# Power Systems

## Quantitative Energy Economics

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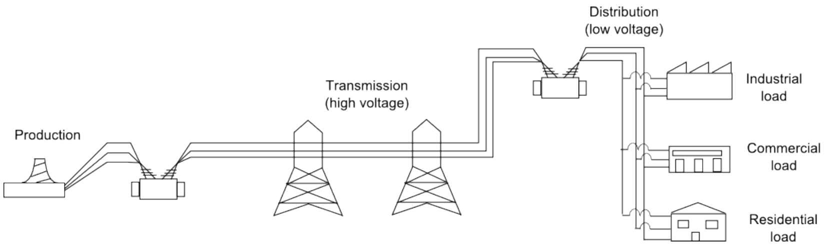
# Power Systems

1. Production
2. Transmission and Distribution
3. Consumption

# Power System Supply Chain




Components:

- production
- transmission and distribution
- consumption

# Outline

1. Production
2. Transmission and Distribution
3. Consumption

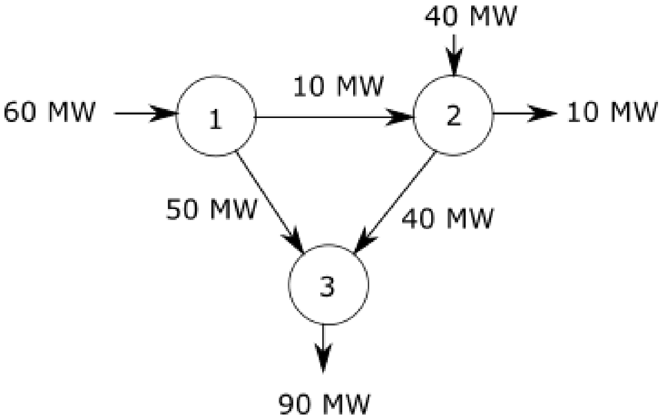
# Production

- Non-renewable energy sources
  - Fossil (coal, oil, natural gas)
  - Nuclear
- Renewable energy sources
  - Hydroelectric (run of river, dams, pumped storage)
  - Geothermal
  - Wind
  - Solar
  - Biomass
  - Other (wave, tidal)

# Units of Measurement

- Energy is measured in megawatt hours (denoted MWh)
- Power is energy per unit time:
  - rate of production of energy
  - rate of consumption of energy
  - flow of energy
- Power is measured in MW

# The Pool Analogy




Who is supplying the load in node 3?

# Production Constraints

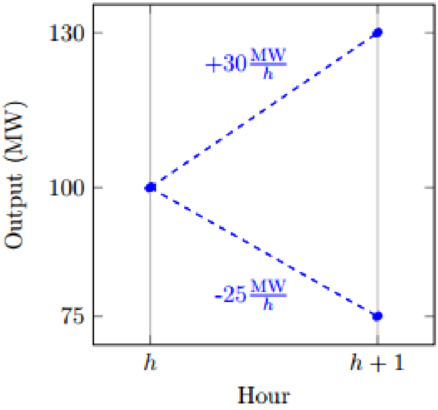
Fossil fuel constraints:

- Min/max production
- Ramp constraints
- Min up/down times

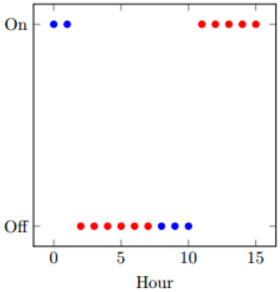
Hydro constraints:

- Max production
- Max storage

# Ramp Constraints



# Min Up/Down Time Constraints




- Min up time: 5 hours
- Min down time: 6 hours
- Red circles: constrained decisions
- Blue circles: free decisions

# Variable and Marginal Cost

**Variable/operating/fuel cost**: cost that depends on amount of output

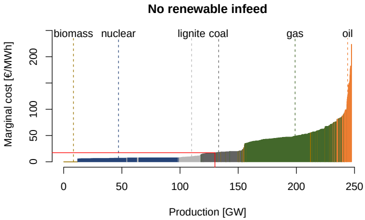
- Measured in \$/h
- Hourly cost of producing a certain amount of power

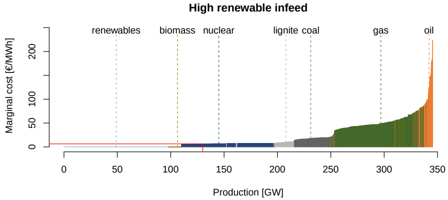
**Marginal cost**: derivative of fuel cost with respect to output

- Measured in \$/MWh
- Increase in fuel cost if an additional MW of power were produced (is this the only interpretation?)

**Merit order curve** = (increasing) system marginal cost curve

Figure: Belgian merit order curve




What is the impact of renewable energy on the system?

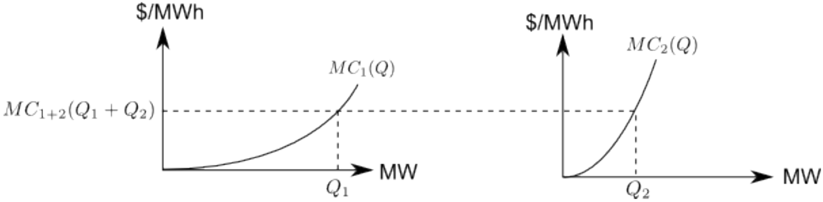
# Marginal Cost Range

If variable cost is non-differentiable, define

- **Left-hand marginal cost**: left-hand side derivative of variable cost
- **Right-hand marginal cost**: right-hand side derivative of variable cost (when is it infinite?)
- **Marginal cost range**: set of values between and including left and right-hand marginal cost

# Horizontal Summation of Marginal Cost

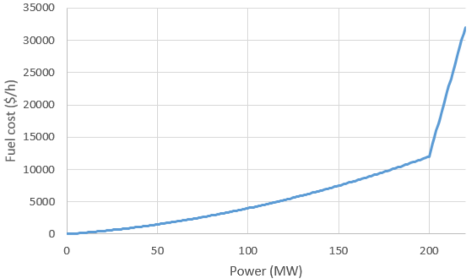
Aggregate marginal cost is obtained by horizontal summation



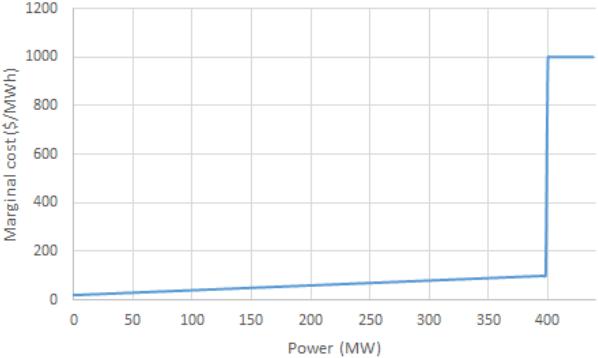
# Example

Consider quadratic fuel cost with

- MC(0 MW) = 20 $/MWh
- MC(200 MW) = 100 $/MWh
- MC(Q) = 1000 $/MWh, 200 MW $\leq$ Q $\leq$ 220 MW
- VC(0 MW) = 0 $/h



# Summation of Marginal Cost Curves




This is the marginal cost of $n$ generators from previous slide, $n =$?

# Investment Cost

Fixed/investment cost: cost that is independent of output

- **Overnight cost** ($/kW): cost that needs to be paid upfront per kW of investment
- **Annualized fixed cost** ($/kWy): cost that needs to be paid per year per kW of investment
- **Hourly fixed cost** ($/MWh): cost that needs to be paid per hour per MW of investment

# Conversion of Investment Cost

Denote

- $T$ (years): investment lifetime
- $r$: interest rate

**Annualized fixed cost** $FC$ (\$/kWy) given *annual discounting*:

$$FC = \frac{r \cdot OC}{1 - 1/(1+r)^T}$$

**Annualized fixed cost** $FC$ (\$/kWy) given *continuous discounting*:

$$FC = \frac{r \cdot OC}{1 - e^{-rT}}$$

**Hourly fixed cost** (\$/MWh): Divide annualized fixed cost by 8.76 (why 8.76?)

# Example

- Gas turbine lifetime: 25 years
- Coal generator lifetime: 45 years
- Continuous discounting with interest rate $r = 12\%$

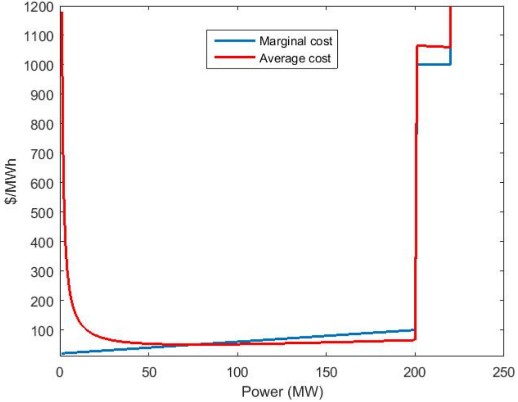
| | *OC* (\$/kW) | *FC* (\$/kWy) | *FC* (\$/MWh) |
|---|---|---|---|
| Gas turbine | 400 | 50.5 | 5.8 |
| Coal | 1200 | 144.7 | 16.5 |

# Average Cost

**Average cost**: total cost per unit of output

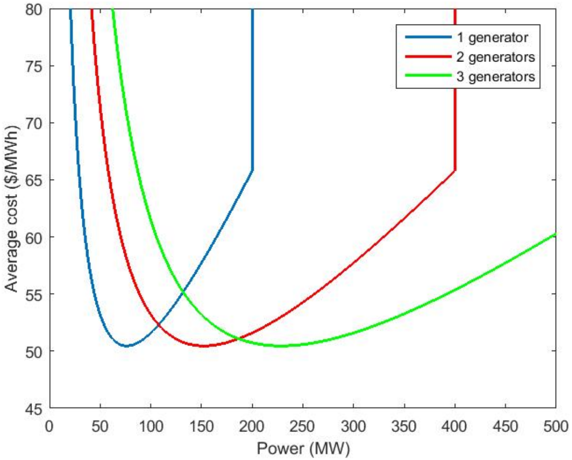
- Definition of average cost generalizes to the case of an *industry* that produces Q at minimum cost
- Economies of scale are realized when average cost decreases
- Average cost influences whether an industry is a **natural monopoly** or not

# Average Cost Curve: Single Unit




Why is there a jump of average cost at Q=0 MW?

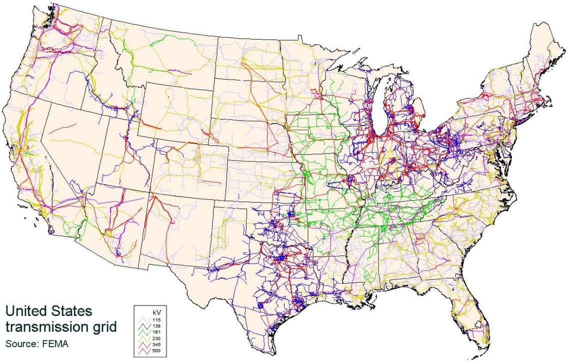
# Average Cost Curve: Multiple Units




Average cost at unit capacity (200 - 220 MW) is lower for $n = 3$ generators

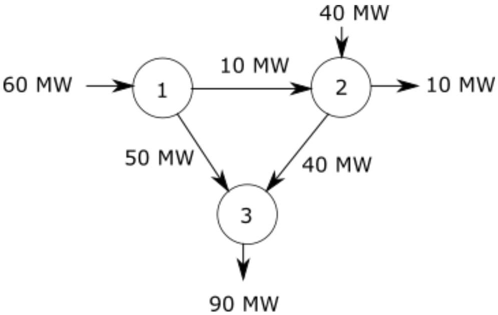
# Outline

# Transmission and Distribution




- Transmission: higher voltage, lower losses
- Distribution: lower voltage, higher losses
- Transformers reduce voltage at the interface

# Power Balance




- **Buses**: nodes of the transmission network
- **Branches/lines**: arcs of the transmission network
- Power balance at each bus (same as transportation models)
- Physical intuition: electricity is ‘lazy’

# Power Flow Equations

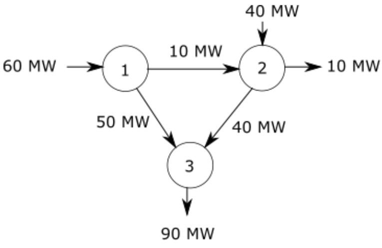
Power networks are more complex than transportation networks

**Kirchhoff's laws**: physical laws that govern flow of electricity in circuits, can be used to derive power flow equations

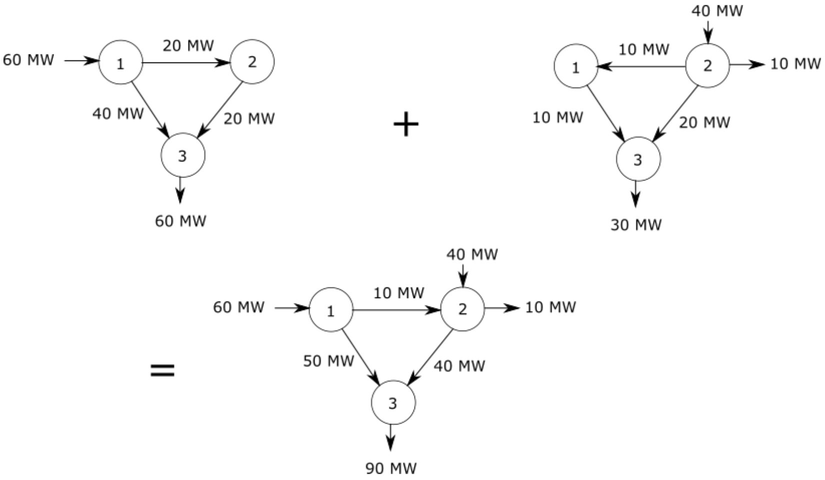
**Power flow equations** determine a mapping $f = P(r)$ of power injections $r$ in buses to power flows $f$ on lines

**Direct current (DC) power flow equations**: Approximation of power flow equations $f = P(r)$ by a *linear* mapping

# Proportionality of Power Flows



# Additivity of Power Flows



# Outline

# Valuation and Benefit

**Consumer benefit**: benefit that depends on amount of consumption

- Measured in \$/h
- Hourly benefit of consuming a certain amount of power

**Marginal benefit/valuation**: derivative of benefit with respect to consumption

- Measured in \$/MWh
- Willingness to pay (why?)

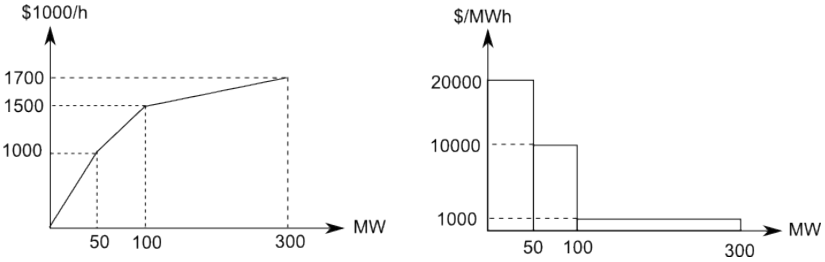
What is the supply-side analog of consumer benefit? of valuation?

# Inverse Demand /Marginal Benefit Function

**Inverse demand / marginal benefit function**: mapping of power consumption $Q$ to marginal benefit $MB(Q)$

Do we expect inverse demand function to be increasing/decreasing?

# Illustration of Marginal Benefit Function



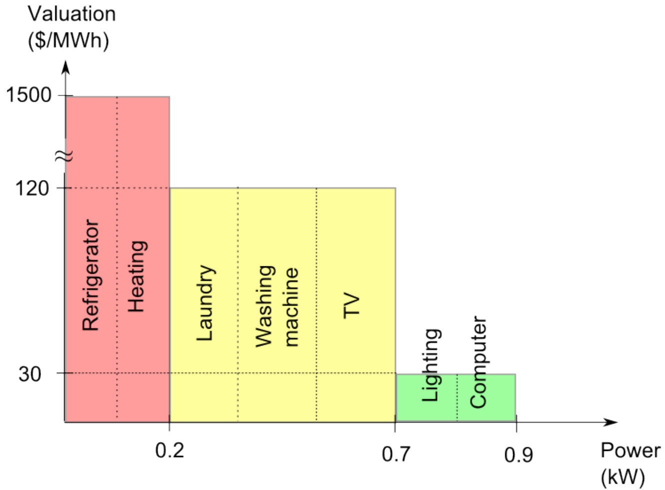

Which graph corresponds to consumer benefit? inverse demand function?

# Example: Household

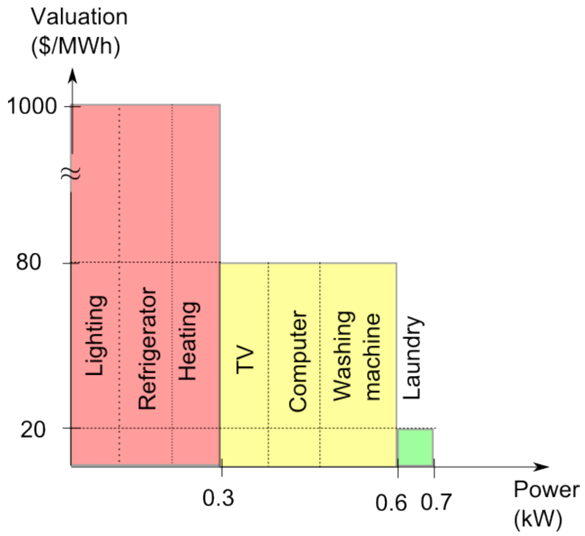
| Trench | Morning (kW) | Morning ($/MWh) | Evening (kW) | Evening ($/MWh) |
|---|---|---|---|---|
| Inflexible | 0.2 | 1500 | 0.3 | 1000 |
| Medium | 0.5 | 120 | 0.3 | 80 |
| Flexible | 0.2 | 30 | 0.1 | 20 |

Devices can move from trench to trench (what devices could belong to the inflexible trench?)

# Morning


Valuation
($/MWh)
1500
120
30
Refrigerator
Heating
Laundry
Washing machine
TV
Lighting
Computer
0.2
0.7
0.9
Power
(kW)

# Evening


Valuation
($/MWh)
1000
80
20
Lighting
Refrigerator
Heating
TV
Computer
Washing
machine
Laundry
0.3
0.6
0.7
Power
(kW)

# Demand Function

**Demand function** $Q(v)$: inverse mapping of inverse demand function

Maps price of power $v$ to quantity $Q$ that would be procured (why?)

# Retail Prices and Elasticity of Demand

**Elasticity**: sensitivity of demand $Q(v)$ to changes in price $v$:

$$\epsilon = \frac{dQ/dv}{Q/v}$$

Steep inverse demand function $\Leftrightarrow$ flat demand function $\Leftrightarrow$ low $\epsilon$ $\Leftrightarrow$ inelastic (insensitive) demand

Suppose household faces retail price of 25 \$/MWh. What is the demand as a function of *wholesale* price? Is it elastic?

# Average Value of Lost Load and VOLL

**Average value of lost load**: long-run average amount of load shed due to random disturbances (failures of generators and lines, forecast errors of renewable resources and load, etc.)

**Value of lost load (VOLL)**: marginal change in average value of lost load due to marginal increase in system capacity, divided by marginal decrease in the amount of shed load

VOLL useful in capacity expansion planning studies for quantifying marginal benefit of investment in generation capacity

# Example: VOLL

Consider the following demand function:

$$Q(v) = 30000 - 2v$$

Lost value from 1% decrease in service with random rationing:

$$\int_{v=0}^{15000} Q(v)dv - \int_{v=0}^{15000} 0.99Q(v)dv$$
$$= 0.01 \cdot \frac{15000 \cdot 30000}{2} = 2.25 \cdot 10^6 \ \$$$

Energy shed from 1% rationing: 300 MWh

$$VOLL = \frac{2250000}{300} = 7500 \ \$/\text{MWh}$$

More sophisticated computation of VOLL via simulation