aggregator who controls consumer appliances. The aggregator has the authority to turn devices on or off anytime there is a need to regulate the power flowing through the network [3]. However, this concept is considered as being too intrusive by consumers in terms of privacy. Instead, this paper focuses on an alternative approach for mobilizing flexible demand, referred to as priority service pricing (PSP) [4], which aims at combining the strengths of real-time pricing and direct load control. According to this paradigm, electricity is considered as a service that can be offered with different levels of quality, here the reliability of electricity supply. Contracts are created for capacity strips of power with a particular reliability and consumers choose among several options that form a menu of price-reliability pairs. An option of higher reliability implies a higher price for that service. The concept is illustrated in a case study of Belgium by Mou [2]. The same idea underlies the approach of Papalexopoulos [5], [6]. According to his method, an aggregator uses a color-tagging system, using priority service pricing with 3 options, based on traffic lights that consumers can set for each of their appliances: **(i)** *Green*: indicates cheap power that can be often interrupted; **(ii)** *Orange*: indicates power that can be interrupted under emergency conditions; **(iii)** *Red*: indicates expensive power that cannot be interrupted. Priority service pricing, as a demand response scheme, works with an aggregator that offers a menu. Consumers have to pick an option that maximize their surplus. Therefore, this is an appealing scheme compared to the two other because it is simpler for consumer to make a one-off selection of a menu instead of actively trading electricity in a real-time market.

A comparison between the real-time pricing scheme with a priority service menu, relying on 3 options (Green, Orange, Red) which is derived using priority service pricing, is made in this paper. In order to compare these two schemes, we assume that a home energy management system is installed in households which can schedule appliances efficiently under both schemes, while placing minimum decision-making requirements on household consumers. The analysis is then focused on the impact of these two schemes in consumer comfort and expenditures. The effect on consumers comfort is measured here by means of a frustration measure due to delays on serving power consumption requests. The scheduling of appliances in a household has already been studied in the context of real-time pricing [7], [8] by means of reinforcement learning algorithms or using mathematical programming formulations [9], [10]. The comparison between priority service pricing and real-time pricing is realized by means of MILP formulations of the device scheduling problem for both schemes.

This paper aims at describing a methodology for comparing real-time pricing and priority service pricing on a consistent basis. The comparison is consistent in the sense that the priority service menu is designed from the real-time pricing profile of the market. In order to compare both schemes, we quantify expenditures for the procurement of power under each scheme and we use a mathematical program for the efficient scheduling of home appliances in order to quantify consumer comfort. The paper is divided into several parts. In section II, the mathematical program used to schedule appliances for a household is presented for both demand response cases. In section III, we describe the data set which we use for the simulation, and we present the results from the comparison of both methods. We conclude in section IV.

## II. Methodology

In this section, we describe a mathematical program for scheduling appliances in the household. We assume that demands for power arrive in the form of 'jobs' with specific execution deadlines and power consumption profiles, which are assumed to be known in advance. There is no uncertainty regarding arrival times, deadlines, consumption profiles, etc. in the present model. Uncertainty in the device scheduling problem will be addressed in future research. We adopt the following assumptions about the behaviour of the consumer and the use of appliances in households:

- **A1** An appliance can change color (i.e. move into a different reliability tier) while in the middle of executing a power consumption profile;
- **A2** An appliance can be interrupted at any stage of its operation and be started on again at the stage it was interrupted;
- **A3** An appliance arrives with a deadline by which the task of the appliance has to be complete in order for the consumer to avoid any frustration;
- **A4** The power consumption footprint of each appliance is known.

A footprint of an appliance is the usual consumption pattern of this appliance. The estimation of these consumption patterns is the focus of an extensive body of literature on non-intrusive load monitoring [11]. Based on these assumptions, the device scheduling mathematical program can now be presented. In section II-A the model for the real-time pricing scheme is exposed. Section II-B is dedicated to the priority service pricing scheme.

### A. Real-Time Pricing Problem

In this section, we describe a mathematical program that schedules appliances under real-time pricing in a household. In this setup, the consumer is facing a real-time price for electricity and chooses appliances turn on or off by reacting to electricity prices. We assume perfect foresight on the realization of real-time prices. The variables of the mathematical program are:

- $end_j$: Binary variable that records if appliance $j$ finishes before the end of the entire computing horizon;
- $y_t$: Continuous variable that represents the total electricity consumption of the household at time period $t$;
- $x_{t,\tau,j}$: Binary variable that records if part $\tau_j$ of the consumption profile of appliance $j$ is served at time $t$;

where the index notation is as follows:

- $j$: index corresponding to each appliance in the household (set $\mathcal{J}$);
- $t$: index representing each time period (set $\mathcal{T}$);
- $\tau$: index representing each part of the power consumption footprint of appliance $j$ (set $\mathcal{T}_j$).

Based on these notations, since the real-time pricing scheme is characterized by the way consumer react to real-time prices, the objective is to maximize the net benefit of the consumer, i.e. maximize the utility of the consumer net of expenditures in the real-time market:

$$\sum_{j\in J}\big[R_j\big(\sum_{\substack{t\in\mathcal{T},\\ t\leq D_j}} x_{t,\tau_{end},j}\big) - F_j\big(\sum_{\substack{t\in\mathcal{T},\\ t>D_j}} (t-D_j)\, x_{t,\tau_{end},j}\big) - NSC_j(1-end_j)\big] - \sum_{t\in\mathcal{T}}\lambda_t y_t \tag{1}$$

Here, $R_j$ is a measure of the benefit of the consumer if appliance $j$ finishes before its deadline, $F_j$ the frustration of the consumer for each one hour delay of the end of the appliance, $NSC_j$ is the frustration of the consumer if the appliance does not finish before the end of the computation horizon, $\lambda_t$ is the real-time price at time period $t$, and $D_j$ the deadline corresponding to appliance $j$.

We now describe the device scheduling problem constraints. The total consumption of the household is the sum of the consumption of each appliance at every time period:

$$y_t = \sum_{j\in\mathcal{J}}\sum_{\tau\in\mathcal{T}_j} f_{\tau,j}x_{t,\tau,j} \;\; \forall t\in\mathcal{T} \tag{2}$$

where $f_{\tau,j}$ represents the consumption pattern of appliance $j$ at part $\tau$ of its footprint. Moreover, the model can only serve one part of the profile of an appliance $j$ during a certain time period, as represented by (3):

$$1 \geq \sum_{\tau\in\mathcal{T}_j} x_{t,\tau,j} \;\; \forall t\in\mathcal{T},\; j\in\mathcal{J} \tag{3}$$

Furthermore, the order of the parts of the consumption profile of each appliance must also be respected, see (4). For example, the first hour of the washing machine has to be served before the second one.

$$\sum_{\substack{t\in\mathcal{T},\\ t<t_{ON}}} x_{t,\tau,j} \geq x_{t_{ON},\tau+1,j} \quad \forall t_{ON}\in\mathcal{T}, \tau\in\mathcal{T}_j\backslash\{\tau_{end}\},\; j\in\mathcal{J} \tag{4}$$

Finally, it remains to define variable $end_j$ (5) and to impose boundaries on the total consumption of the household at each time period in (6).

$$end_j = \sum_{t\in\mathcal{T}} x_{t,\tau_{end},j} \;\; \forall j \in \mathcal{J} \tag{5}$$

$$0 \le y_t \le P_t^{max} \;\; \forall t \in \mathcal{T} \tag{6}$$

where $P_t^{max}$ is the maximum consumption allowed for the household at time period $t$. After bundling all the constraints together along with the objective function, the following mathematical program is obtained:

$$\begin{aligned} \underset{x_{t,\tau,j},y_t,end_j}{\text{maximize}} \quad & (1) \\ \text{subject to} \quad & (2)-(6) \\ & x_{t,\tau,j} \in \{0,1\} \;\; \forall t \in \mathcal{T},\ \tau \in \mathcal{T}_j,\ j \in \mathcal{J} \\ & end_j \in \{0,1\} \;\; \forall j \in \mathcal{J} \end{aligned}$$

This mathematical program computes the optimal schedule for appliances in the household based on real-time pricing.

### *B. Priority Service Problem*

In this section, the mathematical program which schedules appliances by means of priority service pricing is presented. As explained in section I, a price menu containing three options corresponding to colors is used. Each color corresponds to a different level of reliability for serving electricity. Therefore, an additional index is required: $i$ represents each color contained in set $\mathcal{I}$. The variables of the problem now become:

- $end_j$;
- $y_{t,i}$: Continuous variable that represents the total electricity consumption of the household using color $i$ at time period $t$;
- $x_{t,\tau,i,j}$: Binary variable that records if part $\tau_j$ of appliance $j$ was served at time $t$ using color $i$.

The priority service pricing scheme is different from the real-time approach because the consumer subscribes for a certain amount of power to an option of the price menu at the beginning of the horizon instead of facing variable prices at each time period. For example, a consumer can subscribe to 2 kW of red power and 4 kW of orange power. After subscribing, the consumer has the requested amount of power in each color and is facing the reliability of the color. This means that if orange power has 60% reliability in the price menu, then this color will be on 60% of the entire computation horizon and orange power can only be used when the color is on. Therefore, the objective for this approach accounts for the expenditures of the consumer by subscribing to an amount of power from each option at the beginning of the horizon. The model consequently aims at maximizing the utility of the consumer by scheduling appliances for the horizon, while accounting for the interruption of colors and selecting the optimal amount of power from each option in the priority service menu at the beginning of the horizon. The new objective function is now given by (7).

$$\begin{aligned} \sum_{j\in J} \big[ & R_j \big( \sum_{i\in\mathcal{I}} \sum_{\substack{t\in\mathcal{T},\\ t\le D_j}} x_{t,\tau_{end},i,j} \big) \\ & - F_j \big( \sum_{i\in\mathcal{I}} \sum_{\substack{t\in\mathcal{T},\\ t> D_j}} (t-D_j)\, x_{t,\tau_{end},i,j} \big) \\ & - NSC_j(1-end_j) - |\mathcal{T}| \big( \sum_{i\in\mathcal{I}} \lambda_i P_i^{max} \big) \big] \end{aligned} \tag{7}$$

Here, $P_i^{max}$ represents the amount of power subscribed by the consumer to color $i$ and $\lambda_i$ the price of that color in the priority service menu. $P_i^{max}$ is added to the variables of the problem because the consumer needs to decide to pick a certain amount of power for each color from the menu before the beginning of the horizon in order to receive power from the grid.

Another difference with the real-time pricing scheme is that a summation over the different colors is now added to every equation present in the mathematical program. Moreover, boundaries on the total consumption (previously (6)) for each color now become:

$$0 \le y_{t,i} \le profile_{t,i} P_i^{max} \;\; \forall i \in \mathcal{I},\ t \in \mathcal{T} \tag{8}$$

where $profile_{t,i}$ is a binary parameter that records if color $i$ is interrupted or not at time period $t$.

The mathematical program that schedules appliances in the house based on priority service is therefore given by the following problem:

$$\underset{\substack{x_{t,\tau,i,j},y_{t,i},\\ end_j}}{\text{maximize}} \quad \sum_{j\in J} \big[ R_j \big( \sum_{i\in\mathcal{I}} \sum_{\substack{t\in\mathcal{T},\\ t\le D_j}} x_{t,\tau_{end},i,j} \big) - F_j \big( \sum_{i\in\mathcal{I}} \sum_{\substack{t\in\mathcal{T},\\ t> D_j}} (t-D_j)\, x_{t,\tau_{end},i,j} \big) - NSC_j(1-end_j) - |\mathcal{T}| \big( \sum_{i\in\mathcal{I}} \lambda_i P_i^{max} \big) \big]$$

$$\text{subject to} \quad y_{t,i} = \sum_{j\in\mathcal{J}} \sum_{\tau\in\mathcal{T}_j} f_{\tau,j} x_{t,\tau,i,j} \;\; \forall i \in \mathcal{I}, t \in \mathcal{T} \tag{9}$$

$$1 \ge \sum_{i\in\mathcal{I}} \sum_{\tau\in\mathcal{T}_j} x_{t,\tau,i,j} \;\; \forall t \in \mathcal{T},\ j \in \mathcal{J} \tag{10}$$

$$\sum_{i\in\mathcal{I}} \sum_{\substack{t\in\mathcal{T},\\ t< t_{ON}}} x_{t,\tau,i,j} \ge \sum_{i\in\mathcal{I}} x_{t_{ON},\tau+1,i,j} \quad \forall t_{ON} \in \mathcal{T}, \tau \in \mathcal{T}_j \backslash \{\tau_{end}\}\ j \in \mathcal{J} \tag{11}$$

$$end_j = \sum_{i\in\mathcal{I}} \sum_{t\in\mathcal{T}} x_{t,\tau_{end},i,j} \;\; \forall j \in \mathcal{J} \tag{12}$$

$$0 \le y_{t,i} \le profile_{t,i} P_i^{max} \;\; \forall i \in \mathcal{I},\ t \in \mathcal{T}$$

$$x_{t,\tau,i,j} \in \{0,1\} \;\; \forall t \in \mathcal{T},\ \tau \in \mathcal{T}_j,\ i \in \mathcal{I},\ j \in \mathcal{J}$$

$$end_j \in \{0,1\} \;\; \forall j \in \mathcal{J}$$

TABLE I
REAL-TIME PRICES FACED BY THE CONSUMER DURING THE 10-HOUR PERIOD AND THE INTERRUPTION PATTERN OF COLORS FOR THE PRIORITY SERVICE PRICING SCHEME.

| Hour | Real-time Price [€/kWh] | Green | Orange | Red |
|---|---|---|---|---|
| 1 | 0.02 | ON | ON | ON |
| 2 | 0.05 | OFF | OFF | ON |
| 3 | 0.086 | OFF | OFF | OFF |
| 4 | 0.06 | OFF | OFF | ON |
| 5 | 0.04 | OFF | ON | ON |
| 6 | 0.09 | OFF | OFF | OFF |
| 7 | 0.032 | OFF | ON | ON |
| 8 | 0.055 | OFF | OFF | ON |
| 9 | 0.08 | OFF | OFF | ON |
| 10 | 0.01 | ON | ON | ON |

## III. RESULTS

In this section, the real-time pricing and the priority service demand response schemes are compared based on their performance in terms of impact on consumer comfort and expenditures. Section III-A details the data set that we used in order to compare these two schemes in a particular case study. In section III-B, we compute a price menu for the data set in order to compare both approaches in terms of consumer expenditures on the procurement of power. Finally, section III-C is dedicated to the results obtained for each scheme by means of the two mathematical programs presented in section II.

### A. Data

A particular case was chosen in order to compare the performance of each scheme. The selected example is created for a period of 10 hours, with a time step of one hour. During this 10-hour period, the real-time prices faced by the consumer for each hour are presented in Table I.

As we explain in section I, the three different colors correspond to three different reliability levels for the priority service scheme. We choose the following reliability levels for each color:

- Green: 20% reliability
- Orange: 40% reliability
- Red: 80% reliability

The third, fourth and fifth columns of Table I record if a certain color is interrupted or not during a certain time period, given the levels of reliability.

Finally, both mathematical programs presented in section II are applied to a single household that contains only one appliance. This appliance has a duration of 4 hours and the deadline imposed by the consumer for its end is hour 5. The footprint of the appliance is given in Table II. If this appliance finishes on time, then the consumer achieves a reward equal to 1 €. On the other hand, for every hour that the appliance is delayed beyond its deadline, the consumer incurs a cost of 0.5 €. If the appliance job is not complete before the end of the 10-hour horizon then the consumer incurs a cost of 3 €.

TABLE II
FOOTPRINT REPRESENTING THE ELECTRICAL CONSUMPTION OF THE SINGLE APPLIANCE OF THE HOUSEHOLD.

| Part Footprint [h] | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Footprint Consumption [kW] | 1.1 | 3.05 | 2.5 | 0.43 |

TABLE III
PRICE MENU OBTAINED FOR 3 OPTIONS OF PRIORITY SERVICE.

| Option Color | Price [€/kWh] | Reliability [%] | Range Valuation |
|---|---|---|---|
| Green | 0.004 | 20 | [0.02;0.0305[ |
| Orange | 0.01010 | 40 | [0.0305;0.058[ |
| Red | 0.0333 | 80 | [0.058; → |

These three values quantify the frustration or satisfaction of the consumer.

### B. Price Menu Computation

In order to compare both schemes in a meaningful way and manage to solve the priority service pricing problem, we construct a menu which is consistent with the real-time price profile presented in Table I and the chosen reliability given in section III-A. We follow the approach that is developed by Chao [4] and applied by Mou [2] in a case study of Belgium.

In the interest of computing a price menu with different options for the consumer, a function $R(v)$ has to be computed, which represents the reliability level that a consumer with valuation $v$ should enjoy under an efficient dispatch [2]. This function is given by (13) and is presented in Figure 1 by the blue curve.

$$R(v) = Pr\{p \leq v\} \tag{13}$$

Here, $p$ is the spot price of the market (the real-time price in our case). In the case of a finite number of classes (3 in our example), the reliability used for each option is computed using (14), as explained in [4]:

$$r_k = \int_{v_k}^{v_{k+1}} \left[\frac{D(v) - D(v_{k+1})}{D(v_k) - D(v_{k+1})}\right] dR(v) + R(v_k) \tag{14}$$

In this equation, $D(v)$ represents the demand function. For our example, we assume that consumer types are uniformly distributed, which means that the demand function is affine. Equation (14) can be used to solve for the breakpoints in valuation ($v_k$ and $v_{k+1}$), given that we know from section III-A what are the reliability target level of each tier. These breakpoints give the range of valuation of consumers that will choose this option in the menu. The range of valuation for each option is given in Table III along with the computed price menu.

Finally, the prices of each option are obtained by (15) and the price menu for our example is given in Table III.

$$p(v) = p_k = v_0 r_0 + \sum_{l=1}^{k} v_l (r_l - r_{l-1}) \tag{15}$$

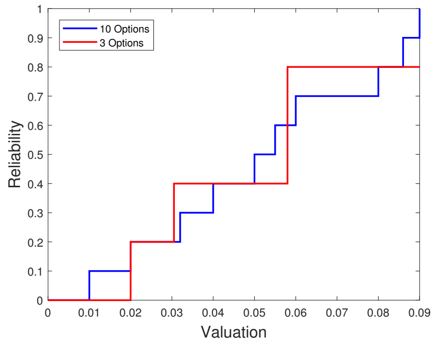


Fig. 1. Reliability as a function of the valuation of consumers for 10 or 3 priority service options.

TABLE IV
OBJECTIVE FUNCTION VALUE AND OPTIMAL POLICY OBTAINED BY SOLVING THE MATHEMATICAL PROGRAM BELONGING TO EACH APPROACH.

| | Objective Value [€] | Best Policy |
|---|---|---|
| Real-time | 0.6583 | Serve-Serve-Not-Serve-Serve |
| Priority Service | -0.01565 | Red-Red-Not-Red-Red |

### C. Real-Time and Priority Service Pricing Results

Given the assumptions presented in the previous section and having derived a priority service menu which is consistent with the prices of the real-time market, the two mathematical programs are solved. The objective value obtained for each scheme along with the best set of actions at a given time period are presented in Table IV. It is important to note that the optimal policy for each approach is identical. This is the case because, in time period 3, the red color is off so the consumer can not use it to serve his appliance and the real-time pricing scheme chooses also not to use the appliance considering high price of that time period.

Before comparing the objective value of both approaches, we first note that the consumer will subscribe to 3.05 kW of red power for 10 hours. To see why this is the case, note that this amount of power corresponds to the highest power consumption of the appliance (the second part). Therefore, if the second part of the appliance's footprint is not served at time period 2 or with red color, this means that the appliance can not finish before at least time period 8 which is far beyond the deadline. Moreover, the loss of finishing the job after the deadline for the consumer is not balanced by a subscription to a less reliable option.

Consumer net benefits can now be compared for the two different approaches. According to table IV, we observe that the priority service pricing scheme is causing a notable reduction in the net benefit of the consumer. This is largely due to the fact that priority service implies that the consumer is obliged to procure a capacity strip. This is how the service is defined: according to priority service, consumers procure increments of capacity. As a result, the consumer subscribes for 10 hours of power and only uses his subscription for 5 hours. A generalization of priority service which also includes an energy charge is described in [12] and will be investigated in future research.

## IV. CONCLUSION

We already note that, although Chao [4] establishes an equivalence between real-time pricing and priority service pricing from the point of view of the menu designer (Proposition III in [4]), we demonstrate, by means of this simple example, that this equivalence does not hold for the household. Mathematically, this is due to the fact that the total benefit of the consumer is non-concave. Intuitively, the reason is that the consumer needs to procure strips of service, and these strips need to be 'filled in' with jobs. The introduction of multiple devices, local storage and local renewable (e.g. rooftop solar) supply are expected to mitigate this effect and improve the performance of priority service relative to real-time pricing.

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