A Generalized Nash Equilibrium Analysis of Electric Power Transmission-Distribution Coordination

Ilyès Mezghani
CORE, UCLouvain
ilyes.mezghani@uclouvain.be

Anthony Papavasiliou
CORE, UCLouvain
anthony.papavasiliou@uclouvain.be

Hélène Le Cadre
EnergyVille Research Center
helene.lecadre@gmail.com

CCS CONCEPTS

• **Hardware → Power and energy**; • **Theory of computation** → *Convex optimization*; *Algorithmic game theory*; *Solution concepts in game theory*; *Exact and approximate computation of equilibria*; *Market equilibria*;

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1 INTRODUCTION

The integration of renewable energy resources leads to an important change in the way electricity markets are operated and organized. The common approach to the optimization of electric power system operations has focused on the high-voltage Transmission Network (TN), while the Distribution Network (DN) is typically not accounted for in detail. Nevertheless, the proliferation of distributed renewable resources (for example, solar panels and electric vehicles) in the DN, coupled with the presence of a substantial amount of load flexibility in the residential and commercial sector, implies that a considerable amount of intelligence will have to be integrated at the distribution level of electric power systems. Consequently, Distribution System Operators (DSOs) will have a more active role in the operation of electric power systems and electricity markets in the future.

The current paradigm of power system operations places all the intelligence in resources that are connected to the TN. Given the vast amount of unexploited flexible resources that are connected to the DN, the existing power system paradigm puts an important part of the system aside by only approximating the distribution system. The TN is the only part of the electricity supply chain that is currently optimized. The flexibility in the DN is mainly originating from active residential and commercial demand-side management, which we will need to exploit effectively in the coming decades if we wish to maintain the quality of service that we currently enjoy [3]. However, the DN is, in itself, a system of massive scale which presents a host of operational challenges. On the one hand, the amount of renewable resources that are located in the DN, mainly in the form of solar panels, has been growing and becoming an increasingly important component of the electric power supply chain. On the other hand, due to distribution constraints and the unpredictability of renewable resources, a certain amount of this renewable power needs to be consumed locally [8]. Coordination of operations in electricity markets has also been discussed in [19], [14], [20].

This work draws inspiration from [19], where the authors focus on the counter-trading of re-dispatching resources between two Transmission System Operators (TSOs), in the context of congestion management. The authors investigate whether there should exist a separate market for transmission capacity by resorting to Generalized Nash Equilibrium (GNE), due to the influence of each TSO's action on the other TSO's decisions. We transpose this framework to the context of TSO-DSO coordination, where the activation of distribution system reserves[1] by the TSO has an impact on the feasible actions of DSOs. We specifically focus on two coordination schemes inspired by the EU SmartNet project on TSO-DSO coordination [12], [1]. Even if we will only provide preliminary results a on a small example in this paper, the SmartNet initiative is willing to implement these schemes on pilot test cases in Denmark, Italy and Spain. Possible inefficiencies due to decentralization then need to be quantified. Although we envision the trading of real power at the transmission-distribution system interface as a viable approach towards TSO-DSO coordination, the SmartNet coordination schemes are not all aligned with such a setup. We therefore aim at comparing the efficiency of the schemes set forth by SmartNet, by relying on a GNE approach.

The focus of our paper is (i) to model various TSO-DSO coordination schemes which have been proposed in the SmartNet project as non-cooperative games, (ii) to propose a method for solving these problems, (iii) to interpret the solutions, and (iv) to compare the relative strengths and weaknesses of the different schemes.

For our modeling, we resort to Generalized Nash Equilibrium, which is a computationally difficult problem [16]. We propose a solution strategy which is based on the theory of Nabetani, Tseng and Fukushima [15]. Our simple example unveils multiple equilibria, a phenomenon which has been well-studied in the literature (see for example [11], [2]), and we comment on the quality of these equilibria in our numerical example.

The rest of the paper is organized as follows: we present the context of TSO-DSO coordination and our notation in section 2. We present the Generalized Nash Equilibrium models of two TSO-DSO coordination schemes in section 3. The implementation of the different schemes is illustrated through numerical results presented on a toy example in section 4. Section 5 concludes the paper.

[1]The only ancillary services we consider in this paper are activation of reserves.

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2 GENERAL ASSUMPTIONS AND MARKET STRUCTURE

2.1 Topology of the network

In this work, we follow the literature [4] in considering the following decomposition of the power grid:

- The TN hosts conventional generators, industrial loads, and large-scale renewable resource installations such as wind farms. TNs are typically meshed. Kirchhoff's power flow equations, which govern the flow of power on these networks, are adequately represented through a linearization called the Direct Current (DC) approximation.
- The low-voltage DN hosts residential and commercial consumers. DNs are typically radial, and the nonlinearity of power flows cannot be ignored due to the role of reactive power and voltage constraints. The nonlinear physics of power flow are in principle represented through a set of nonlinear and non-convex power flow equations. Nevertheless, since the DN is radial, and assuming that power flow is balanced, it has been shown that a Second Order Cone (SOC) relaxation of the power flow equations is exact under reasonable assumptions [10], [7].

We consider a set $\mathcal{K} := \{1, \ldots, K\}$ of K local (distribution) markets. The set of distribution nodes in local market network k is denoted as DN_k, where $k \in \mathcal{K}$. We assume that aggregations of producers and consumers at each node of the DN are represented by a single marginal supply function for reserve activation.

The set of nodes at the interface of the transmission and distribution grids is denoted by N_∞. Note that $|N_\infty| = K$. We will assume that $N_\infty = \mathcal{K}$, and we will denote an interface node by $k \in N_\infty$, and by DN_k the DN having k as interface node.

The set of transmission nodes is denoted as TN. Similarly, we assume that there is a single generator at each node of the TN. Interface nodes $k \in N_\infty$ have the same characteristics as a transmission node in the sense that there is a single high-voltage generator in this type of nodes.

2.2 Market structure

In this paper, we focus on the interaction of a TSO with a group of DSOs in the real-time market, where reserves are activated in order to balance random disturbances. It is assumed there exists a previously determined dispatch from day-ahead or intraday operations. The decisions of each operator can be seen as adjustments on this dispatch. The following schemes are inspired by the SmartNet project [1].

We also assume that adjusting power generation in the DN is cheaper than in the TN. Indeed, the TN includes power plants with high capacity in practice and adjusting power generation in these resources might be very costly. On the contrary, adjusting generation in the DN can be done through demand response or using local renewable resources which are considered to be cheap adjustment actions (we do not discuss the investment of renewable resources in this paper). Currently, the use of distributed resources is limited and uncoordinated operations can then lead to very bad behaviors in terms of welfare. These assumptions and the schemes we propose in this paper are in line with what can be found at the EU level in [12].

In a decentralized market structure, each DSO resolves local grid issues at the lowest possible cost. The role of the TSO, which ignores distribution system constraints in order to allow for scalability of operations, can vary according to the type of coordination scheme that we consider. In this paper, we assume that the TSO may either partially access distribution system resources, or rely exclusively on transmission-level resources in order to balance transmission-level imbalances.

NOTATIONS

The following notation is used:

- TN, set of transmission nodes.
- N_∞, set of interface nodes and $|N_\infty|$ designates the number of DNs.
- DN_k, set of distribution nodes in DN $k \in N_\infty$. Note that since we assume that the DN is radial, DN_k is also the set of lines of the same DN: line $i \in DN_k$ is the line connecting node $i \in DN_k$ to its ancestor node.
- L, set of lines of the TN linking nodes $(n, m) \in (TN \cup N_\infty)^2$.
- $\mathcal{X}^t$ / $\mathcal{X}_k^d$, set gathering all bounds on the variables used in the transmission (except for the bound on power generation) / distribution network (except for the bound on real power generation)..
- B_l, susceptance of transmission line $l \in L$.
- ΔD_n / ΔD_i, power demand at transmission node $n \in TN \cup N_\infty$ / real power demand at distribution node $i \in DN_k, k \in N_\infty$.
- R_i / X_i /G_i / B_i, resistance / reactance / shunt conductance / shunt susceptance of distribution line $i \in DN_k, k \in N_\infty$.
- S_i, complex power limit of distribution line $i \in DN_k, k \in N_\infty$.
- $\overline{\Delta p_i^d}$ / $\overline{\Delta p_n^t}$, capacity of generator $i \in DN_k, k \in N_\infty$ / capacity of generator $n \in TN \cup N_\infty$.
- $C_n(.)$ / $C_i(.)$ is the marginal cost of reserve activation at node $n \in TN \cup N_\infty$ / $i \in DN_k, k \in N_\infty$
- Δp_n^t /Δp_i^d, balancing power production at transmission node $n \in TN \cup N_\infty$ / real balancing power production at distribution node $i \in DN_k, k \in N_\infty$.
- θ_n, bus angle of transmission bus $n \in TN \cup N_\infty$.
- f_l / f_i^p / f_i^q, flow of power of transmission line $l \in L$ / real / reactive flow of power of distribution line $i \in DN_k, k \in N_\infty$.
- q_i reactive power net injection at distribution node $i \in DN_k, k \in N_\infty$.
- v_i voltage magnitude squared at distribution node $i \in DN_k, k \in N_\infty$.
- l_i current magnitude squared of distribution line $i \in DN_k, k \in N_\infty$.

Also, we introduce the dual variable (in Greek letter) of a constraint by placing it in parenthesis at the left-hand side of the constraint.

3 MODELING

3.1 Generalized Nash Equilibrium

We model two decentralized coordination schemes, referred to as Shared Balancing Responsibility (SBR) and TSO Limited Access (TLA), as Generalized Nash Equilibria ([9],[13],[6]). We use the approach of [6] in order to derive solutions to the Generalized Nash Equilibrium problems (GNEP).

A GNEP consists of a game among N players. Without loss of generality and for the sake of simplifying the exposition, we will only consider the case where $N = 2$. Player i controls variables y_i, $i = 1, 2$. y is the vector of all the variables: $y := (y_1, y_2)'$ (superscript $'$ representing the transposition operation). The utility function of player i is denoted as π_i and can depend on the decisions of other players.

To optimize its strategy, each player minimizes its costs assuming that the strategy of the other player is fixed. This can be stated by the two following mathematical programs:
For $(i, j) = (1, 2)$ and $(i, j) = (2, 1)$,

$$\begin{array}{lll} \mathcal{S}_i(y_j) : \min\limits_{y_i} & & \pi_i(y_i, y_j) \\ & s.t. & Y_i(y_i) \geq 0 \\ & (\alpha_i) & A_i(y_i) + A_j(y_j) \geq 0 \\ & (\beta_i) & B_i(y_i) + B_j(y_j) \geq 0 \end{array}$$

$\mathcal{S}_i(y_j)$ is the set of optimal solutions of the problem of player i depending on the decisions of player j. A solution of the GNEP, called equilibrium, is a vector $y^* := (y_1^*, y_2^*)'$ such that $y_1^* \in \mathcal{S}_1(y_2^*)$ and $y_2^* \in \mathcal{S}_2(y_1^*)$.

Note that the GNEP has the following types of shared constraints: the A-type ($A_i(y_i) + A_j(y_j) \geq 0$), and the B-type ($B_i(y_i) + B_j(y_j) \geq 0$). The A-type are the ones for which one requires the dual variables to be equal ($\alpha_1 = \alpha_2$), and can typically be seen as resources that are traded. The B-type constraints are the ones for which there are no restrictions on the dual variables (β_1 and β_2 can be different): the two players do not need see the same price or, as mentioned in [19], "*no internal market has been created for these common constraints*".

One method to solve the GNEP formulated above is proposed by Nabetani, Tseng and Fukushima [15] (we abbreviate this as the NTF in the rest of the paper). The method relies on introducing parameters γ_1, γ_2 and solving the following standard constrained optimization problem:

$$\begin{array}{lll} \mathcal{S}^{\text{NTF}}(\gamma_1, \gamma_2) : \min\limits_{y_1, y_2} & & \pi_1(y_1, y_2) + \pi_2(y_1, y_2) \\ & & + \gamma_1 B_1(y_1) + \gamma_2 B_2(y_2) \\ & s.t. & Y_1(y_1) \geq 0 \\ & & Y_2(y_2) \geq 0 \\ & & A_1(y_1) + A_2(y_2) \geq 0 \\ & & B_1(y_1) + B_2(y_2) \geq 0 \end{array}$$

Ideally, one should solve for all the possible values of γ_1 and γ_2 in order to observe the whole set of equilibria, because $\mathcal{S}^{\text{GNEP}} \subseteq \{\mathcal{S}^{\text{NTF}}(\gamma_1, \gamma_2), (\gamma_1, \gamma_2) \in \mathbb{R}^2\}$ (where $\mathcal{S}^{\text{GNEP}}$ is the set of equilibrium points for the original GNEP). However, solving this optimization problem will not always lead to an equilibrium, and one should still check that the vector y obtained is in fact an equilibrium for the original GNEP.

We now proceed in casting the Shared Balancing Responsibility scheme and the TSO Limited Access scheme as Generalized Nash Equilibria. The two players of the setting is one TSO playing against the set of DSOs.

3.2 Mathematical Formulation

The transmission system is modeled through a linear approximation of the power flow equations. We use the $B - \theta$ formulation [18].

<u>Transmission equations</u>

$$f_l = B_l(\theta_n - \theta_m), \ \forall l = (n, m) \in L \tag{1}$$

$$\Delta p_n^t + \sum_{l=(m,n)} f_l - \sum_{l=(n,m)} f_l = \Delta D_n, \ \forall n \in TN \tag{2}$$

$$0 \leq \Delta p_n^t \leq \overline{\Delta p_n^t}, \ \forall n \in TN \cup N_\infty \tag{3}$$

$$(f, \theta) \in \mathcal{X}^t \tag{4}$$

For each DN, the SOC relaxation of the nonlinear power flow equations is applied.

<u>Distribution equations in DN_k, for $k \in N_\infty$</u>

$$\Delta p_i^d + \sum_{j \in C_i} (f_j^p - l_j R_j) - f_i^p - G_i v_i = \Delta D_i, \ \forall i \in DN_k \tag{5}$$

$$f_i^q - \sum_{j \in C_i} (f_j^q - l_j X_j) + q_i - B_i v_i = 0, \ \forall i \in DN_k \tag{6}$$

$$v_i = v_{A_i} + 2(R_i f_i^p + X_i f_i^q) - l_i(R_i^2 + X_i^2), \ \forall i \in DN_k \tag{7}$$

$$(f_i^p)^2 + (f_i^q)^2 \leq S_i^2, \ \forall i \in DN_k \tag{8}$$

$$(f_i^p)^2 + (f_i^q)^2 \leq v_i l_i, \ \forall i \in DN_k \tag{9}$$

$$(f_i^p - l_i R_i)^2 + (f_i^q - l_i X_i)^2 \leq S_i^2, \ \forall i \in DN_k \tag{10}$$

$$0 \leq \Delta p_i^d \leq \overline{\Delta p_i^d}, i \in DN_k \tag{11}$$

$$(f^p, f^q, q, v, l) \in \mathcal{X}_k^d \tag{12}$$

Transmission and distribution equations are not linked for the moment. The coupling appears at the interface nodes, N_∞, which can be seen as interconnection buses between transmission and DNs.

<u>Interface equations</u>

$$\begin{aligned} \Delta p_k^t + \sum_{l=(m,k)} f_l - \sum_{l=(k,m)} f_l \\ = \Delta D_k(\omega) - \sum_{j \in C_k} (f_j^p - l_j R_j), \ \forall k \in N_\infty \end{aligned} \tag{13}$$

3.2.1 Shared balancing responsibilities (SBR). Each operator decouples the operations of its own network from the operations of the networks of other operators. We formulate this problem as a GNEP. We need to define the decisions of each player. As mentioned previously, the TSO is interacting with the DSOs in a simultaneous game. We then define:

$$y^{\text{TSO}} = (\Delta p^t, f, \theta)$$
$$y^{\text{DSO}} = (\Delta p^d, f^p, f^q, q, v, l)$$

Since the interface constraints (13) will be shared by both players, each problem is parametric on the decisions of the other problem; we then introduce F^{TSO} (resp. F^{DSO}) which represents the optimal value of the TSO (resp. DSOs) problem depending on the DSOs (resp. TSO) decisions. The associated problems are the following:

- TSO problem:

$$\begin{array}{lll} F^{\text{TSO}}(y^{\text{DSO}}) := & \min\limits_{y^{\text{TSO}}} & \sum\limits_{n \in TN} \int_0^{\Delta p_n^t} C_n(x) dx \\ & s.t. & (1) - (4), \\ & & (13) \end{array}$$

- DSOs problem:

$$F^{\text{DSO}}(y^{\text{TSO}}) := \min_{y^{\text{DSO}}} \sum_{\substack{k\in N_\infty \\ i\in DN_k}} \int_0^{\Delta p_i^d} C_i(x)dx$$
$$s.t. \quad (5)-(12),\ \forall k \in N_\infty$$
$$(13)$$

The only shared constraint in this problem is (13), and the SBR scheme does not involve a market for real power flowing at the interface [12]. By introducing γ_k^T (for the TSO coupling constraint (13)) and γ_k^D (for the DSO coupling constraints (13)) for all $k \in N_\infty$, the NTF problem is then:

$$\min_{y^{\text{DSO}}, y^{\text{TSO}}} \sum_{n\in TN} \int_0^{\Delta p_n^t} C_n(x)dx + \sum_{\substack{k\in N_\infty \\ i\in DN_k}} \int_0^{\Delta p_i^d} C_i(x)dx$$
$$+ \sum_{k\in N_\infty} \gamma_k^T \left(\Delta p_k^t + \sum_{l=(m,k)} f_l - \sum_{l=(k,m)} f_l\right)$$
$$+ \sum_{k\in N_\infty} \gamma_k^D \left(\sum_{j\in C_k} (f_j^p - l_j R_j)\right)$$
$$s.t. \quad (1)-(4),$$
$$(5)-(12),\ \forall k \in N_\infty$$
$$(13)$$

Note that this NTF formulation falls in the scope of [15] since (13) is the only shared constraint (of B-type in our case) and the other constraints are individual constraints. From Theorem 3.3 in [15], we also deduce that a solution of NTF is necessarily an equilibrium of the original GNEP since the only shared constraint (13) is an equality.

Note also that the particular case where $\gamma_k^T = \gamma_k^D = 0$ can be seen as the constrained optimization problem where a single operator (probably the TSO) minimizes the total cost on the global system. In other words, this is the case where an operator maximizes the Social Welfare (SW). For more information about a centralized scheme where an operator would maximize SW, the reader can refer to the extended version of the paper[2] or [17].

3.2.2 TSO has limited access to DSO resources (TLA). In this scheme, we allow the TSO to activate resources in the DSOs network. These resources bid in a transmission-level reserve activation market, in which the TSO does not account for DSOs network constraints. The TSO can then activate a certain amount of real power at node $i \in DN_k, k \in N_\infty$, through the decision variable $\Delta p_i^{d,T}$. The DSOs can also activate distributed resources, through the variable Δp_i^d. In order to represent the fact that the TSO may actually receive less power at the interface than what it actually activated (due to ignorance of distribution system losses by the TSO), we introduce a variable $\eta_k, k \in N_\infty$. Thus, the variable variable η_k, which is a decision variable in the DSOs problem, represents the difference between the amount of reserves that the TSO activates, and the physical flow of power resulting from the activation at the TSO-DSO interface.

[2] http://docdro.id/QMGB2KP

The interface equations (13) are then replaced by:

$$(\lambda_k) \quad \Delta p_k^t + \sum_{l=(m,k)} f_l - \sum_{l=(k,m)} f_l = \Delta D_k(\omega) - \sum_{i\in DN_k} \Delta p_i^{d,T} - \eta_k,\ \forall k \in N_\infty \qquad (14)$$

$$(\epsilon_k) \quad \sum_{i\in DN_k} \Delta p_i^{d,T} + \eta_k = \sum_{j\in C_k} (f_j^p - l_j R_j),\ \forall k \in N_\infty \qquad (15)$$

Constraint (14) appears in both the TSO and the DSOs problems, while constraint (15) which defines η_k as the difference of requested and physically delivered power will only appear in the DSOs problem, since it involves distribution topology. Distribution constraints (5), (11) are also modified, in order to represent the fact that the TSO can activate distribution system resources:

$$(\lambda_i) \quad \Delta p_i^d + \Delta p_i^{d,T} + \sum_{j\in C_i} (f_j^p - l_j R_j) - f_i^p - G_i v_i = \Delta D_i,\ \forall i \in DN_k \qquad (16)$$
$$0 \le \Delta p_i^d,\ \forall i \in DN_k \qquad (17)$$
$$0 \le \Delta p_i^{d,T},\ \forall i \in DN_k \qquad (18)$$
$$(\delta_i) \quad \Delta p_i^d + \Delta p_i^{d,T} \le \overline{\Delta p_i^d},\ \forall i \in DN_k \qquad (19)$$

Note that if both players activate from the same distribution node, the DSOs would have access to the cheaper units while the TSO would only be able to activate the more expensive resources which are left over. Implicitly, it gives a priority to the DSOs to use their resources which is a desirable property in practice. This is captured in the objective function of the TSO and DSOs[3], which are presented below. The two problems defining the GNEP are then:

$$y^{\text{TSO}} = (\Delta p^t, f, \theta, \Delta p^{d,T})$$
$$y^{\text{DSO}} = (\Delta p^d, f^p, f^q, q, v, l, \eta)$$

- TSO problem:

$$F^{\text{TSO}}(y^{\text{DSO}}) := \min_{y^{\text{TSO}}} \sum_{n\in TN} \int_0^{\Delta p_n^t} C_n(x)dx + \sum_{\substack{k\in N_\infty \\ i\in DN_k}} \int_{\Delta p_i^d}^{\Delta p_i^d + \Delta p^{d,T}} C_i(x)dx$$
$$s.t. \quad (1)-(4)$$
$$(14)$$
$$(18), (19)$$

- DSOs problem:

$$F^{\text{DSO}}(y^{\text{TSO}}) := \min_{y^{\text{DSO}}} \sum_{\substack{k\in N_\infty \\ i\in DN_k}} \int_0^{\Delta p_i^d} C_i(x)dx$$
$$s.t. \quad (6)-(10), (12), (16), (17), (19)$$
$$(14), (15)$$

We have two shared constraints: (14), and (19). Constraint (19) is the the generation limit at node i, and we assume that both the TSO and DSO face the same dual variable for this constraint, which means that (19) is A-type. On the contrary, like in SBR, we allow for

[3] Especially the bounds when we integrate over $C_i(x)$

the possibility that the TSO and DSOs might not assign the same price for the real power balance constraint at the interface (14). The price deviation is readily interpreted as the bid-ask spread on the real power exchanged between the TSO and the DSOs. The NTF formulation is then:

$$\begin{aligned}\min_{y^{\text{DSO}}, y^{\text{TSO}}} \quad & \sum_{n \in TN} \int_0^{\Delta p_n^t} C_n(x)dx + \sum_{\substack{k \in N_\infty \\ i \in DN_k}} \int_0^{\Delta p_i^d + \Delta p_i^{d,T}} C_i(x)dx \\ & + \sum_{k \in N_\infty} \gamma_k^T \left(\Delta p_k^t + \sum_{l=(m,k)} f_l - \sum_{l=(k,m)} f_l \right. \\ & \left. + \sum_{i \in DN_k} \Delta p_i^{d,T} \right) + \sum_{k \in N_\infty} \gamma_k^D \eta_k \\ s.t. \quad & (1), (2), (3), (4), \\ & (14), (15), \\ & (6), (7), (8), (9), (10), (12), (16), (19)\end{aligned}$$

Note that constraints (15) and (16) only appear in the DSOs problem, but involves TSO variables ($\Delta p_i^{d,T}$). These constraints are neither individual constraints nor shared constraints (since they do not appear in the TSO problem). Our setup is therefore more general than what considered by [15], which implies that there is no guarantee that applying the NTF methodology will yield equilibria.

Notwithstanding, one can still compute solutions to the NTF formulation described above, and check whether the solutions computed are actual equilibria for the original GNEP even if the NTF provides no guarantee of obtaining equilibria. It turns out that this idea delivers equilibria for the test case presented in section 4.

Although there is no theoretical guarantee that the NTF method will furnish equilibria, we can characterize the solutions of the NTF optimization problem. These two propositions go in that direction:

Proposition 1. *Consider the NTF formulation of TLA. If $\gamma^D \neq \gamma^T$, then for all $i \in DN_k$, $k \in N_\infty$,*

$$\Delta p_i^d \Delta p_i^{d,T} = 0.$$

Proposition 2. *Consider the NTF formulation of TLA. If $\gamma^D > \gamma^T$ then $\Delta p_i^d = 0, \forall i \in DN_k, k \in N_\infty$. If $\gamma^D < \gamma^T$ then $\Delta p_i^{d,T} = 0, \forall i \in DN_k, k \in N_\infty$.*

The proofs of Propositions 1 and 2 can be found in the appendix of the extended version of the paper.There is a simple interpretation of these results: if the NTF formulation is to furnish equilibria for the TLA coordination scheme, these equilibria will be such that at most one of the players (either the TSO, or the DSO, but not both) activates a resource at a given node. Therefore, we concede if there exist equilibria to the TLA scheme in which both agents activate reserve on a given node simultaneously, our proposed method will not be able to detect them. Developing a method for detecting such equilibria, if they exist, is left as a topic for future research.

Obviously, in the case where $\Delta p_i^{d,T} = 0, \forall i \in DN_k, k \in N_\infty$, we fall back to the equilibria of SBR. This implies that the SBR scheme may in fact be subsumed by the TLA scheme, and this suggests that SBR may be a subset of TLA. It turns out from the numerical example that this is not the case, and that there exist SBR equilibria which are not TLA equilibria. On the other hand, in the case where $\Delta p_i^d = 0, \forall i \in DN_k, k \in N_\infty$, we unravel TLA equilibria that are not SBR equilibria, so that the two coordination schemes are truly distinct, even if they share some common solutions.

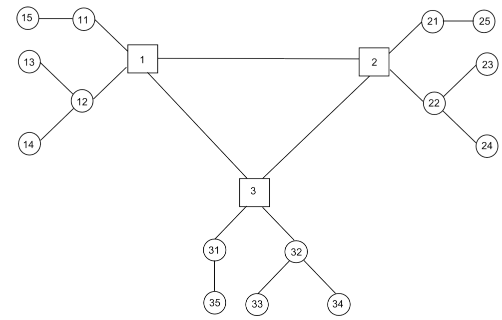

Figure 1: The toy example considered for testing the different coordination schemes.

4 RESULTS

We show preliminary results on a toy example[4]. The topology of the toy example on which the schemes have been tested is shown in Fig. 1. We have the TN with nodes 1-2-3 and 3 topologically identical DNs. The data is available in the following link[5]. Since our goal is to unveil multiple equilibria, we rely on the corollary 3.2 of [15]. The corollary implies that we obtain the entire set of equilibria if we solve the NTF problem for $(\gamma^T, \gamma^D) \in \{\gamma^T \geq 0, \gamma^D \geq 0, \text{ and } \forall k \in N_\infty, \gamma_k^T \gamma_k^D = 0\}$. In order to focus our analysis, we consider the same perturbation at each interface, which means that $\gamma_k^T = \gamma^T, \forall k \in N_\infty$. We also assume that $\gamma_k^D = \gamma^D, \forall k \in N_\infty$.

The results are summarized in Fig. 2. All the points represented are equilibria for their respective schemes and we reported only physically meaningful points, in other words, tight for the constraint (9).

4.1 SBR equilibria

The interpretation of the SBR scheme is fairly straightforward. Once the flow of real power is fixed at the T&D interface, if this flow does not correspond to the optimal flow it will result in sub-optimal dispatch. By fixing the interface flow, the SBR scheme decouples the problems of the agents and therefore results in an equilibrium, unless the value of the interface flow cannot be supported by a physically feasible dispatch at any of the sub-networks. The specific choice of flow at the interface will determine the extent of efficiency losses of the SBR scheme.

4.2 TLA equilibria

For this coordination scheme, as mentioned before, the theory does not guarantee that the solution of the NTF problem is an equilibrium, and one therefore needs to check that explicitly. It turns out that for the specific example of this paper, the SW point coincides with an equilibrium for this scheme. As we show in section 3.2.2, the equilibria to the left of the welfare solution are identical to the corresponding equilibria of SBR. The interpretation

[4]Larger test cases could have been considered. The idea here was to provide a proof of concept. NICTA/NESTA ([5]) test cases can be a good basis for further testing.
[5]http://docdro.id/dUQ0bRb

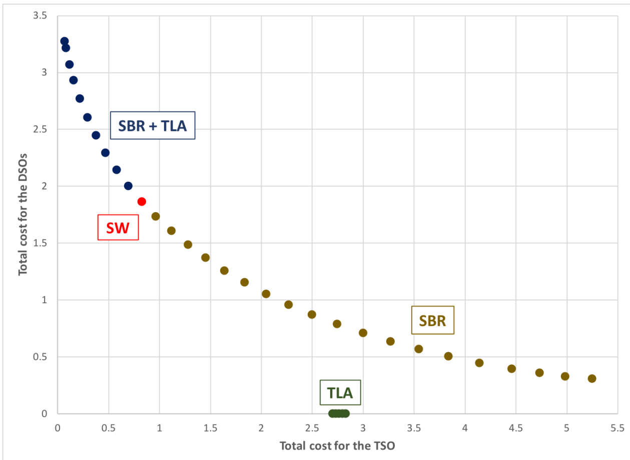

Figure 2: Equilibria for SBR and TLA. SW point is indicated in red and obtained for $\gamma_n^T = \gamma_n^D = 0$ in SBR. Note that the points at the left-hand side of SW point are the ones obtained for $\gamma^T > 0$ and $\gamma^D = 0$, and SBR and TLA points coincide as expected. At the right-hand side (for $\gamma^T = 0$ and $\gamma^D > 0$), we have 'continuity' for SBR and we obtain equilibria for TLA for values $\gamma^T = 0$ and $8.5 \geq \gamma^D \geq 8$.

of the right-hand side cluster of equilibria is more delicate. These are equilibria for which the TSO finds it worthwhile to activate such a large quantity of DN reserves, that these reserves are sufficient for both covering the DN disturbances, while also supporting the TN disturbances, even if part of the power is foregone as resistive losses on the DSO network (with equilibrium points further to the right corresponding to increasing real power losses on the DSO network). This turns out to be an equilibrium because (i) clearly for the DSOs there is no reason to deviate, since the TSO is paying for reserves that cover the DSOs' imbalance, while (ii) even when covering the DSOs imbalance and facing distribution system losses, the TSO still finds it preferable to activate distributed resources due to the fact that the transmission system reserve activation is a more expensive alternative.

In terms of social welfare, the equilibria furnished by the SBR scheme result in welfare values that might be lower in general than those of the TLA scheme. This is due to the number of equilibria that can generate SBR and how far they can get from the social welfare reference. It therefore appears that, for this specific numerical example, the SBR scheme seems not preferable to the TLA scheme even if TLA provides extremely interesting equilibria for the DSOs.

5 CONCLUSION

This paper is focused on the modeling of TSO-DSO coordination schemes. A centralized dispatch of the entire system is challenging due to the large size of the network. Decentralized models, where the computational effort would be separated and the privacy of information of each operator would be preserved, may be more viable in practice. We employ GNE in order to quantify the efficiency losses of two alternative TSO-DSO coordination schemes and analyze the results on a small-scale example. We tackle the resulting GNEP, which is known to be computationally challenging, using an algorithm inspired by the NTF method, and we unveil a multiplicity of equilibria for each scheme. Using our methodology, we unveil a free-riding effect in the TLA coordination scheme, whereby for some equilibria the DSO imbalances are entirely covered by the TSO, provided that the marginal cost of the distributed resources in the DSO network are substantially lower than the marginal cost of transmission-level reserves. Within the SmarNet project, such decentralized scheme would be, at least, tested in zonal markets (like Italy) or where local distributed resources are already important (like Denmark).

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