Pricing Transmission

Quantitative Energy Economics

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Pricing Transmission

1. Locational Marginal Pricing
2. Congestion Rent and Congestion Cost
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4. Zonal Pricing
   - Zonal Pricing with Re-Dispatch
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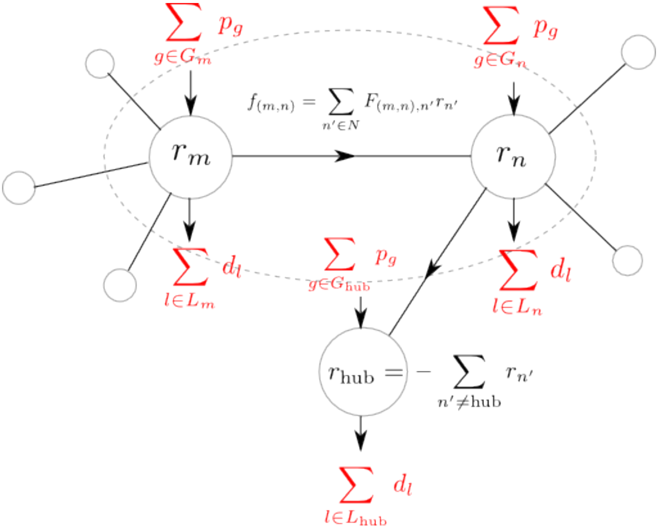
Recall DCOPF

$$
\begin{aligned}
(DCOPF): \quad & \max \sum_{l \in L} \int_0^{d_l} MB_l(x)dx - \sum_{g \in G} \int_0^{p_g} MC_g(x)dx \\
(\lambda_k^+): \quad & f_k \leq T_k \\
(\lambda_k^-): \quad & -f_k \leq T_k \\
(\psi_k): \quad & f_k - \sum_{n \in N} F_{kn} r_n = 0 \\
(\rho_n): \quad & r_n - \sum_{g \in G_n} p_g + \sum_{l \in L_n} d_l = 0 \\
(\phi): \quad & \sum_{n \in N} r_n = 0 \\
& p_g, d_l \geq 0
\end{aligned}
$$


$\sum_{g\in G_m} p_g$
$\sum_{g\in G_n} p_g$
$f_{(m,n)} = \sum_{n'\in N} F_{(m,n),n'} r_{n'}$
r_m
r_n
$\sum_{l\in L_m} d_l$
$\sum_{g\in G_{\text{hub}}} p_g$
$\sum_{l\in L_n} d_l$
$r_{\text{hub}} = -\sum_{n'\neq \text{hub}} r_{n'}$
$\sum_{l\in L_{\text{hub}}} d_l$

Complicating Constraints of OPF

Energy balance constraint:

$$\sum_{l \in L} d_l - \sum_{g \in G} p_g = 0.$$

Transmission network limits:

$$-T_k \leq \sum_{n \in N} F_{kn} \sum_{g \in G_n} p_g - \sum_{n \in N} F_{kn} \sum_{l \in L_n} d_l \leq T_k$$

Locational Marginal Pricing

Locational marginal pricing/nodal pricing: uniform price auction conducted as follows:

- Sellers and buyers submit price-quantity pairs
- Market operator solves (*DCOPF*) and announces ρ_n as market clearing price for bus n

Efficiency of LMP Auction

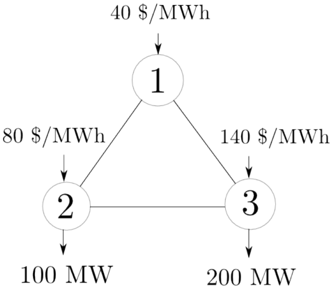
If agents bid truthfully, LMP auction reproduces optimal solution of OPF

Proof: Follows from KKT conditions of OPF

Locational marginal prices (LMPs): Prices ρ_n produced for bus n from OPF

Example

All lines have identical electrical characteristics (reactance)



Price Splitting in Neighboring Nodes

Suppose $T_{1-2} = T_{2-3} = T_{1-3} = 50$ MW

Lines 1-3, 2-3 should be used fully (why?)

Optimal dispatch: $p_1 = 50$ MW, $p_2 = 150$ MW, $p_3 = 100$ MW
Optimal flows: $f_{1-2} = 0$ MW, $f_{2-3} = f_{1-3} = 50$ MW

$\rho_1 = 40$ \$/MWh, $\rho_2 = 80$ \$/MWh, $\rho_3 = 140$ \$/MWh (why?)

Observe: $f_{1-2} < T_{1-2}$, but $\rho_2 > \rho_1$

Settlement of the LMP auction:

	Bid	Cleared	Payment ($/h)
G1	$+\infty$ MW at 40 $/MWh	50 MW at 40 $/MWh	2000
G2	$+\infty$ MW at 80 $/MWh	150 MW at 80 $/MWh	12000
G3	$+\infty$ MW at 140 $/MWh	100 MW at 140 $/MWh	14000
L2	100 MW at $+\infty$ $/MWh	100 MW at 80 $/MWh	-8000
L3	200 MW at $+\infty$ $/MWh	200 MW at 140 $/MWh	-28000

How much surplus is left over to the auctioneer?

LMP Can Be Different From Fuel Cost

Suppose $T_{1-2} = 50$ MW, $T_{2-3} = 100$ MW, $T_{1-3} = 120$ MW

Optimal dispatch: $p_1 = 160$ MW, $p_2 = 140$ MW, $p_3 = 0$ MW
Optimal flows: $f_{1-2} = 40$ MW, $f_{2-3} = 80$ MW, $f_{1-3} = 120$ MW

$\rho_3 = 120$ \$/MWh (use sensitivity)

Observe: ρ_3 is different from marginal cost of *all* generators

Non-Uniqueness of LMPs

Suppose $T_{1-2} = 50$ MW, $T_{2-3} = 100$ MW, $T_{1-3} = 100$ MW

Optimal dispatch: $p_1 = 100$ MW, $p_2 = 200$ MW, $p_3 = 0$ MW
Optimal flows: $f_{1-2} = 0$ MW, $f_{2-3} = f_{1-3} = 100$ MW

$\rho_3 = 140$ \$/MWh is a valid LMP (use sensitivity)
$\rho_3 = 120$ \$/MWh is a valid LMP (use sensitivity)

Observe: 120 \$/MWh $\leq \rho_3 \leq$ 140 \$/MWh are all valid LMPs

Efficiency of LMP Pricing

If agents bid truthfully,

1. locational marginal pricing maximizes welfare, and
2. the resulting allocation maximizes the profit of agents *given* the market clearing price

Proof of item 1: LMP auction is solving welfare maximization problem

Proof of item 2: Decomposition of KKT conditions of DCOPF

	Transmission
Producers $0 \le p_g \perp MC_(p_g) - \rho_{n(g)} + \mu_g \ge 0$ $0 \le \mu_g \perp P_g - p_g \ge 0$ $\Leftrightarrow$ $\max \rho_{n(g)} p_g - \int_0^{p_g} MC_g(x)dx$ $(\mu_g): \quad p_g \le P_g$ $p_g \ge 0$	**Transmission** $f_k - \sum_{n \in N} F_{kn} r_n = 0$ $r_n - \sum_{g \in G_n} p_g + \sum_{l \in L_n} d_l = 0$ $\sum_{n \in N} r_n = 0$ $\lambda_k^+ - \lambda_k^- + \psi_k = 0$ $-\sum_{k \in K} F_{kn} \psi_k + \rho_n + \phi = 0$ $0 \le \lambda_k^+ \perp T_k - f_k \ge 0$ $0 \le \lambda_k^- \perp T_k + f_k \ge 0$
Consumers $0 \le d_l \perp -MB_l(d_l) + \rho_{n(l)} + \nu_l \ge 0$ $0 \le \nu_l \perp D_l - d_l \ge 0$ $\Leftrightarrow$ $\max \int_0^{d_l} MB_l(x)dx - \rho_{n(l)} d_l$ $(\nu_l): d_l \le D_l$ $d_l \ge 0$	

Nodal Pricing in PJM (February 15, 2014)

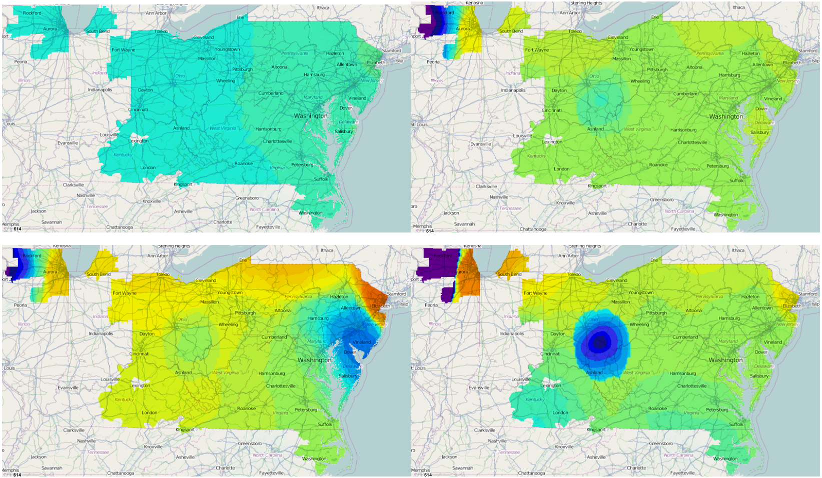


Figure: 05:40 (upper left), 08:40 (upper right), 09:20 (lower left), 09:55 (lower right).

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Congestion Rent

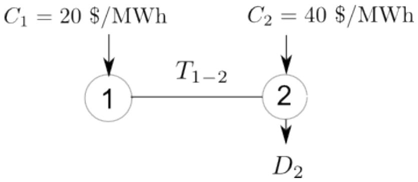
Congestion rent: Surplus from locational price differences

$$\sum_{n \in N} \rho_n \Big(\sum_{l \in L_n} d_l - \sum_{g \in G_n} p_g \Big)$$

Congestion cost: excess cost due to finite capacity of transmission lines

$$\text{Congestion rent} \neq \text{Congestion cost}$$

Example: Congestion Rent $\geq$ Congestion cost



Suppose $D_2 = 50$ MW, $T_{1-2} = 50$ MW

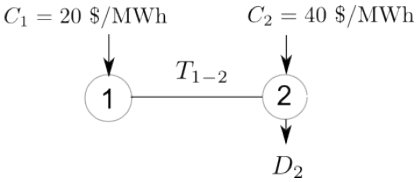
Competitive market clearing prices:

$\rho_1 = 20$ \$/MWh, 20 \$/MWh $\leq \rho_2 \leq$ 40 \$/MWh

Congestion rent: 0 \$/h - 1000 \$/h

Congestion cost: 0 \$/h

Example: Congestion Rent > Congestion cost



Suppose $D_2 = 60$ MW, $T_{1-2} = 50$ MW

Competitive market clearing prices:

$\rho_1 = 20$ \$/MWh, $\rho_2 = 40$ \$/MWh

Congestion rent: 1000 \$/h

Congestion cost: 200 \$/h

Congestion Rent Is Non-Negative

Congestion rent is non-negative, and given by the following expression:

$$\sum_{n \in N} \rho_n \left(\sum_{l \in L_n} d_l - \sum_{g \in G_n} p_g\right) = \sum_{k \in K} (\lambda_k^+ + \lambda_k^-) T_k$$

Proof: If identity is true, then since $\lambda_k^+, \lambda_k^- \geq 0$, congestion rent is non-negative

$$
\begin{aligned}
\sum_{n\in N}\rho_n\Big(\sum_{l\in L_n} d_l - \sum_{g\in G_n} p_g\Big) = \qquad & \text{definition of } r_n \\
-\sum_{n\in N}\rho_n r_n = \qquad & \text{from } \rho_n = -\phi + \sum_{k\in K} F_{kn}(\lambda_k^- - \lambda_k^+) \\
& \text{and } \sum_{n\in N} r_n = 0 \\
\sum_{k\in K}(\lambda_k^+ - \lambda_k^-)\sum_{n\in N} F_{kn} r_n = \qquad & \text{definition of } f_k \\
\sum_{k\in K}(\lambda_k^+ - \lambda_k^-) f_k = \qquad & \text{from } 0 \le \lambda_k^+ \perp T_k - f_k \ge 0 \\
& \text{and } 0 \le \lambda_k^- \perp T_k + f_k \ge 0 \\
\sum_{k\in K}(\lambda_k^+ + \lambda_k^-) T_k \qquad &
\end{aligned}
$$

Congestion Rent and FTR Payments

Financial transmission rights (coming later) pay to their holders

$$-\sum_{n \in N} \rho_n \tilde{r}_n$$

where $\tilde{r}_n$ is a feasible (not necessarily optimal) dispatch

Congestion rent is adequate to cover FTR payments:

$$-\sum_{n \in N} \rho_n r_n \geq -\sum_{n \in N} \rho_n \tilde{r}_n$$

Proof: From previous proof,

$$-\sum_{n\in N}\rho_n(r_n-\tilde{r}_n)=\sum_{k\in K}(\lambda_k^+-\lambda_k^-)(f_k-\tilde{f}_k)$$

where

- λ_k^+, λ_k^- are dual optimal multipliers,
- f_k are flows corresponding to r_n
- $\tilde{f}_k$ are flows corresponding to $\tilde{r}_n$

Consider three cases:

- $f_k = T_k$ (which implies $\lambda_k^- = 0$)
- $f_k = -T_k$ (which implies $\lambda_k^+ = 0$)
- $-T_k < f_k < T_k$ (which implies $\lambda_k^+ = \lambda_k^- = 0$)

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Separable Optimization

$$
\begin{aligned}
(Sep): \quad & \max_{x} \sum_{i=1}^{n} f_i(x_i) \\
(\rho_i): \quad & g_i(x_i) \leq 0 \\
(\lambda): \quad & \sum_{i=1}^{n} h_i(x_i) \leq 0
\end{aligned}
$$

- n agents, action variables $x_i \in \mathbb{R}^{n_i}$
- Coupling/complicating constraint $\sum_i h_i(x_i) \leq 0$ with $h_i : \mathbb{R}^{n_i} \to \mathbb{R}^m$ convex
- $g_i : \mathbb{R}^{n_i} \to \mathbb{R}^{a_i}$ is a convex private constraint function
- $f_i : \mathbb{R}^{n_i} \to \mathbb{R}$ is a concave private benefit function

Competitive Equimibrium

Competitive market equilibrium: pair of prices and quantities $(\lambda^\star, x_i^\star, q_i^\star)$ such that:

- $(x_i^\star, q_i^\star)$ maximize profit given $\lambda^\star$:

$$
\begin{aligned}
&\text{(Profit-i)} : \max_{x_i, q_i} (f_i(x_i) - (\lambda^\star)^T q_i) \\
&g_i(x_i) \leq 0, \\
&h_i(x_i) = q_i,
\end{aligned}
$$

- market clearing (supply $\geq$ demand): $\sum_{i=1}^{n} q_i^\star \leq 0$

Competitive Market Model with Transmission

1. Agents: power producers, power consumers
2. Scarce resources (commodities): power, transmission
3. Profit maximization (quantity adjustment) of agents
4. Market clearing (price adjustment) of commodities

Complication: usage of line capacity depends on location of producer *and* consumer

Insight:

- producers responsible for shipping power *to* hub
- consumers responsible for shipping power *from* hub

Denote

- ϕ: price of power
- λ_k^+, (λ_k^-): price of transmission rights in (opposite to) reference direction

Producer profit maximization:

$$\max \phi \cdot p_g - \sum_{k \in K} \lambda_k^+ F_{kn} p_g + \sum_{k \in K} \lambda_k^- F_{kn} p_g - \int_0^{p_g} MC_g(x) dx$$

$$p_g \leq P_g$$

$$p_g \geq 0$$

Consumer profit maximization:

$$\max \int_0^{d_l} MB_l(x)dx - \phi \cdot d_l + \sum_{k \in K} \lambda_k^+ F_{kn} d_l - \sum_{k \in K} \lambda_k^- F_{kn} d_l$$

$$d_l \leq D_l$$

$$d_l \geq 0$$

Market clearing for power:

$$\sum_{g \in G} p_g = \sum_{l \in L} d_l,$$

Market clearing for transmission capacity:

$$0 \leq \lambda_k^+ \perp T_k - f_k \geq 0$$

$$0 \leq \lambda_k^- \perp T_k + f_k \geq 0.$$

Efficiency of LMP Pricing

Nodal pricing produces an allocation of power and market clearing prices that correspond to a competitive market equilibrium. The converse is also true.

Proof: Compare KKT conditions of (*DCOPF*) to competitive market model

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Criticisms of Nodal Pricing

Criticisms of nodal pricing

- Too complicated, i.e. too many prices
- Local markets are too small $\Rightarrow$ reduced liquidity
- Local markets are too small $\Rightarrow$ opportunities for manipulation of prices

Alternative proposal: **zonal pricing**

Formulation of Zonal Pricing Model

$$
\begin{aligned}
(ZP): \quad & \max \sum_{l \in L} \int_0^{d_l} MB_l(x)dx - \sum_{g \in G} \int_0^{p_g} MC_g(x)dx \\
(\rho_z): \quad & -\sum_{g \in G_z} p_g - \sum_{a=(\cdot,z)} f_a + \sum_{l \in L_z} d_l + \sum_{a=(z,\cdot)} f_a = 0, z \in Z \\
& -ATC_a \leq f_a \leq ATC_a, a \in A \\
& p_g \geq 0, g \in G, d_l \geq 0, l \in L
\end{aligned}
$$

- Z: set of zones
- A: set of links between zones
- G_z: generators located in zone z
- L_z: loads located in zone z
- ATC: capacity of links connecting zones

Zonal Pricing

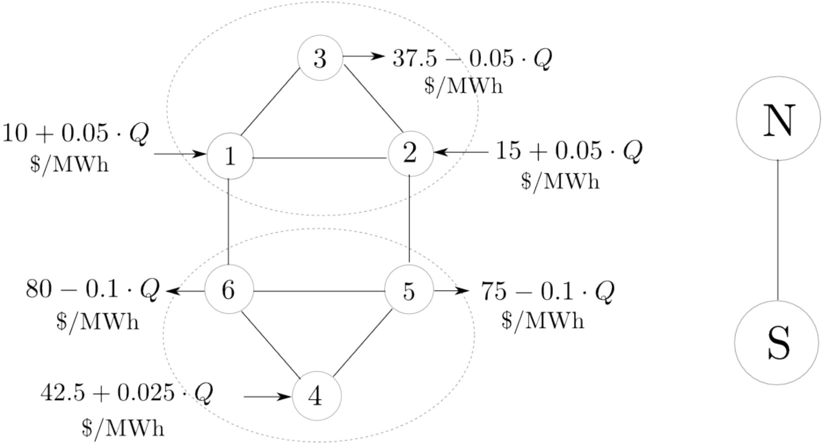
Zonal pricing: uniform price auction conducted as follows

- Sellers and buyers submit price-quantity pairs
- Market operator solves (ZP) and announces ρ_z as market clearing price for zone z

Features:

- Kirchhoff's laws are ignored
- Congestion within zones is ignored
- Flows within a zone assumed not to influence flows on interconnections among zones

6-Node Example



6-Node Example with Flow-Based Pricing

	Node 1	Node 2	Node 3	Node 4	Node 5
Link 1-6	0.625	0.5	0.5625	0.0625	0.125
Link 2-5	0.375	0.5	0.4375	-0.0625	-0.125

Suppose $T_{1-6} = 200$ MW, $T_{2-5} = 250$ MW

LMP pricing

- Welfare: 23000 \$/h
- Different price at each node: $\rho_1 = 25$ \$/MWh, $\rho_2 = 30$ \$/MWh, $\rho_3 = 27.5$ \$/MWh, $\rho_4 = 47.5$ \$/MWh, $\rho_5 = 45$ \$/MWh, $\rho_6 = 50$ \$/MWh
- Lines flows: $f_{1-6} = f_{2-5} = 200$ MW

Zonal model definition:

- $Z = \{\text{N, S}\}$
- $A = \{\text{N-S}\}$
- North zone includes nodes 1, 2, 3
- South zone includes nodes 4, 5, 6

- Zonal pricing with $ATC_{\text{N-S}} = 200$ MW
  - Welfare: 18520 $/h
  - $\rho_{\text{N}} = 24.17$ \$/MWh, $\rho_{\text{S}} = 50.83$ \$/MWh
  - Flows: $f_{1-6} = 109.38$ MW, $f_{2-5} = 90.63$ MW
- Zonal pricing with $ATC_{\text{N-S}} = 450$ MW
  - Welfare: 24145 $/h
  - $\rho_{\text{N}} = 28.33$ \$/MWh, $\rho_{\text{S}} = 46.77$ \$/MWh
  - Flows: $f_{1-6} = 234.38$ MW, $f_{2-5} = 215.63$ MW

How would you verify the correctness of these prices?

Zonal model is either:

- too conservative ($ATC = 200$ MW)
  - Flow constraints are respected
  - ... but zonal pricing welfare < nodal pricing welfare
- too aggressive ($ATC = 450$ MW)
  - Zonal pricing welfare > nodal pricing welfare
  - ... but flow constraints are violated

Flow-Based Zonal Pricing

Consider a link $a \in A$ of the zonal model, denote K_a as set of lines that correspond to link a

$K_A = \cup_{a \in A} K_a$

Use PTDFs to account for Kirchhoff laws on K_A:

$$-T_k \leq \sum_{n \in N} F_{kn} (\sum_{g \in G_n} p_g - \sum_{l \in L_n} d_l) \leq T_k, k \in K_A$$

Flow-based zonal pricing: uniform price auction that maximizes welfare subject to

- Zonal prices
- Flow-based constraints

$$
\begin{aligned}
(FBP): \quad & \max \sum_{l \in L} \int_0^{d_l} MB_l(x)dx - \sum_{g \in G} \int_0^{p_g} MC_g(x)dx \\
& 0 \leq p_g \perp MC_g(p_g) - \rho_{z(g)} \geq 0, g \in G \\
& 0 \leq d_l \perp -MB_l(d_l) + \rho_{z(l)} \geq 0, l \in L \\
& \sum_{g \in G} p_g - \sum_{l \in L} d_l = 0 \\
& -T_k \leq \sum_{n \in N} F_{kn}(\sum_{g \in G_n} p_g - \sum_{l \in L_n} d_l) \leq T_k, k \in K_A
\end{aligned}
$$

6-Node Example with Flow-Based Pricing

Recall $T_{1-6} = 200$ MW, $T_{2-5} = 250$ MW

- Welfare: 22806.6 \$/h
- $\rho_{\text{N}} = 27.19$ \$/MWh, $\rho_{\text{S}} = 47.81$ \$/MWh
- Flows: $f_{1-6} = 200$ MW, $f_{2-5} = 181.25$ MW

How do these results compare to LMP pricing? zonal pricing without flow-based constraints?

Re-Dispatch

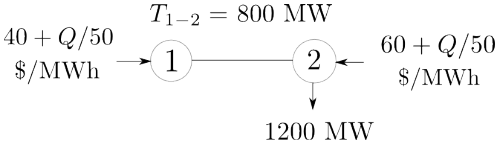
Re-dispatch: Pay-as-bid auction conducted after zonal pricing

- Sellers submit increment (inc) and decrement (dec) bids
- Inc bids: price producers are asking to provide additional power relative to zonal pricing auction
- Dec bids: price producers are willing to *pay* to market operator for decreasing production relative to zonal pricing auction
- Inc bids cleared to minimize payment to bidders
- Dec bids cleared to maximize payment to market operator

Example

Under truthful bidding, zonal pricing followed by re-dispatch achieves the same result as LMP pricing with

- fewer prices
- lower charges to consumers <u>above</u> generator costs



LMP solution:

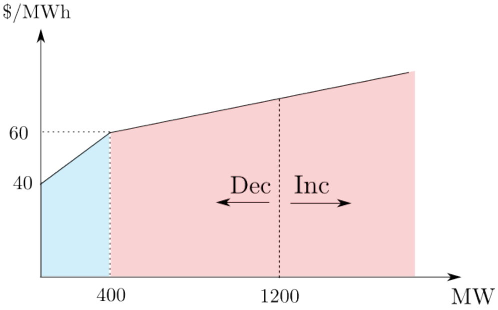
- $p_1 = 800$ MW, $p_2 = 400$ MW
- $\rho_1 = 56$ \$/MWh, $\rho_2 = 68$ \$/MWh
- 9600 \$/h left to market operator

Zonal pricing (single zone):

- $p_1 = 1100$ MW, $p_2 = 100$ MW (violates line limit)
- $\rho = 62$ \$/MWh
- Zero surplus for market operator

Re-dispatching under truthful bidding:

- 300 MW of inc cleared from node 2
- 300 MW of dec cleared from node 1
- Payment *to* market operator from dec bids: 17700 $/h
- Payment *from* operator to cleared inc bids: 19500 $/h
- Difference: 1800 $/h



Gaming Zonal Pricing

Zonal pricing with re-dispatch can be gamed

Dec Game, known by Enron traders as *Death Star*