Hedging Risk

Quantitative Energy Economics

Anthony Papavasiliou

Contents

Table of Contents

Forward Contracts

Forward contracts: financial instruments for trading a commodity in a price fixed in advance

Characterized by

- selling price f_t
- quantity x of traded commodity
- delivery time T of commodity / **expiration date** of forward contract.

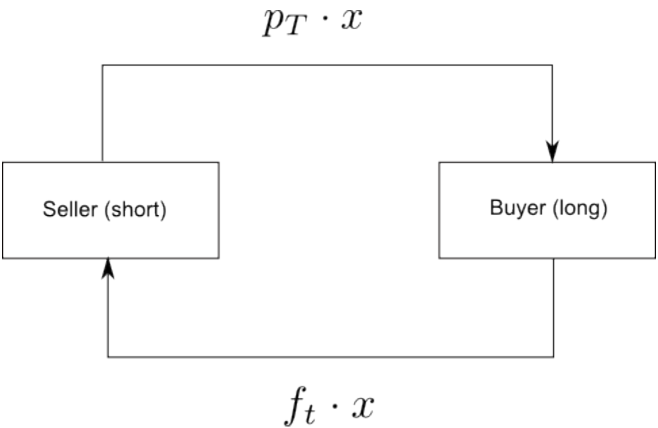
Definition

Seller. Seller of a forward contract with expiration date T sells contract at $t < T$ for a price f_t. Seller has a **short position.**

Buyer. Buyer of a forward contract with expiration date $t = T$ buys contract at $t < T$ for a price f_t. Buyer has a **long position.**

Obligations and payoffs. At time $t < T$, buyer pays seller $f_t \cdot x$. At time $t = T$, seller pays buyer $p_T \cdot x$. p_T is real-time price of the underlying commodity.

Payments



Virtues of Forward Contracts

- Hedging
- Forward contracts do not distort real-time incentives
- Forward contracts can be traded

Trading at Fixed Prices through Forwards

Producers: sell forward, produce in real time

- $+f_t \cdot x$ (from selling forward contract)
- $+p_T \cdot x$ (from producing in real-time market)
- $-p_T \cdot x$ (from settling forward contract)

Consumers: buy forward, consume in real time

- $-f_t \cdot x$ (from buying forward contract)
- $-p_T \cdot x$ (from consuming in real-time market)
- $+p_T \cdot x$ (from settling forward contract)

Hedging Risk without Distorting Real-Time Incentives

Suppose producer buys forward contract for x units at price f_t and produces q in real time

Producer is paid

$$R = f_t \cdot x + p_T \cdot (q - x)$$

where p_T is real-time price

- At T, producer only influences $p_T \cdot q \Rightarrow$ correct incentives, because the real-time price p_T is applied to the real-time decision q
- By producing $q = x$, producer receives price $f_t \Rightarrow$ hedging

Futures Contracts

Futures contracts: standardized forward contracts with rigid terms that are exchanged in a clearing house

- Default risk is reduced, carried by clearing house (+)
- Liquidity is enhanced (+)
- No concerns of credit-worthiness for traders (+)
- Less flexibility (-)

Integration with Power System/Market Operations

- Forward contracts
  - Suppliers and consumers can enter a forward contract *in advance*
  - In *real time*
    - Suppliers submit zero supply bid
    - Consumers submit ceiling demand bid
- Future contracts can be traded with the system operator
  - Sellers of futures pay system operator
  - Buyers of futures get paid by system operator
  - System operator gets information about supply-demand balance from the contracts

Price of a Forward Contract

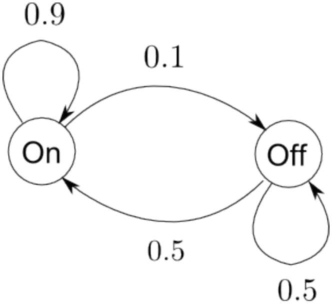
Given risk neutral market agents with same beliefs about the distribution of future real-time price p_T,

$$f_t = \mathbb{E}[p_T | \xi_{[t]}]$$

$\xi_{[t]}$: state of the world at time t

Example

- Inverse demand function: $D(p) = 1620 - 4p$
- Generator 1
  - Capacity: 295 MW
  - Marginal cost: 65.1 \$/MWh
- Generator 2
  - Capacity: 1880 MW
  - Marginal cost: 11.8 \$/MWh
  - Failures described by Markov chain


0.9
0.1
On
Off
0.5
0.5

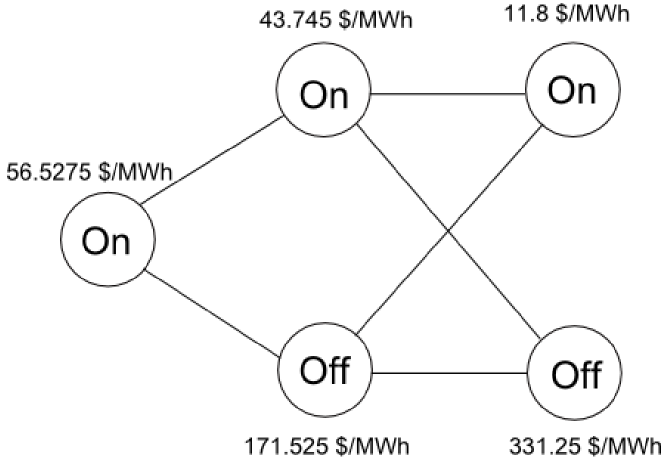
Computing Forward Prices

- Period 2 (you should compute this)
  - Generator 2 off: 295 MW at 331.25 $/MWh
  - Generator 2 on: 1572.8 MW at 11.8 $/MWh
- Period 1

$$f_1 = \begin{cases} 0.9 \cdot 11.8 + 0.1 \cdot 331.25 = 43.745 \text{ \$/MWh}, & \xi_1 = \text{On} \\ 0.5 \cdot 11.8 + 0.5 \cdot 331.25 = 171.525 \text{ \$/MWh}, & \xi_1 = \text{Off} \end{cases}$$

- Period 0 (assuming generator 2 is on)

$$f_0 = 0.9 \cdot 43.745 + 0.1 \cdot 171.525 = 56.5275 \text{ \$/MWh}$$


43.745 $/MWh
11.8 $/MWh
56.5275 $/MWh
On
On
On
Off
Off
171.525 $/MWh
331.25 $/MWh

Contracts for Differences

Contracts for differences (CfD): Alternative derivatives that serve same function as forward contract

Seller. A seller sells a CfD with expiration date T at time $t < T$ for x units of a commodity

Buyer. A buyer buys a CfD with expiration date T at time $t < T$ for x units of a commodity

Obligations and payoffs. At time T the buyer pays the seller $(f_t - p_T) \cdot x$, where p_T is the price of the commodity at T

Trading at Fixed Prices through CfDs

Buyer of CfD (consumer) consumes x at T:

- Pays $(f_t - p_T) \cdot x$ for CfD
- Pays $p_T \cdot x$ to spot market

Seller of CfD (supplier) produces x at T:

- Paid $(f_t - p_T) \cdot x$ for CfD
- Paid $p_T \cdot x$ from spot market

Table of Contents

The Need for Financial Transmission Rights

Forward contracts are adequate for trading electricity at a fixed price in a market without congestion

What happens if there is congestion?

Example

Generator A wants to trade 400 MW with consumer B at 40 $/MWh

Generator sells forward contract for 400 MW at 40 $/MWh to load

Suppose $p_A = p_B = 50$ $/MWh

Cash flows to producer:

- $+40 \cdot 400 = +16000$ $ (sell forward)
- $+50 \cdot 400 = +20000$ $ (produce in real-time market)
- $-50 \cdot 400 = -20000$ $ (settle forward)

Cash flows to load: $-40 \cdot 400 - 50 \cdot 400 + 50 \cdot 400 = -16000$ $

Result: parties trade at 40 $/MWh

Suppose $p_A = 36$ \$/MWh, $p_B = 45$ \$/MWh

Suppose generator sells forward contract for 400 MW in location A

Cash flows to producer:
$+40 \cdot 400 + 36 \cdot 400 - 36 \cdot 400 = +16000$ \$

Cash flows to load: $-40 \cdot 400 - 45 \cdot 400 + 36 \cdot 400 = -19600$ \$

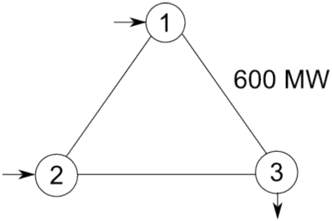
Result: generator paid 40 \$/MWh, load pays 49 \$/MWh $\Rightarrow$ load pays $p_B - p_A = 9$ \$/MWh

Transmission Rights

In order to develop financial instruments that hedge against locational price differences it is necessary to define *rights* for the usage of lines

- **Contract paths**: right to ship power between zones
  - Ignores physical reality (Kirchoff laws)
  - Failed
- **Financial transmission rights** (Hogan, 1992): right to ship power *between buses*

Failure of Contract Paths (Hogan, 1992)



- Line 1-3 limit: 600 MW
- Lines have identical characteristics

Why Contract Paths Fail

Define contract path from zone G (nodes 1, 2) to zone L (node 3)

How many rights?

- Option 1: 900 MW
  - Advantage: line 1-3 will never be overloaded (why?)
  - Disadvantage: inefficient trade (suppose cheapest generators in node 2)
- Option 2: 1800 MW
  - Advantage: maximize opportunities for trade
  - Disadvantage: line 1-3 may be destroyed (why?)

Conclusion: contract paths may either (i) limit trade to inefficient levels, or (ii) violate line limits

Financial Transmission Rights

Seller. At time T the seller sells a **financial transmission right** for shipping power from location A to location B for x MW with expiration date T

Buyer. At time $t < T$ the buyer of an FTR with expiration date T buys the contract

Obligations and payoffs. At time T the seller pays the buyer of the FTR $(p_B - p_A) \cdot x$ (p_A, p_B are the LMPs)

Example Revisited

Load B buys forward contract from generator A and FTR from A to B

Cash flows to load:

- $-40 \cdot 400 = -16000$ \$ (buying forward)
- $-45 \cdot 400 = -18000$ \$ (consuming in real-time market)
- $+36 \cdot 400 = +14400$ \$ (settling forward)
- $+9 \cdot 400 = +3600$ \$ (settling FTR)

Result: Load pays 40 $/MWh

FTR Auctions

Default seller of FTRs: system operator (why?)

Simultaneous feasibility of FTRs: Allocation of FTRs must respect transmission constraints

Recall **congestion rent**: LMP auction payments

Revenue adequacy: LMP auction payments are enough to cover FTR payments if FTRs are simultaneously feasible

Proof: we first recall that congestion rent is non-negative, then show it exceeds FTR payments

Bilateral Trade at Fixed Prices

Producer sells forward contract to load and load buys FTR from generator location (A) to load location (B)

Cash flows to producer:

- $+f_t \cdot x$ (selling forward)
- $+p_A \cdot x$ (producing in real-time market)
- $-p_A \cdot x$ (settling forward)

Cash flows to consumer:

- $-f_t \cdot x$ (buying forward)
- $-p_B \cdot x$ (consuming in real-time market)
- $+p_A \cdot x$ (settling forward)
- $+(p_B - p_A) \cdot x$ (settling FTR)

Result: Trade in fixed price f_t which is known in advance

Recall OPF

$$\max \sum_{l \in L} \int_0^{d_l} MB_l(x)dx - \sum_{g \in G} \int_0^{p_g} MC_g(x)dx$$

$$(\lambda_k^+): \quad f_k \leq T_k$$

$$(\lambda_k^-): \quad -f_k \leq T_k$$

$$(\psi_k): \quad f_k - \sum_{n \in N} F_{kn} r_n = 0$$

$$(\rho_n): \quad r_n - \sum_{g \in G_n} p_g + \sum_{l \in L_n} d_l = 0$$

$$(\phi): \quad \sum_{n \in N} r_n = 0$$

$$p_g, d_l \geq 0$$

Congestion Rent Is Non-Negative

Congestion rent is non-negative, and given by the following expression:

$$\sum_{n \in N} \rho_n \left(\sum_{l \in L_n} d_l - \sum_{g \in G_n} p_g \right) = \sum_{k \in K} (\lambda_k^+ + \lambda_k^-) T_k$$

Proof: If identity is true, then since $\lambda_k^+, \lambda_k^- \geq 0$, congestion rent is non-negative

$$\sum_{n \in N} \rho_n (\sum_{l \in L_n} d_l - \sum_{g \in G_n} p_g) = \qquad \text{definition of } r_n$$

$$- \sum_{n \in N} \rho_n r_n = \qquad \text{from } \rho_n = -\phi + \sum_{k \in K} F_{kn}(\lambda_k^- - \lambda_k^+)$$

$$\text{and } \sum_{n \in N} r_n = 0$$

$$\sum_{k \in K} (\lambda_k^+ - \lambda_k^-) \sum_{n \in N} F_{kn} r_n = \qquad \text{definition of } f_k$$

$$\sum_{k \in K} (\lambda_k^+ - \lambda_k^-) f_k = \qquad \text{from } 0 \leq \lambda_k^+ \perp T_k - f_k \geq 0$$

$$\text{and } 0 \leq \lambda_k^- \perp T_k + f_k \geq 0$$

$$\sum_{k \in K} (\lambda_k^+ + \lambda_k^-) T_k$$

Congestion Rent and FTR Payments

Financial transmission rights pay to their holders

$$-\sum_{n \in N} \rho_n \tilde{r}_n$$

where $\tilde{r}_n$ is a feasible (not necessarily optimal) dispatch

Congestion rent is adequate to cover FTR payments:

$$-\sum_{n \in N} \rho_n r_n \geq -\sum_{n \in N} \rho_n \tilde{r}_n$$

Proof: From previous proof,

$$-\sum_{n \in N} \rho_n (r_n - \tilde{r}_n) = \sum_{k \in K} (\lambda_k^+ - \lambda_k^-)(f_k - \tilde{f}_k)$$

where

- λ_k^+, λ_k^- are dual optimal multipliers,
- f_k are flows corresponding to r_n
- $\tilde{f}_k$ are flows corresponding to $\tilde{r}_n$

Consider three cases:

- $f_k = T_k$ (which implies $\lambda_k^- = 0$)
- $f_k = -T_k$ (which implies $\lambda_k^+ = 0$)
- $-T_k < f_k < T_k$ (which implies $\lambda_k^+ = \lambda_k^- = 0$)

Physical Transmission Rights

Physical transmission rights (PTRs): provide *exclusive* access to the holder of the rights, no financial payoff

FTRs are purely financial, do not interfere with efficient dispatch $\neq$ PTRs can lead to inefficiencies

Table of Contents

Call Options

Seller. Seller of a call option with expiration date T and **strike price** k sells option at $t < T$ for amount x of underlying commodity

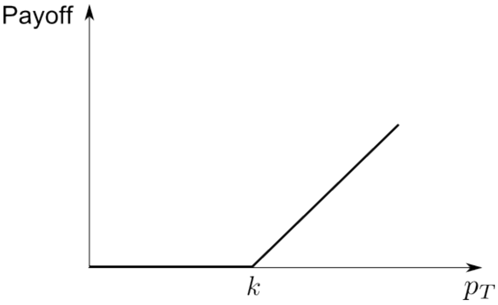
Buyer. Buyer of call option with expiration date T and strike price k buys contract at $t < T$ for amount x of underlying commodity

Obligations and payoffs. At $t < T$ buyer pays seller the price of the call option. At T seller pays buyer $\max(p_T - k, 0) \cdot x$, where p_T is spot price of the underlying commodity.

The Function of Call Options

The buyer of the option has the right, but not the obligation, to buy the commodity at strike price k at expiration

- $p_T \leq k$: no value from call option
- $p_T > k$: buyer receives $p_T - k$, can buy the commodity in the spot market with net expense of k



Callable Forward

Seller: Seller of a callable forward with expiration date T and strike price k sells contract at $t < T$ for amount x of underlying commodity

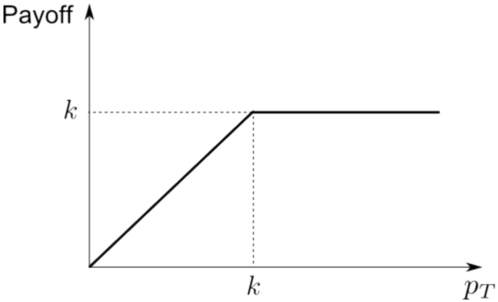
Buyer: Buyer of callable forward buys contract at $t < T$ for amount x of underlying commodity

Obligations and payoffs: At $t < T$ buyer pays seller the price of the callable forward, at T seller pays buyer $\min(p_T, k) \cdot x$, where p_T is spot price of the underlying commodity

The Function of Callable Forward Contracts

Curtail the provision of a commodity to the buyer of the contract when $p_T \geq k$:

- If $p_T \leq k$, buyer receives p_T from seller and can buy the commodity in the spot market
- If $p_T > k$, buyer receives k



Price of Callable Forward Forward Contracts

Define

$$Q_t(p) = \mathbb{P}[p_T \leq p | \xi_{[t]}]$$

where $\xi_{[t]}$ is information in time t

Assuming density for $Q_t(p)$ exists,

$$q_t(p) = \frac{\partial}{\partial p} Q_t(p).$$

Price for forward, callable forward in time t:

$$f_t = \mathbb{E}[f_T | \xi_{[t]}] = \int_0^\infty p q_t(p) dp \tag{1}$$

$$j_t(k) = \mathbb{E}[\min(p_T, k) | \xi_{[t]}] = \int_0^\infty \min(p, k) q_t(p) dp \tag{2}$$

$q_t(p)$ Determines $j_t(k)$ and Vice Versa

Integrating by parts:

$$\begin{aligned} j_t(k) &= k - \int_0^k Q_t(p)dp \\ &= \int_0^k (1 - Q_t(p))dp \end{aligned} \tag{3}$$

Differentiating with respect to k:

$$\frac{\partial}{\partial k} j_t(k) = 1 - Q_t(k) \tag{4}$$

Differentiating again with respect to k:

$$q_t(k) = -\frac{\partial^2}{\partial k^2} j_t(k) \tag{5}$$

Properties of Callable Forward Price

- $j_t(k)$ nondecreasing, concave in k
  - Proof: follows from equations 4, 5
  - Intuitive: higher strike price increases payoff for holder
- $j_t(k) \leq k$ for all k
  - Proof: follows from equation 3
  - Intuitive: callable forward cannot pay more than k
- $\lim_{k \to \infty} j_t(k) = f_t$
  - Proof: follows from equations 1, 2
  - Intuitive: as k increases, likelihood of $p_T \leq k$ decreases

Virtues of Callable Forward Contracts

- Useful for integrating demand response
- Consumers self-select the ‘right’ contract
- Callable forwards can be traded

Integration of Demand Response

Mutual benefits from callable forwards for loads and system operator

- Loads with valuation v *always* receive full value of power supply, regardless of real-time price of electricity, by selecting $k = v$
  - If $p_T \leq v$, loads consume power
  - If $p_T > v$, loads receive compensation $k = v$ (equivalent to consuming power)
- System operator receives information about demand function, beneficial for capacity planning with renewable resources

Consumer Self-Selection

Assuming risk neutral consumers, callable forwards priced according to expected payoff

$$\begin{aligned}\mathbb{E}[B_t(k)|\xi_{[t]}] &= Q_t(k)\cdot v + (1-Q_t(k))\cdot k - j_t(k) \\ &= k + Q_t(k)\cdot(v-k) - j_t(k) \qquad (6)\end{aligned}$$

where $B_t(k)$ is benefit of consumer

From equation 4 we get

$$\begin{aligned}\frac{\partial}{\partial k}\mathbb{E}[B_t(k)|\xi_{[t]}] &= 1 - \frac{\partial}{\partial k}j_t(k) - Q_t(k) + (v-k)\cdot q_t(k) \\ &= (v-k)\cdot q_t(k) \qquad (7)\end{aligned}$$

Suppose $q_t(k) > 0$ for all $k > 0$

- $k = v$ is unique maximizer of expected benefit
  - $\frac{\partial}{\partial k}\mathbb{E}[B_t(k)|\xi_{[t]}] = 0$ for $k = v$
  - $\frac{\partial}{\partial k}\mathbb{E}[B_t(k)|\xi_{[t]}] > 0$ for $k < v$
  - $\frac{\partial}{\partial k}\mathbb{E}[B_t(k)|\xi_{[t]}] < 0$ for $k > v$
- Buying callable forward is better than not buying
  - From equation 6, expected payoff for $k = v$ is $v - j_t(v)$
  - From equation 3 and $q_t(k) > 0$, $v - j_t(v) > 0$