Application of Multilevel Demand Subscription Pricing for Mobilizing Residential Demand Response in Belgium

Yuting Mou
Center for Operations Research and Econometrics
Université catholique de Louvain
Voie du Roman Pays 34
Louvain la Neuve, Belgium
Email: yuting.mou@uclouvain.be

Anthony Papavasiliou
Center for Operations Research and Econometrics
Université catholique de Louvain
Voie du Roman Pays 34
Louvain la Neuve, Belgium
Email: anthony.papavasiliou@uclouvain.be

Philippe Chevalier
Center for Operations Research and Econometrics
Université catholique de Louvain
Voie du Roman Pays 34
Louvain la Neuve, Belgium
Email: philippe.chevalier@uclouvain.be

***Abstract*—The large-scale integration of renewable energy is progressing at an unprecedented pace. The highly variable, non-controllable and unpredictable variation of renewable energy supply requires fast-response generators for balancing, which results in increased operating costs and emissions. This ultimately undermines the economic and environmental benefits of renewable energy integration. These adverse effects can be mitigated by mobilizing the flexibility on the demand side. In this paper, we revisit a multilevel demand subscription pricing policy for electric power, in which a menu of service contracts for assigning different reliability combined with a non-linear energy price schedule is offered. The motivation of this paradigm is to provide differentiated products that can mobilize massive residential demand response resources while respecting the requirement of consumers for simplicity, privacy and control. In this paper, we apply multilevel demand subscription pricing on Belgium to design such a contract for residential consumers.**

I. Introduction

The increasing integration of renewable energy sources poses challenges to the operation of power systems. It is well-known that there exists substantial flexibility potential on the demand side [1], and with the advances of ICT, it is feasible to implement large-scale demand response for overcoming these challenges. The LINEAR project in Belgium unveils substantial demand response potential in the residential sector [2]. Flexible household appliances include white goods (i.e., dish washers, washing machines and tumble dryers), electric vehicles, electric water heaters, etc. In order to take advantage of the flexibility of these appliances, differentiated electricity products are necessary. By offering differentiated products, the heterogeneity among the population of consumers can be exploited, thereby mobilizing their flexibility and achieving greater allocative efficiency.

Following this stream, a deadline differentiated pricing policy for deferrable electric loads is proposed in [3], which mainly applies to electric vehicle charging parks equipped with variable renewable supply [4]. Another differentiation scheme based on the duration of service and the power level is presented in [5], which is referred to as duration differentiated energy service. The authors study a forward market where loads require a fixed power for a specified duration and the supply is deterministic. Inspired by these two studies, a duration-deadline jointly differentiated energy service is put forward in [6], where the authors consider a group of flexible loads with each load requiring a constant power level for a specified duration before a specified deadline. In addition, rate-constrained energy services [7] are also a recent approach for mobilizing flexible demand, where service is characterized by a delivery window, the total amount of energy that must be supplied, and the maximum rate at which this energy may be delivered. Nevertheless, these approaches mainly cater for storage-like appliances and they are not suitable for mobilizing demand response in a household because consumers should retain their authority over electricity consumption and control should only be imposed behind the meter.

Another paradigm based on priority service is proposed in [8], with the underlying economic theory dating back to [9]. In this paradigm, electricity supply is perceived as a service that can be offered with various degrees of reliability. Priority service pricing is demonstrated with the Belgian case study in [10]. In this paper, we implement a richer rate structure called multilevel demand subscription pricing, that differentiates electric power according to both service reliability and load pattern, using the theory proposed in [11]. In this scheme, the price is decomposed into energy charges and capacity charges based on the duration of load slices and reliability, respectively. This pricing induces customers to flatten their load pattern over time. Additionally, it induces customers to select lower reliability levels for the peak portion of their load.

The goal of this paper is to apply multilevel demand subscription pricing for designing a contract for residential consumers in Belgium, and to illuminate certain implementation challenges that emerge. We first revisit multilevel demand subscription pricing theory in Section II. Section III presents the process for designing such a price menu for Belgium. Section IV compares the new price menu with the random rationing policy using a case study and describes the practical implementation of the menu. Section V concludes the paper and provides directions for future research.

II. Multilevel Demand Subscription Pricing Theory

This section presents multilevel demand subscription pricing theory based on [11]. According to [11], the system load duration curve is treated as a collection of load slices and the authors consider a price plan that offers the option of selecting a different duration time and reliability level for each load slice. The general form of the price plan is $p(r,t)$, which is the total charge for a load slice of duration t at specified reliability level r. The reliability r is defined as the long-run average fraction of a load slice that will be served when reliability r is selected. Each load slice is priced independently. This section proceeds with the consumer choice model and the cost model, and then describes how one designs such a price menu with the objective of maximizing social welfare.

A. Consumer model

Aggregate consumer response to price is defined by the function $L(p,t)$, which describes a family of load duration curves, parametric on a uniform price p. We consider the current average price p_0 and random conditions ω, then the actual load duration under realization on ω is denoted by $h(\omega)L(t)$, where $L(t)$ is the average load duration curve and $h(\omega)$ is a scaling function assumed to be increasing with ω. We also define the maximum willingness to pay for the load slice at level L when its duration is t as $v(L,t)$. Then the optimal pair $\{r(L),t(L)\}$ is determined by solving the consumers' surplus maximization problem

$$\max_{0\leq r\leq 1, 0\leq t\leq T}\{S(r,t,L)=H(r)[v(L,t)-p(r,t)]\}, \quad (1)$$

where

$$H(r)=\int_0^r h(\omega)d\omega, \text{with } H(0)=0, H(1)=1 \quad (2)$$

B. Cost model

The operating cost attributed to an individual load slice is denoted by $c(r,t,L)$, which is the average cost for serving a load slice of level L with duration t and service reliability r. The function $c(r,t,L)$ is assumed to be linear in t.

C. Determining the optimal $\{r(L),t(L)\}$ trajectory

Using L to denote the index of each load slice, the social welfare maximization problem[1] can be formulated as

$$\max_{r(L),t(L),L_0}\sum_{L=1}^{L0}\{H(r(L))v(L,t(L))-c(r(L),t(L),L)\} \quad (3)$$

$$s.t.\quad 0\leq r(L)\leq R(L) \quad (4)$$

$$0\leq t(L)\leq T \quad (5)$$

$$r(L+1)\leq r(L) \quad (6)$$

$$t(L+1)\leq t(L) \quad (7)$$

[1]The authors in [11] use integration whereas we discretize the formulas for the sake of implementation.

where L_0 is the cutoff level, beyond which no service is offered to consumers. $R(L)$ is the maximum reliability that the system can offer to a given load level L and T is the horizon of service (e.g. 8760 hours for a one-year contract). We explain how this function can be estimated numerically in Section III.

D. Determining the optimal price function

Given the optimal trajectory $\{r(L),t(L)\}$, we need to derive the optimal price function that induces consumers to choose the optimal trajectory when they maximize their surplus. Denote the optimal price function as $P(L)=p[r(L),t(L)]$, then as shown in [11], $P(L)$ is calculated as

$$P(L)=v[L,t(L)]+\{1/H(r(L))\}\sum_{l=L+1}^{L_0}H(r(l))v_L(l,t(l)), \quad (8)$$

where v_L is the partial derivative of $v(L,t)$ with respect to L. Furthermore, $p(r,t)$ can be represented as an additively separable function $p(r,t)=f(t)+g(r)$, which decomposes the price function into an energy price $f(t)$ and a capacity price $g(r)$. Assuming $g(r)=0$ at the lowest reliability level, $f(L)=f(t(L))$ and $g(L)=g(r(L))$ are computed by the following formulas [11]:

$$f(L)=v(L_0,t(L0)) - \sum_{\tau=t(L+1)}^{t(L_0)}\{v_t(L(\tau),\tau)\cdot(t(L+1)-t(L))\} \quad (9)$$

$$g(L)=P(L)-f(L) \quad (10)$$

III. Applying Multilevel Demand Subscription Pricing in Belgium

In this section, we apply multilevel demand subscription pricing theory, as presented in the previous section, to the Belgian residential sector. We consider a scenario of year 2050 based on [12], which corresponds to a high renewable penetration scenario. Firstly, we calibrate the value function for residential consumers and describe how we estimate cost functions on the supply side. Then the social welfare maximization problem is solved in order to obtain the optimal $\{r(L),t(L)\}$ trajectory defined in the previous section. Subsequently, we develop price menus with a finite number of options.

A. Calibration of value function

One of the main ingredients for designing the tariff is the value function $v(L,t)$. According to [11], it can be estimated from the system load duration curve $L(t)$ that results from consumers' response to a uniform energy price. As a result, the first step towards estimating the value function is to approximate the system load duration curve. Given a constant price p_0, we use the following closed-form expression to approximate the load duration curve

$$L(p_0,t)=-\frac{a_1}{b}\Big(\frac{t}{T}\Big)^{\frac{1}{\lambda}}-\frac{p_0}{b}+\frac{a_0}{b},$$

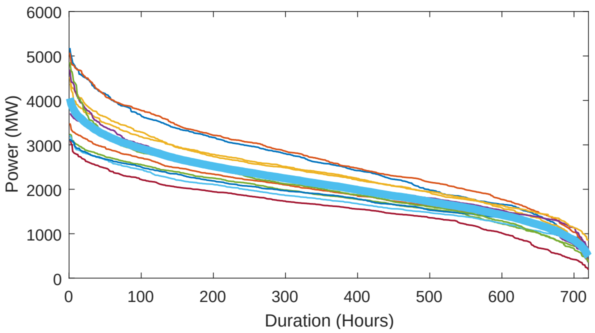

Fig. 1. Thin curves show the net residential load duration curve of each month, while the thick curve describes the average net residential load duration curve.

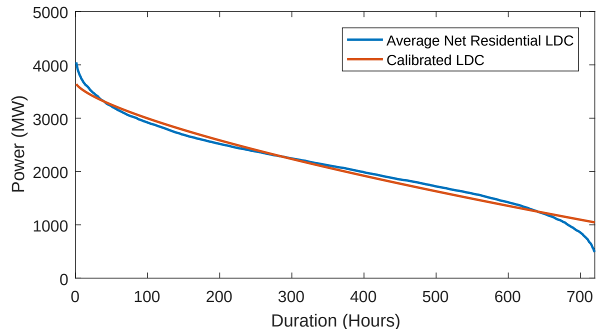

Fig. 2. Comparison between the actual average net residential load duration curve and the calibrated one for the year 2050.

where parameters a_0, a_1, b and λ can be estimated using the average elasticity of demand. The details are available in the appendix.

From the expression of the system load duration curve, we can calculate the valuation function. According to [11],

$$v_t(L,t) = p_0 = -a_1\Big(\frac{t}{T}\Big)^{\frac{1}{\lambda}} - bL + a_0. \tag{11}$$

taking the antiderivative of this partial derivative yields

$$v(L,t) = -a_1\Big(\frac{1}{T}\Big)^{\frac{1}{\lambda}}\Big(\frac{1}{\frac{1}{\lambda}+1}\Big)\cdot t^{\frac{1}{\lambda}+1} + (a_0 - bL)t + f(L). \tag{12}$$

Clearly, $v(L,0) = 0$ for all values of L. Hence,

$$\begin{aligned} v(L,t) &= -a_1\Big(\frac{1}{T}\Big)^{\frac{1}{\lambda}}\Big(\frac{1}{\frac{1}{\lambda}+1}\Big)\cdot t^{\frac{1}{\lambda}+1} + (a_0 - bL)\cdot t \\ &= -a_1\Big(\frac{1}{T}\Big)^{\frac{1}{\lambda}}\Big(\frac{\lambda}{\lambda+1}\Big)\cdot t^{\frac{1}{\lambda}+1} + (a_0 - bL)\cdot t \end{aligned} \tag{13}$$

$v(L,t)$ can be interpreted as the valuation of a slice of power (1 MW) indexed by L, with a duration of t (hours).

As we only consider residential consumers, the system load duration curve actually refers to the residential load duration curve. We use synthetic load profiles (SLP) [13] to estimate the residential load profile and the parameter p_0 is set equal to the average electricity price for the year 2015 and is available from [14]. Renewable production is assumed to be behind the meter[2], and in order to calculate a scaling function $h(\omega)$, we use the average of monthly net load duration curve to calibrate the function $L(p_0,t)$. Fig. 1 shows the monthly net load duration curves. The scaling function is computed as the ratio of energy consumption in a certain month to the average monthly electricity consumption, so $h(\omega)$ takes 12 different values[3].

[2]This implies that we consider uncertainty in renewable production as emanating purely from the demand side [11]. In contrast, uncertainty in the available capacity of conventional generators is represented by the expected capacity.

[3]This implies that we model the state of world ω on the basis of the 12 monthly load duration curves that we observe in the historical data of Belgian consumption.

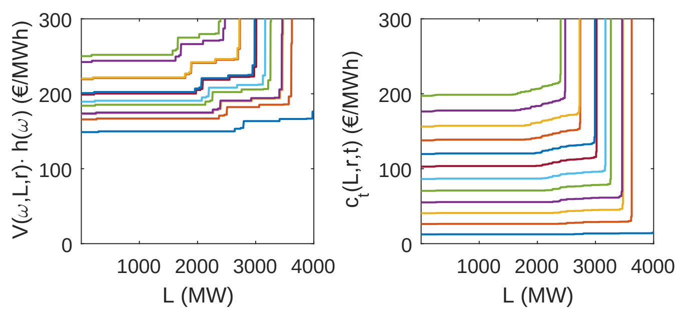

Fig. 3. Cost in each scenario ω and $c_t(r,L)$, for different load levels and different reliability levels. Cost is higher when reliability is higher.

Fig. 2 compares the actual average net load duration curve to the calibrated one which is obtained using the approach in the appendix.

B. Estimation of cost functions

In our framework, industrial and commercial (IC) demand is assumed to be served first, so IC demand is fixed at its average historical value. In addition, following the development in [11], uncertainty only comes from demand side, i.e., scaling the thickness of load slice dL using scaling function $h(\omega)$. The uncertainty in renewable production is considered on the demand side since we use the net load duration curve.

The entire conventional generator fleet consists of 55 units, with a capacity of 15.78 GW. The long-term maintenance schedule of units is accounted for by derating the maximum capacity of the units. Run-of-river resources and imports follow average historical values. We run economic dispatch in order to obtain $c(r(L),t(L),L)$, which is assumed to be linear in t. The cost functions are shown in Fig. 3. Transmission and distribution costs, and taxes are added on top of the production cost since these charges are included in p_0.[4]

[4]The cost of load level L in scenario ω is denoted by $C(\omega,L)$ and is calculated as follows. First, run economic dispatch with demand equal to $h(\omega)(L-1)$ and we obtain the corresponding production cost $prodCost(\omega, L-1)$. Similarly, we can obtain $prodCost(\omega,L)$. Let TD denote other charges including transmission and distributing costs. Then $C(\omega,L) = prodCost(\omega,L) - prodCost(\omega,L-1) + h(\omega)TD$. Subsequently, $c(r(L),t(L),L)$ is calculated as the average of $C(\omega,L)$ multiplied by t.

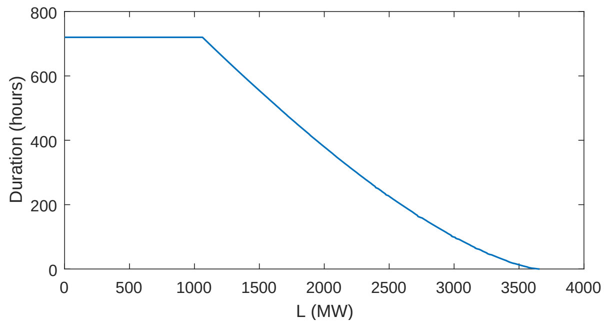

Fig. 4. Optimal duration of each load level.

C. Design the menu

There are two steps towards deriving the menu as presented in the previous section. The first step is to determine the optimal $\{r(L), t(L)\}$ trajectory with the objective of maximizing social welfare.

To simplify problem (3) - (7), we can ignore constraints (6) and (7) first and come back to check them later. Then the problem is decoupled by L. Moreover, considering that there are only 12 cases of reliability ($h(\omega)$), the problem can be easily solved by traversing over $r_i(L), i = 1...12$, i.e., enumerating all cases of $r_i(L)$, solving for t, comparing the value of the objective function (social welfare) and choosing the best value for $r_i(L)$. For each L, we need to solve

$$\max_{t(L)} \quad H(r_i) \cdot v(L, t(L)) - c(r_i, t(L), L) \tag{14}$$

$$s.t. \quad 0 \leq t(L) \leq T \tag{15}$$

However, the constraint $t(L+1) \leq t(L)$ is sometimes violated when the reliability jumps to a lower level. We implement an 'ironing' procedure to make the the optimal duration non-increasing [15], i.e., replacing the increasing part with a non-increasing piece-wise linear function. The optimal $t(L)$ and $r(L)$ are demonstrated in Fig. 4 and Fig. 5.

With the optimal $\{r(L), t(L)\}$ trajectory in hand, the optimal prices for each load level L can be calculated according to Eqs. (8), (9) and (10)[5]. The price menu functions $P(L)$, $f(L)$ and $g(L)$ are illustrated in Fig. 6.

Fig. 7 illustrates cost $c(r(L), t(L), L)$, price $P(L)$ and valuation $v(L, t(L))$ for each slice, which demonstrates how the designed price menu can achieve maximization of social welfare while respecting the desire of consumers to maximize their surplus.

After calculating the price functions, we can compute the actual charge imposed on consumers as follows:

$$\text{total charge of } L = H(r(L))g(L) \quad \text{(capacity charge)} \tag{16}$$

$$+ H(r(L))f(L) \quad \text{(energy charge)}. \tag{17}$$

We then discretize load level L into several classes, leading to a price menu with several options that can be chosen by

[5]The calculation is performed in MATLAB using *cumtrapz* function, which is based on cumulative trapezoidal numerical integration. The readers can refer to [16] for the scripts used in this paper

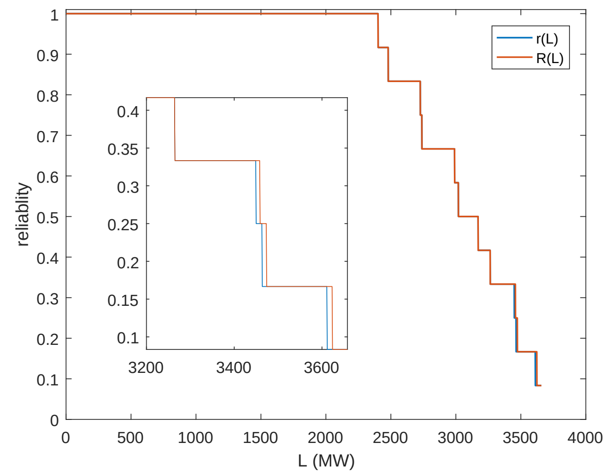

Fig. 5. Comparison between the optimal reliability $r(L)$ and the highest reliability that the system can offer $R(L)$.

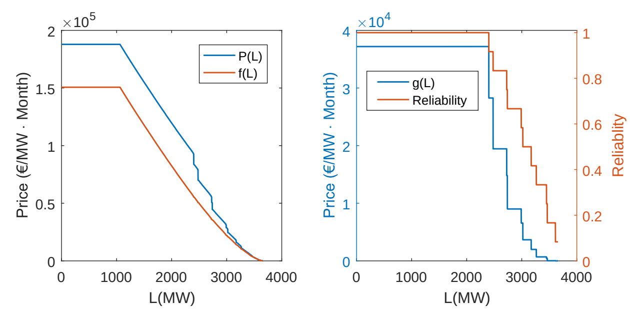

Fig. 6. The left panel illustrates the total price and energy price of the corresponding power slice. The right panel shows the capacity price and the corresponding reliability level.

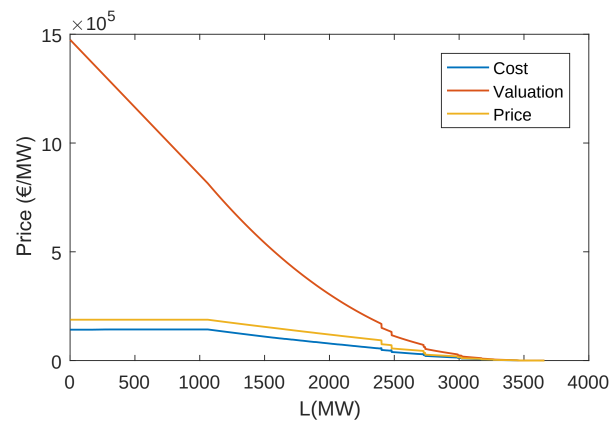

Fig. 7. Cost $c(r(L), t(L), L)$, price $P(L)$ and valuation $v(L, t(L))$ for each slice. The difference between the valuation curve and the cost curve shows social welfare, whereas the difference between the price curve and the valuation curve describes consumers' surplus. Since the objective is to maximize social welfare, at cut-off level L_0, the valuation curve should crosses the cost curve. Since consumers wish to maximize their surplus, the valuation curve crosses the price curve at L_0.

consumers. The price menu is shown in Table I. L^- and

TABLE I
PRICE MENU FOR YEAR 2050 WITH 10 OPTIONS

L^-	L^+	r	t	Total Charge (€/kW·mo.)	Capacity Charge (€/kW·mo.)	Energy Charge (€/kWh)
1	1124	1	1	187.845	37.2	0.209
1125	1311	1	0.9327	178.220	37.2	0.210
1312	1561	1	0.8023	159.563	37.2	0.212
1562	1890	1	0.6766	141.522	37.2	0.214
1891	2346	1	0.4802	112.495	37.2	0.218
2347	2698	0.85	0.2891	55.479	17.8	0.222
2699	2989	0.67	0.1844	23.689	5.7	0.222
2990	3252	0.48	0.1024	8.513	1.5	0.227
3253	3499	0.26	0.0307	1.345	0.1	0.262
3500	3644	0.08	0.0013	0.013	0.0	0.221

L^+ show the lowest and highest load level that will choose this option. The variables r and t describe the reliability and duration (normalized to 1) that this option can offer. The total charge and capacity charge are monthly. It can be seen that capacity charge is relatively high, which is due to the system being stressed. In contrast, the energy charge is similar to the current average electricity price in Belgium, which amounts to 0.213€/kWh.

IV. PRACTICAL IMPLEMENTATION AND ANALYSIS OF RESULTS

A. Practical implementation

This price menu should be implemented intuitively in practice, so that the information processing burden on consumers is minimized. Assuming that consumer loads are synchronized with the total system load, then the menu can be offered as a time-of-use rate with capacity charges. This scheme is illustrated in Fig. 8 with an example of a price menu with 3 options. In Fig. 8-(1), 3 options with different duration, reliability and price are offered to a consumer. Then the consumer can subscribe to each option with different quantities of capacity (kW) based on the load duration curve shown in Fig. 8-(2), which is a mapping from the consumption profile in Fig. 8-(3). So the real time intervals corresponding to each option are deducted as in Fig. 8-(4).

At each interval, a certain amount of capacity in each option (reliability) is available to the consumer and a smart meter records the capacity consumed so that the consumer is constrained at the meter. In case of shortage, the utility company can choose a reliability level r and the capacity with a lower reliability level than r is curtailed, in order to balance total supply and demand. However, within the household, the consumer can assign different reliability levels to different appliances in a dynamic way, possibly using a smart plug as shown in Fig 9.

The color code can be associated with the menu as follows. The consumer subscribes to option 1 with the capacity of ΔL_1, corresponding to the green color. This option has the lowest reliability level and least capacity charge. Thus, any appliances assigned to the green color can be potentially shut down at any time. In contrast, the consumer subscribes to option 3 with a

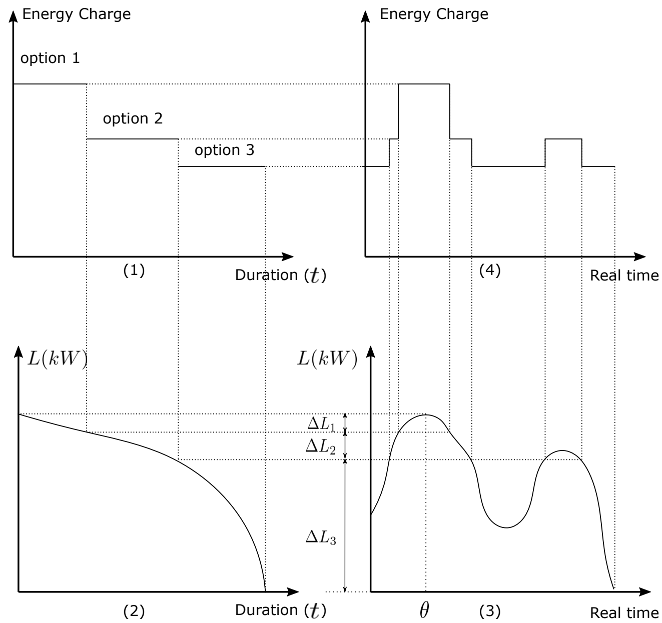

Fig. 8. Implementation of the price menu as a time-of-use tariff with capacity charges.

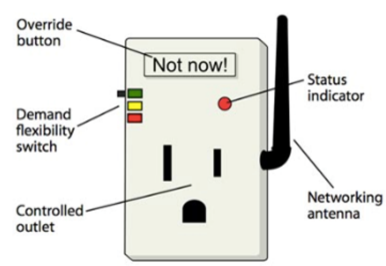

Fig. 9. Illustration of a smart plug. Different colors of the buttons refer to different reliability levels. Source: http://mitei.mit.edu/news/tomorrows-power-grid.

capacity of ΔL_3, corresponding to the red color and appliances are only turned off to prevent a blackout or other emergency.

B. Analysis of results

In this section, we follow the price menu presented in Table I and compare the following three policies in Table II.

(1) Optimal economic dispatch: each load slice L is assigned with the optimal duration and reliability.
(2) Multilevel demand subscription price menu: given several options of duration and reliability pair in the price menu, each load slice L chooses the option that maximizes its surplus.
(3) Random rationing: each load level L is rationed with the same proportion in case of scarcity.

Compared with random rationing, the social welfare increases by 4.48% under the optimal policy and by 3.84% if the proposed price menu is offered to consumers. The served energy also increases slightly. Regarding the social welfare per MWh energy, the menu achieves an increase of 3.58%.

TABLE II
COMPARISON OF THREE POLICIES

Policy	Optimal	Menu	Rationing
Served Energy (MWh)	17878761	17878953	17828829
Production Cost (M€)	3597	3656	3588
Consumer benefits (M€)	22324	22269	21512
Social Welfare (M€)	18726	18612	17924
Social Welfare (€/MWh)	1047	1041	1005

V. CONCLUSION

In this paper, we review multilevel demand subscription pricing and present its application to the Belgian market. We illustrate how this menu can be put into practice so as to mobilize residential demand response and the case study shows significant efficiency benefits of the price menu compared to random rationing.

There are two directions that we plan to consider in future research. Firstly, the effect of this new tariff on day-ahead market is worth evaluating. Secondly, we are interested in developing a model for a single household and investigating how a single household would react to the new tariff.

APPENDIX

Due to the page limitation, the appendix is available on the author's website [16].

ACKNOWLEDGMENT

The authors would also like to thank the Fair Isaac Corporation FICO for providing licenses for Xpress.

This research has been supported by the ENGIE Chair in Energy Economics and Energy Risk Management and by the ENGIE-Electrabel ColorPower grant.

REFERENCES

[1] H. C. Gils, "Assessment of the theoretical demand response potential in europe," *Energy*, vol. 67, pp. 1–18, 2014.

[2] R. Dhulst, W. Labeeuw, B. Beusen, S. Claessens, G. Deconinck, and K. Vanthournout, "Demand response flexibility and flexibility potential of residential smart appliances: Experiences from large pilot test in belgium," *Applied Energy*, vol. 155, pp. 79–90, 2015.

[3] E. Bitar and S. Low, "Deadline differentiated pricing of deferrable electric power service," in *Decision and Control (CDC), 2012 IEEE 51st Annual Conference on*. IEEE, 2012, pp. 4991–4997.

[4] E. Bitar and Y. Xu, "Deadline differentiated pricing of deferrable electric loads," *IEEE Transactions on Smart Grid*, vol. 8, no. 1, pp. 13–25, 2017.

[5] A. Nayyar, M. Negrete-Pincetic, K. Poolla, and P. Varaiya, "Duration-differentiated energy services with a continuum of loads," *IEEE Transactions on Control of Network Systems*, vol. 3, no. 2, pp. 182–191, 2016.

[6] W. Chen, L. Qiu, and P. Varaiya, "Duration-deadline jointly differentiated energy services," in *Decision and Control (CDC), 2015 IEEE 54th Annual Conference on*. IEEE, 2015, pp. 7220–7225.

[7] M. Negrete-Pincetic, A. Nayyar, K. Poolla, F. Salah, and P. Varaiya, "Rate-constrained energy services in electricity," *IEEE Transactions on Smart Grid*, 2016.

[8] S. S. Oren, "A historical perspective and business model for load response aggregation based on priority service," in *System Sciences (HICSS), 2013 46th Hawaii International Conference on*. IEEE, 2013, pp. 2206–2214.

[9] H.-p. Chao and R. Wilson, "Priority service: Pricing, investment, and market organization," *The American Economic Review*, pp. 899–916, 1987.

[10] Y. Mou, A. Papavasiliou, and P. Chevalier, "Application of priority service pricing for mobilizing residential demand response in Belgium," in *European Energy Market (EEM), 2017 14th International Conference on the*. IEEE, 2017, pp. 1–5.

[11] H.-p. Chao, S. S. Oren, S. A. Smith, and R. B. Wilson, "Multilevel demand subscription pricing for electric power," *Energy Economics*, vol. 8, no. 4, pp. 199–217, 1986.

[12] European Commission, "EU reference scenario 2016," October 2017. [Online]. Available: https://ec.europa.eu/energy/en/data-analysis/energy-modelling

[13] Synergrid. (2017, Oct.) Synthetic load profiles (SLP). [Online]. Available: http://www.synergrid.be/index.cfm?PageID=16896

[14] Eurostat. (2017, Jan.) Electricity price statistics. [Online]. Available: http://ec.europa.eu/eurostat/statistics-explained/index.php/Electricity_price_statistics

[15] R. B. Wilson, *Nonlinear pricing*. Oxford University Press, 1993.

[16] Y. Mou, "Source code for submission to energycon2018," October 2017. [Online]. Available: https://sites.google.com/site/yutingmouchina/source-code/energycon2018