Solving Large-Scale Optimization Problems under Uncertainty and Non-Convexity in Electric Power Systems

Andy Sun

Georgia Institute of Technology

H. Milton Stewart School of Industrial and Systems Engineering

Joint work with **Burak Kocuk, Alvaro Lorca, Jikai Zou (doctoral students)**

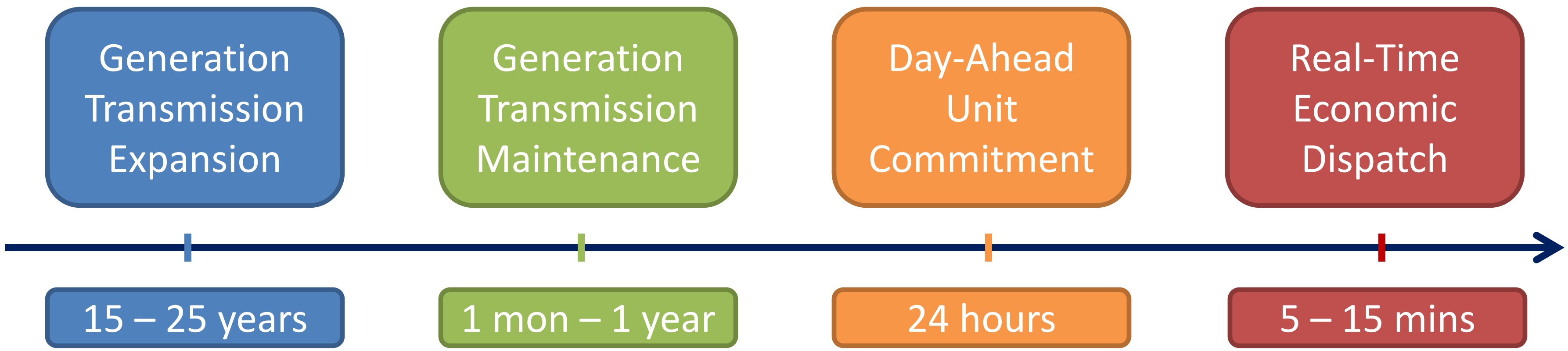
Shabbir Ahmed, Santanu Dey (GT), Tongxin Zheng, Eugene Litvinov (ISO-NE)

CORE50 **Bridging the Gap**

May 27, 2016

Electric Power Systems Problems

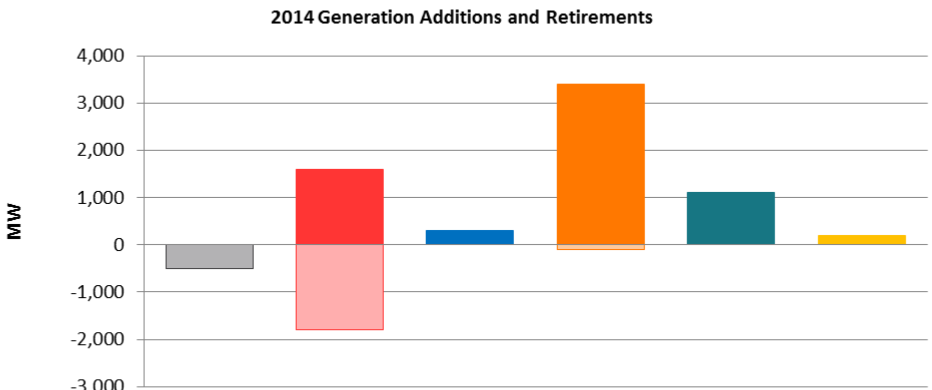
- Key Optimization Problems in power system operations from System perspective:



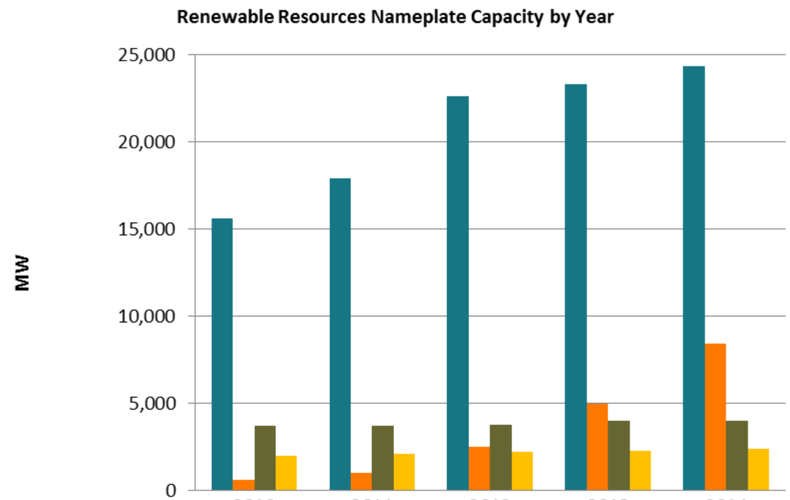
  - Real-Time Economic Dispatch:
    - Hourly bidding and ISO 5, 15 min dispatch
  - Day-Ahead Unit Commitment:
    - A day prior to operation to determine unit commitment
  - Yearly generation/transmission maintenance:
  - Long-term generation/transmission expansion:

Challenge: Renewable Integration

- **Renewable Energy** Integration in Western Interconnection



	Coal	Gas	Hydro	Solar	Wind	Other Renewable
Commissioned	0	1,600	300	3,400	1,100	200
Retired	-500	-1,800	0	-100	0	0
Net	-500	-200	300	3,300	1,100	200

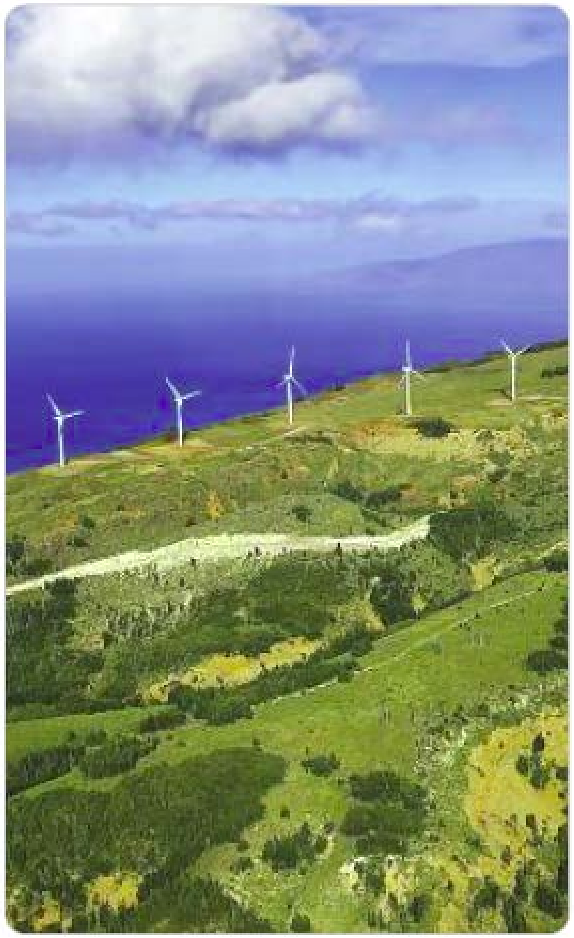


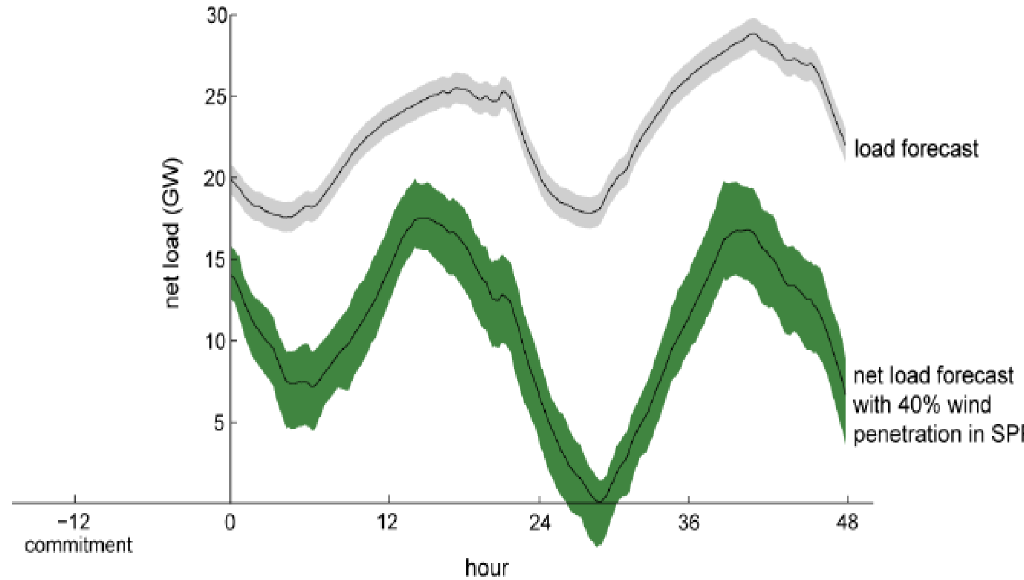
	2010	2011	2012	2013	2014
Wind	15,600	17,900	22,600	23,300	24,300
Solar	600	1,000	2,500	5,000	8,400
Geothermal	3,700	3,700	3,800	4,000	4,000
Other Renewable	2,000	2,100	2,200	2,300	2,400

- WECC's Largest generation addition in 2014: **3,400 MW utility-scale solar**
- **Behind-the-meter solar** at least 3,200 MW
- Since 2010, nearly **10,000 MW wind** and **8,000 MW solar** added

Challenge: Supply/Demand Uncertainty

- Renewable Integration





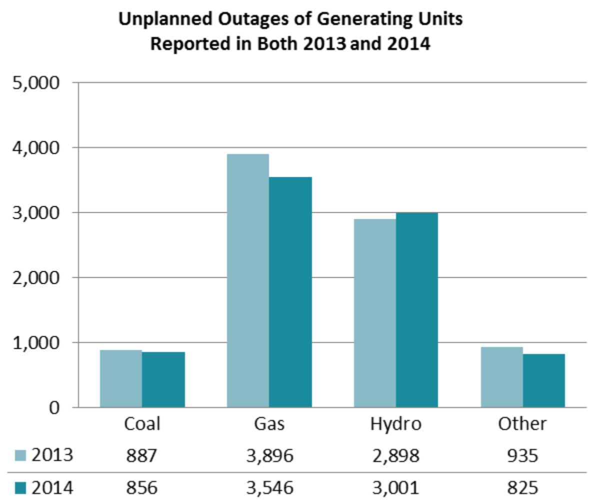
[Ruiz, Philbrick 10]

Supply Variation:
Wind/Solar Power Penetration
Behind-the-Meter installation

Net Load Uncertainty
Can be Huge!

Challenge: Unplanned Outages

- Unplanned Generator Outages:

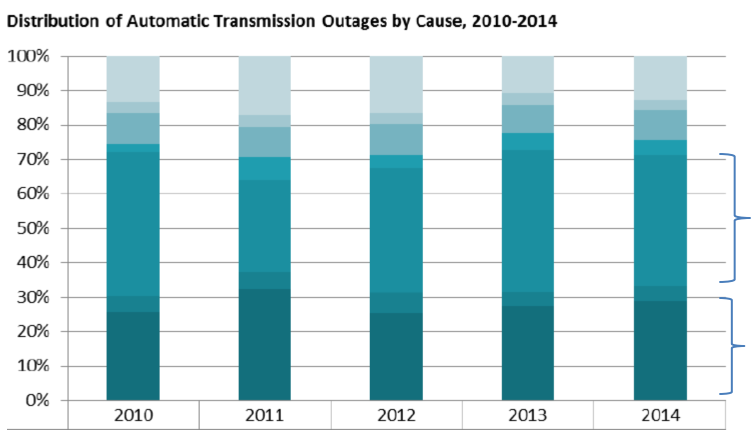


Median Unplanned Outages Per Unit Per Year, 2013-2014

	2013	2014
Coal	9	9
Gas	5	5
Hydro	3	3

Lack of monitoring
Entails high economic Cost and threaten system security

- Unplanned Transmission Outages:

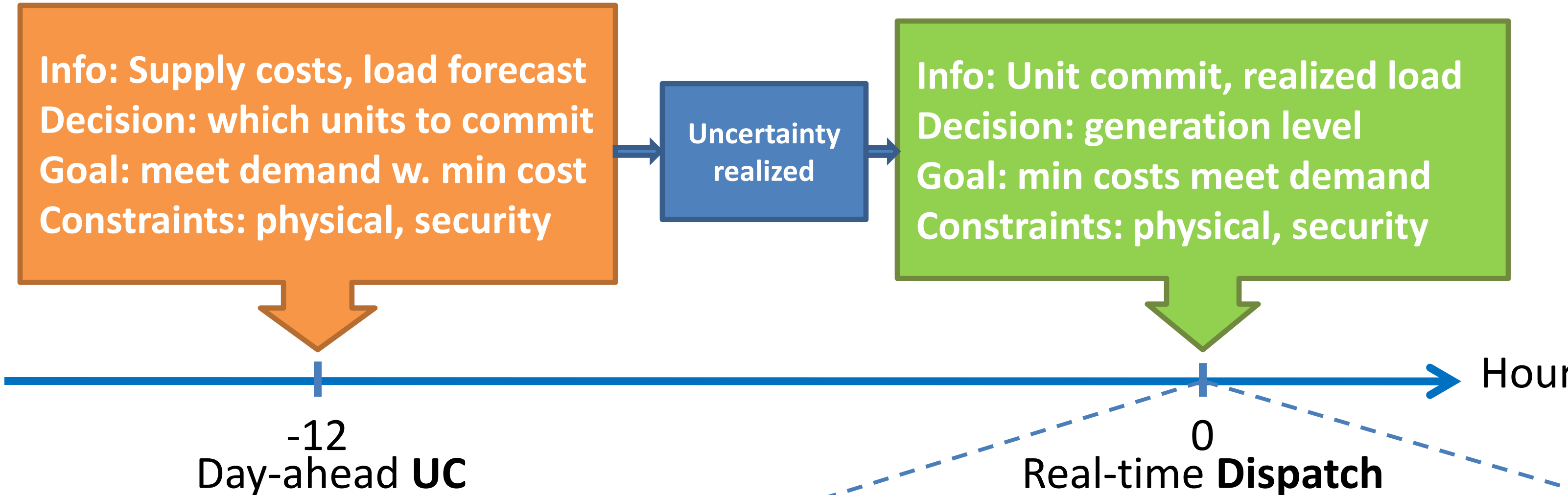


Environmental & Weather: 558/1471=38%

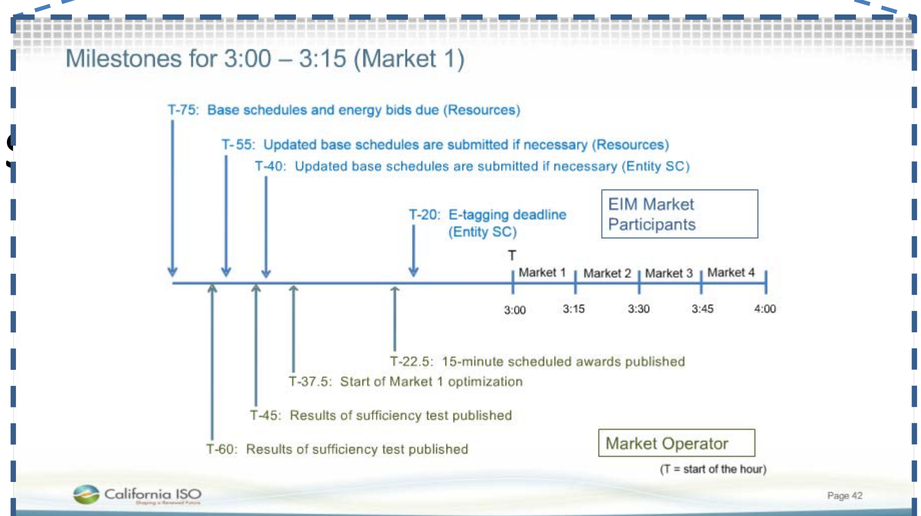
Unknown causes: 425/1471 = 29%!

Challenge: Dynamic Decision Making

- Uncertainty in Dynamic Decision Making



  - Wind/solar rapid changes
  - Limited ramping



Challenge: Non-convex Models

- Non-convexity: Discrete nature
  - **commitment** decisions
  - Transmission line/Capacitor **switching** decisions
  - **Maintenance** decisions
  - Generation/transmission **expansion** decisions

Modeled As 0-1 variables

- Non-convexity: Continuous nature
  - **Power Flow** Physics: non-convex quadratics
  - **Control** devices: FACTS
  - **Hydrology**: water flow vs power

Convexify

Outline

- Some projects on dealing with **Uncertainty**
  1. Stochastic dual dynamic programming with binary recourse for generation expansion planning
  2. Multistage robust optimization with decision rule for unit commitment

- Some project on dealing with **Non-convexity:**
  3. Optimal Power Flow (OPF) and Optimal Transmission Switching (OTS) problem

Act 1

Nested Decomposition and SDDP with Binary Recourse

Multistage Stochastic Integer Program

- Extensive form of an MSIP:

$$\min_{x_n, y_n} \left\{ \sum_{n \in \mathcal{T}} p_n f_n(x_n, y_n) : \ (x_{a(n)}, x_n, y_n) \in X_n \ \forall n \in \mathcal{T} \right\}$$

- State variables: $\{x_n\}_{n \in \mathcal{T}}$ binary (key assumption)
- Local variables: $\{y_n\}_{n \in \mathcal{T}}$ mixed integer
- Assumptions:
  - $f_n(x_n, y_n)$ linear in x_n, y_n
  - X_n compact, linear mixed-integer set
  - Complete recourse

A Key Reformulation

- A reformulation of MSIP:

$$\begin{aligned}
\min_{x_n, y_n, z_n} \quad & \sum_{n \in \mathcal{T}} p_n f_n(x_n, y_n) \\
\text{s.t.} \quad & \forall n \in \mathcal{T} \\
& (z_n, x_n, y_n) \in X_n \\
& \boxed{\begin{aligned} & z_n = x_{a(n)} \\ & z_n \in [0,1]^d \end{aligned}} \\
& x_n \in \{0,1\}^d
\end{aligned}$$

Dynamic Programming Recursion

- Dynamic programming recursion: at node n

$$
\begin{aligned}
Q_n(x_{a(n)}) := \min_{x_n, y_n, z_n} \quad & f_n(x_n, y_n) + \sum_{m \in \mathcal{C}(n)} q_{nm} Q_m(x_n) \\
\text{s.t.} \quad & (z_n, x_n, y_n) \in X_n \\
& z_n = x_{a(n)} \\
& z_n \in [0,1]^d \\
& x_n \in \{0,1\}^d
\end{aligned}
$$

- Nested decomposition algorithm:
  - Approximate cost-to-go $\Psi_n^i(x_n) \leq \sum_m q_{nm} Q_m(x_m)$
  - Iteratively strengthen Ψ_n^i by linear cuts

Nested Decomposition Algorithm

In iterations i

- **Forward:**
  - solve the lower approximation $P_n^i(x_{a(n)}^i, \psi_n^i)$

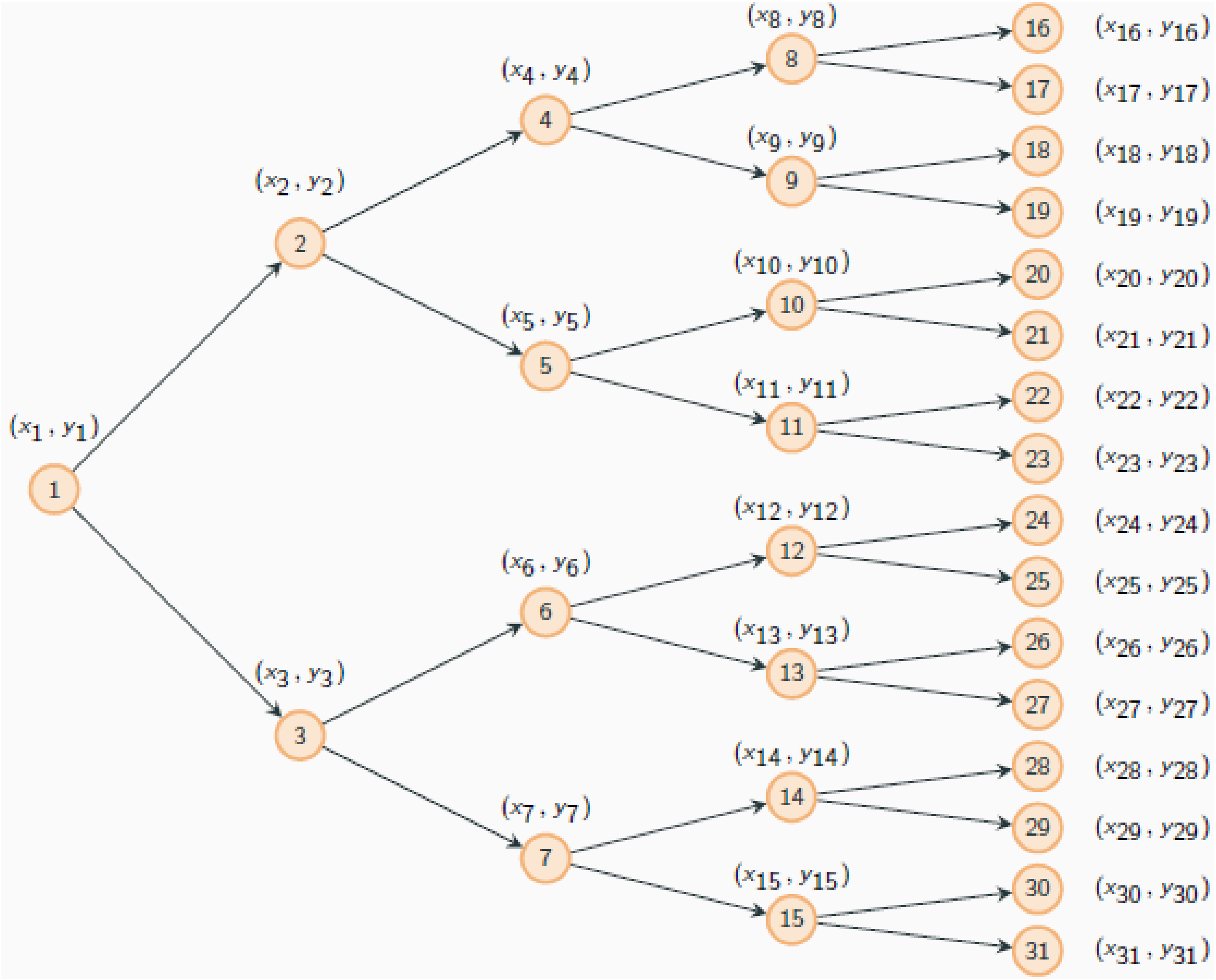
$$\underline{Q}_n^i(x_{a(n)}^i, \psi_n^i) := \min_{x_n, y_n, z_n} \quad f_n(x_n, y_n) + \psi_n^i(x_n)$$
$$\text{s.t.} \quad (z_n, x_n, y_n) \in X_n$$
$$z_n = x_{a(n)}^i, \;\; x_n \in \{0, 1\}^d$$

  where $\psi_n^i(x_n) = \max\{L_n, \ell_n^k(x_n) : k \leq i-1\}$, $\ell_n^k(\cdot)$ are linear cuts

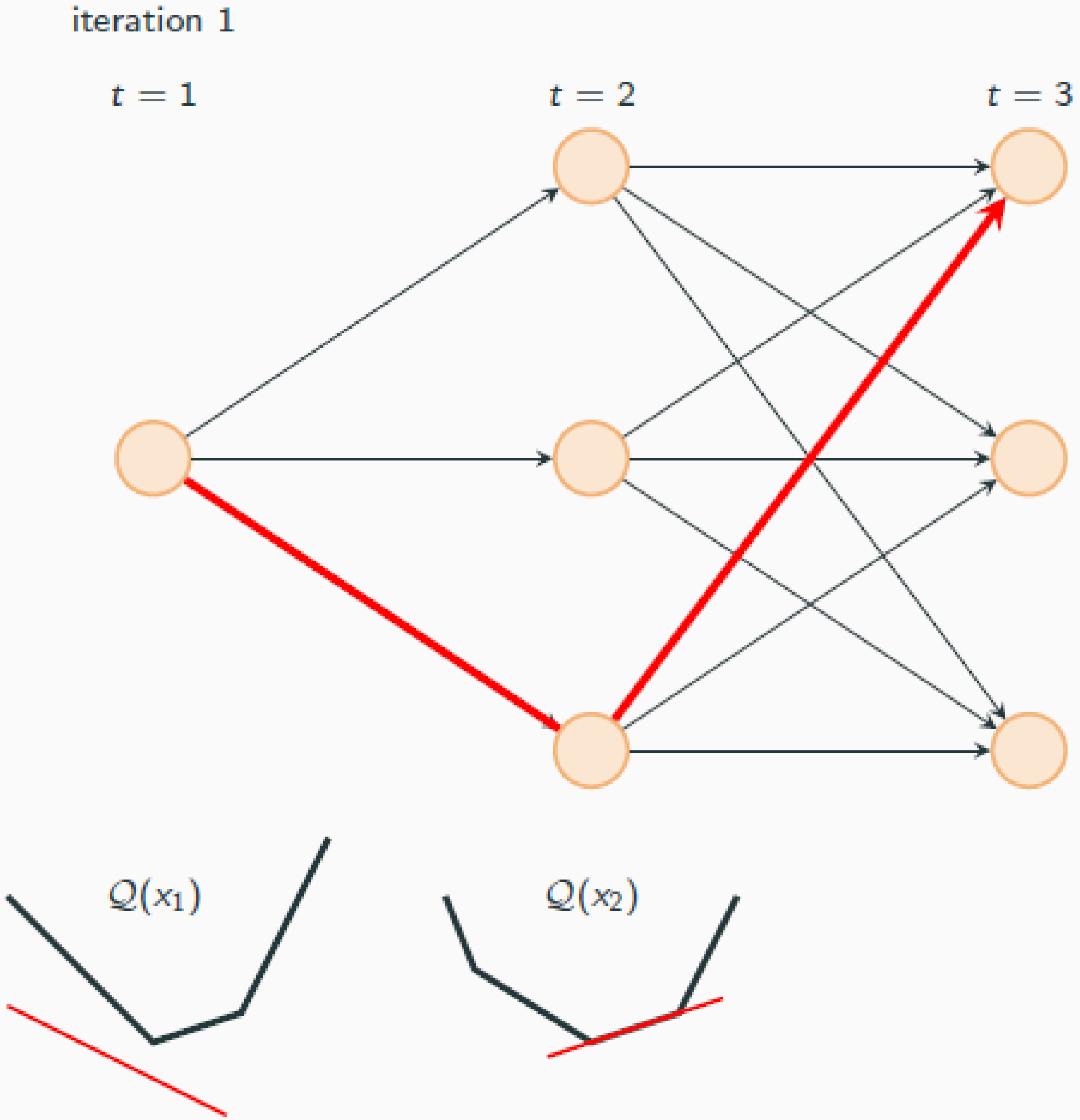
  - upper bound

- **Backward:**
  - solve $P_m^i(x_n^i, \psi_m^{i+1})$ (or relaxation) and collect cut coefficients (v_m^i, π_m^i) for all $m \in \mathcal{C}(n)$
  - set $\ell_n^i(x_n) = \sum_{m \in \mathcal{C}(n)} q_{nm}(v_m^i + (\pi_m^i)^\top x_n)$, and $\psi_n^{i+1}(x_n) \leftarrow \max\{\psi_n^i(x_n), \ell^i(x_n)\}$
  - lower bound

Nested Decomposition Algorithm



SDDP



FORWARD:

- Independent sampling across stages
- Compute candidate solutions
- Obtain statistical UB

BACKWARD:

- Evaluate candidate solution at every outcome
- Generate cuts
- Obtain exact LB

Properties of Linear Cuts

Recall (v_n^i, π_n^i) is collected from solving

$$\begin{aligned} \underline{Q}_n^i(x_{a(n)}^i, \psi_n^{i+1}) := \min_{x_n, y_n, z_n} \quad & f_n(x_n, y_n) + \psi_n^{i+1}(x_n) \\ \text{s.t.} \quad & (z_n, x_n, y_n) \in X_n \\ & z_n = x_{a(n)}^i, \; x_n \in \{0,1\}^d \end{aligned}$$

or its relaxation, we say the cut is

- Valid. $Q_n(x_{a(n)}) \geq v_n^i + (\pi_n^i)^\top x_{a(n)}, \; \forall \; x_{a(n)} \in \{0,1\}^d$
- Tight. $\underline{Q}_n^i(x_{a(n)}^i, \psi_n^{i+1}) = v_n^i + (\pi_n^i)^\top x_{a(n)}^i$
- Finite. only produce finitely many different (v_n^i, π_n^i)

Convergence

- **Theorem 2**:

If the cuts used in Nested Decomposition (ND) are valid, tight, and finite, then ND terminates in a finite number of iterations with an optimal solution.

- **Theorem 3:**

Suppose the sampling step is done with replacement, and the cuts generated in the backward steps are valid, tight, and finite, then SDDP converges to an optimal solution in a finite number of steps with probability 1.

- Convergence is what you would expect. However, a rigorous proof is not entirely trivial for Theorem 3.

Existing Cuts

- Benders' cut (Benders, 1962) (valid but *not* tight)

$$\ell_n^i(x_n) = \sum_{m \in \mathcal{C}(n)} q_{nm} v_m^{\mathrm{LP},i} + \sum_{m \in \mathcal{C}n} q_{nm} (\pi_m^{\mathrm{LP},i})^\top (x_n - x_n^i)$$

- Integer optimality cut (Laporte and Louveaux, 1993) (valid and tight)

$$\ell_n^i(x_n) = (\bar{v}_n^{i+1} - L_n) \left(\sum_{j \in S(x_n^i)} x_{n,j} - \sum_{j \notin S(x_n^i)} x_{n,j} - |S(x_n^i)| \right) + \bar{v}_n^{i+1}$$

where $S(x_n^i) = \{j : x_{n,j}^i = 1\}$ and $\bar{v}_n^{i+1} = \sum_{m \in \mathcal{C}(n)} q_{nm} \underline{Q}_n^i(x_{a(n)}^i, \psi_n^{i+1})$

Our Proposal: Lagrangian Cuts

In the BACKWARD step, $\forall m \in \mathcal{C}(n)$, we solve

$$\begin{aligned}\mathcal{L}_m^i(\pi_m) = \min_{x_m, y_m, z_m} \quad & f_m(x_m, y_m) + \psi_m^{i+1}(x_m) - \pi_m^\top(z_m - x_n^i) \\ \text{s.t.} \quad & (z_m, x_m, y_m) \in X_m \\ & z_m \in [0,1]^d \\ & x_m \in \{0,1\}^d\end{aligned}$$

- Lagrangian cut: let $v_m^{\mathrm{LG},i} = \max_{\pi_m} \mathcal{L}_m^i(\pi_m)$

$$\ell_n^i(x_n) = \sum_{m \in \mathcal{C}(n)} q_{nm} v_m^{\mathrm{LG},i} + \sum_{m \in \mathcal{C}n} q_{nm} (\pi_m^{\mathrm{LG},i})^\top (x_n - x_n^i)$$

Theorem 1: Given any binary $\{x_n^i\}_{n \in T}$, the collection of Lagrangian cuts $\{(v_n^i, \pi_n^i)\}_{n \in T}$ is **valid** and **tight**.

Computational Results

T	# branch	cuts	best LB ($MM)	# iter	stat. UB ($MM)	gap	time (hrs)
6	50	B + I	2818.8	237	2840.6	0.77%	2.24
		SB + I	2818.9	74	2855.8	1.29%	0.60
		B + L	2818.9	63	2848.5	1.04%	0.96
		SB + L	2818.9	56	2849.2	1.06%	0.70
		SB + I + L	2818.9	50	2820.7	0.06%	1.03
7	50	B + I	3564.5	239	3614.8	1.39%	8.10
		SB + I	3564.4	111	3588.9	0.68%	1.08
		B + L	3564.5	100	3569.1	0.13%	2.48
		SB + L	3564.5	66	3576.9	0.35%	2.37
		SB + I + L	3564.5	69	3577.6	0.37%	1.95
8	30	B + I	4159.4	340	4254.2	2.23%	7.78
		SB + I	4159.4	152	4207.5	1.14%	1.53
		B + L	4159.6	147	4227.7	1.61%	4.00
		SB + L	4159.6	87	4218.9	1.41%	2.55
		SB + I + L	4159.6	103	4278.0	2.77%	2.72
9	30	B + I	5058.0	520	5081.5	0.46%	19.85
		SB + I	5058.6	230	5102.0	0.85%	2.57
		B + L	5058.7	218	5108.6	0.98%	8.01
		SB + L	5058.7	120	5145.3	1.68%	2.96
		SB + I + L	5058.9	119	5079.4	0.40%	4.39

References

- J. Zou, S. Ahmed, A. Sun. “Nested decomposition for multistage stochastic integer programming with binary state variables”, submitted to **Mathematical Programming**, 2016.
- J. Zou, S. Ahmed, A. Sun. “Partially adaptive stochastic optimization for electric power generation expansion planning”, **INFORMS Journal on Computing**, minor revision, 2016.

Act 2

Multistage Robust Unit Commitment: Decision rules for **Uncertainty**

Act 2: Multistage Robust UC

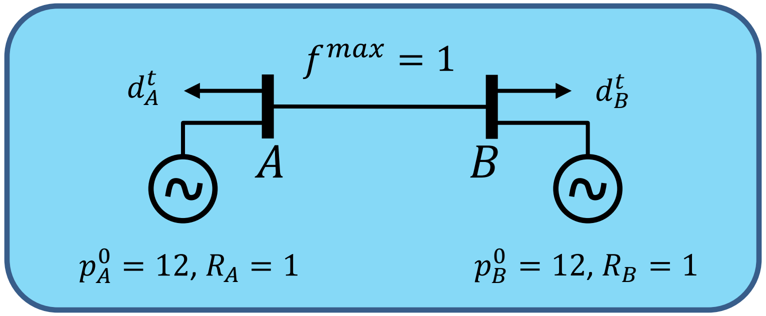
- Motivation
- Model, Affine Policy, and Algorithm
- Computational Results:
  - Reasonable computation time 2718-bus
  - Near-optimal performance
  - Advantage over two-stage models
  - Average performance

Recent Works on Robust UC and ED

- Robust Optimization for unit commitment
  - Adaptive two-stage robust SCUC models
    - [Jiang et. al. 2012], [Zhao, Zeng 2012],
    - [Bertsimas, Litvinov, Sun, Zhao, Zheng 2013] (joint w. ISO-NE)
  - RO for security optimization
    - [Street et. al. 2011], [Wang et. al. 2013]
  - Unifying RO with Stochastic UC
    - [Wang et. al. 2013]
  - New types uncertainty set
    - [Guan Wang 2014] [Lorca Sun 2014] [Chen et. al. 2015]
- Robust Optimization for economic dispatch
  - AGC control (two-stage: dispatch + AGC)
    - [Zheng et. al. 2012]
  - Affine policy (dispatch as linear function of total load)
    - [Jabr 2013][Warrington2012,2013]

Issues with Two-Stage Robust UC

- A simple two-bus two-period example:



Demand uncertainty sets:
$D^1 = \{(12,12)\}$,
$D^2 = \left\{(d_A^2, d_B^2): d_A^2 + d_B^2 = 25, d_i^2 \in [10,15]\right\}$

- Claim: **Two-stage robust UC is feasible**
  - UC solution: $(x_A^t, x_B^t) = (1,1)$ for $t = 1,2$
  - Feasible dispatch solution:
    - $p_A^1(\boldsymbol{d}) = 12 + \frac{2}{5}(d_A^2 - 12.5), p_B^1(\boldsymbol{d}) = 12 - \frac{2}{5}(d_A^2 - 12.5)$
    - $p_A^2(\boldsymbol{d}) = 12.5 + \frac{3}{5}(d_A^2 - 12.5), p_B^2(\boldsymbol{d}) = 12.5 - \frac{3}{5}(d_A^2 - 12.5)$
  - Satisfy $p_A^t(\boldsymbol{d}) + p_B^t(\boldsymbol{d}) = d_A^t + d_B^t, f_{AB}(\boldsymbol{d}) \le f^{max}, \forall \boldsymbol{d} \in D$

Capture Multistage Nature is Critical

- Can we find a policy $p(\cdot)$ that does not look into the future? i.e. $\boldsymbol{p}^1(\boldsymbol{d}^1), \boldsymbol{p}^2(\boldsymbol{d}^1, \boldsymbol{d}^2)$?
  - Because real-time dispatch cannot depend on future
- No feasible **non-anticipative** policy exists!
  - No feasible $\boldsymbol{p}^1$ s.t. for any $\boldsymbol{d}^2 \in D^2$ there exists $\boldsymbol{p}^2$
  - If $p_A^1 \in [11,12]$: $p_A^2 \leq 13$, impossible to satisfy $\boldsymbol{d}^2 = (15,10)$
  - If $p_A^1 \in [12,13]$: $p_B^2 \leq 13$, impossible to satisfy $\boldsymbol{d}^2 = (10,15)$
- Bottleneck: Ramping constraint

Multistage Robust UC

$$\min_{\boldsymbol{x},\boldsymbol{u},\boldsymbol{v},\boldsymbol{p}(\cdot)} \left\{ \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{N}_g} (G_i x_i^t + S_i u_i^t) + \max_{\boldsymbol{d}\in\mathcal{D}} \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{N}_g} C_i\, p_i^t(\boldsymbol{d}^{[t]}) \right\}$$

s.t.

constraints for $\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{v}$

$$p_i^{min} x_i^t \le p_i^t(\boldsymbol{d}^{[t]}) \le p_i^{max} x_i^t \qquad \forall\, \boldsymbol{d}\in\mathcal{D},\ i\in\mathcal{N}_g,\ t\in\mathcal{T}$$

$$-RD_i x_i^t - SD_i v_i^t \le \boxed{p_i^t(\boldsymbol{d}^{[t]})} - p_i^{t-1}(\boldsymbol{d}^{[t-1]}) \le RU_i x_i^{t-1} + SU_i u_i^t$$

$$\forall\, \boldsymbol{d}\in\mathcal{D},\ i\in\mathcal{N}_g,\ t\in\mathcal{T}$$

$$-f_l^{max} \le \alpha_l^\top \left(B^p\, \boldsymbol{p}^t(\boldsymbol{d}^{[t]}) - B^d\, \boldsymbol{d}^t \right) \le f_l^{max} \qquad \forall\, \boldsymbol{d}\in\mathcal{D},\ t\in\mathcal{T},\ l\in\mathcal{N}_l$$

$$\sum_{i\in\mathcal{N}_g} p_i^t(\boldsymbol{d}^{[t]}) = \sum_{j\in\mathcal{N}_d} d_j^t \qquad \forall\, \boldsymbol{d}\in\mathcal{D},\ t\in\mathcal{T}$$

Notation: $\boldsymbol{d}^{[t]} = (\boldsymbol{d}^1, \dots, \boldsymbol{d}^t)$

Affine Multistage Robust UC

- Tractable alternative for $p(\cdot)$:

$$p_i^t(\boldsymbol{d}^1, \ldots, \boldsymbol{d}^t) = w_i^t + \sum_{s \in \{1,\ldots,t\}} \sum_{j \in \mathcal{N}_d} W_{itjs}\, d_j^s$$

- Multistage robust UC with affine policy:

$$\min_{\boldsymbol{x},\boldsymbol{u},\boldsymbol{v},\boldsymbol{w},\boldsymbol{W}} \left\{ \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{N}_g} (G_i x_i^t + S_i u_i^t) + \max_{\boldsymbol{d} \in \mathcal{D}} \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{N}_g} C_i \left(w_i^t + \sum_{s \in \{1,\ldots,t\}} \sum_{j \in \mathcal{N}_d} W_{itjs} d_j^s \right) \right\}$$

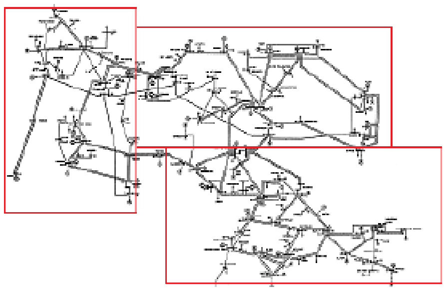
s.t.

constraints for $\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{v}$

$$p_i^{min} x_i^t \leq w_i^t + \sum_{s \in \{1,\ldots,t\}} \sum_{j \in \mathcal{N}_d} W_{itjs} d_j^s \leq p_i^{max} x_i^t \qquad \forall\, \boldsymbol{d} \in \mathcal{D},\ i \in \mathcal{N}_g,\ t \in \mathcal{T}$$

$\ldots$

Simplified Affine Policies



Spatial Aggregation

Temporal Aggregation

General affine policy: $$p_i^t(\boldsymbol{d}^{[t]}) = w_i^t + \sum_{s \in \{1,\dots,t\}} \sum_{j \in \mathcal{N}_d} W_{itjs}\, d_j^s$$

Simpler information basis: $$p_i^t(\boldsymbol{d}^{[t]}) = w_i^t + \sum_{j \in \mathcal{N}_d} W_{itj}\, d_j^t$$

All loads aggregated: $$p_i^t(\boldsymbol{d}^{[t]}) = w_i^t + W_{it} \sum_{j \in \mathcal{N}_d} d_j^t$$

Loads and time periods aggregated: $$p_i^t(\boldsymbol{d}^{[t]}) = w_i^t + W_i \sum_{j \in \mathcal{N}_d} d_j^t$$

Solution Method

- **Dualization** approach does not work:
  - Traditionally, robust constraints are dualized
  - Resulting problem is too large for power systems

- **Constraint generation** makes sense:

$$p_i^{min} x_i^t \leq w_i^t + W_{it} \sum_{j \in \mathcal{N}_d} d_j^t \leq p_i^{max} x_i^t \qquad \forall\, \boldsymbol{d} \in \mathcal{D},\ i \in \mathcal{N}_g,\ t \in \mathcal{T}$$

- However, naïve CG also does not work

Solution Method

- **Valid inequalities** for x and specific d's for ramping, generating limits, and line flow
- **Fixing** binary decisions and finding cuts by CG with an LP master
- **Iteratively improving** policy structure (e.g. $W_i \to W_{it}$) with approximate warm-start (not solving W_i fully)
- **Exploiting structure** of special policy form: e.g. pre-computing all needed constraints for ramping and generation limit constraints for W_{it}-policy.

Computational Study

- How good is the proposed algorithm?
  - Effectiveness of various algorithmic improvements
- How good is the simplified affine policy?
  - Compared to the “true” multi-stage robust UC
- Why should we use multi-stage formulation?
  - Worst case infeasibility of two-stage robust UC
  - Managing Ramping capability
- How good is affine UC “on average”?
  - Rolling-horizon Monte-Carlo simulation
  - Average performance in cost, std, reliability

How Good is the Algorithm?

Solution time (s) for three test systems using W_{it} policy:

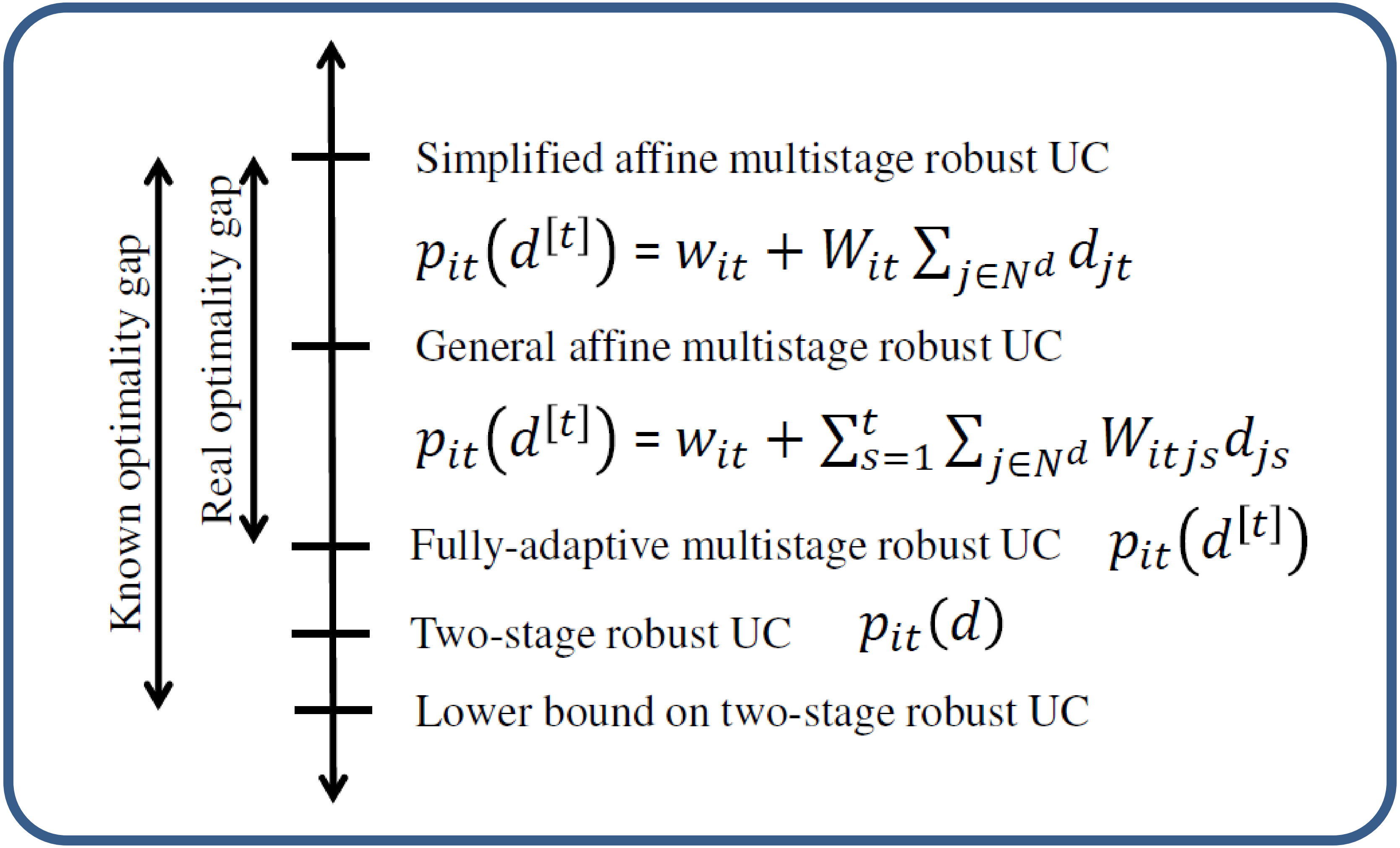
System	$\Gamma = 0.25$	$\Gamma = 0.5$	$\Gamma = 1$	$\Gamma = 2$	$\Gamma = 4$
30 bus	6s	3s	8s	6s	29s (inf)
118 bus	66s	64s	47s	63s	178s
2718 bus	3.6h	3.2h	2.3h	2.0h	0.4h (inf)

Note: "inf" indicates that the problem is infeasible

MIP optimality gap used for 30, 118, 2718 bus systems: 0.1%, 0.1%, 1%

How Good is the Simplified Affine Policy?

- How good is the simplified affine policy?



How Good is the Simplified Affine Policy?

Table : Opt. gap under different policy structures, for the 118 bus system.

$(n_g, n_\mathcal{T}, n_d, L)$	$\Gamma = 0.5$	$\Gamma = 1$	$\Gamma = 2$	$\Gamma = 4$
(10,1,1,0)	0.03%	0.06%	0.11%	0.95%
(21,1,1,0)	0.03%	0.05%	0.11%	0.77%
(31,1,1,0)	0.02%	0.04%	0.10%	0.74%
(54,1,1,0)	0.02%	0.04%	0.10%	0.67%
(54,4,1,0)	0.02%	0.03%	0.10%	0.52%
(54,24,1,0)	0.02%	0.03%	0.07%	0.35%

Table : Opt. gap for the 2718 bus system under the "W_{it}" policy.

$(n_g, n_\mathcal{T}, n_d, L)$	$\Gamma = 0.25$	$\Gamma = 0.5$	$\Gamma = 1$	$\Gamma = 1.5$	$\Gamma = 2$
(289,1,1,0)	0.09%	0.22%	0.42%	0.55%	1.05%
(289,24,1,0)	0.07%	0.11%	0.25%	0.35%	0.53%

Why Multistage? Worst-Case

- Worst-case (US$) of multistage robust dispatch under two-stage and Multistage UC solutions for the 2718-bus system.

	$\Gamma = 0.5$	$\Gamma = 1$	$\Gamma = 1.5$	$\Gamma = 2$	$\Gamma = 3$
Affine multistage UC solutions					
Total Cost	9,445,069	9,596,788	9,746,685	9,905,527	10,234,459
Penalty	0	0	0	0	0
Two-stage UC solutions					
Total Cost	9,505,651	9,745,889	10,183,433	10,975,403	12,864,719
Penalty	96,313	224,952	591,661	1,165,324	2,703,522
Rel Diff	0.64%	1.55%	4.49%	10.80%	25.70%

How Good is Affine UC on Average?

- Average performance over independent demand

Affine multistage robust UC with policy-enforcement robust ED

Γ	0.25	0.5	1	1.5	2	3
Cost Avg ($)	9,397,528	9,319,396	9,342,754	9,360,359	9,379,464	9,442,858
Cost Std ($)	113,725	15,970	12,828	12,509	12,363	12,092
Penalty Cost Avg ($)	93,552	3497	727	61	5	0
Penalty Freq Avg	10.00%	1.47%	0.40%	0.01%	0.00%	0.00%

0.46%

Two-stage robust UC with look-ahead ED

Γ	0.25	0.5	1	1.5	2	3
Cost Avg ($)	9,398,109	9,456,599	9,408,732	9,383,569	9,407,290	9,362,379
Cost Std ($)	93,470	195,774	173,884	144,698	162,469	45,584
Penalty Cost Avg ($)	80,127	152,637	98,113	66,801	82,864	6,103
Penalty Freq Avg	9.93%	12.26%	7.80%	5.11%	5.57%	0.37%

Deterministic UC with reserve and look-ahead ED **0.95%**

Reserve	2.5%	5%	10%	15%	20%	30%
Cost Avg ($)	9,556,549	9,575,446	9,424,678	9,561,024	9,408,173	9,411,741
Cost Std ($)	261,464	288,777	121,122	196,354	92,268	69,050
Penalty Cost Avg ($)	254,627	271,672	119,127	248,658	83,938	51,907
Penalty Freq Avg	15.93%	13.37%	14.31%	18.16%	10.03%	7.22%

How Good is Affine UC on Average?

- Average performance over wind power and persistent demand

Affine multistage robust UC with policy-enforcement robust ED

Γ	0.25	0.5	1	1.5	2	3
Cost Avg ($)	10,996,931	9,459,785	8,502,923	8,581,532	8,646,665	9,415,693
Cost Std ($)	3,665,301	2,007,317	490,457	466,999	424,801	458,865
Penalty Cost Avg ($)	2,679,299	1,110,032	101,234	81,834	27,344	218
Penalty Freq Avg	18.84%	14.44%	1.67%	0.47%	0.18%	0.01%

1.23%

Two-stage robust UC with look-ahead ED

Γ	0.25	0.5	1	1.5	2	3
Cost Avg ($)	10,390,214	11,365,568	8,734,840	8,863,975	8,609,160	8,947,959
Cost Std ($)	1,831,279	1,059,427	620,301	802,441	522,881	793,447
Penalty Cost Avg ($)	2,064,045	1,032,109	380,451	490,562	195,681	443,401
Penalty Freq Avg	12.73%	3.68%	7.37%	5.19%	2.07%	2.66%

24.52%

Deterministic UC with reserve and look-ahead ED

Reserve	2.5%	5%	10%	15%	20%	30%
Cost Avg ($)	13,186,705	14,272,477	13,110,030	13,617,194	11,879,817	11,248,546
Cost Std ($)	5,557,309	7,023,964	5,596,039	6,082,173	4,095,780	3,113,902
Penalty Cost Avg ($)	4,905,635	6,003,861	4,827,766	5,334,746	3,578,986	2,912,186
Penalty Freq Avg	30.45%	29.94%	33.00%	32.43%	23.61%	15.03%

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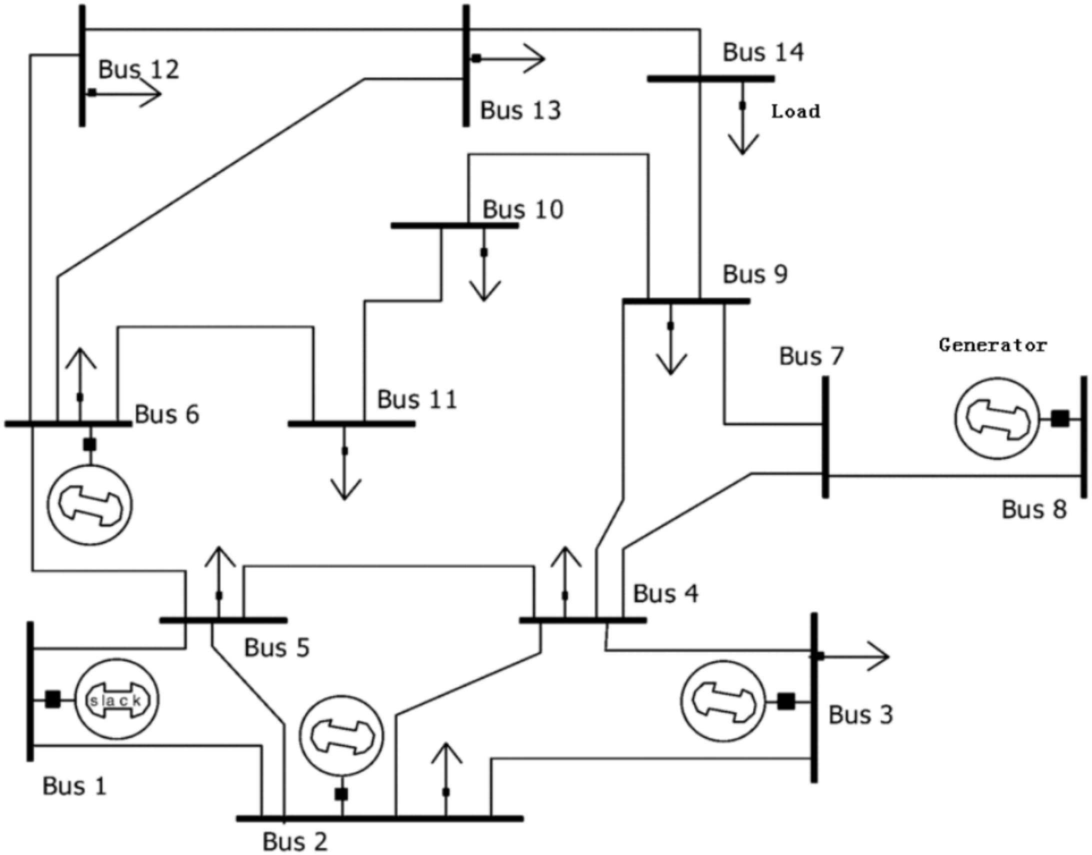
Act 3

Optimal Power Flow (OPF) and Optimal Transmission Switching (OTS) problem: Fast Convexification and Cutting Plane Method to deal with **non-convexity**

AC Optimal Power Flow



<u>Data</u>:

- Network:

$$\mathcal{N} = (\mathcal{B}, \mathcal{L})$$

- Load at bus i:

$$p_i^d,\ q_i^d$$

- Generator at bus i:

$$[p_i^{\min}, p_i^{\max}]$$

$$[q_i^{\min}, q_i^{\max}]$$

- Voltage bounds at bus i:

$$[V_i^{\min}, V_i^{\max}]$$

- Network line admittance:

$$(G_{ij}, B_{ij})_{(i,j)\in\mathcal{L}}$$

- Line flow limit:

$$\bar{S}_{ij}$$

AC OPF Formulation: Variables and Objective

Variables:

1. Active and reactive **power** at generator i: (p_i^g, q_i^g)
2. Active and reactive **power flow** on line (i,j): (p_{ij}, q_{ij})
3. Complex **voltage** at bus i: $V_i = |V_i|(\cos\theta_i + \mathrm{i}\sin\theta_i) = e_i + \mathrm{i} f_i$

Objective:

$$\min \sum_{i \in \mathcal{G}} C_i(p_i^g)$$

Usually a separable increasing function.

AC OPF Formulation: Constraints

$$p_i^g - p_i^d = \sum_{j \in \delta(i)} p_{ij} \qquad \text{(active flow balance)}$$

$$q_i^g - q_i^d = \sum_{j \in \delta(i)} q_{ij} \qquad \text{(reactive flow balance)}$$

$$p_{ij}^2 + q_{ij}^2 \leq \overline{S}_{ij}^2 \qquad \text{(apparent flow limit)}$$

$$p_i^{\min} \leq p_i^g \leq p_i^{\max} \qquad \text{(active power limits)}$$

$$q_i^{\min} \leq q_i^g \leq q_i^{\max} \qquad \text{(reactive power limits)}$$

Power flow equations and voltage bounds in **polar coordinates**

$$\begin{cases} p_{ij} = -G_{ij}|V_i|^2 + G_{ij}|V_i||V_j|\cos(\theta_i - \theta_j) + B_{ij}|V_i||V_j|\sin(\theta_i - \theta_j) \\ q_{ij} = B_{ij}|V_i|^2 - B_{ij}|V_i||V_j|\cos(\theta_i - \theta_j) + G_{ij}|V_i||V_j|\sin(\theta_i - \theta_j) \\ \underline{V}_i \leq |V_i| \leq \overline{V}_i \end{cases}$$

Power flow equations and voltage bounds in **rectangular coordinates**

$$\begin{cases} p_{ij} = -G_{ij}(e_i^2 + f_i^2) + G_{ij}(e_i e_j + f_i f_j) - B_{ij}(e_i f_j - e_j f_i) \\ q_{ij} = B_{ij}(e_i^2 + f_i^2) - B_{ij}(e_i e_j + f_i f_j) - G_{ij}(e_i f_j - e_j f_i) \\ \underline{V}_i^2 \leq e_i^2 + f_i^2 \leq \overline{V}_i^2 \end{cases}$$

AC OPF Reformulation

- Introduce Hermitian matrix $X = (e + if)(e + if)^H$:

$$p_{ij} = -G_{ij}X_{ii} + G_{ij}\mathcal{R}(X_{ij}) + B_{ij}\mathcal{I}(X_{ij})$$

$$q_{ij} = B_{ij}X_{ii} - B_{ij}\mathcal{R}(X_{ij}) + G_{ij}\mathcal{I}(X_{ij})$$

$$\underline{V}_i^2 \leq X_{ii} \leq \overline{V}_i^2$$

$$X \text{ is hermitian}$$

$$X \succeq 0$$

$$\boxed{\operatorname{rank}(X) = 1}$$

- Standard SDP relaxation: Ignore rank constraint

Recent Literature on OPF

- **Local solvers** by Newton-Raphson and Interior-Point methods
- **Convex relaxations** using **semidefinite programming** (SDP) and Lasserre hierarchy: (Lavaei and Low, 2012; Madani et. al., 2013; Zhang and Tse, 2012; Lavaei et al., 2014, Molzahn et al. 2013, Molzahn and Hiskens, 2014, Chen et al. 2015)
- **Second order cone program** (SOCP) relaxation: (Jabr 2006, Hijazi et al., 2014)
- **Approximate LPs** with guaranteed bounds for the AC-OPF problem on graphs with bounded tree-width (Bienstock and Munoz, 2015)
- **Global optimal solutions** based on branch-and-bound (Phan, 2012)

Non-Convexities in SOCP reformulation

Change of variables:

$$\left.\begin{array}{rcl} (e_i^2 + f_i^2) & = & c_{ii} \\ (e_i e_j + f_i f_j) & = & c_{ij} \\ (e_i f_j - f_i e_j) & = & s_{ij}. \end{array}\right\}$$

Non-convex quadratic mapping:

$$\left(e_i, f_i, e_j, f_j\right) \to (c_{ii}, c_{jj}, c_{ij}, s_{ij})$$

is not injective (so not invertible anywhere).

Non-convexity modeled:
Surface of SOCP cone
within bounds on c, s

$$\boxed{c_{ij}^2 + s_{ij}^2 \quad = \quad c_{ii}^j c_{jj}^i}$$

Non-convexity NOT modeled:
Angle sum to 0 (KVL)

What we should have:

$$\boxed{\sum_{(i,j)\in\mathcal{C}} \theta_{ij} = 0}$$

Relaxation of Surface of Cone

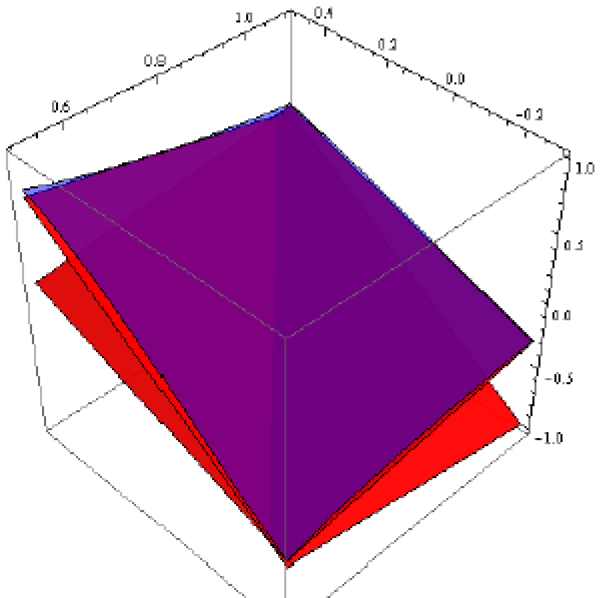
- **Surface of SOCP cone**: $c_{ij}^2 + s_{ij}^2 = c_{ii}^j c_{jj}^i$
  - One direction is convex: $c_{ij}^2 + s_{ij}^2 \leq c_{ii}^j c_{jj}^i$
  - Other direction is reverse convex:
    - $f(c_{ij}, s_{ij}) = \sqrt{c_{ij}^2 + s_{ij}^2} \geq \sqrt{c_{ii}^j c_{jj}^i} = g(c_{ii}^j, c_{jj}^i)$
    - f is convex, g is concave
    - Overestimate of f and underestimate of g by hyperplanes
- Angle-sum-to-zero constraints:
  - Arctangent envelopes
  - Trigonometric reformulation
  - SDP separation

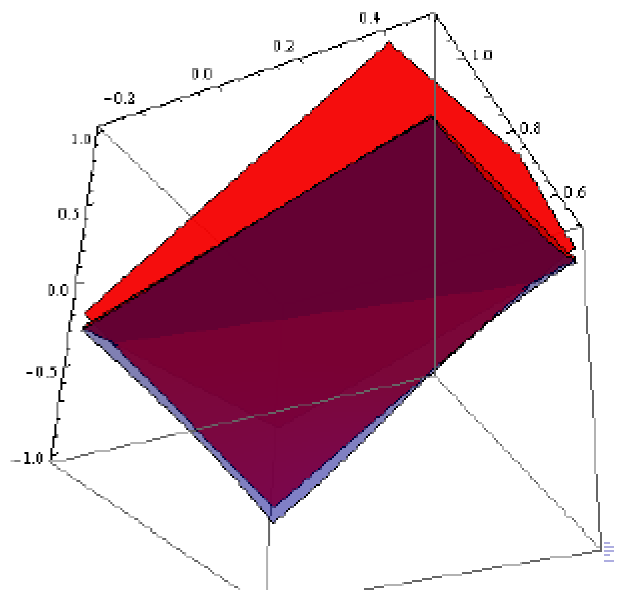
Arctangent Envelopes

1. For each edge (i, j), we want to enforce the arctan constraint for $x_{ij} = 1$:

$$\mathcal{AT} := \left\{ (c_{ij}, s_{ij}, \theta_{ij}) \in \mathbb{R}^3 : \theta_{ij} = \arctan\left(\frac{s_{ij}}{c_{ij}}\right), (c_{ij}, s_{ij}) \in [\underline{c}_{ij}, \bar{c}_{ij}] \times [\underline{s}_{ij}, \bar{s}_{ij}] \right\}$$

2. Outer approximation of the above set by 4 linear inequalities: Need to solve four simple global optimization problems to obtain these inequalities.





Cycle Constraints

For a cycle C with all edges on, instead of satisfying:

$$\sum_{(i,j)\in C} \arctan\left(\frac{\mathbf{s_{ij}}}{\mathbf{c_{ij}}}\right) = 0,$$

We approximate "angles sum to zero over the cycle" by the following relaxation:

$$\sum_{(i,j)\in C} \theta_{ij} = 2\pi k, \quad \text{for some } k \in \mathbb{Z}. \tag{1}$$

We enforce (1) over cycles in a *cycle basis* (instead of *all* cycles).

Condition (1) is equivalent to:

$$\textit{Cycle constraint:} \quad \cos\Big(\sum_{(i,j)\in C} \theta_{ij}\Big) = 1. \tag{2}$$

Cycle constraint (2) can be reformulated as a degree $|C|$ homogeneous polynomial $p_C = 0$ in $\mathbf{s_{ij}}$ and $\mathbf{c_{ij}}$ for $(i,j) \in C$.

3-Cyle, 4-Cycle, and Larger Cycles

- 3-cycle:

For a 3-cycle: $\cos(\theta_{12}+\theta_{23}+\theta_{31})=1$ can be written as

$$s_{12}c_{33}+c_{23}s_{31}+s_{23}c_{31}=0$$
$$c_{12}c_{33}-c_{23}c_{31}+s_{23}s_{31}=0.$$

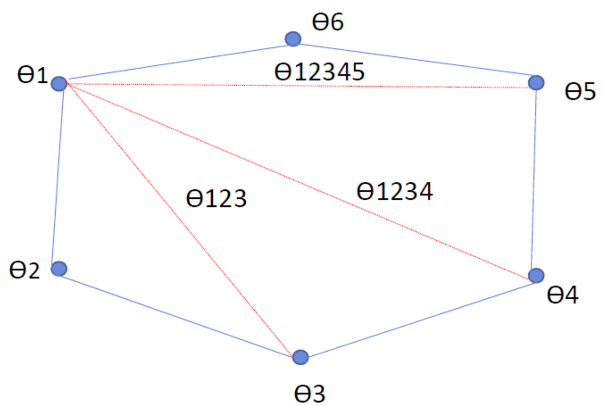
- 4-cycle:

For a 4-cycle: $\cos(\theta_{12}+\theta_{23}+\theta_{34}+\theta_{41})=1$ can be written as

$$s_{12}c_{34}+c_{12}s_{34}+s_{23}c_{41}+c_{23}s_{41}=0$$
$$c_{12}c_{34}-s_{12}s_{34}+c_{23}c_{41}-s_{23}s_{41}=0.$$

- Larger-cycle:

Decomposition



SDP Separation

Given a solution (p^*, q^*, c^*, s^*) of SOCP relaxation,

1. If there exists a matrix $W^* \succeq 0$, s.t. (c^*, s^*, W^*) satisfies:

$$\begin{aligned} c_{ij} &= W_{ij} + W_{i'j'} & (i,j) &\in \mathcal{L} \\ s_{ij} &= W_{ij'} - W_{ji'} & (i,j) &\in \mathcal{L} \\ c_{ii} &= W_{ii} + W_{i'i'} & i &\in \mathcal{B}, \end{aligned}$$

   where $i' = i + |\mathcal{B}|$ and $j' = j + |\mathcal{B}|$,
   then (c^*, s^*, W^*) is feasible for SDP relaxation.

2. Otherwise, we can separate $z = (c^*, s^*)$ from the following SDP set $\mathcal{S}$:

$$\mathcal{S} := \left\{ z \in \mathbb{R}^{2|C|} : \exists W \in \mathbb{R}^{2|C| \times 2|C|} \text{ s.t. } -z_l + A_l \bullet W = 0 \; \forall l \in L, \; W \succeq 0 \right\}$$

by solving a small SDP over each cycle C in a cycle basis,
which produces a linear constraint $\alpha^T z \leq 0$ to be added to SOCP relaxation.

Our Strategy for Solving AC-OPF

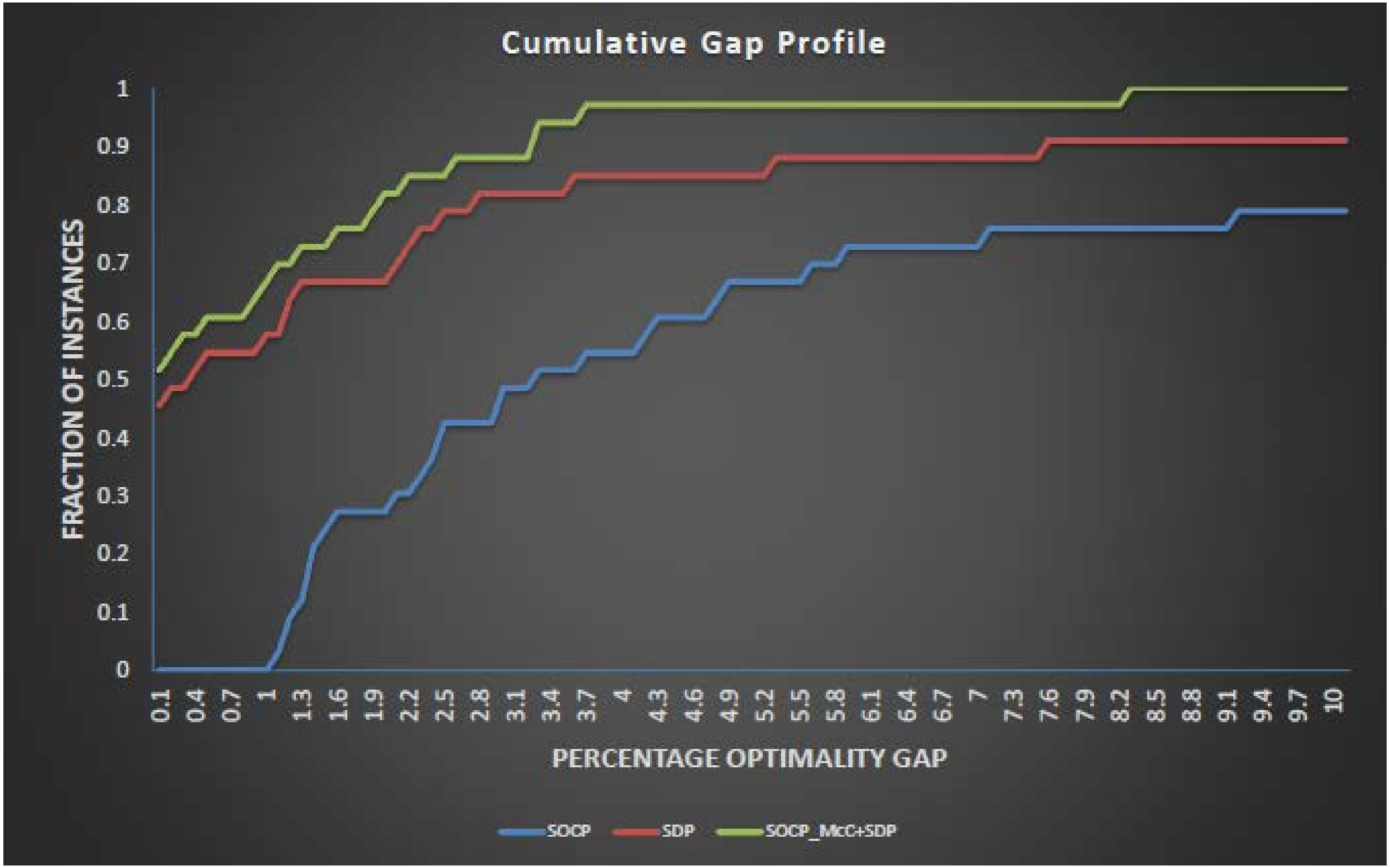
- Workhorse: SOCP relaxation for fast computation
- Strengthen SOCP relaxation for key non-convexities:
  - **Type 1**: Characterize convex hull and linear outer envelope
  - **Type 2**: Three approaches to convexify KVL:
    - **Cycle constraints:** polynomial equations →McCormick Linearization
    - **Arctangent envelope:** Linear upper/lower approximation
    - **SDP separation:** Lift-and-project
- **Results:**
  - IEEE instances (Easy):

	%gap	Time (s)
SOCP	0.43	2.62
SOCP_cuts	0.08	207.81
SDP	0.04	380.37

  - NESTA (Hard):

	Plain SOCP		SOCP with Cuts		SDP	
	%gap	time	%gap	time	%gap	time
Typical	5.14	1.97	0.56	454.38	1.15	817.31
Congested	9.83	3.27	1.15	393.09	3.76	631.24
Small Angle	5.91	2.53	1.13	559.74	3.53	979.63

Our Strategies for Solving AC-OPF



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Some Concluding Remarks

- Significant challenges:
  - AC Optimal Switching Problem (up to 300-bus)
  - AC OPF Global Optimization (up to 3375-bus)
  - Multiple-phase AC OPF (need new techniques)
  - Robust UC with AC OPF
  - Multistage stochastic UC
  - Sensor-driven real-time operation and maintenance scheduling
- Many more challenging computational problems!
- "Bridging the Gap" between OR and Engineering is so important!

Happy Birthday to CORE!

Andy Sun
ISyE @ Georgia Tech
andy.sun@isye.gatech.edu