Optimal Dispatch of Wind Farms Facing Market Prices

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***Abstract*—At present, wind power producers (WPP) are paid following feed-in tariffs in Belgium. This system will come to an end soon due to its high cost and the producers will have to bid in the day-ahead market. As wind owners cannot forecast their production perfectly, they will face imbalance costs or revenues. Imbalance price forecasting is therefore a critical problem. In this paper, we implement a machine learning model to assess the usefulness of introducing exogenous variables in imbalance price forecasting. This method shows improved results compared to classical methods. Since the imbalance price is obtained by the marginal cost of producing the missing energy, the strategic behaviour of a WPP will influence the imbalance price. In this paper, we propose a way to represent this influence as well as a formulation of a model to obtain the optimal bidding strategy in that situation. This model has been cast as a convex quadratic program that can readily be solved using a commercial solver.**

***Index Terms*—Wind power, electricity market, imbalance market, optimization, machine learning.**

I. Introduction

A. Motivation

The share of renewable energy supply has increased substantially in recent years and will continue in the future due to the ambitious goal [1] of the European Commission to cover 20% of European energy consumption by renewable energy supply by 2020 and 27% by 2030. To this end, Belgium has used feed-in tariffs, but this will change in the future. This change of remuneration mechanism induces the need for new bidding strategies for Belgian wind producers. In order to develop these strategies, models of imbalance prices are needed. Moreover, due to the increasing size of wind power portfolios, the owner cannot make anymore the assumption that he does not influence the imbalance price. Therefore modelling this influence should be an important concern for WPPs.

B. Literature review

One of the first papers in which the problem of bidding strategies for wind farms is considered is [2]. A Markov chain is used to model the transition between the production outputs at two consecutive periods, afterwards a policy is implemented and compared with several classical strategies for wind forecasting (e.g. persistence, persistence with scaling factor) and the revenue is computed with respect to different values of imbalance prices. In subsequent literature, bidding strategies are proposed for the two-settlement market in the Nordic system [3]. An improved formulation of the model has been given in [4] in which the authors cast the model as a linear program. Additionally, risk aversion has been introduced in [4] and [5]. Some work has been done to include an intraday market in the model in [4] and [6]. The influence of the dependence between wind production and imbalance price has been studied in [7]. A closed form solution and sensitivity analysis have been performed in [8].

C. Contribution

Our contribution is twofold. Firstly, we implement a model for imbalance price forecasting based on machine learning methods to test the usefulness of exogenous variables for imbalance price forecasting. This model has been compared to other forecasting methods for a hypothetical wind farm site in Belgium. Secondly, we develop a procedure to model the influence of our wind farm producer on the imbalance market and cast this problem as a convex quadratic program. We find an approximate closed form solution and compare it with the solution of the price taker model.

D. Paper organization

In order to analyse optimal bidding strategies, we use the approach shown in Fig. 1. The methodology includes generating production scenarios, price scenarios and using these scenarios to obtain the optimal bids by solving a stochastic program. In section 2, we describe briefly the Belgian market. Section 3 explains the model used to generate power scenarios. In section 4, we present different methods for forecasting imbalance prices. Section 5 recalls the price-taking model for obtaining the bids. Section 6 presents a model that takes into account the influence of the producer on the imbalance market. In section 7 we present our results on a test case.

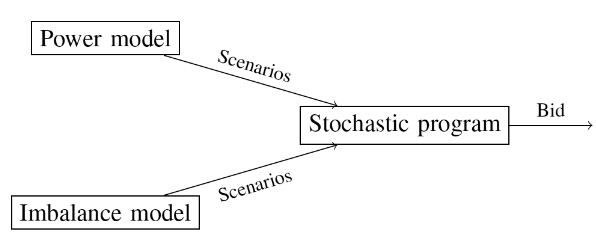

Fig. 1. Model decomposition in principal subtasks and interaction between these subtasks.

II. Overview of the Belgian market

In Belgium, there are two markets in which a wind farm producer can participate:

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- In the day-ahead market, producers need to submit their bids before 12AM the day before.
- In the intraday market, which is not modelled in this paper due to its lack of liquidity.

After these markets, if the producer does not produce its cleared quantity, it will have to pay the difference at the imbalance price. The imbalance settlement scheme is explained in table I, inspired from [9].

The price that will be charged to the producer depends on two properties

1) the state position: of the system either long (too much production) or short (not enough production)
2) the state of the producer: either over or underproduction.

Let us first define the notation used in table I, afterwards we will present an example to illustrate how to read this table:

- **MIP** : Highest price of activated bid to increase production.
- **MDP** : Lowest price of activated bid to decrease production.
- α : Additional incentive applied on top of the regulation costs in case of major system imbalances. It is equal to zero if the system imbalance is lower than 140MW.

Let us present an example in order to illustrate how to use the table. We consider the case in which the market is short in production and the producer produces less than its day-ahead cleared quantity. As the market requires more power, it activates upward regulation and since the producer produces less than its promised quantity, it needs to buy the difference in megawatts at MIP $+\,\alpha$. It is worth noting that the producer always pays more for being in underproduction than what it is paid for being in overproduction.

TABLE I
IMBALANCE SETTLEMENT RULES.

	Net downward regulation	Net upward regulation
Overproduction	MDP $-\,\alpha$	MIP
Underproduction	MDP	MIP $+\,\alpha$

III. Wind speed model

We have at our disposal a day-ahead forecast $\hat{y}$ and the realization of wind speed y. Moreover, we have the power curve (curve that maps wind speed to power) of the wind farm. As the wind forecast is not perfect, we will generate error scenarios to model the different possible outcomes. Let us explain the procedure:

- First we obtain the error time series: $e = y - \hat{y}$.
- Then we suppress the seasonality: $\frac{e-\mu_s}{\sigma_s}$ with s a given period (a month and an hour).
- After that, we fit an ARMA model. The general formulation of an ARMA(p,q) is

$$X_t = \sum_{i=1}^{p} \alpha_i x_{t-i} + \sum_{j=1}^{q} \theta_j \epsilon_{t-j} + \epsilon_t$$

  It makes sense to use an autoregressive process to generate the error scenarios because we expect a certain continuity in the error process. Indeed, if we overestimate the wind speed at a quarter, it is likely the case that we will also overestimate it for the next quarters.
  The problem is that, as shown in Fig. 2, we have discontinuity in the series from the last quarter of a given day to the first quarter of the following day. This figure shows the wind speed forecast for two consecutive days and the moment at which these forecasts are made. The forecast of the first day is made at hour 11 and the forecast of the second day is made at hour 35. In a given day, we know that the forecast error of a quarter will likely be propagated to the following quarters. On the contrary, it is not the case between the last quarter of a day and the first quarter of the following day because for the second day, the producer has received more information which can be used for forecasting. For this reason, in order to model the first quarter of the day, we will draw random points from past realizations (at the first quarter, due to the discontinuity, we do not have any starting point for an autoregressive model). To forecast the second quarter we only have access to one lag of the process (X_{t-1}) and zero lag from the error. Indeed, we do not know the value of two quarters ago. Therefore, we can only use the best possible AR(1). For the third period, we can use the best ARMA(1,2). This is represented in Fig. 3 for the case in which the best possible model is an ARMA(3,3). After determining the order of the ARMA model, we can learn the parameters of the model by using maximum likelihood estimation.
- Finally, we convert wind speed to production by using the power curve of the wind farm.

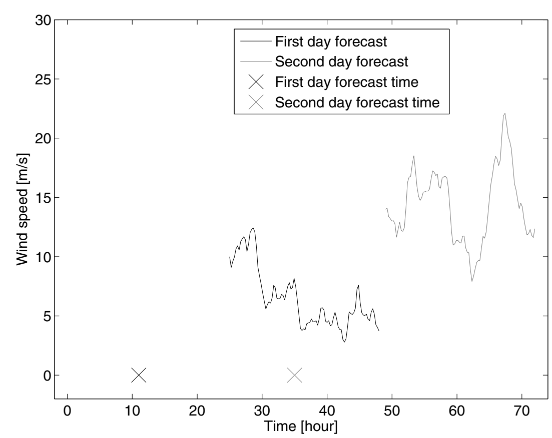

Fig. 2. Wind forecast of two days and the time at which there are made.

IV. Imbalance price forecasting

In this section, we will describe different methods we test for imbalance price forecasting. The aim of these methods is to generate a point forecast[1]. We will denote this forecast as $\bar{p_{\text{imb}}}$. Initially, we introduce three base cases using only

[1]The reason for which we do not generate scenarios will be explained in the optimization model part.

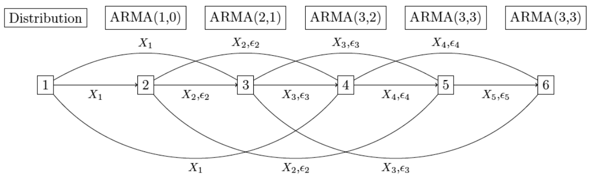

Fig. 3. Available data for the producer for each quarter and the best associated model.

past information of the series, afterwards we propose some relevant features for forecasting imbalance prices, next we propose a procedure for selecting the useful features. Finally, we propose two models for forecasting imbalance prices with these exogenous variables.

A. Base cases

The aim of these base cases is to have a reference in order to check if we get improvements by using exogenous variables.

- The value of the same quarter the day before (two days before if the quarter is after 12 AM).

$$\bar{p}_{\text{imb},t,q} = \begin{cases} p_{\text{imb},t-1,q} & \forall q \leq 44 \\ p_{\text{imb},t-2,q} & \forall q > 44 \end{cases}$$

- The mean value of the day before.

$$\bar{p}_{\text{imb},t,q} = \frac{1}{96}\left(\sum_{l=1}^{44} p_{\text{imb},t-1,l} + \sum_{l=1}^{52} p_{\text{imb},t-2,97-l}\right)$$

- The mean value of the same quarter in the last 50 days.

$$\bar{p}_{\text{imb},t,q} = \begin{cases} \frac{1}{50}\sum_{k=1}^{50} p_{\text{imb},t-k,q} & \forall q \leq 44 \\ \frac{1}{50}\sum_{k=1}^{50} p_{\text{imb},t-k-1,q} & \forall q > 44 \end{cases}$$

B. Exogenous data

We now proceed with describing two alternatives to the base cases that rely on explanatory factors: (i) a linear model, (ii) radial basis function networks. To build our model we require exogenous variables. We consider the following explanatory factors for these models:

- r_1, the index of the quarter in the year of the price we want to forecast (a number between 1 and 35040).
- r_2, the index of the quarter in the day of the price we want to forecast (a number between 1 and 96).
- r_3, the available generation forecast for the considered quarter.
- r_4, the load forecast for the considered quarter.
- r_5, the solar forecast expectation for the considered quarter.
- r_6, the wind forecast expectation for the considered quarter.
- r_7, the wind forecast 10^{th} quantile for the considered quarter.
- r_8, the wind forecast 90^{th} quantile for the considered quarter.
- r_9, the day-ahead price for the hour in which the quarter is.
- r_{10}, the last available gas price at Zeebrugge hub.
- r_{11}, the imbalance price mean of the the last day.

C. Feature selection

From this initial set of features, we will try to find the minimal subset of features that maximizes mutual information. The mutual information is defined as:

$$I(p_{\text{imb}}^{-}, x_i) = H(p_{\text{imb}}^{-}) - H(p_{\text{imb}}^{-} \mid x_i)$$

with $x_i \in P(\{r_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8, r_9, r_{10}, r_{11}\})$.[2]
The mutual information could be understood as the difference between the entropy of p_{imb}^{-} and the entropy of p_{imb}^{-} when x_i is known. Consequently, mutual information is always positive and is equal to 0 if the two variables are independent.
In our case, for every subset of features, we compute the mutual information and we take the subset of the smallest size that gives the maximum mutual information.

Regression model

Having selected relevant features, our goal now is to fit a parametric function f_α that links the input x (which is the optimal set of features obtained by the feature selection process) and the output p_{imb}^{-} by the relation $p_{\text{imb}}^{-} = f_\alpha(x)$. These parameters are estimated from historical data using the following two alternative approaches.

1) Linear model: The regression function is defined as:

$$f_w = w^T x + w_0$$

To compute the weights w and w_0, we minimize the sum of the quadratic error for the N historical data.

$$\min_{w,w_0} \sum_{k=1}^{N} \|\bar{p}_{\text{imb},k} - w^T x_k - w_0\|_2^2$$

2) Radial basis function network: This method is nonlinear, the regression function is defined by:

$$f_{c,\sigma,w}(x) = \sum_{i=1}^{n} w_i \phi(x, c_i, \sigma_i)$$

The idea of this method is to take a weighted sum of basis function $\phi(x, c_i, \sigma_i)$. In our case, we use the following function as our basis function:

$$\phi(x, c_i, \sigma_i) = \exp\left(\frac{- \mid\mid x - c_i \mid\mid_2^2}{2\sigma_i^2}\right)$$

Each Gaussian function is centred at a centroid c_i and has a standard deviation σ_i to represent its influence range. For more details about RBFN, the reader may refer to [10].

V. PRICE TAKER MODEL

As we consider a wind farm, we do not have any coupling constraints between different quarters. Therefore we can solve the problem independently for each quarter. The aim is to obtain a function $f(p_{\text{DA}})$ that will indicate for a given day-ahead price which quantity should be bid. We use the following notation.

- q_{DA}: the bid quantity in the day-ahead market.
- $q_{imb,p,i}$: the excess production in real time.
- $q_{imb,n,i}$: the missing production in real time.

[2] $P()$ represents the power set.

- p_{DA}: the day-ahead price.
- $p_{imb,p,j}$: the price paid for $1MW$ of excess production in scenario j.
- $p_{imb,n,j}$: the price paid by the wind producer for a missing megawatt in scenario j.
- Prod_i: the wind farm production in scenario i.
- r_i: the probability of wind power production scenario i.
- s_j: the probability of imbalance price scenario j.
- I: the set of wind scenarios.
- J: the set of imbalance price scenarios.
- P_{WF}: the wind farm maximal production.

The optimization problem that wind power producers are called to solve in the day-ahead frame can be formulated as follows:

$$\begin{aligned}
\max_{q_{\text{DA}},q_{\text{imb},p,i},q_{\text{imb},n,i}} & [p_{\text{DA}}q_{\text{DA}} + \sum_{i\in I} r_i q_{\text{imb},p,i}\sum_{j\in J} s_j p_{\text{imb},p,j} \\
& - \sum_{i\in I} r_i q_{\text{imb},n,i}\sum_{j\in J} s_j p_{\text{imb},n,j} \quad] \qquad (1)\\
\text{st} \quad & q_{\text{DA}} + q_{\text{imb},p,i} - q_{\text{imb},n,i} = \text{Prod}_i \quad \forall i \in I\\
& q_{\text{DA}} \geq 0\\
& q_{\text{DA}} \leq P_{\text{WF}}\\
& q_{\text{imb},p,i} \geq 0 \quad \forall i \in I\\
& q_{\text{imb},n,i} \geq 0 \quad \forall i \in I
\end{aligned}$$

This formulation is similar to the one proposed by [4].

In fact there exists an analytical solution to this problem. To prove this, we will use the fact that the problem is concave. By analysing the problem, we see that when q_{DA} is fixed, the other variables are also fixed. We can study 3 cases:

- The maximum is at $q_{\text{DA}} = 0$. It is the case if the derivative at 0 is negative. This derivative is $p_{\text{DA}} - \bar{p}_{\text{imb},p}$. It means that if the day-ahead price is lower than the positive imbalance prices, the producer prefers to bid 0 and be paid at the imbalance price.
- The maximum is at $q_{\text{DA}} = P_{\text{WF}}$. It is the case if the derivative at P_{WF} is positive. The derivative at P_{WF} is $p_{\text{DA}} - \bar{p}_{\text{imb},n}$. It means that if the day ahead price is higher than the negative imbalance prices, the producer prefers to bid its maximum capacity. By doing so, for the unserved quantity, he will be paid at the day-ahead price and will have to pay it back at the imbalance price which is lower.
- There is a maximum in the interval $0 < q_{\text{DA}} < P_{\text{WF}}$. We obtain (the proof is presented in Appendix A.)

$$q_{\text{DA}} = H^{-1}(\kappa) \qquad (2)$$
$$\text{with } \kappa = \frac{\bar{p}_{\text{imb},p} - p_{\text{DA}}}{\bar{p}_{\text{imb},p} - \bar{p}_{\text{imb},n}}$$

with $H()$ the cumulative distribution of **Prod**.

The results justify that we do not generate imbalance price scenarios. Indeed, in a risk neutral case, the results of the model only depend of the mean of the imbalance price scenarios.

It is also interesting to notice that we are in the first two cases if p_{DA} is not in the interval $[\bar{p}_{\text{imb},p}, \bar{p}_{\text{imb},n}]$, which is very likely (between 2013 and 2015, the realization of the day-ahead price was between the realization of the positive and the negative imbalance prices for only 0.08% of the quarters). Moreover, for these two first cases, the wind producer bid is independent of the production scenarios.

VI. Modelling influence on imbalance price

In this section, we develop a model which accounts for the producer influence on the imbalance price.

A. Procedure

To model the dependence between the output of wind power producers and the imbalance price, we consider that the producer influences the imbalance price linearly as a function of his imbalance quantity. Mathematically, the optimum bidding problem can be described by the following model:

$$\begin{aligned}
\max_{q_{\text{DA}},q_{\text{imb},p,i},q_{\text{imb},n,i}} & [p_{\text{DA}}q_{\text{DA}}\\
& + \sum_{i\in I} r_i q_{\text{imb},p,i}\sum_{j\in J} s_j[p_{\text{imb},p,j} + \beta q_{\text{imb},p,i}]\\
& - \sum_{i\in I} r_i q_{\text{imb},n,i}\sum_{j\in J} s_j[p_{\text{imb},n,j} - \beta q_{imb,n,i}]] \qquad (3)\\
\text{st} \quad & q_{\text{DA}} + q_{\text{imb},p,i} - q_{\text{imb},n,i} = \text{Prod}_i \quad \forall i \in I\\
& q_{\text{DA}} \geq 0\\
& q_{\text{DA}} \leq P_{\text{WF}}\\
& q_{\text{imb},p,i} \geq 0 \quad \forall i \in I\\
& q_{\text{imb},n,i} \geq 0 \quad \forall i \in I
\end{aligned}$$

This model requires the estimation of the β coefficient. For this purpose, we represent the imbalance price against the system imbalance. The result is shown in figure 4. Next, we estimate the coefficient of the linear regression and obtain a value of -0.2429. This can be interpreted as follows: if we increase voluntarily our output by $1MW$, the imbalance price will decrease by $0.2429\frac{\text{euro}}{MWh}$. Notice that the producer's influence is always unfavourable. Indeed, if it is in overproduction, the price decreases and if it is in underproduction it increases. Consequently, this model could only reduce the gain of the WPP with respect to the case in which the WPP is assumed not to influence the imbalance price. However, the model represents better the revenue that the producer will obtain from the market.

B. Analytical solution

We make the following approximations in order to obtain an analytical solution to this problem:

- $p_{\text{imb},p,j} = p_{\text{imb},n,j}$
- The WPP always delivers its entire available quantity.

With these assumptions we obtain the following optimal day-ahead bid:

$$q_{\text{DA}} = \bar{\text{Prod}} + \frac{\bar{p}_{\text{imb}} - p_{\text{DA}}}{2\beta} \qquad (4)$$

The proof is provided in Appendix B. The result can be interpreted as follows: the first term is the expected production of the wind farm. The second term could be decomposed as follows:

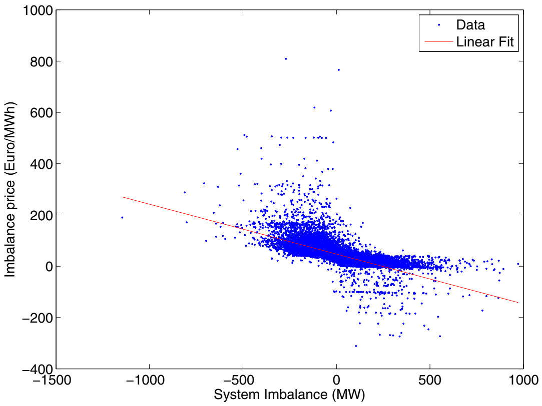

Fig. 4. Positive imbalance price versus system imbalance between 2013 and 2015.

- The numerator is the difference between the imbalance price mean and the day-ahead price. It is clear that this difference is the expected reward when the WPP decides to move away from the expected value.
- The denominator represents the influence of the WPP on the market. The greater the influence on the market, the closer to its expected production the WPP has to bid. This is logic because as we explained earlier, the WPP can only influence the market in an unfavourable way.

It is possible that the optimal solution is not in the interval $[0, P_{\mathrm{WF}}]$. In that case, we project the solution on this interval.

VII. Case study

The aim of this section is to:

- assess the quality of the different price forecasting methods.
- compare the revenue between the two optimization models.

A. Imbalance forecasting assessment

For this purpose, we use three years of data from 2013 to 2015 of the imbalance prices and the relative features. This data is available at the site of the Belgian transmission system operator Elia[3]. Table II presents results for each model. We begin by describing the different columns:

- The second column contains the root mean square error (RMSE) of the imbalance model. It seems that it does not explain completely the results for the revenue shown in the last column (some methods have better RMSE but worse revenue).
- The third column contains the percentage of right classifications of the imbalance price model. We count a right classification if either both the forecast and the realization of imbalance are lower or higher than the day-ahead price. Consequently, the percentage of right classifications is clearly related to the revenue. However it does not explain completely the results. For instance, the method using the value of the same quarter the day before (line 1) has one of the best percentages but one of the worst final revenues.
- The fourth column contains the average absolute difference between the day-ahead price and the imbalance price when the classification is correct. In fact it represents the average surplus obtained by deviating from the expectation of 1MW if the classification is correct.
- The fifth column presents the average absolute difference between the day-ahead price and the imbalance price when the classification is incorrect. It represents the average surplus lost by deviating from the expectation of 1MW if the classification is incorrect.
- The sixth column is the expected revenue for a deviation of 1 MW from the expected value. It is the expected sum of the two last columns with the probability of column 2. This criterion predicts almost perfectly the ranking of the method. Therefore it provides a quality criterion of the imbalance model without solving any optimization problem.
- The last column is the revenue on the three year period.

From the last column of this table, we can conclude that using the optimization model with RBFN improves the revenue by:

- 41% with respect to the case in which we bid the expected wind power production. It proves that developing the optimization model is useful.
- 5.4% with respect to the best base case (mean value of the last 50 days).
- 5.7% with respect to the linear model.

TABLE II
Results for different models.

Method/param	RMSE	True %	Mean win	Mean loss	Crit	Revenue
Previous quarter	60.82	0.589	33.34	28.07	8.1	4.0765e+06
Mean previous day	48.03	0.567	33.98	27.49	7.36	4.0446e+06
Mean 50 quarter	45.61	0.595	34.19	26.74	9.51	**4.2931e+06**
Linear	44.98	0.588	33.85	27.36	8.63	**4.2805e+06**
RBFN	51.78	0.602	34.39	26.31	10.23	**4.5263e+06**
Expectation						**3.2089e+06**

B. Market influence assessment

We compare the revenue for the models of sections V and VI. For this purpose, we use the data available at the site of the Belgian transmission system operator Elia. For each quarter, the data give the volumes of reserve that can be activated and the marginal prices of the bids for activation of those volumes (notice that between two points of this table, we consider a linear relation). With this procedure, we can estimate the imbalance price for a given value of imbalance of our producer. Finally, we compute the revenue for three different sizes of wind farms. We perform this evaluation for every quarter between October 2013 (beginning of the data) and December 2015 for three sizes of wind farms (in order to obtain the 60MW and 120MW results, we solve the optimization model with both the forecast and the realization of the 12 MW multiplied by 5 and 10). The results are presented in table III. For a small wind farm, the hypothesis that the producer is a price-taker is valid, therefore taking into account that we

[3]Available at http://www.elia.be/en/grid-data/data-download

influence the market is not necessary. Nevertheless, for a wind farm of higher capacity, if the producer does not account for its influence on the market, it will face huge losses. Indeed, for the 120 MW case, the producer earns less than 20% of its income if it takes into account that it influences the market. Finally, it is clear that the case in which the WPP accounts for its influence performs at least as well as the other methods and in some cases far better.

TABLE III
REVENUE FOR DIFFERENT METHODS AND INSTALLED CAPACITY (IN MILLIONS OF EUROS).

Model/Installed capacity	12	60	120
Initial	3.06	10.26	3.90
Influence on the price	3.07	11.99	22.57
Expected value	2.25	11.15	22.11

VIII. CONCLUSION

In this paper, we propose a set of features in order to forecast imbalance prices. We prove that using these features as input of a radial basis function network can improve the revenue of a WPP bidding in the day-ahead market. We also propose a procedure for modelling the influence of the producer on imbalance prices. We then develop a model for obtaining the optimal bid. We cast that model as a convex quadratic program. We show that for producers of important size, modelling their influence on the imbalance prices can improve their profit substantially.

APPENDIX A

In this section, we prove equation 2. We start by computing the derivative of the objective function of model (1). This reasoning is inspired from [8] and [11]:

$$F^{'}(q_{\text{DA}}) = p_{\text{DA}} - \bar{p}_{\text{imb},p}\Big(\sum_{q_{\text{DA}}<\text{Prod}_i} r_i\Big) - \bar{p}_{\text{imb},n}\Big(\sum_{q_{\text{DA}}\geq\text{Prod}_i} r_i\Big)$$
$$= p_{\text{DA}} - \bar{p}_{\text{imb},p} + (\bar{p}_{\text{imb},p} - \bar{p}_{\text{imb},n})\Big(\sum_{q_{\text{DA}}\geq\text{Prod}_i} r_i\Big)$$
$$= p_{\text{DA}} - \bar{p}_{\text{imb},p} + (\bar{p}_{\text{imb},p} - \bar{p}_{\text{imb},n})\Pr(q_{\text{DA}} \geq \mathbf{Prod})$$

In what follow, we denote as **Prod**: the random variable representing the production distribution.

$$= p_{\text{DA}} - \bar{p}_{\text{imb},p} + (\bar{p}_{\text{imb},p} - \bar{p}_{\text{imb},n})H(q_{\text{DA}})$$

where H() denotes the cumulative distribution of **Prod**. We equate this derivative to 0:

$$0 = p_{\text{DA}} - \bar{p}_{\text{imb},p} + (\bar{p}_{\text{imb},p} - \bar{p}_{\text{imb},n})H(q_{\text{DA}})$$
$$\Leftrightarrow q_{\text{DA}} = H^{-1}(\kappa)$$
$$\text{with } \kappa = \frac{\bar{p}_{\text{imb},p} - p_{\text{DA}}}{\bar{p}_{\text{imb},p} - \bar{p}_{\text{imb},n}}$$

APPENDIX B

In this section, we prove equation (4). We start by computing the derivative of the objective function of model (3).

$$f(q_{\text{DA}}) = p_{\text{DA}}q_{\text{DA}} + \sum_{i\in I} r_i(\text{Prod}_i - q_{\text{DA}}) \sum_{j\in J} s_j[p_{\text{imb},j} + \beta(\text{Prod}_i - q_{\text{DA}})]$$
$$= p_{\text{DA}}q_{\text{DA}} + \sum_{i\in I} r_i(\text{Prod}_i - q_{\text{DA}})[\bar{p}_{\text{imb}} + \beta(\text{Prod}_i - q_{\text{DA}})]$$
$$= p_{\text{DA}}q_{\text{DA}} + \bar{p}_{\text{imb}}(\bar{\text{Prod}} - q_{\text{DA}}) + \sum_{i\in I} r_i(\text{Prod}_i - q_{\text{DA}})\beta(\text{Prod}_i - q_{\text{DA}})$$
$$= p_{\text{DA}}q_{\text{DA}} + \bar{p}_{\text{imb}}(\bar{\text{Prod}} - q_{\text{DA}}) + \beta\sum_{i\in I} r_i(\text{Prod}_i^2 - 2q_{\text{DA}}\text{Prod}_i + q_{\text{DA}}^2)$$
$$= p_{\text{DA}}q_{\text{DA}} + \bar{p}_{\text{imb}}(\bar{\text{Prod}} - q_{\text{DA}}) + \beta(\mathbf{E}[\text{Prod}_i^2] - 2q_{\text{DA}}\bar{\text{Prod}} + q_{\text{DA}}^2)$$

Let us take the derivative of f with respect to q_{DA}:

$$f'(q_{\text{DA}}) = p_{\text{DA}} - \bar{p}_{\text{imb}} - 2\beta\bar{\text{Prod}} + 2\beta q_{\text{DA}}$$

We equate the derivative to 0:

$$0 = p_{\text{DA}} - \bar{p}_{\text{imb}} - 2\beta\bar{\text{Prod}} + 2\beta q_{\text{DA}}$$
$$\Leftrightarrow q_{\text{DA}} = \bar{\text{Prod}} + \frac{\bar{p}_{\text{imb}} - p_{\text{DA}}}{2\beta}$$

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